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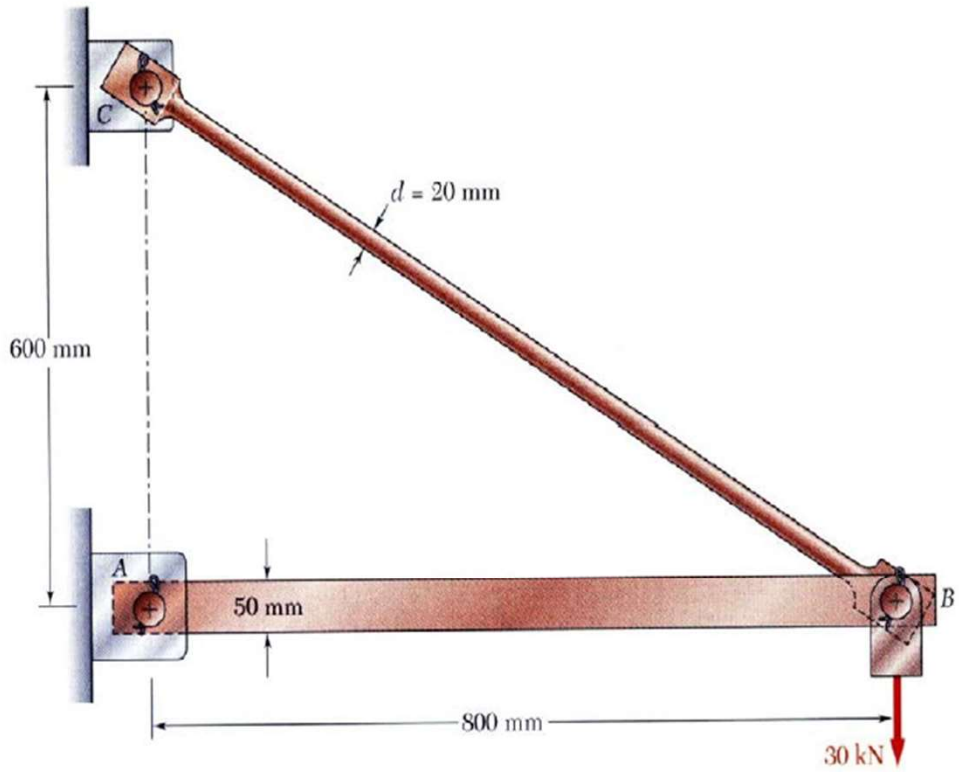
Concept of Stress

Dr. Md Saiful Islam

Learning Objectives

- The main objective of the study of the mechanics of solids is to provide future engineers with the means of analyzing and designing various load-bearing structures and machines.
- Both the analysis and design of a given structure involve the determination of stresses and deformations.

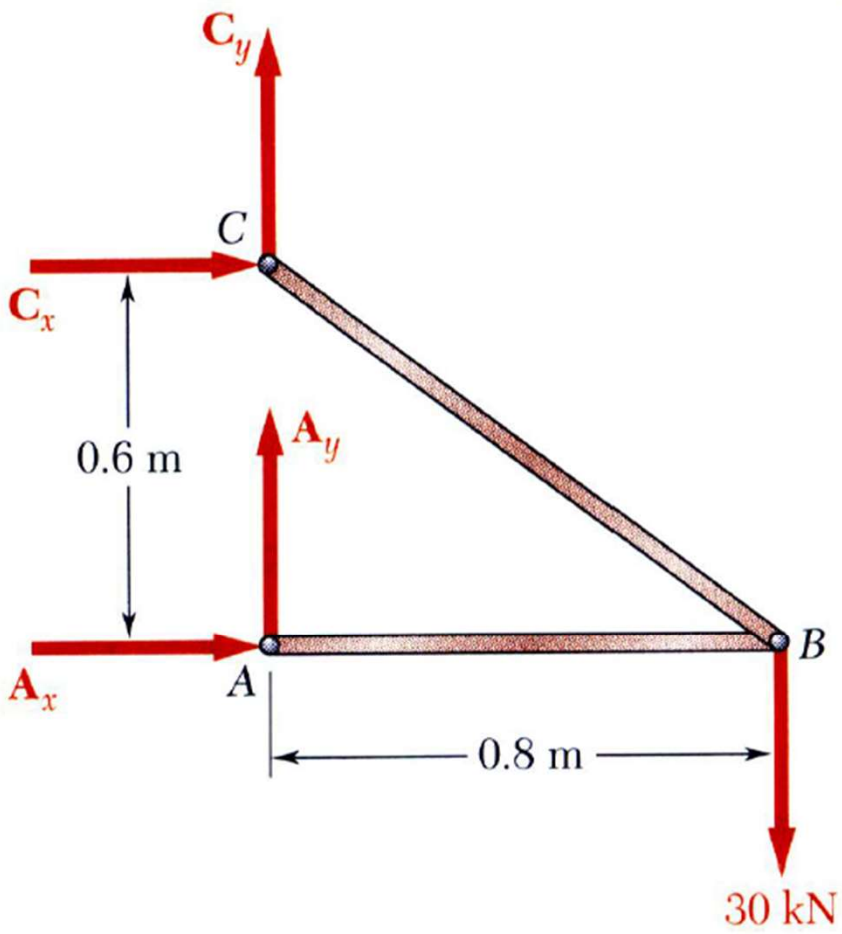
REVIEW OF STATICS



Ref: Beer et al., 2011

- **The structure is designed to support a 30 kN load**
- **The structure consists of a boom and rod joined by pins (zero moment connections) at the junctions and supports**
- **Perform a static analysis to determine the internal force in each structural member and the reaction forces at the supports**

STRUCTURE FREE-BODY DIAGRAM



- **Structure is detached from supports and the loads and reaction forces are indicated**
- **Conditions for static equilibrium:**

$$\sum M_C = 0 = A_x(0.6\,\text{m}) - (30\,\text{kN})(0.8\,\text{m})$$

$$A_x = 40\,\text{kN}$$

$$\sum F_x = 0 = A_x + C_x$$

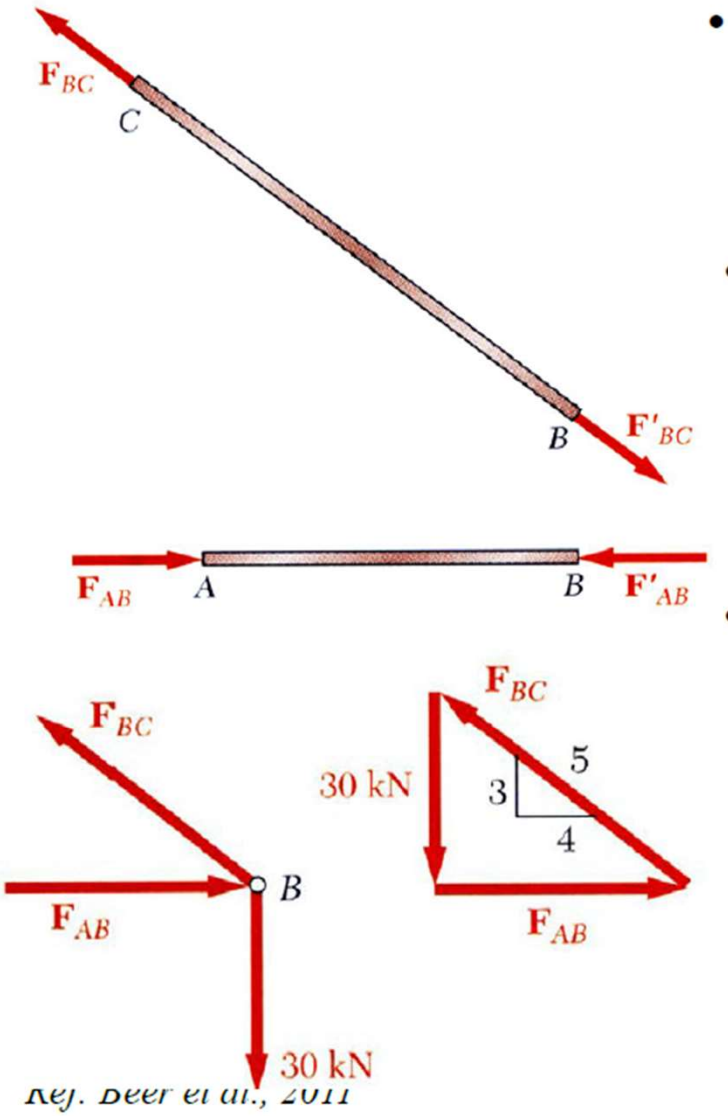
$$C_x = -A_x = -40\,\text{kN}$$

$$\sum F_y = 0 = A_y + C_y - 30\,\text{kN} = 0$$

$$A_y + C_y = 30\,\text{kN}$$

- **A_y and C_y can not be determined from these equations**

Ref: Beer et al., 2011



Ref. Beer et al., 2011

- **The boom and rod are 2-force members, i.e., the members are subjected to only two forces which are applied at member ends**
- **For equilibrium, the forces must be parallel to an axis between the force application points, equal in magnitude, and in opposite directions**
- **Joints must satisfy the conditions for static equilibrium which may be expressed in the form of a force triangle:**

$$\sum \vec{F}_B = 0$$

$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30\,\text{kN}}{3}$$

$$F_{AB} = 40\,\text{kN} \qquad F_{BC} = 50\,\text{kN}$$

Stress: **Stress is defined as the intensity of force per unit area at any point. To designate stress notations σ (normal stress) and τ (shear stress) are used.**

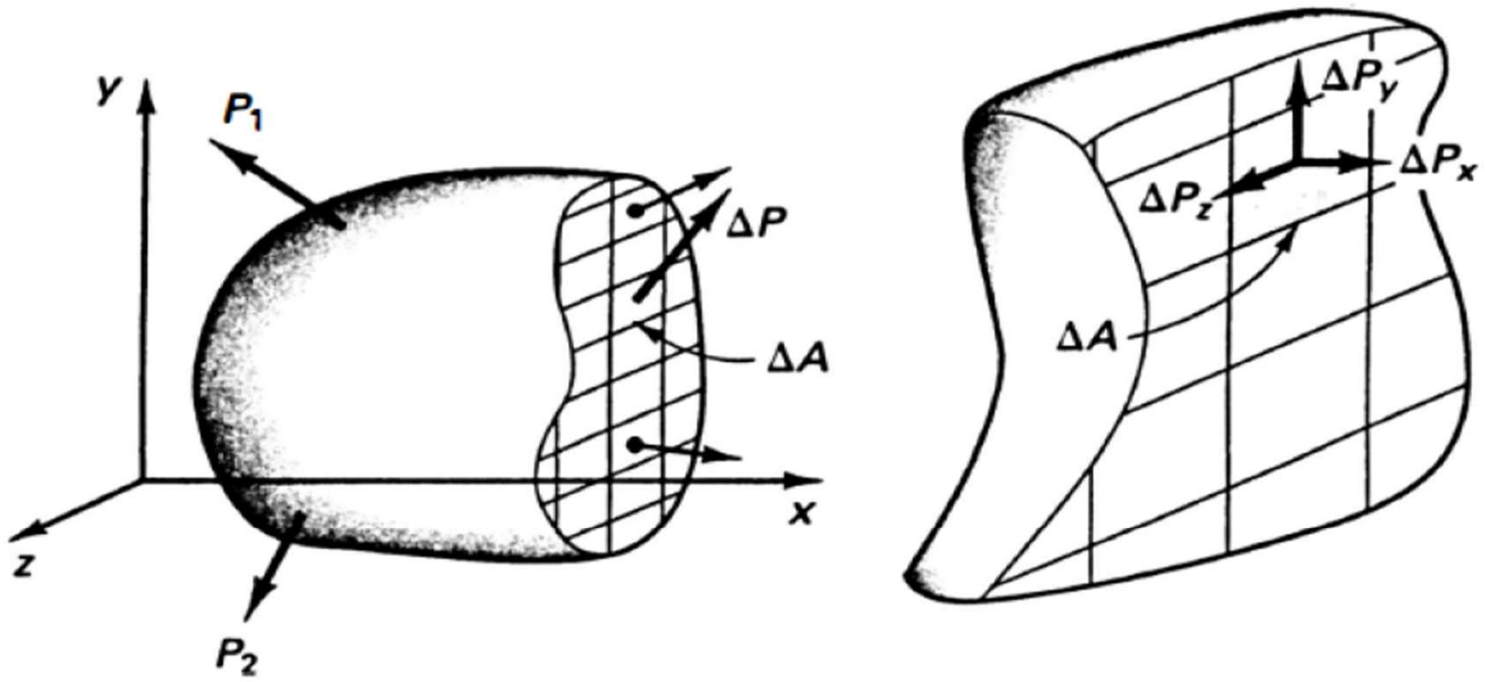


Fig. 1-2 Sectioned body: (a) free body with some internal forces, (b) enlarged view with components of ΔP.

AXIAL LOADING: NORMAL STRESS

The force per unit area, or intensity of the forces distributed over a given section, is called the *stress* on that section and is denoted by the Greek letter σ (sigma). The stress in a member of cross-sectional area A subjected to an axial load **P** (Fig. 1.8) is therefore obtained by dividing the magnitude P of the load by the area A:

$$\sigma = \frac{P}{A} \tag{1.5}$$

A positive sign will be used to indicate a tensile stress (member in tension) and a negative sign to indicate a compressive stress (member in compression).

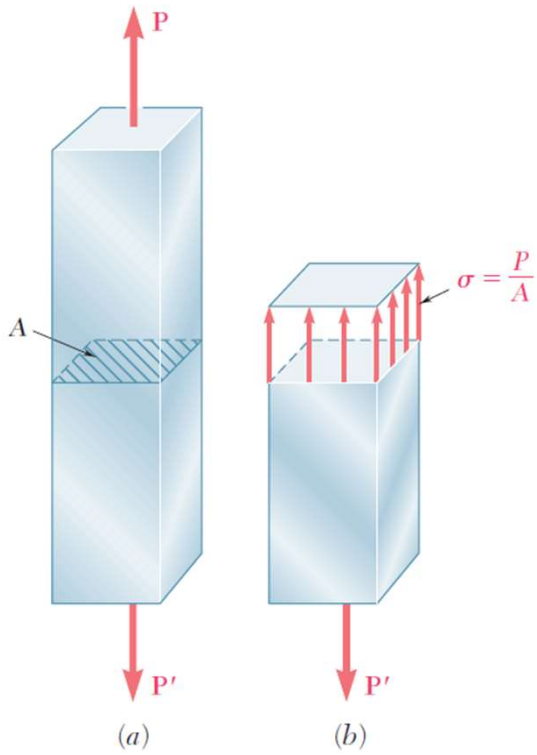


Fig. 1.8 Member with an axial load.

Since SI metric units are used in this discussion, with P expressed in newtons (N) and A in square meters (m^2), the stress σ will be expressed in N/m^2. This unit is called a *pascal* (Pa). However, one finds that the pascal is an exceedingly small quantity and that, in practice, multiples of this unit must be used, namely, the kilopascal (kPa), the megapascal (MPa), and the gigapascal (GPa). We have

$$1 \text{ kPa} = 10^3 \text{ Pa} = 10^3 \text{ N/m}^2$$

$$1 \text{ MPa} = 10^6 \text{ Pa} = 10^6 \text{ N/m}^2$$

$$1 \text{ GPa} = 10^9 \text{ Pa} = 10^9 \text{ N/m}^2$$

When U.S. customary units are used, the force P is usually expressed in pounds (lb) or kilopounds (kip), and the cross-sectional area A in square inches (in^2). The stress σ will then be expressed in pounds per square inch (psi) or kilopounds per square inch (ksi).†

To define the stress at a given point Q of the cross section, we should consider a small area ΔA (Fig. 1.9). Dividing the magnitude of $\Delta\mathbf{F}$ by ΔA, we obtain the average value of the stress over ΔA. Letting ΔA approach zero, we obtain the stress at point Q:

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \tag{1.6}$$

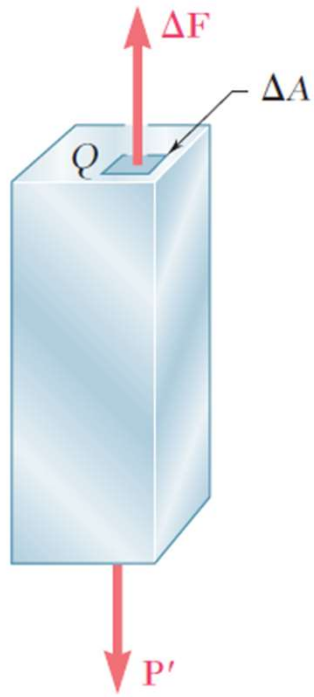


Fig. 1.9

In general, the value obtained for the stress σ at a given point Q of the section is different from the value of the average stress given by formula (1.5), and σ is found to vary across the section. In a slender rod subjected to equal and opposite concentrated loads **P** and **P′** (Fig. 1.10*a*), this variation is small in a section away from the points of application of the concentrated loads (Fig. 1.10*c*), but it is quite noticeable in the neighborhood of these points (Fig. 1.10*b* and *d*).

It follows from Eq. (1.6) that the magnitude of the resultant of the distributed internal forces is

$$\int dF = \int_A \sigma \, dA$$

But the conditions of equilibrium of each of the portions of rod shown in Fig. 1.10 require that this magnitude be equal to the magnitude P of the concentrated loads. We have, therefore,

$$P = \int dF = \int_A \sigma \, dA \tag{1.7}$$

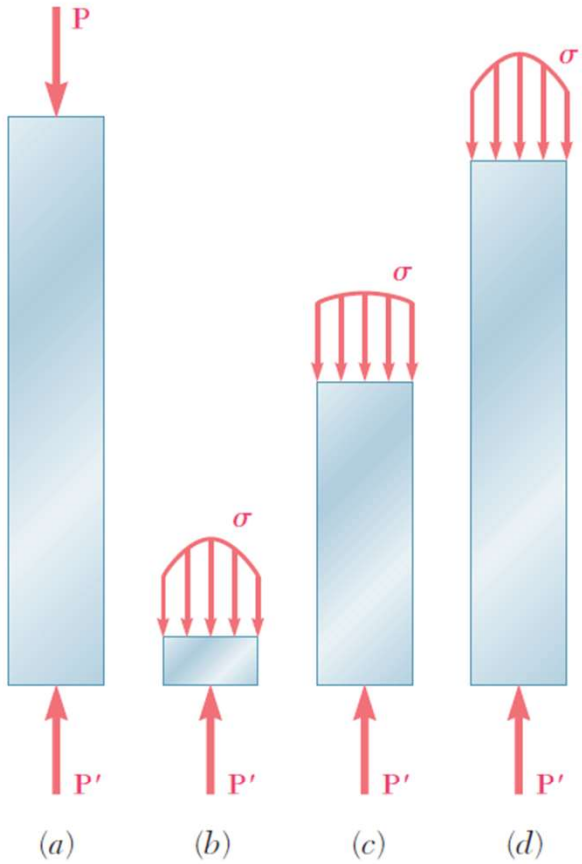


Fig. 1.10 Stress distributions at different sections along axially loaded member.

In practice, it will be assumed that the distribution of normal stresses in an axially loaded member is uniform, except in the immediate vicinity of the points of application of the loads. The value σ of the stress is then equal to σ_{ave} and can be obtained from formula (1.5). However, we should realize that, when we assume a uniform distribution of stresses in the section, i.e., when we assume that the internal forces are uniformly distributed across the section, it follows from elementary statics† that the resultant **P** of the internal forces must be applied at the centroid C of the section (Fig. 1.11). This means that *a uniform distribution of stress is possible only if the line of action of the concentrated loads* **P** *and* **P′** *passes through the centroid of the section considered* (Fig. 1.12). This type of loading is called *centric loading* and will be assumed to take place in all straight two-force members found in trusses and pin-connected structures, such as the one considered in Fig. 1.1. However, if a two-force member is loaded axially, but *eccentrically* as shown in Fig. 1.13*a*, we find from the conditions of equilibrium of the portion of member shown in Fig. 1.13*b* that the internal forces in a given section must be

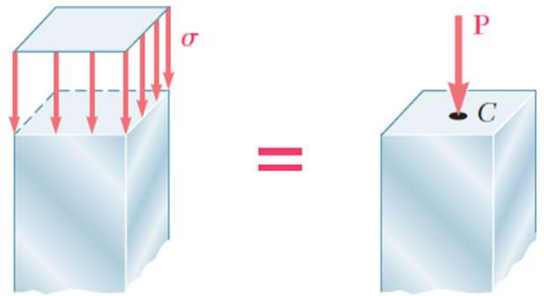


Fig. 1.11

equivalent to a force **P** applied at the centroid of the section and a couple **M** of moment $M = Pd$. The distribution of forces—and, thus, the corresponding distribution of stresses—*cannot be uniform*. Nor can the distribution of stresses be symmetric as shown in Fig. 1.10. This point will be discussed in detail in Chap. 4.

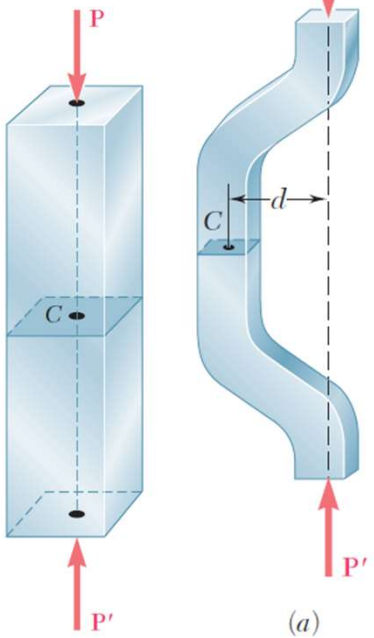


Fig. 1.12

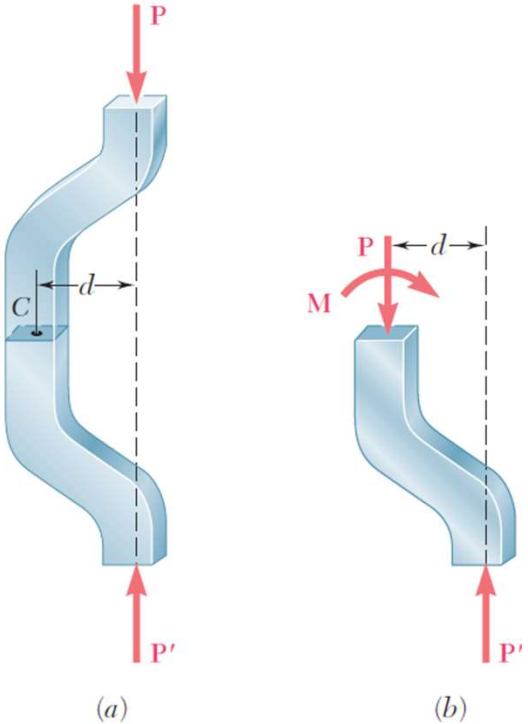


Fig. 1.13 Eccentric axial loading.

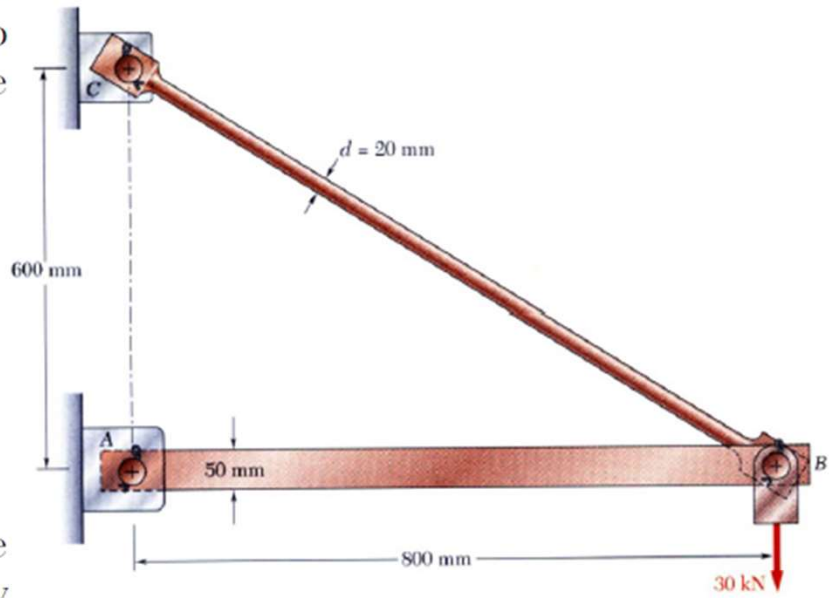
1.4 ANALYSIS AND DESIGN

Considering again the structure of Fig. 1.1, let us assume that rod BC is made of a steel with a maximum allowable stress $\sigma_{all} = 165$ MPa. Can rod BC safely support the load to which it will be subjected? The magnitude of the force F_{BC} in the rod was found earlier to be 50 kN. Recalling that the diameter of the rod is 20 mm, we use Eq. (1.5) to determine the stress created in the rod by the given loading. We have

$$P = F_{BC} = +50 \text{ kN} = +50 \times 10^3 \text{ N}$$

$$A = \pi r^2 = \pi\left(\frac{20 \text{ mm}}{2}\right)^2 = \pi(10 \times 10^{-3} \text{ m})^2 = 314 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{+50 \times 10^3 \text{ N}}{314 \times 10^{-6} \text{ m}^2} = +159 \times 10^6 \text{ Pa} = +159 \text{ MPa}$$



Since the value obtained for σ is smaller than the value σ_{all} of the allowable stress in the steel used, we conclude that rod BC can safely support the load to which it will be subjected. To be complete, our analysis of the given structure should also include the determination of the compressive stress in boom AB, as well as an investigation of the stresses produced in the pins and their bearings. This will be discussed later in this chapter. We should also determine whether

- ***Normal Stress:*** Stress resulting from force that is normal to the surface is called normal stress. It can be either tensile stress or compressive stress. If the normal force is away from the surface, the resulting stress is **tensile normal stress** and if the force acts on an inward direction to the surface the resulting stress is **compressive normal stress**. Notations $\boldsymbol{\sigma}$ is used to designate normal stress.

- ***Shear Stress:*** Stress that results from forces that are along the surface is called shear stress. Notations $\boldsymbol{\tau}$ is used to designate shear stress.

1.6 SHEARING STRESS

The internal forces and the corresponding stresses discussed in Secs. 1.2 and 1.3 were normal to the section considered. A very different type of stress is obtained when transverse forces **P** and **P′** are applied to a member *AB* (Fig. 1.14). Passing a section at *C* between the points of application of the two forces (Fig. 1.15*a*), we obtain the diagram of portion *AC* shown in Fig. 1.15*b*. We conclude that internal forces must exist in the plane of the section, and that their resultant is equal to **P.** These elementary internal forces are called *shearing forces*, and the magnitude *P* of their resultant is the *shear* in the section. Dividing the shear *P* by the area *A* of the cross section, we obtain the *average shearing stress* in the section. Denoting the shearing stress by the Greek letter τ (tau), we write

$$\tau_{\text{ave}} = \frac{P}{A} \tag{1.8}$$

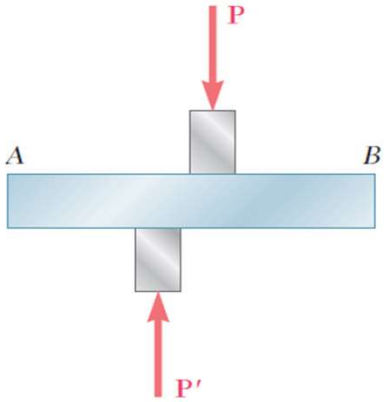


Fig. 1.14 Member with transverse loads.

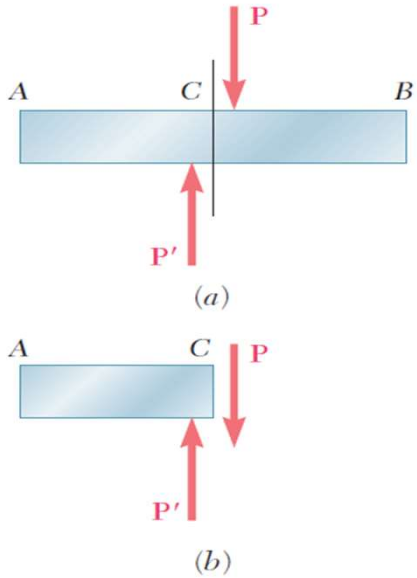


Fig. 1.15

It should be emphasized that the value obtained is an average value of the shearing stress over the entire section. Contrary to what we said earlier for normal stresses, the distribution of shearing stresses across the section *cannot* be assumed uniform. As you will see in Chap. 6, the actual value τ of the shearing stress varies from zero at the surface of the member to a maximum value τ_{max} that may be much larger than the average value τ_{ave}.



Photo 1.2 Cutaway view of a connection with a bolt in shear.

Shearing stresses are commonly found in bolts, pins, and rivets used to connect various structural members and machine components (Photo 1.2). Consider the two plates A and B, which are connected by a bolt CD (Fig. 1.16). If the plates are subjected to tension forces of magnitude F, stresses will develop in the section of bolt corresponding to the plane EE'. Drawing the diagrams of the bolt and of the portion located above the plane EE' (Fig. 1.17), we conclude that the shear P in the section is equal to F. The average shearing stress in the section is obtained, according to formula (1.8), by dividing the shear $P = F$ by the area A of the cross section:

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A} \tag{1.9}$$

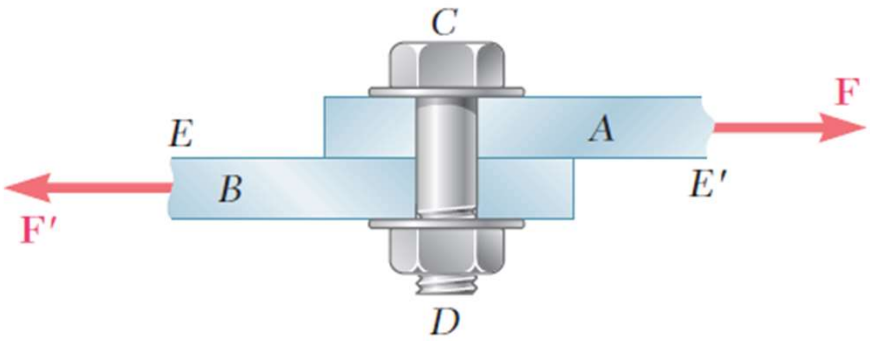


Fig. 1.16 Bolt subject to single shear.

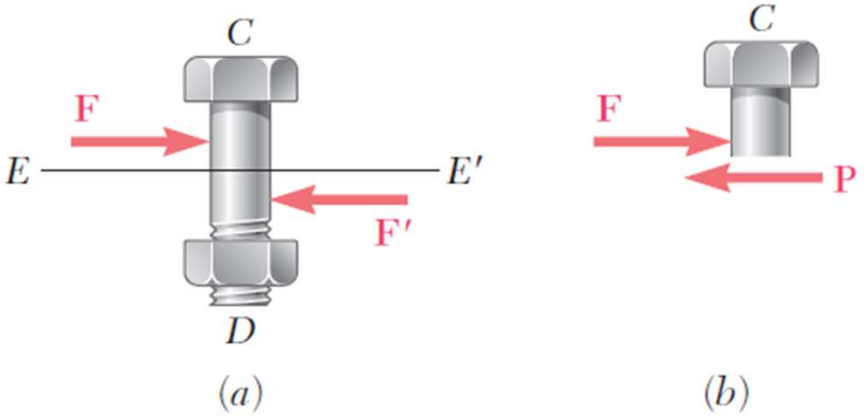


Fig. 1.17

The bolt we have just considered is said to be in *single shear*. Different loading situations may arise, however. For example, if splice plates C and D are used to connect plates A and B (Fig. 1.18), shear will take place in bolt HJ in each of the two planes KK' and LL' (and similarly in bolt EG). The bolts are said to be in *double shear*. To determine the average shearing stress in each plane, we draw free-body diagrams of bolt HJ and of the portion of bolt located between the two planes (Fig. 1.19). Observing that the shear P in each of the sections is $P = F/2$, we conclude that the average shearing stress is

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F/2}{A} = \frac{F}{2A} \tag{1.10}$$

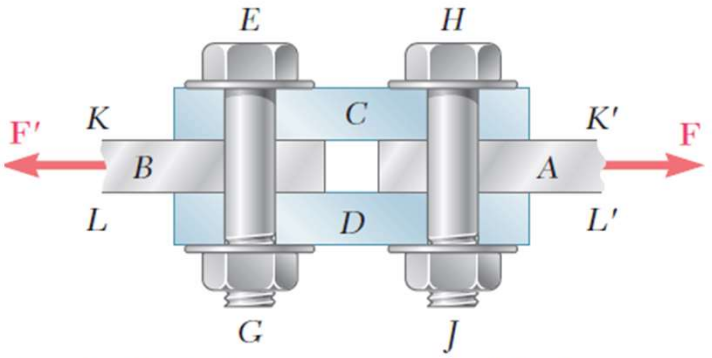


Fig. 1.18 Bolts subject to double shear.

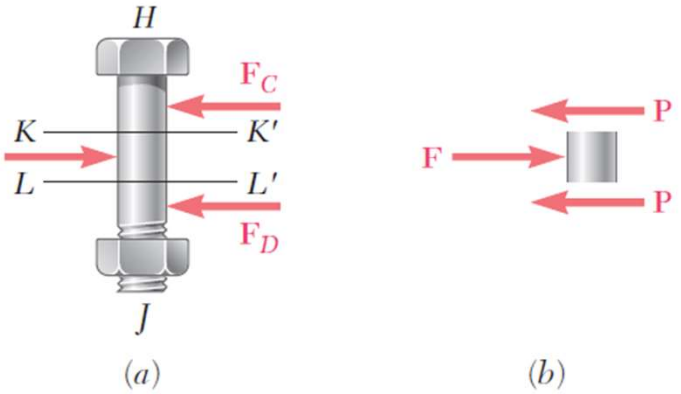


Fig. 1.19

BEARING STRESS

Sometimes compressive stresses arise where one body is supported by another. If the resultant of the applied forces coincides with the centroid of the contact area between the two bodies, the intensity of force, or stress, between the two bodies can again be determined from Eq. 1-6. It is customary to refer to this normal stress as a **bearing stress**. Figure 1-9, where a short block bears on a concrete pier and the latter bears on the soil, illustrates such a stress.

These bearing stresses can be approximated by dividing the applied force P by the corresponding contact area giving a useful nominal bearing stress.

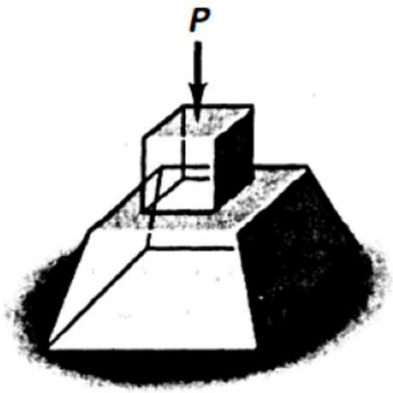


Fig. 1-9 Bearing stresses occur between the block and pier, as well as between the pier and soil.

1.7 BEARING STRESS IN CONNECTIONS

Bolts, pins, and rivets create stresses in the members they connect, along the *bearing surface*, or surface of contact. For example, consider again the two plates A and B connected by a bolt CD that we have discussed in the preceding section (Fig. 1.16). The bolt exerts on plate A a force **P** equal and opposite to the force **F** exerted by the plate on the bolt (Fig. 1.20). The force **P** represents the resultant of elementary forces distributed on the inside surface of a half-cylinder of diameter d and of length t equal to the thickness of the plate. Since the distribution of these forces—and of the corresponding stresses—is quite complicated, one uses in practice an average nominal value σ_b of the stress, called the *bearing stress*, obtained by dividing the load P by the area of the rectangle representing the projection of the bolt on the plate section (Fig. 1.21). Since this area is equal to td, where t is the plate thickness and d the diameter of the bolt, we have

$$\sigma_b = \frac{P}{A} = \frac{P}{td} \tag{1.11}$$

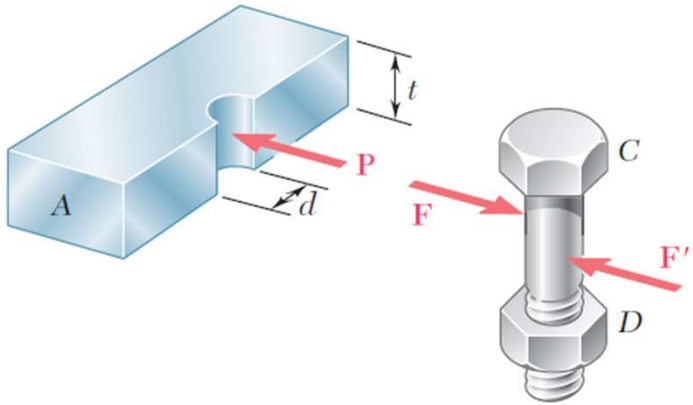


Fig. 1.20

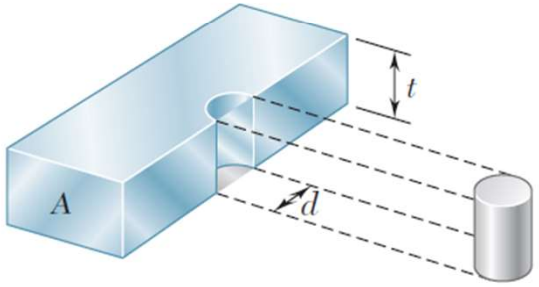
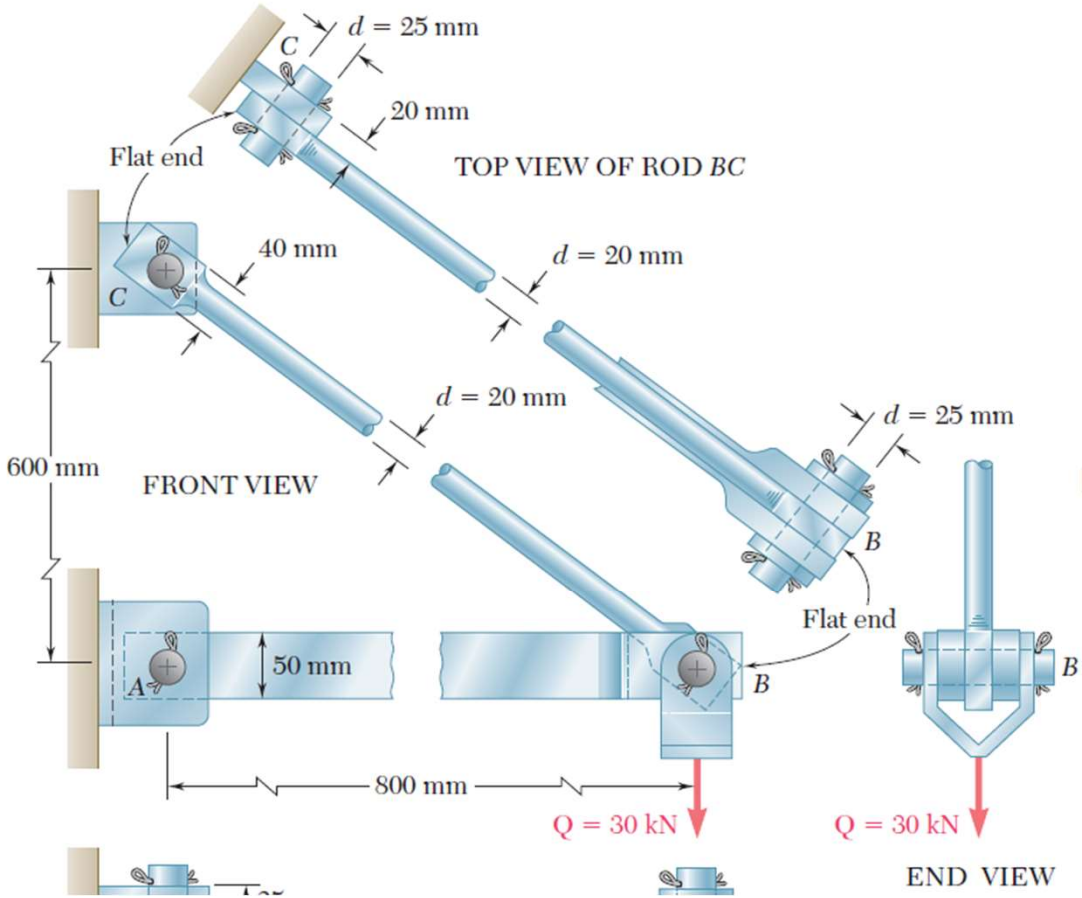


Fig. 1.21

APPLICATION TO THE ANALYSIS AND DESIGN OF SIMPLE STRUCTURES



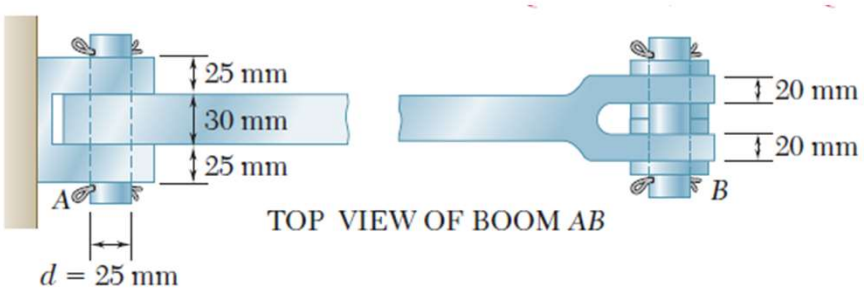
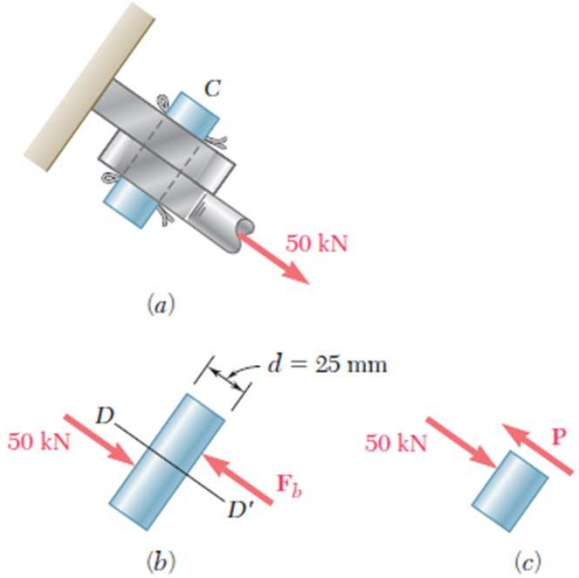
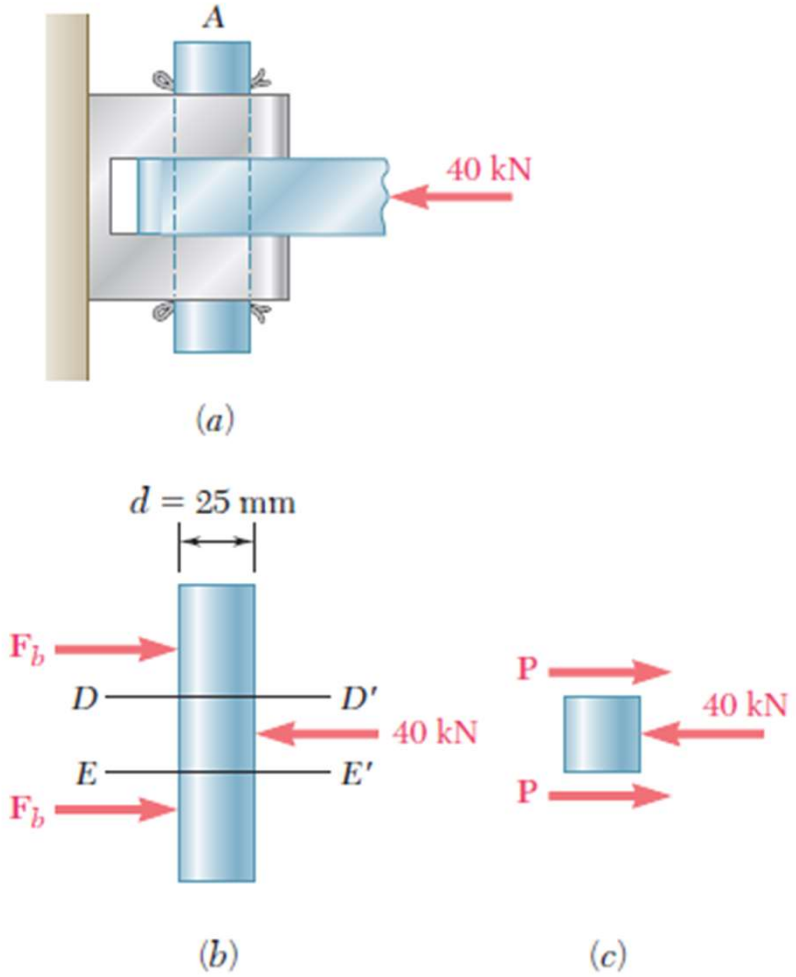
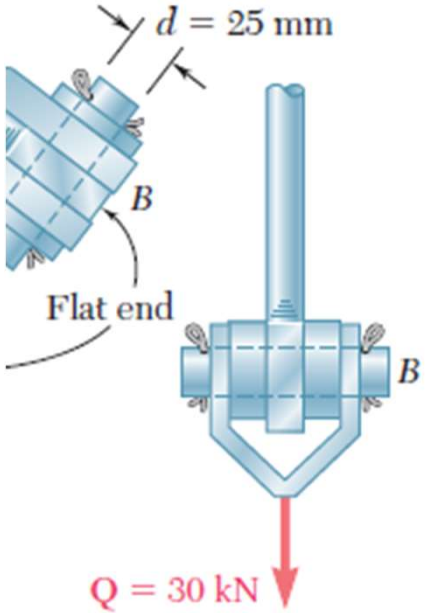
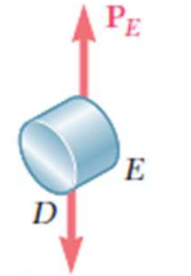


Fig. 1.22

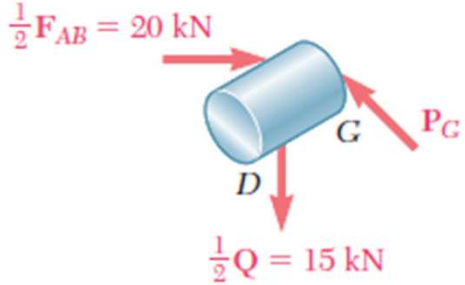

C
50 kN
(a)
d = 25 mm
D
50 kN
F_b
D′
(b)
50 kN
P
(c)


A
40 kN
(a)
d = 25 mm
F_b
D
D′
40 kN
E
E′
F_b
(b)
P
40 kN
P
(c)

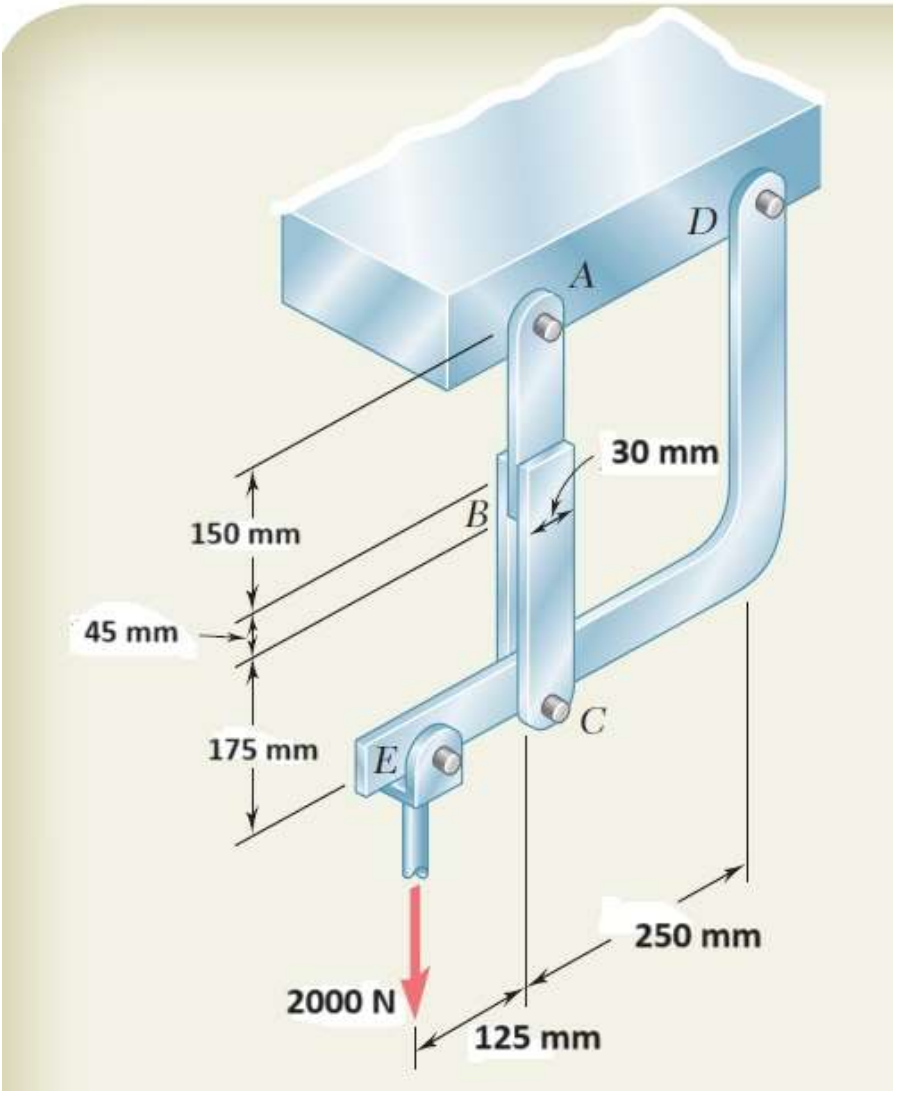




(b)



- In the hanger shown, the upper portion of link ABC is 10 mm thick and the lower portions are each 6 mm thick. Epoxy resin is used to bond the upper and lower portions together at B. The pin at A is of 10 mm diameter while a 6 mm diameter pin is used at C. Determine (a) the shearing stress in pin A, (b) the shearing stress in pin C, (c) the largest normal stress in link ABC, (d) the average shearing stress on the bonded surfaces at B, (e) the bearing stress in the link at C



- The steel tie bar shown is to be designed to carry a tension force of magnitude P =120 kN when bolted between double brackets at A and B. The bar will be fabricated from 20-mm-thick plate stock. For the grade of steel to be used, the maximum allowable stresses are: σ=175 MPa, τ=100 MPa, σ_b = 350 MPa. Design the tie bar by determining the required values of (a) the diameter d of the bolt, (b) the dimension b at each end of the bar, (c) the dimension h of the bar.

