Buckling of Slender Column

Objective

- To determine critical/ buckling load of slender column (experimental).
- To determine critical/ buckling load of slender column (theoretical).
- To compare the experimental and theoretical results.

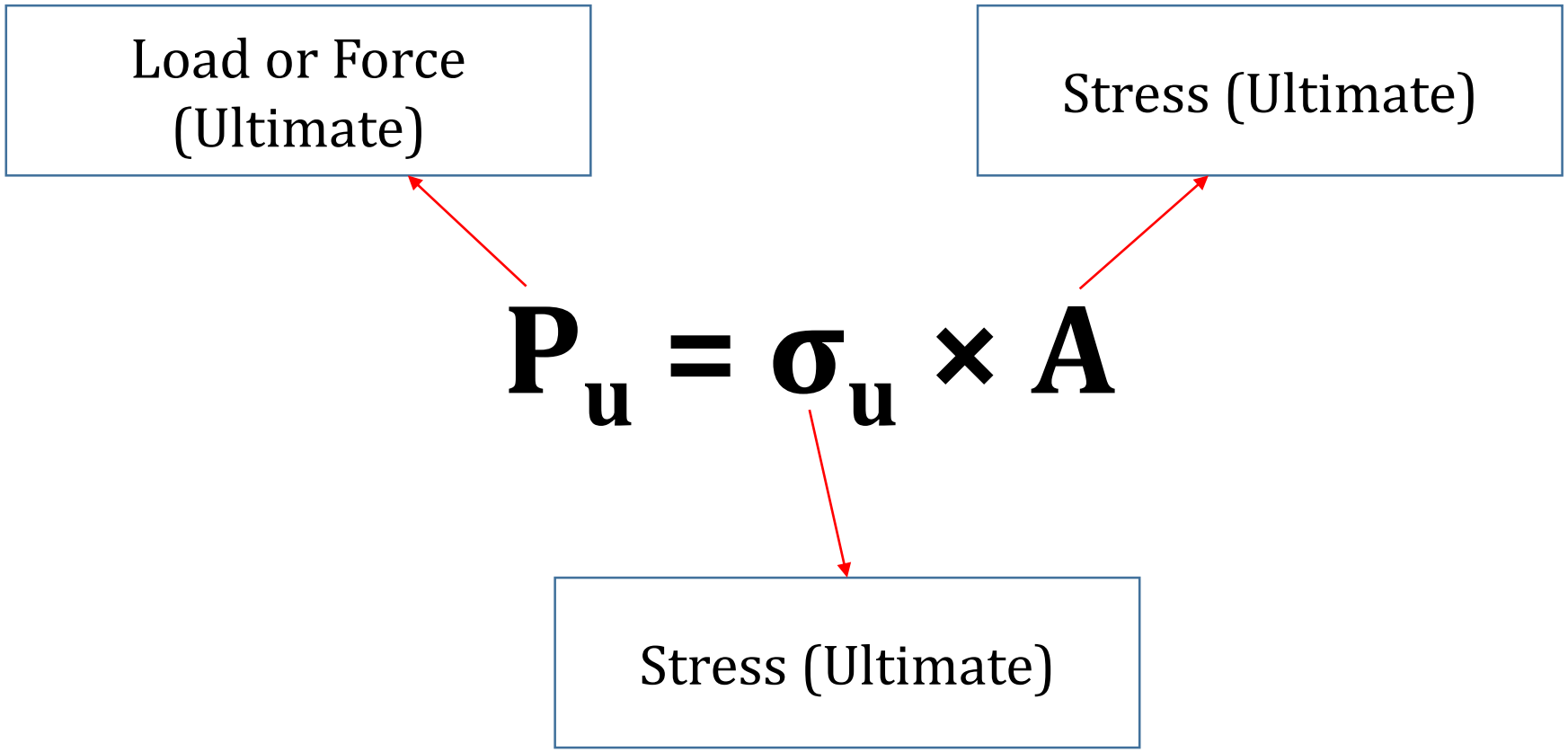
Preliminaries





Apparatus

- Digital slide calipers
- Column testing apparatus
- Steel scale
- Support systems


Load or Force
(Ultimate)
Stress (Ultimate)
$P_u = \sigma_u \times A$
Stress (Ultimate)

In case of Short Column previous equation holds good for determining strength.

In Case of Slender Column, column will fail at a lower load than that given by previous equation and buckling effect is to be considered. A column will buckle when it is subjected to a load greater than the critical load. That is, instead of remaining straight, it will suddenly become sharply curved as illustrated in Figure 1.

If ratio of effective length to the least lateral dimension is greater or equal to 12 , the column will be considered as slender column.

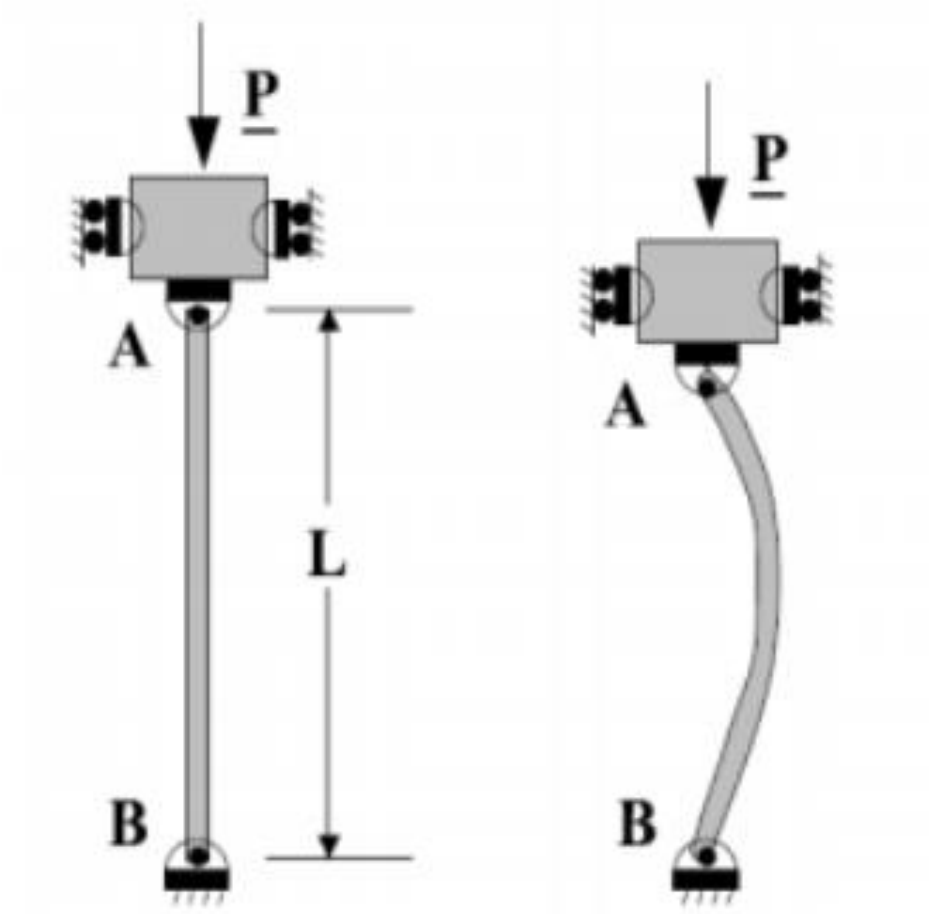


Figure 1: Buckling of column due to critical load

Eulers Theory for Slender Column

$$P_{cr} = \frac{\pi^2 E I}{L^2}$$

or

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2}$$

σ_{cr} = Critical load (stress)

P_{cr} = Critical load or force

$r = (I_{min}/A)^{1/2}$

E = Modulus of elasticity (stress unit)

I = Moment of Inertia

L = Length of the column

r = Radius of Gyration

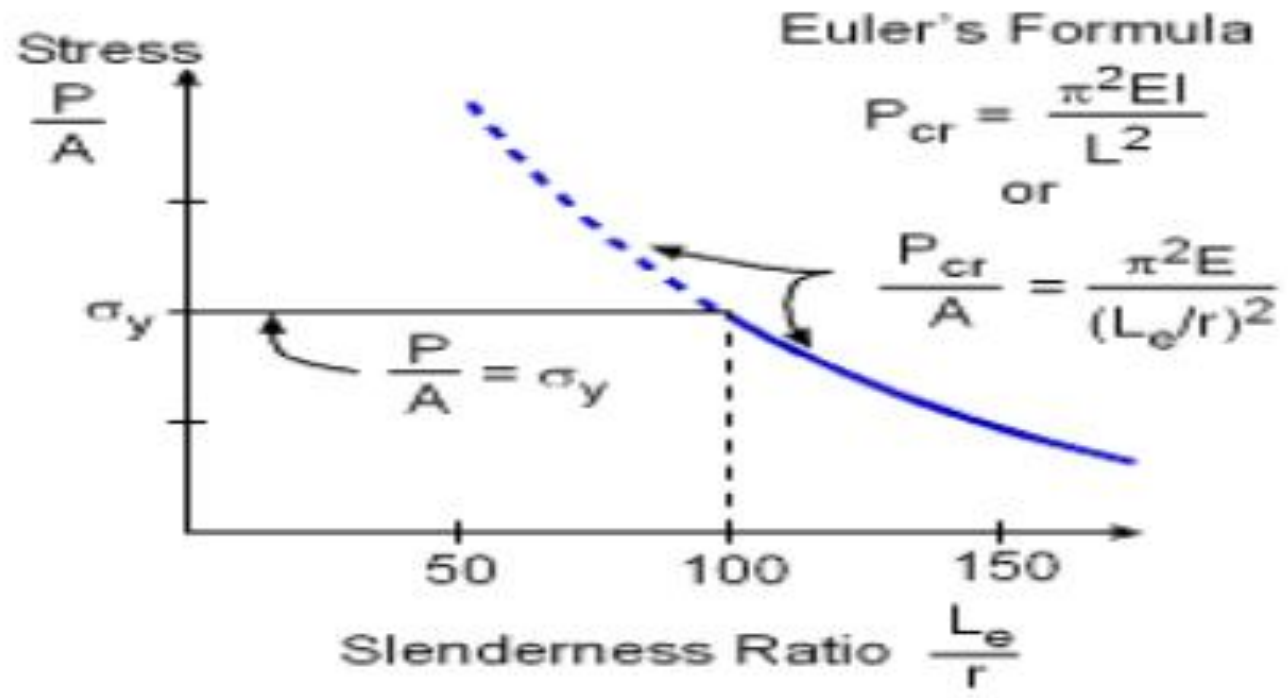


Figure 2: Euler Column Curve of a Given Material

Column End Conditions and Effective Length Concept

Column support condition can be –

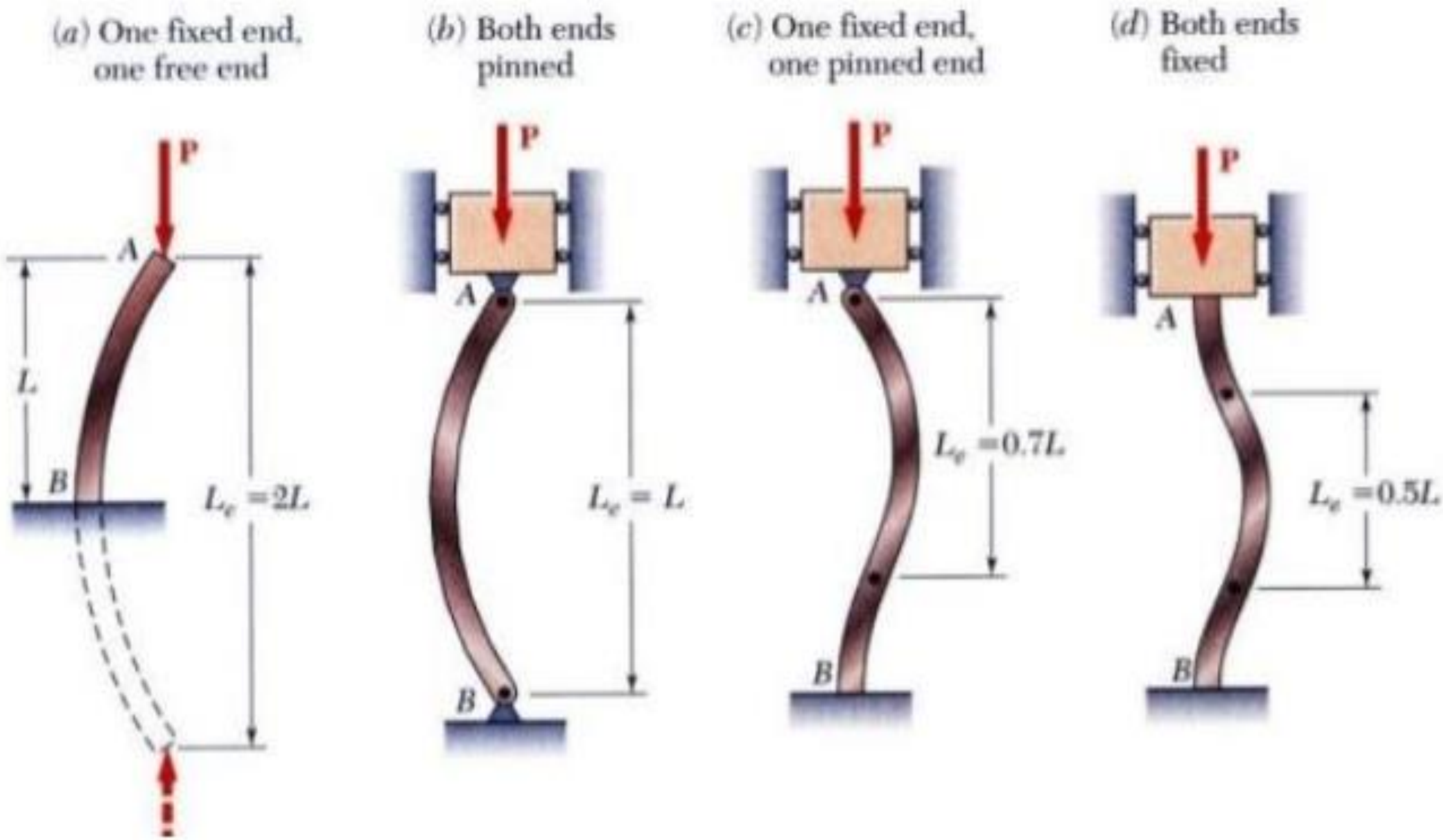
- Fixed
- Hinge
- Roller

Effective length is defined as the length of the equivalent pin-ended column that would have the same elastic critical load as the actual end-restrained column.

$$L_e = K \times L$$

- L_e: Effective length
- K: Effective length factor
- L: Actual length

End condition of the column	Effective Length
Both ends hinged	1.0 ℓ
One end hinged and one end fixed	0.707 ℓ
Both ends fixed	0.50 ℓ
One end fixed and one end free (i.e. cantilever)	2.0 ℓ


(a) One fixed end, one free end
(b) Both ends pinned
(c) One fixed end, one pinned end
(d) Both ends fixed
P
A
B
L
$L_e = 2L$
$L_e = L$
$L_e = 0.7L$
$L_e = 0.5L$

Eulers Theory for Slender Column

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

or

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

σ_{cr} = Critical load (stress)

P_{cr} = Critical load or force

E = Modulus of elasticity (stress unit)

I = Moment of Inertia

KL = Effective Length of the column (L_e)

r = Radius of Gyration

K = Effective Length Factor

$$r = (I_{min}/A)^{1/2}$$

$\frac{KL}{r}$ = slenderness ratio

Test Procedure

- At first, measure the geometric dimensions of the column (L, b, h, r)
- Then place the column in the testing apparatus between the end supports.
- Apply the compressive load axially.
- Record the critical buckling load from the display.
- Record deflection value for all support conditions.
- Perform the test for all support conditions (H-H, H-F, F-F)

Column Test

Support Condition	Effective Length Factor, K	Length, L (mm)	Radius of gyration, r (mm)	Slenderness ratio, KL/r	Analytical Critical Load, P_{cr} (N)	Experimental Critical Load, P_{cr} (N)	Critical Stress, σ_{cr} (Analyt.)	Critical Stress, σ_{cr} (Exp.)
F-F	0.5	750	0.932	402.36	793.94	500	12.81	8.079
H-F	0.7	750	0.9324	563.063	404.65	300	6.54	4.85
H-H	1	750	0.9324	804.376	198.25	90	3.203	1.454

*H= Hinge, F= Fixed.

Both End Hinge

Load (N)	30	60	90	120						
Deflection (mm)	0.2	0.93	1.1	3.8						

One End Hinge Another End Fixed

Load (N)	50	100	150	200	250	300	350			
Deflection (mm)	0	0.03	0.08	0.18	0.25	0.30	0.80			

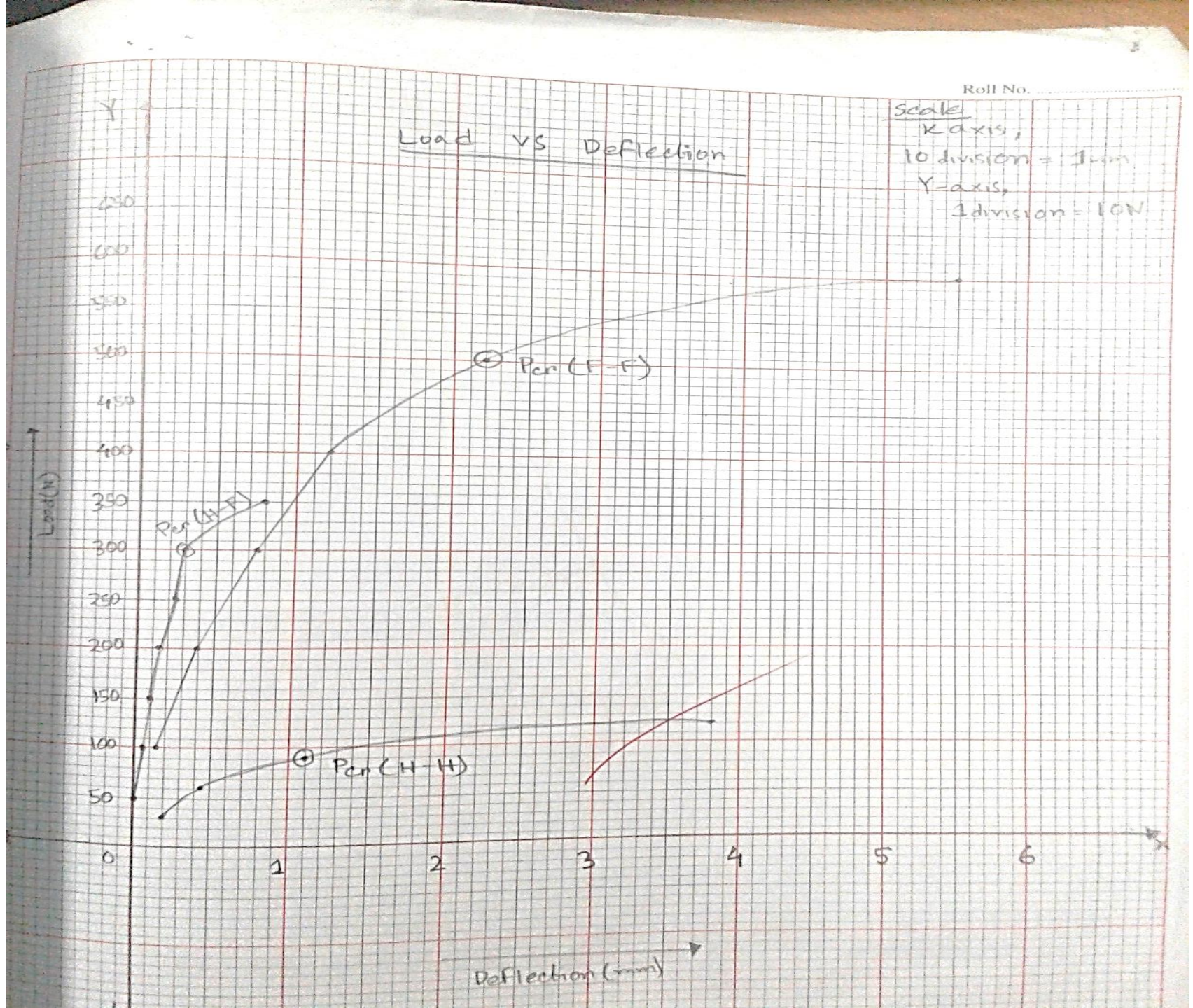
Both End Fixd

Load (N)	100	200	300	400	500	600				
Deflection (mm)	0.13	0.4	0.74	1.18	2.23	5.50				

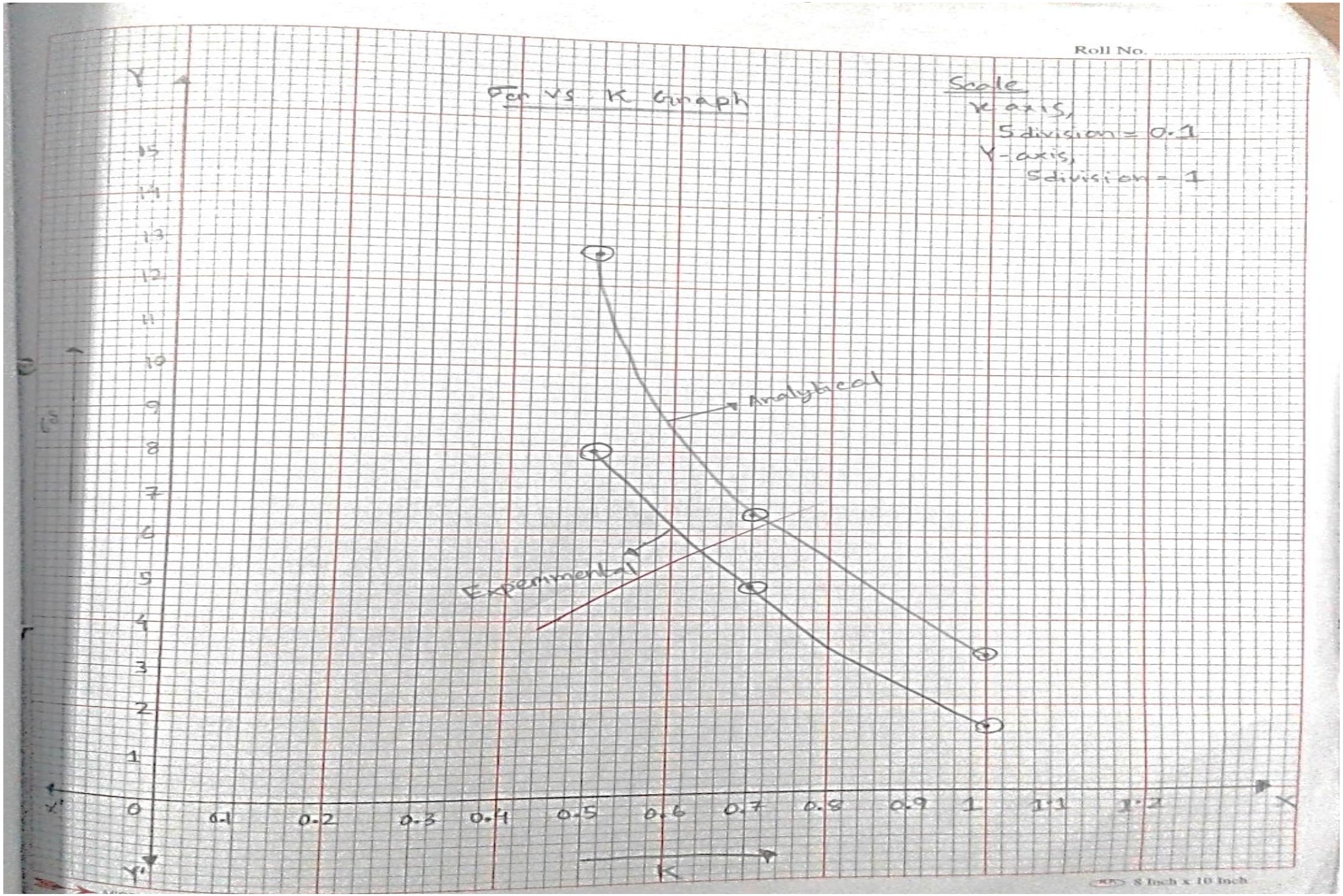
Calculation and Result

- Calculate critical/ buckling stress for all support conditions using Euler's formula and compare it with the experimental critical stress value.
- Plot applied load vs deflection in a graph paper for all support conditions.
- Plot critical stress vs slenderness ratio in a graph paper for all support conditions (both experimental and theoretical).

Roll No. .....


Load VS Deflection
Scale
X axis,
10 division = 1 mm
Y-axis,
1 division = 10N
Y
Load (N)
450
400
350
300
250
200
150
100
50
0
Pcr (F-F)
Pcr (H-H)
1
2
3
4
5
6
X
Deflection (mm)

Roll No.



8 Inch x 10 Inch

Calculation and Result

Theoretical critical stress for,

- H-H condition:
- H-F condition:
- F-F condition:

Experimental critical stress for,

- H-H condition:
- H-F condition:
- F-F condition:

Discussion