# EWCE-211
# Concept of Stress

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- So far, we have seen axial forces exerted on a two-force member (Fig. 1a) were found to cause normal stresses in that member (Fig. 1), while transverse forces exerted on bolts and pins (Fig. 2) were found to cause shearing stresses in those connections. The reason such a relation was observed between axial forces and normal stresses on one hand, and transverse forces and shearing stresses on the other, was because stresses were being determined only on planes perpendicular to the axis of the member or connection.

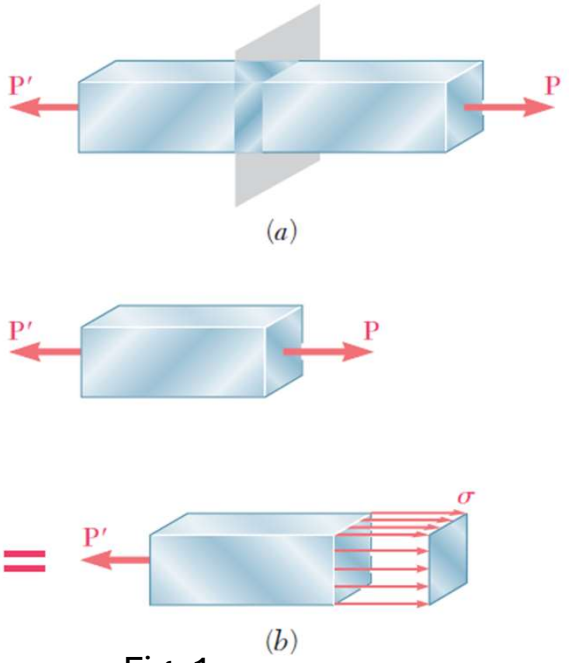


Fig. 1

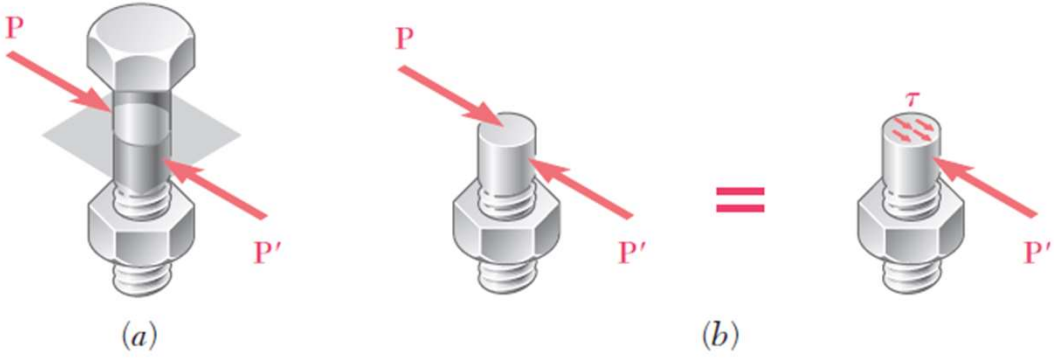
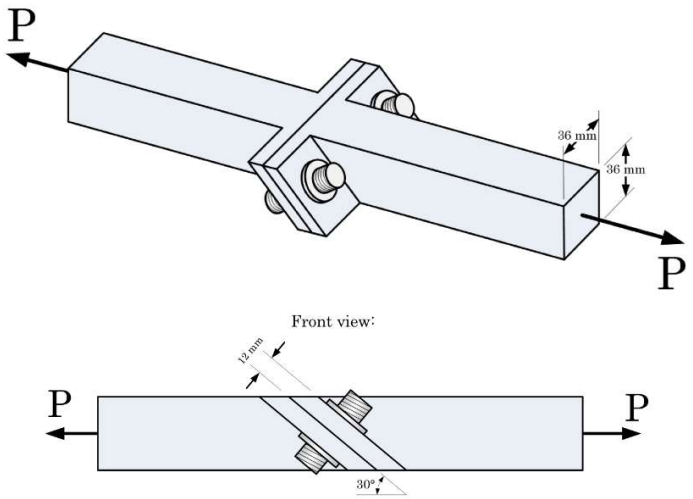
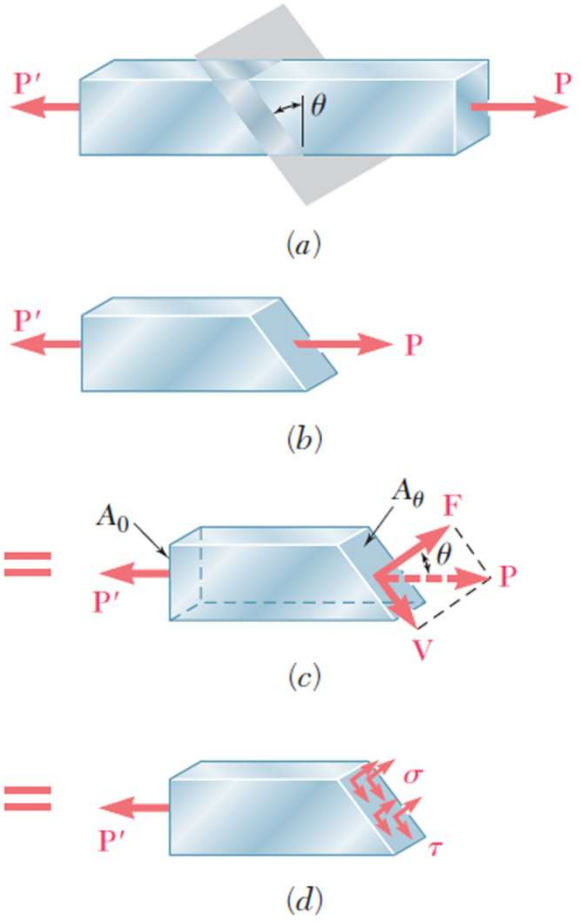


Fig. 2

# STRESS ON AN OBLIQUE PLANE UNDER AXIAL LOADING

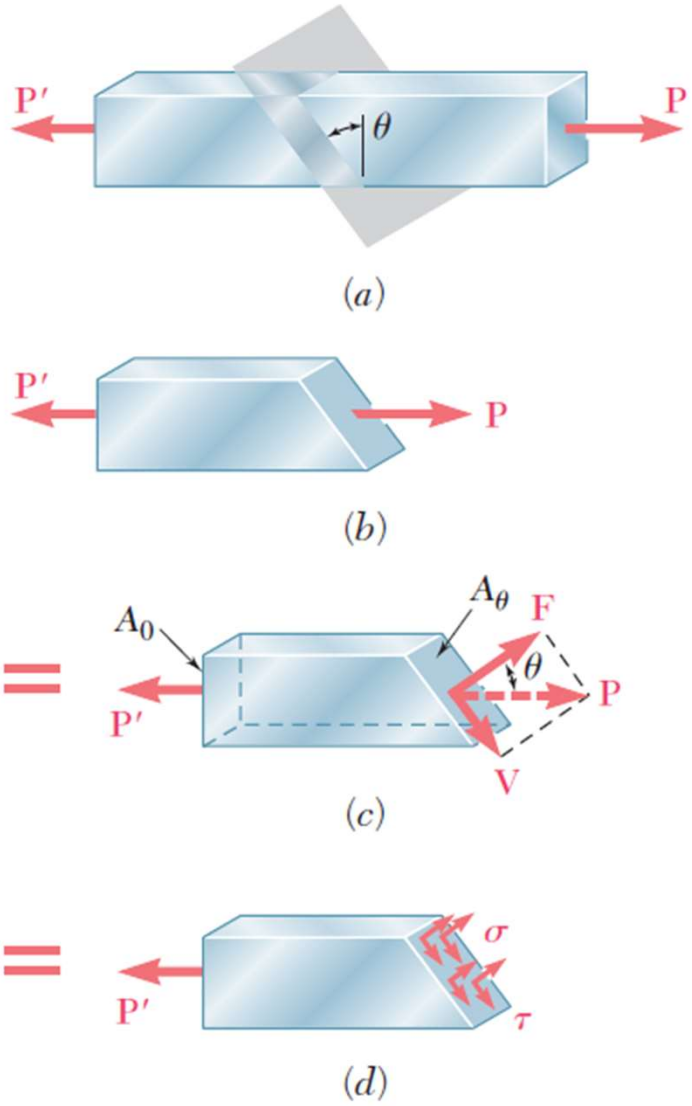
- Axial forces cause both normal and shearing stresses on planes which are not perpendicular to the axis of the member. Similarly, transverse forces exerted on a bolt or a pin cause both normal and shearing stresses on planes which are not perpendicular to the axis of the bolt or pin.



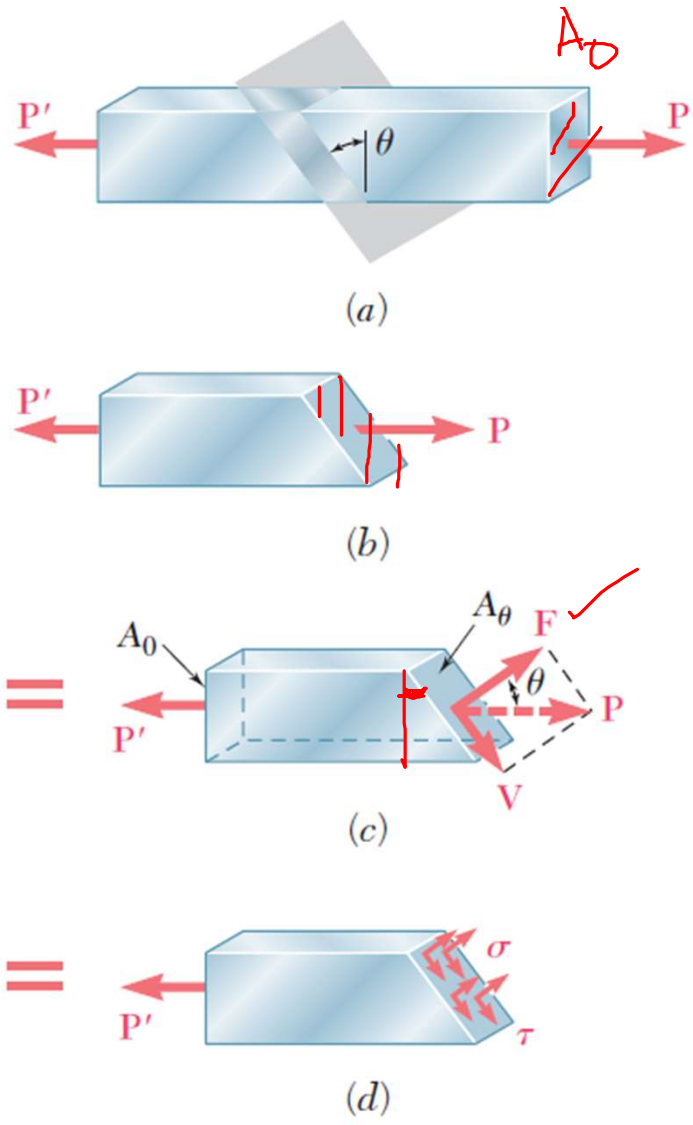
- If we pass a section forming an angle θ with a normal plane (Fig.) and draw the free-body diagram of the portion of members located to the left of that section (Fig.), we find from the equilibrium conditions of the free body that the distributed forces acting on the section must be equivalent to the force P. Resolving P into components F and V, respectively normal and tangential to the section (Fig), we have

$$F = P\cos\theta \qquad V = P\sin\theta$$



- The force F represents the resultant of normal forces distributed over the section, and the force V the resultant of shearing forces (Fig.). The average values of the corresponding normal and shearing stresses are obtained by dividing, respectively, F and V by the area $A_\theta$ of the section:

$$\sigma = \frac{F}{A_\theta} \qquad \tau = \frac{V}{A_\theta}$$



- Substituting for F and V , and observing from
- Fig. that $A_0 = A_\theta \cos\theta$, or $A_\theta = A_0/\cos\theta$, where $A_0$ denotes the area of a section perpendicular to the axis of the member, we obtain

$$\sigma = \frac{P\cos\theta}{A_0/\cos\theta} \qquad \tau = \frac{P\sin\theta}{A_0/\cos\theta}$$

$$\sigma = \frac{P}{A_0}\cos^2\theta \qquad \tau = \frac{P}{A_0}\sin\theta\cos\theta$$



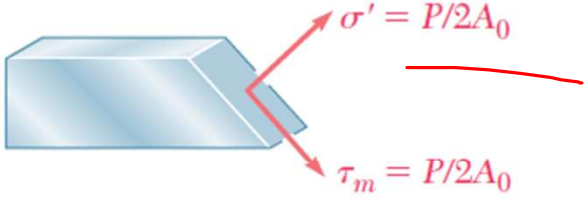

(*a*) Axial loading

$$\sigma_m = \frac{P}{A_0}$$



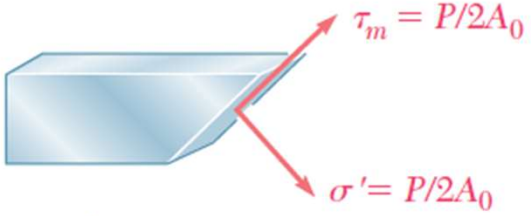

(*b*) Stresses for $\theta = 0$

$$\tau_m = \frac{P}{A_0} \sin 45° \cos 45° = \frac{P}{2A_0}$$




(*c*) Stresses for $\theta = 45°$

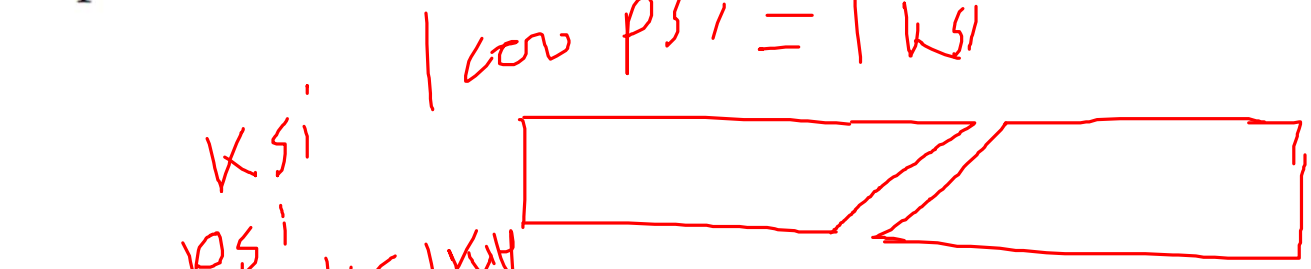
$$\sigma' = \frac{P}{A_0} \cos^2 45° = \frac{P}{2A_0}$$

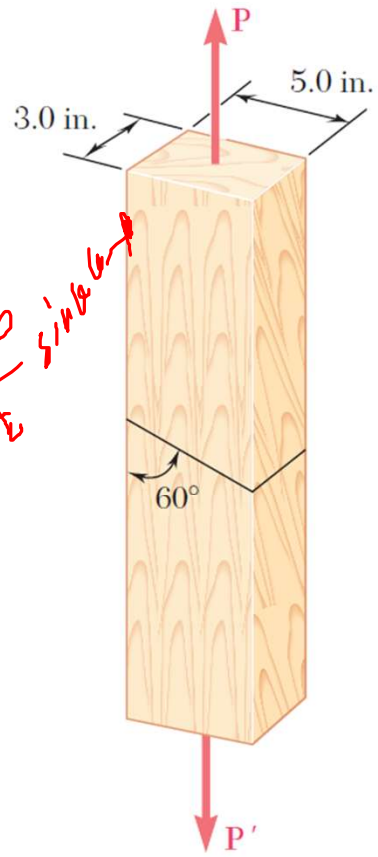



(*d*) Stresses for $\theta = -45°$

The 1.4-kip load **P** is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

$A_0 = 3 \times 5 \text{ in}^2$






$$\sigma = \frac{P}{A_0}\cos\theta = \frac{1.4}{A_0}\cos 60$$

$$\tau = \frac{P}{A_0}\sin\theta$$

## 1.12 STRESS UNDER GENERAL LOADING CONDITIONS; COMPONENTS OF STRESS

The examples of the previous sections were limited to members under axial loading and connections under transverse loading. Most structural members and machine components are under more involved loading conditions.

Consider a body subjected to several loads $\mathbf{P}_1$, $\mathbf{P}_2$, etc. (Fig. 1.30). To understand the stress condition created by these loads at some point $Q$ within the body, we shall first pass a section through $Q$, using a plane parallel to the $yz$ plane. The portion of the body to the left of the section is subjected to some of the original loads, and to normal and shearing forces distributed over the section. We shall denote by $\Delta\mathbf{F}^x$ and $\Delta\mathbf{V}^x$, respectively, the normal and the shearing

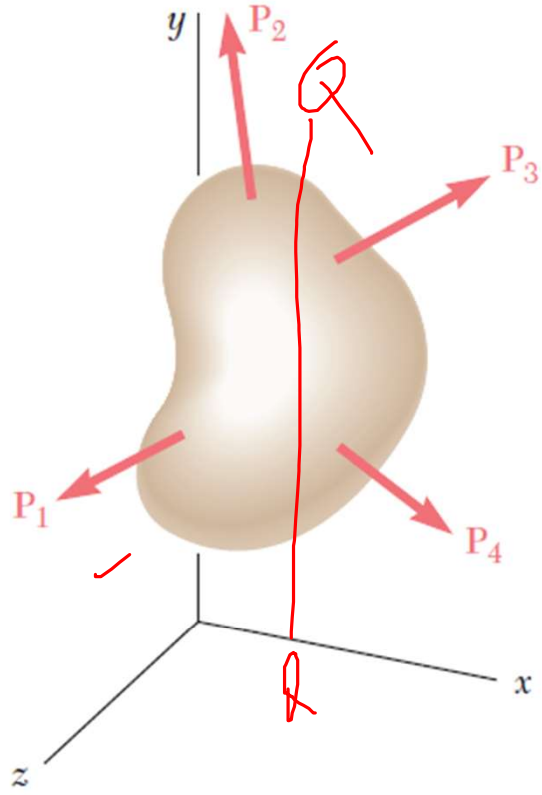


**Fig. 1.30**

forces acting on a small area $\Delta A$ surrounding point $Q$ (Fig. 1.31*a*). Note that the superscript $x$ is used to indicate that the forces $\Delta\mathbf{F}^x$ and $\Delta\mathbf{V}^x$ act on a surface perpendicular to the $x$ axis. While the normal force $\Delta\mathbf{F}^x$ has a well-defined direction, the shearing force $\Delta\mathbf{V}^x$ may have any direction in the plane of the section. We therefore resolve $\Delta\mathbf{V}^x$ into two component forces, $\Delta\mathbf{V}^x_y$ and $\Delta\mathbf{V}^x_z$, in directions parallel to the $y$ and $z$ axes, respectively (Fig. 1.31*b*). Dividing now the magnitude of each force by the area $\Delta A$, and letting $\Delta A$ approach zero, we define the three stress components shown in Fig. 1.32:

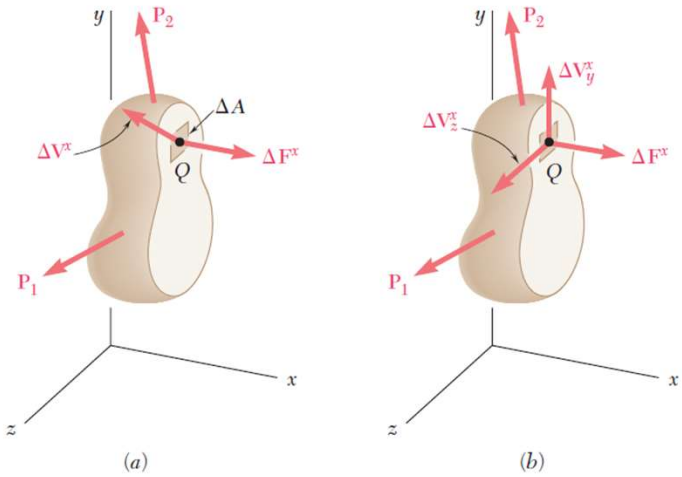



Fig. 1.31

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F^x}{\Delta A} \tag{1.18}$$

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V^x_y}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta V^x_z}{\Delta A}$$

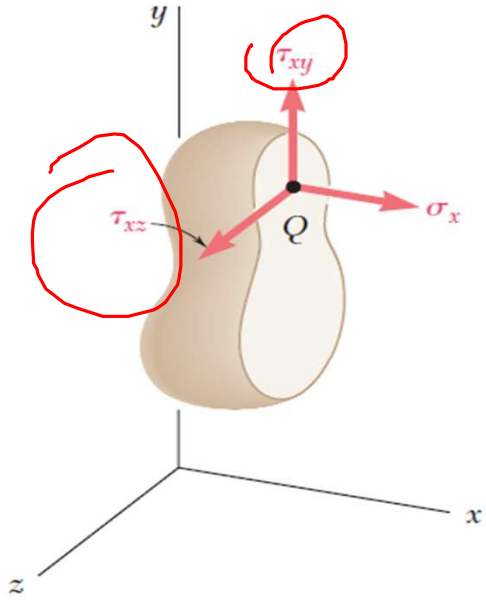



Fig. 1.32

We note that the first subscript in $\sigma_x$, $\tau_{xy}$, and $\tau_{xz}$ is used to indicate that the stresses under consideration are exerted *on a surface perpendicular to the x axis.* The second subscript in $\tau_{xy}$ and $\tau_{xz}$ identifies *the direction of the component.* The normal stress $\sigma_x$ is positive if the corresponding arrow points in the positive $x$ direction, i.e., if the body is in tension, and negative otherwise. Similarly, the shearing stress components $\tau_{xy}$ and $\tau_{xz}$ are positive if the corresponding arrows point, respectively, in the positive $y$ and $z$ directions.

The above analysis may also be carried out by considering the portion of body located to the right of the vertical plane through $Q$ (Fig. 1.33). The same magnitudes, but opposite directions, are obtained for the normal and shearing forces $\Delta\mathbf{F}^x$, $\Delta\mathbf{V}_y^x$, and $\Delta\mathbf{V}_z^x$. Therefore, the same values are also obtained for the corresponding stress components, but since the section in Fig. 1.33 now faces the *negative x axis,* a positive sign for $\sigma_x$ will indicate that the corresponding arrow points *in the negative x direction.* Similarly, positive signs for $\tau_{xy}$ and $\tau_{xz}$ will indicate that the corresponding arrows point, respectively, in the negative $y$ and $z$ directions, as shown in Fig. 1.33.

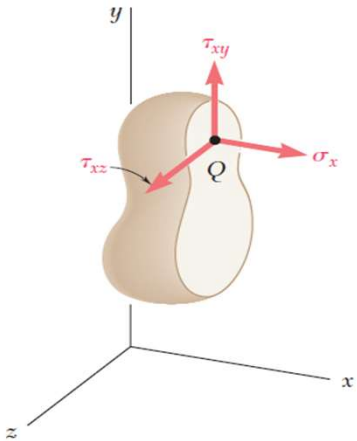



**Fig. 1.32**

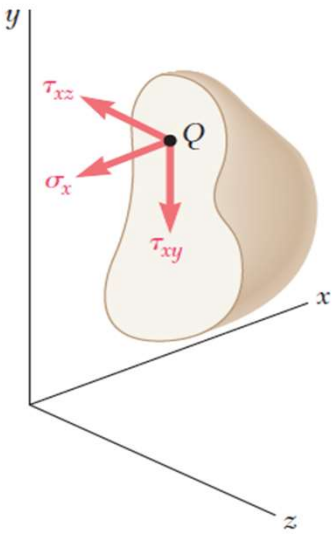



**Fig. 1.33**

Passing a section through $Q$ parallel to the $zx$ plane, we define in the same manner the stress components, $\sigma_y$, $\tau_{yz}$, and $\tau_{yx}$. Finally, a section through $Q$ parallel to the $xy$ plane yields the components $\sigma_z$, $\tau_{zx}$, and $\tau_{zy}$.

To facilitate the visualization of the stress condition at point $Q$, we shall consider a small cube of side $a$ centered at $Q$ and the stresses exerted on each of the six faces of the cube (Fig. 1.34). The stress components shown in the figure are $\sigma_x$, $\sigma_y$, and $\sigma_z$, which represent the normal stress on faces respectively perpendicular to the $x$, $y$, and $z$ axes, and the six shearing stress components $\tau_{xy}$, $\tau_{xz}$, etc. We recall that, according to the definition of the shearing stress components, $\tau_{xy}$ represents the $y$ component of the shearing stress exerted on the face perpendicular to the $x$ axis, while $\tau_{yx}$ represents the $x$ component of the shearing stress exerted on the face perpendicular to the $y$ axis. Note that only three faces of the cube are actually visible in Fig. 1.34, and that equal and opposite stress components act on the hidden faces. While the stresses acting on the faces of the cube differ slightly from the stresses at $Q$, the error involved is small and vanishes as side $a$ of the cube approaches zero.

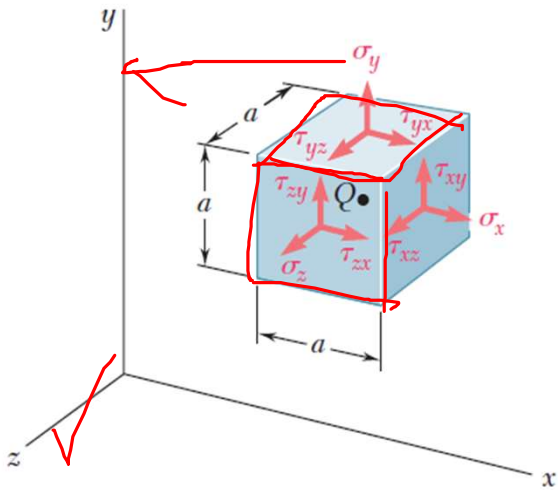



**Fig. 1.34**

Important relations among the shearing stress components will now be derived. Let us consider the free-body diagram of the small cube centered at point $Q$ (Fig. 1.35). The normal and shearing forces acting on the various faces of the cube are obtained by multiplying the corresponding stress components by the area $\Delta A$ of each face. We first write the following three equilibrium equations:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \tag{1.19}$$

Since forces equal and opposite to the forces actually shown in Fig. 1.35 are acting on the hidden faces of the cube, it is clear that Eqs. (1.19) are satisfied. Considering now the moments of the forces about axes $x'$, $y'$, and $z'$ drawn from $Q$ in directions respectively parallel to the $x$, $y$, and $z$ axes, we write the three additional equations

$$\Sigma M_{x'} = 0 \qquad \Sigma M_{y'} = 0 \qquad \Sigma M_{z'} = 0 \tag{1.20}$$

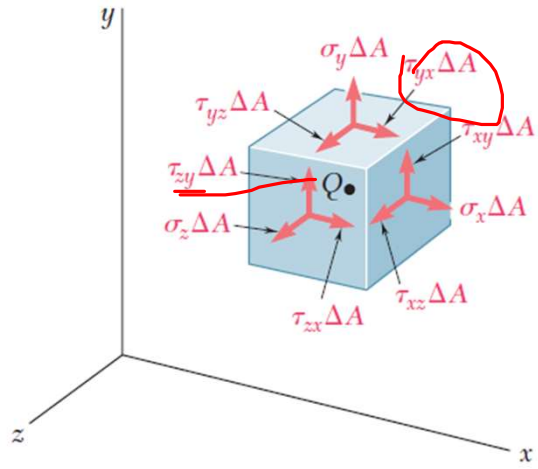



**Fig. 1.35**

Using a projection on the $x'y'$ plane (Fig. 1.36), we note that the only forces with moments about the $z$ axis different from zero are the shearing forces. These forces form two couples, one of counterclockwise (positive) moment $(\tau_{xy}\,\Delta A)a$, the other of clockwise (negative) moment $-(\tau_{yx}\,\Delta A)a$. The last of the three Eqs. (1.20) yields, therefore,

$$+\circlearrowleft \Sigma M_z = 0: \qquad (\tau_{xy}\,\Delta A)a - (\tau_{yx}\,\Delta A)a = 0$$

from which we conclude that

$$\tau_{xy} = \tau_{yx} \tag{1.21}$$

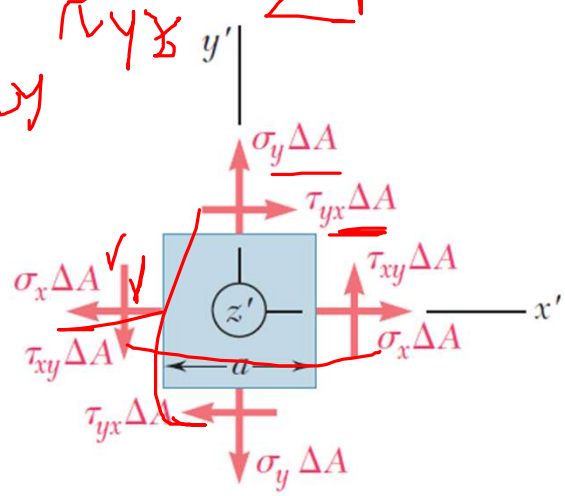



**Fig. 1.36**

The relation obtained shows that the $y$ component of the shearing stress exerted on a face perpendicular to the $x$ axis is equal to the $x$ component of the shearing stress exerted on a face perpendicular to the $y$ axis. From the remaining two equations (1.20), we derive in a similar manner the relations

$$\tau_{yz} = \tau_{zy} \qquad \tau_{zx} = \tau_{xz} \tag{1.22}$$

We conclude from Eqs. (1.21) and (1.22) that only six stress components are required to define the condition of stress at a given point $Q$, instead of nine as originally assumed. These six components are $\sigma_x$, $\sigma_y$, $\sigma_z$, $\tau_{xy}$, $\tau_{yz}$, and $\tau_{zx}$. We also note that, at a given point, *shear cannot take place in one plane only;* an equal shearing stress must be exerted on another plane perpendicular to the first one. For example, considering again the bolt of Fig. 1.27 and a small cube at the center $Q$ of the bolt (Fig. 1.37*a*), we find that shearing stresses of equal magnitude must be exerted on the two horizontal faces of the cube and on the two faces that are perpendicular to the forces **P** and **P′** (Fig. 1.37*b*).

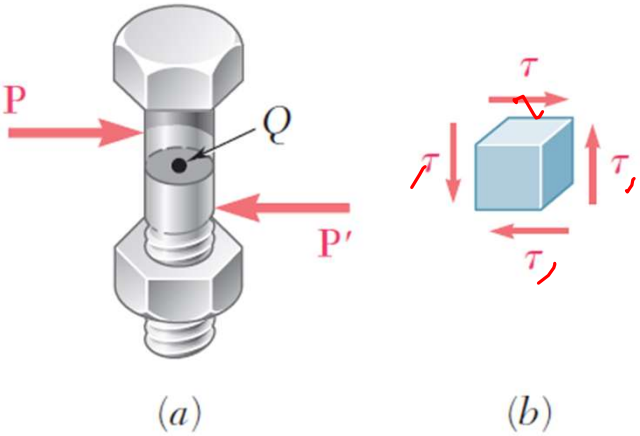



**Fig. 1.37**

Before concluding our discussion of stress components, let us consider again the case of a member under axial loading. If we consider a small cube with faces respectively parallel to the faces of the member and recall the results obtained in Sec. 1.11, we find that the conditions of stress in the member may be described as shown in Fig. 1.38*a*; the only stresses are normal stresses $\sigma_x$ exerted on the faces of the cube which are perpendicular to the $x$ axis. However, if the small cube is rotated by 45° about the $z$ axis so that its new orientation matches the orientation of the sections considered in Fig. 1.29*c* and *d*, we conclude that normal and shearing stresses of equal magnitude are exerted on four faces of the cube (Fig. 1.38*b*). We thus observe that the same loading condition may lead to different interpretations of the stress situation at a given point, depending upon the orientation of the element considered. More will be said about this in Chap 7.

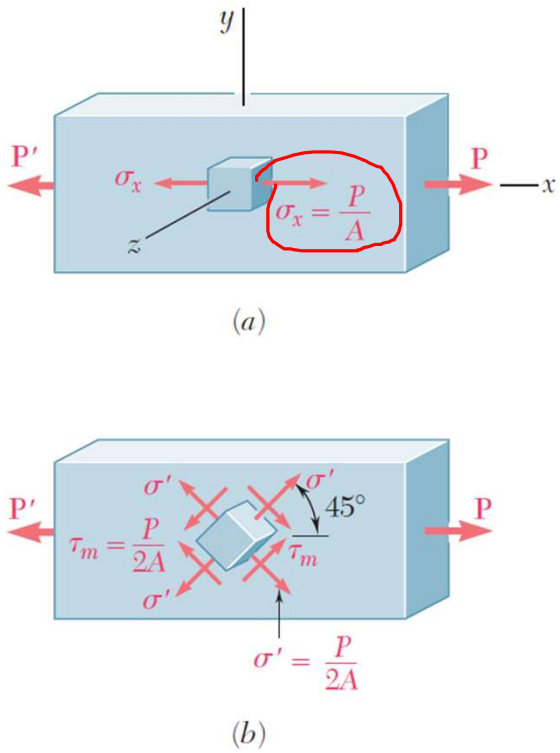



**Fig. 1.38**

## 1.13 DESIGN CONSIDERATIONS

In the preceding sections you learned to determine the stresses in rods, bolts, and pins under simple loading conditions. In later chapters you will learn to determine stresses in more complex situations. In engineering applications, however, the determination of stresses is seldom an end in itself. Rather, the knowledge of stresses is used by engineers to assist in their most important task, namely, the design of structures and machines that will safely and economically perform a specified function.

**a. Determination of the Ultimate Strength of a Material.** An important element to be considered by a designer is how the material that has been selected will behave under a load. For a given material, this is determined by performing specific tests on prepared samples of the material. For example, a test specimen of steel may be prepared and placed in a laboratory testing machine to be subjected to a known centric axial tensile force, as described in Sec. 2.3. As the magnitude of the force is increased, various changes in the specimen are measured, for example, changes in its length and its diameter.

Eventually the largest force which may be applied to the specimen is reached, and the specimen either breaks or begins to carry less load. This largest force is called the *ultimate load* for the test specimen and is denoted by $P_U$. Since the applied load is centric, we may divide the ultimate load by the original cross-sectional area of the rod to obtain the *ultimate normal stress* of the material used. This stress, also known as the *ultimate strength in tension* of the material, is

$$\sigma_U = \frac{P_U}{A} \tag{1.23}$$

Several test procedures are available to determine the *ultimate shearing stress*, or *ultimate strength in shear*, of a material. The one most commonly used involves the twisting of a circular tube (Sec. 3.5). A more direct, if less accurate, procedure consists in clamping a rectangular or round bar in a shear tool (Fig. 1.39) and applying an increasing load $P$ until the ultimate load $P_U$ for single shear is obtained. If the free end of the specimen rests on both of the hardened dies (Fig. 1.40), the ultimate load for double shear is obtained. In either case, the ultimate shearing stress $\tau_U$ is obtained by dividing the ultimate load by the total area over which shear has taken place. We recall that, in the case of single shear, this area is the cross-sectional area $A$ of the specimen, while in double shear it is equal to twice the cross-sectional area.

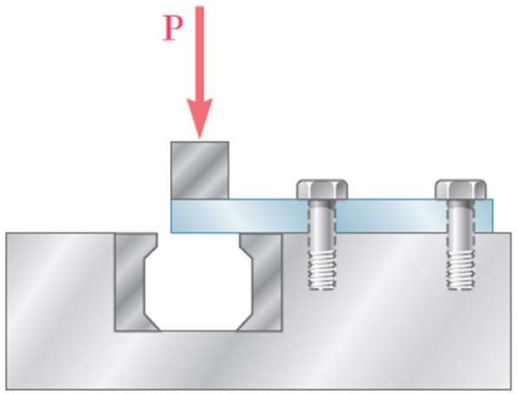



**Fig. 1.39** Single shear test.

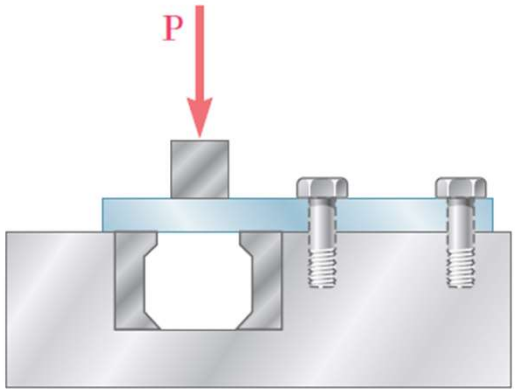



**Fig. 1.40** Double shear test.

**b. Allowable Load and Allowable Stress; Factor of Safety.** The maximum load that a structural member or a machine component will be allowed to carry under normal conditions of utilization is considerably smaller than the ultimate load. This smaller load is referred to as the *allowable load* and, sometimes, as the *working load* or *design load*. Thus, only a fraction of the ultimate-load capacity of the member is utilized when the allowable load is applied. The remaining portion of the load-carrying capacity of the member is kept in reserve to assure its safe performance. The ratio of the ultimate load to the allowable load is used to define the *factor of safety*.† We have

$$\text{Factor of safety} = F.S. = \frac{\text{ultimate load}}{\text{allowable load}} \tag{1.24}$$

An alternative definition of the factor of safety is based on the use of stresses:

$$\text{Factor of safety} = F.S. = \frac{\text{ultimate stress}}{\text{allowable stress}} \tag{1.25}$$

**c. Selection of an Appropriate Factor of Safety.** The selection of the factor of safety to be used for various applications is one of the most important engineering tasks. On the one hand, if a factor of safety is chosen too small, the possibility of failure becomes unacceptably large; on the other hand, if a factor of safety is chosen unnecessarily large, the result is an uneconomical or nonfunctional design. The choice of the factor of safety that is appropriate for a given design application requires engineering judgment based on many considerations, such as the following:

1. *Variations that may occur in the properties of the member under consideration*. The composition, strength, and dimensions of the member are all subject to small variations during manufacture. In addition, material properties may be altered and residual stresses introduced through heating or deformation that may occur during manufacture, storage, transportation, or construction.
2. *The number of loadings that may be expected during the life of the structure or machine*. For most materials the ultimate stress decreases as the number of load applications is increased. This phenomenon is known as *fatigue* and, if ignored, may result in sudden failure (see Sec. 2.7).

3. *The type of loadings that are planned for in the design, or that may occur in the future.* Very few loadings are known with complete accuracy—most design loadings are engineering estimates. In addition, future alterations or changes in usage may introduce changes in the actual loading. Larger factors of safety are also required for dynamic, cyclic, or impulsive loadings.
4. *The type of failure that may occur.* Brittle materials fail suddenly, usually with no prior indication that collapse is imminent. On the other hand, ductile materials, such as structural steel, normally undergo a substantial deformation called *yielding* before failing, thus providing a warning that overloading exists. However, most buckling or stability failures are sudden, whether the material is brittle or not. When the possibility of sudden failure exists, a larger factor of safety should be used than when failure is preceded by obvious warning signs.
5. *Uncertainty due to methods of analysis.* All design methods are based on certain simplifying assumptions which result in calculated stresses being approximations of actual stresses.

6. *Deterioration that may occur in the future because of poor maintenance or because of unpreventable natural causes*. A larger factor of safety is necessary in locations where conditions such as corrosion and decay are difficult to control or even to discover.
7. *The importance of a given member to the integrity of the whole structure*. Bracing and secondary members may in many cases be designed with a factor of safety lower than that used for primary members.

For the majority of structural and machine applications, factors of safety are specified by design specifications or building codes written by committees of experienced engineers working with professional societies, with industries, or with federal, state, or city agencies. Examples of such design specifications and building codes are

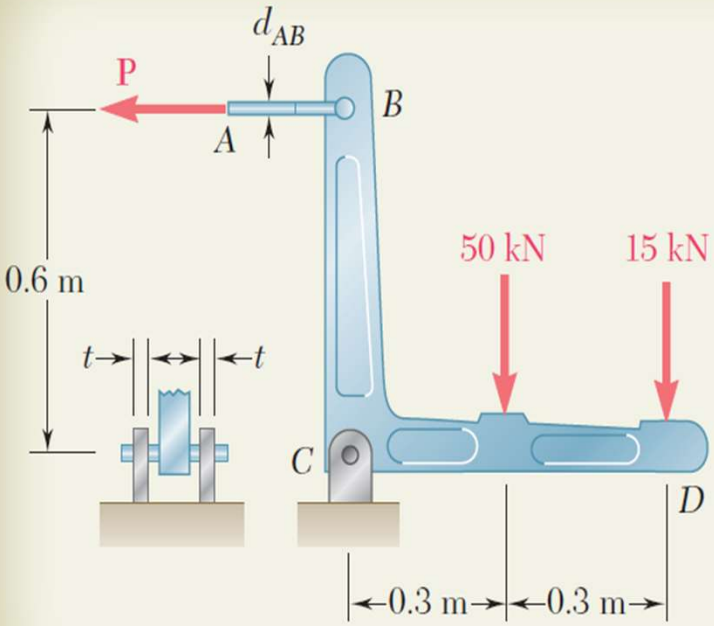
1. *Steel:* American Institute of Steel Construction, Specification for Structural Steel Buildings
2. *Concrete:* American Concrete Institute, Building Code Requirement for Structural Concrete
3. *Timber:* American Forest and Paper Association, National Design Specification for Wood Construction
4. *Highway bridges:* American Association of State Highway Officials, Standard Specifications for Highway Bridges

***d. Load and Resistance Factor Design.** As we saw previously, the allowable-stress method requires that all the uncertainties associated with the design of a structure or machine element be grouped into a single factor of safety. An alternative method of design, which is gaining acceptance chiefly among structural engineers, makes it possible through the use of three different factors to distinguish between the uncertainties associated with the structure itself and those associated with the load it is designed to support. This method, referred to as *Load and Resistance Factor Design (LRFD)*, further allows the designer to distinguish between uncertainties associated with the *live load*, $P_L$, that is, with the load to be supported by the structure, and the *dead load*, $P_D$, that is, with the weight of the portion of structure contributing to the total load.

When this method of design is used, the *ultimate load*, $P_U$, of the structure, that is, the load at which the structure ceases to be useful, should first be determined. The proposed design is then acceptable if the following inequality is satisfied:

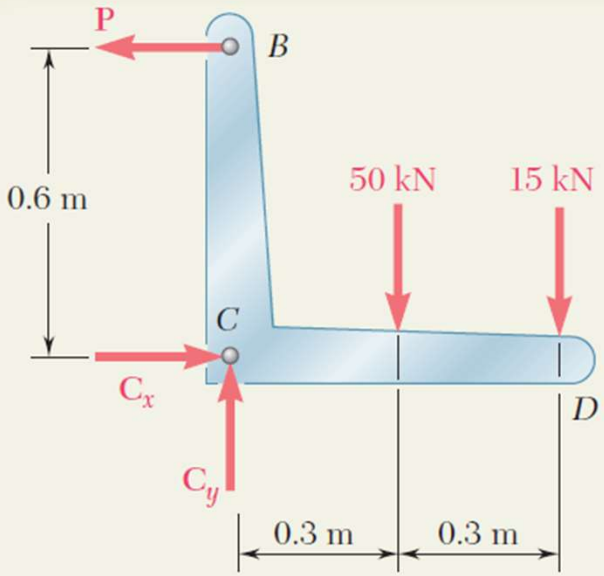
$$\gamma_D P_D + \gamma_L P_L \leq \phi P_U \tag{1.26}$$

The coefficient $\phi$ is referred to as the *resistance factor;* it accounts for the uncertainties associated with the structure itself and will normally be less than 1. The coefficients $\gamma_D$ and $\gamma_L$ are referred to as the *load factors;* they account for the uncertainties associated, respectively, with the dead and live load and will normally be greater than 1, with $\gamma_L$ generally larger than $\gamma_D$. While a few examples or assigned problems using LRFD are included in this chapter and in Chaps. 5 and 10, the allowable-stress method of design will be used in this text.




## SAMPLE PROBLEM 1.3

Two forces are applied to the bracket *BCD* as shown. (*a*) Knowing that the control rod *AB* is to be made of a steel having an ultimate normal stress of 600 MPa, determine the diameter of the rod for which the factor of safety with respect to failure will be 3.3. (*b*) The pin at *C* is to be made of a steel having an ultimate shearing stress of 350 MPa. Determine the diameter of the pin *C* for which the factor of safety with respect to shear will also be 3.3. (*c*) Determine the required thickness of the bracket supports at *C* knowing that the allowable bearing stress of the steel used is 300 MPa.




## SOLUTION

**Free Body: Entire Bracket.** The reaction at $C$ is represented by its components $\mathbf{C}_x$ and $\mathbf{C}_y$.

$+\circlearrowleft \Sigma M_C = 0$: $P(0.6\text{ m}) - (50\text{ kN})(0.3\text{ m}) - (15\text{ kN})(0.6\text{ m}) = 0$ $P = 40\text{ kN}$

$\Sigma F_x = 0$: $C_x = 40\text{ k}$

$\Sigma F_y = 0$: $C_y = 65\text{ kN}$ $C = \sqrt{C_x^2 + C_y^2} = 76.3\text{ kN}$

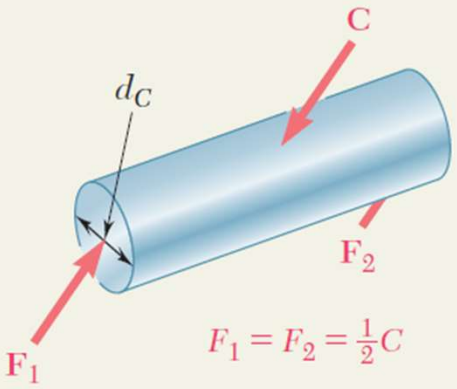
**a. Control Rod *AB*.** Since the factor of safety is to be 3.3, the allowable stress is

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{600\text{ MPa}}{3.3} = 181.8\text{ MPa}$$

For $P = 40$ kN the cross-sectional area required is

$$A_{\text{req}} = \frac{P}{\sigma_{\text{all}}} = \frac{40\text{ kN}}{181.8\text{ MPa}} = 220 \times 10^{-6}\text{ m}^2$$

$$A_{\text{req}} = \frac{\pi}{4} d_{AB}^2 = 220 \times 10^{-6}\text{ m}^2 \qquad d_{AB} = 16.74\text{ mm}$$



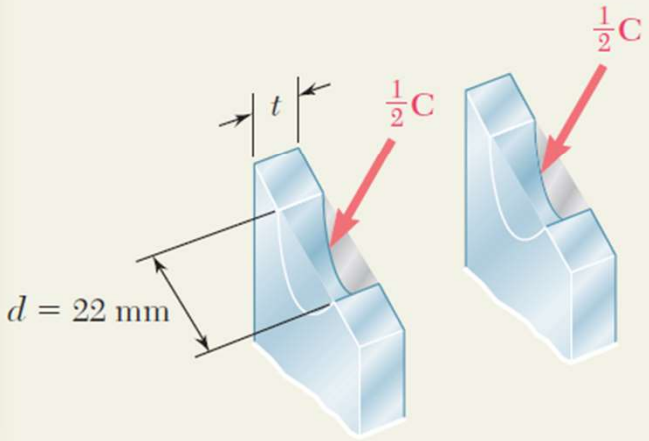

**b. Shear in Pin C.** For a factor of safety of 3.3, we have

$$\tau_{\text{all}} = \frac{\tau_U}{F.S.} = \frac{350 \text{ MPa}}{3.3} = 106.1 \text{ MPa}$$

Since the pin is in double shear, we write

$$A_{\text{req}} = \frac{C/2}{\tau_{\text{all}}} = \frac{(76.3 \text{ kN})/2}{106.1 \text{ MPa}} = 360 \text{ mm}^2$$

$$A_{\text{req}} = \frac{\pi}{4} d_C^2 = 360 \text{ mm}^2 \qquad d_C = 21.4 \text{ mm} \qquad \text{Use: } d_C = 22 \text{ mm}$$

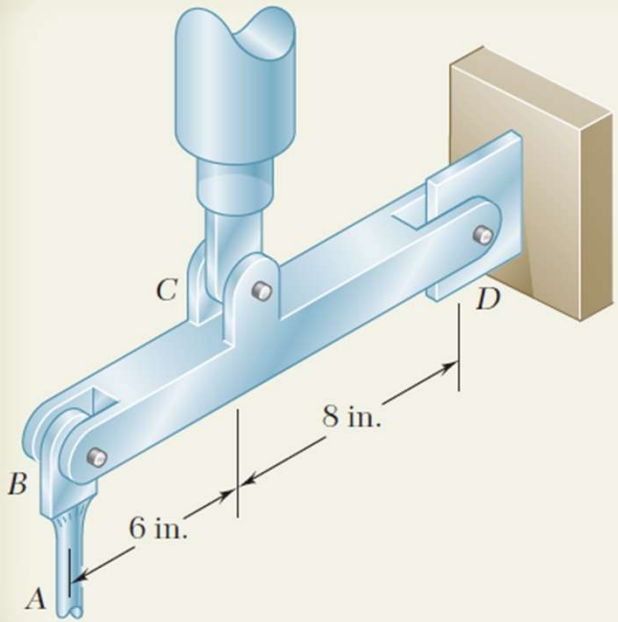



The next larger size pin available is of 22-mm diameter and should be used.

**c. Bearing at C.** Using $d = 22$ mm, the nominal bearing area of each bracket is $22t$. Since the force carried by each bracket is $C/2$ and the allowable bearing stress is 300 MPa, we write

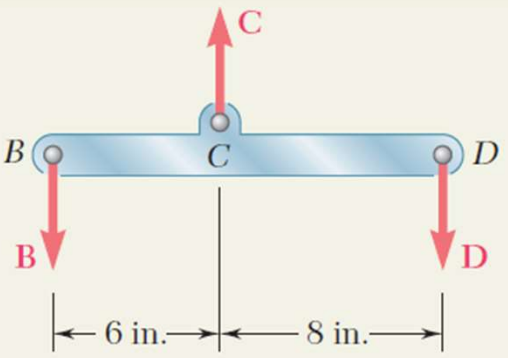
$$A_{\text{req}} = \frac{C/2}{\sigma_{\text{all}}} = \frac{(76.3 \text{ kN})/2}{300 \text{ MPa}} = 127.2 \text{ mm}^2$$

Thus $22t = 127.2$ $\quad t = 5.78$ mm $\qquad$ Use: $t = 6$ mm




## SAMPLE PROBLEM 1.4

The rigid beam $BCD$ is attached by bolts to a control rod at $B$, to a hydraulic cylinder at $C$, and to a fixed support at $D$. The diameters of the bolts used are: $d_B = d_D = \frac{3}{8}$ in., $d_C = \frac{1}{2}$ in. Each bolt acts in double shear and is made from a steel for which the ultimate shearing stress is $\tau_U = 40$ ksi. The control rod $AB$ has a diameter $d_A = \frac{7}{16}$ in. and is made of a steel for which the ultimate tensile stress is $\sigma_U = 60$ ksi. If the minimum factor of safety is to be 3.0 for the entire unit, determine the largest upward force which may be applied by the hydraulic cylinder at $C$.




## SOLUTION

The factor of safety with respect to failure must be 3.0 or more in each of the three bolts and in the control rod. These four independent criteria will be considered separately.

**Free Body: Beam *BCD*.** We first determine the force at $C$ in terms of the force at $B$ and in terms of the force at $D$.

$$+\circlearrowleft \Sigma M_D = 0: \qquad B(14 \text{ in.}) - C(8 \text{ in.}) = 0 \qquad C = 1.750B \tag{1}$$

$$+\circlearrowleft \Sigma M_B = 0: \qquad -D(14 \text{ in.}) + C(6 \text{ in.}) = 0 \qquad C = 2.33D \tag{2}$$

**Control Rod.** For a factor of safety of 3.0 we have

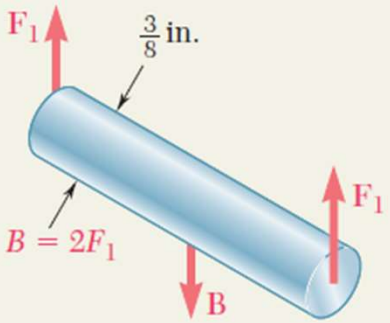
$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{60 \text{ ksi}}{3.0} = 20 \text{ ksi}$$

The allowable force in the control rod is

$$B = \sigma_{\text{all}}(A) = (20 \text{ ksi})\tfrac{1}{4}\pi(\tfrac{7}{16} \text{ in.})^2 = 3.01 \text{ kips}$$

Using Eq. (1) we find the largest permitted value of $C$:

$$C = 1.750B = 1.750(3.01 \text{ kips}) \qquad C = 5.27 \text{ kips}$$



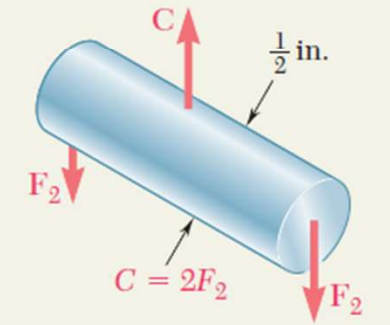

**Bolt at *B*.** $\tau_{\text{all}} = \tau_U/F.S. = (40 \text{ ksi})/3 = 13.33 \text{ ksi}$. Since the bolt is in double shear, the allowable magnitude of the force **B** exerted on the bolt is

$$B = 2F_1 = 2(\tau_{\text{all}} A) = 2(13.33 \text{ ksi})(\tfrac{1}{4}\pi)(\tfrac{3}{8} \text{ in.})^2 = 2.94 \text{ kips}$$

From Eq. (1): $C = 1.750B = 1.750(2.94 \text{ kips})$ $C = 5.15 \text{ kips}$

**Bolt at *D*.** Since this bolt is the same as bolt $B$, the allowable force is $D = B = 2.94$ kips. From Eq. (2):

$$C = 2.33D = 2.33(2.94 \text{ kips}) \qquad C = 6.85 \text{ kips}$$

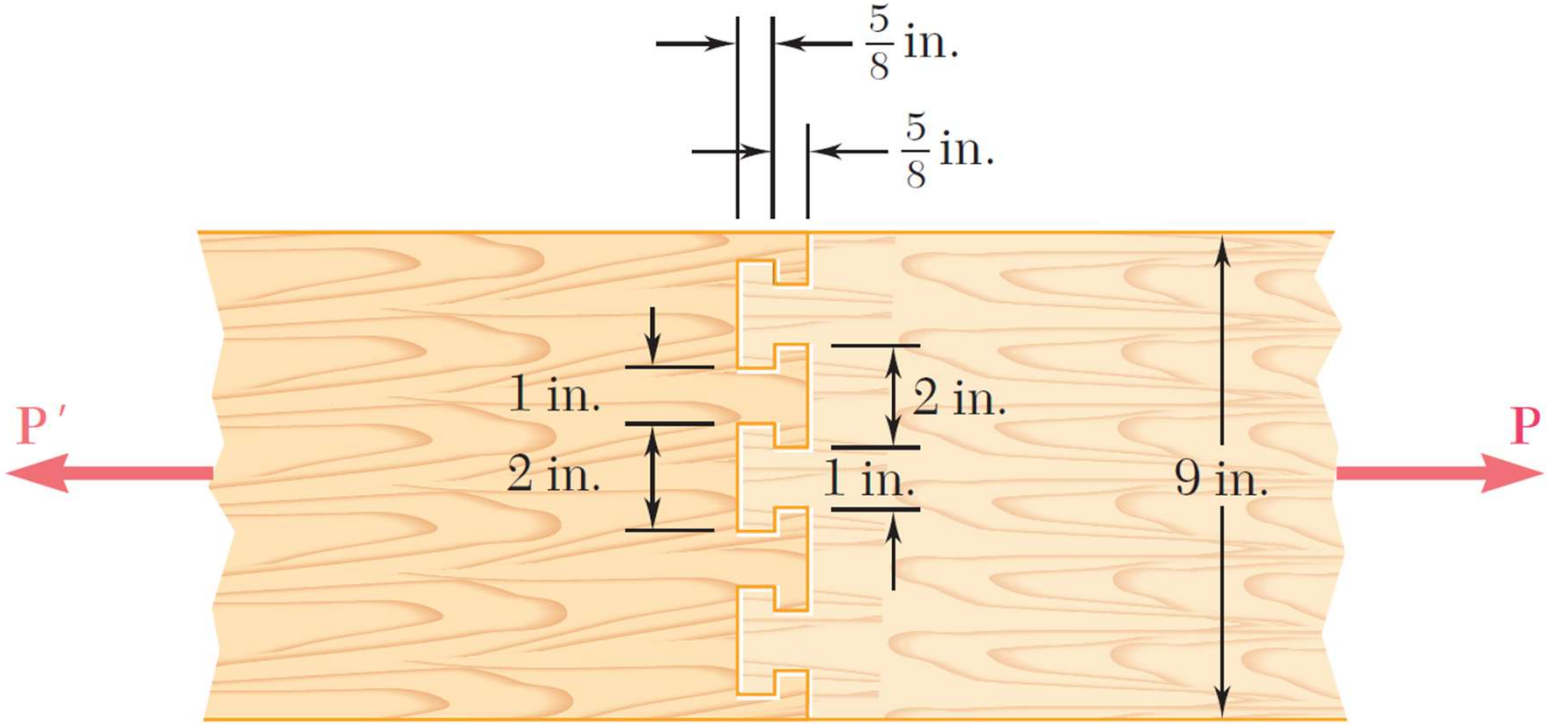



**Bolt at *C*.** We again have $\tau_{\text{all}} = 13.33$ ksi and write

$$C = 2F_2 = 2(\tau_{\text{all}} A) = 2(13.33 \text{ ksi})(\tfrac{1}{4}\pi)(\tfrac{1}{2} \text{ in.})^2 \qquad C = 5.23 \text{ kips}$$

**Summary.** We have found separately four maximum allowable values of the force $C$. In order to satisfy all these criteria we must choose the smallest value, namely: $C = 5.15$ kips

Two wooden planks, each $\frac{1}{2}$ in. thick and 9 in. wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 1.20 ksi, determine the magnitude $P$ of the axial load that will cause the joint to fail.



A steel plate $\frac{5}{16}$ in. thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when $P = 2.5$ kips, determine $(a)$ the required width $a$ of the plate, $(b)$ the minimum depth $b$ to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the bottom edge of the plate.)

