Numerical Differentiation

Newton's Divided Difference formula

Newton's Divided Difference formula
1. Find equation using Newton's Divided Difference Interpolation formula $f(x) = y_0 + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2] + (x - x_0)(x - x_1)(x - x_2)f[x_0, x_1, x_2, x_3] + \ldots$ 2. Now, differentiate f(x) with respect to x to get f'(x) and f''(x) 3. Now, substitute value of x in f'(x) and f''(x)

Example: Use Newton's Divided Difference formula to find solution when x = 5

x	f(x)
2	4
4	56
9	711
10	980

Solution:

The value of table for x and y

x	2	4	9	10
y	4	56	711	980

Newton's divided difference table is

x	y	1st order	2nd order	3rd order
2	4			
		26		
4	56		15	
		131		1
9	711		23	
		269		
10	980			

Newton's divided difference interpolation formula is

$f(x)=y0+(x-x0)f[x0,x1]+(x-x0)(x-x1)f[x0,x1,x2]+(x-x0)(x-x1)(x-x2)f[x0,x1,x2,x3]$

$f(x)=4+(x-2)\times 26+(x-2)(x-4)\times 15+(x-2)(x-4)(x-9)\times 1$

$f(x)=4+(x-2)\times 26+(x^2-6x+8)\times 15+(x^3-15x^2+62x-72)\times 1$

$f(x)=4+(26x-52)+(15x^2-90x+120)+(x^3-15x^2+62x-72)$

$f(x)=x^3-2x$

Now, differentiate with x

$f'(x)=3x^2-2$

$f''(x)=6x$

Now, substitute $x=5$

$f'(5)=3\times(5^2)-2=73$

$f''(5)=6\times5=30$

Example 2: Use Newton's Divided Difference formula to find solution when
x = 2

x	f(x)
0	4
1	3
4	24
5	39

Newton's divided difference table is

x	y	1st order	2nd order	3rd order
0	4			
		-1		
1	3		2	
		7		0
4	24		2	
		15		
5	39			

Newton's divided difference interpolation formula is

$$f(x)=y0+(x-x0)f[x0,x1]+(x-x0)(x-x1)f[x0,x1,x2]+(x-x0)(x-x1)(x-x2)f[x0,x1,x2,x3]$$

$$f(x)=4+(x-0)\times-1+(x-0)(x-1)\times2+(x-0)(x-1)(x-4)\times0$$

$$f(x)=4+(x)\times-1+(x^2-x)\times2+(x^3-5x^2+4x)\times0$$

$$f(x)=4+(-x)+(2x^2-2x)+(0)$$

$$f(x)=2x^2-3x+4$$

Now, differentiate with x

$f'(x)=4x-3$

$f''(x)=4$

Now, substitute $x=2$

$f'(2)=4\times2-3=5$

$f''(2)=4=4$