BANGLADESH UNIVERSITY OF PROFESSIONALS
Military Institute of Science and Technology
B.Sc. in Environmental, Water Resources and Coastal Engineering
Term Final (Spring) Examination 2024: October-November 2024

Student Group: 36<Earned Credit Hours ≤ 72

Subject: MATH -201, Vector Analysis, Laplace Transform and Co-ordinate Geometry

Time: 3.00 hours **Full Marks: 180**

INSTRUCTIONS:

a. Use **SEPARATE** answer scripts for each section.
b. **Question-1** in **Section-A** and **Question-5** in **Section-B** are compulsory
c. Answer any other **TWO** questions out of **THREE** from each section.
d. Figures in the margin indicate full **marks**.
e. Assume reasonable data if necessary.
f. **Symbols** and **abbreviations** used have their usual meanings.

SECTION-A

Question 1 (Compulsory)

a. Define Laplace transform of a periodic function. Find Laplace transform of the triangular wave defined by **10**
$f(t) = \begin{cases} t; & for\ 0 \le t < a \\ 2a - t; & a \le t < 2a \end{cases}$; where $f(2a + t) = f(t)$.

b. Find the inverse Laplace transform of $F(s) = \left\{\frac{2s+1}{(s+2)^2(s-1)^2}\right\}$ using partial fraction. **10**

c. Apply Heaviside expansion theorem to find $\mathcal{L}^{-1}\left\{\frac{5s-1}{(s-1)(s^2+1)}\right\}$. **10**

Question 2

a. Solve the initial value problem **10**
$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y = 0;\ y(0) = 3,\ y'(0) = 6.$

b. Use Laplace transform to solve **10**
$ty'' + 2y' + ty = 0\,; y(0) = 1,\ y(\pi) = 0.$

c. Solve the simultaneous differential equations **10**
$$\frac{dx}{dt} = 2x - 3y\ , \frac{dy}{dt} = y - 2x$$
subject to the conditions $x(0) = 8,\ y(0) = 3.$

Question 3

a. A particle is acted on by constant forces $3\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$ and $2\hat{\imath} + \hat{\jmath} - 3\hat{k}$ is displaced from a point whose position vector is $2\hat{\imath} - \hat{\jmath} - 3\hat{k}$ to a point whose position vector is $4\hat{\imath} - 3\hat{\jmath} + 7\hat{k}$. Calculate the work done using dot product. **8**

b. Find the constants a , b and c so that **8**
$$\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$$
is irrotational.

c. Find the angle between the surfaces **14**
$x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$.

Question 4

a. Evaluate $\int_C \vec{F}.d\vec{r}$ where C is the circle $x^2+y^2=9$ in the xy plane, where $\vec{F}=(2x-y+z)\hat{\imath}+(x+y-z)\hat{\jmath}+(3x-2y+4z)\hat{k}$ **12**

b. Using Gauss divergence theorem, evaluate the surface integral $\iint_S \vec{F}.\vec{n}ds$, where $\vec{F}=4x\hat{\imath}+y^2\hat{\jmath}+z^2\hat{k}$ and S is $x^2+y^2=4;\ z=0,$ $z=2.$ **18**

SECTION-B

Question 5 (Compulsory)

a. Prove that the straight lines represented by the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ will be equidistant from the origin if $,f^4-g^4=c(bf^2-ag^2).$ **15**

b. Find the equation of pair of straight lines through the origin and perpendicular to the lines $5x^2-7xy-3y^2=0.$ **05**

c. Find the relation between the coordinate axes when the axes are rotated through an angle $\theta.$ **10**

Question 6

a. Define radical center of circles. Find the radical center of the three circles $x^2+y^2+x+2y+3=0, x^2+y^2+2x+4y+5=0$ and $x^2+y^2-7x-8y-9=0$. **10**

b. Find the coordinates of the limiting points of the coaxial system determined by the circles $x^2+y^2+4x+2y+5=0$ and $x^2+y^2+2x+4y+7=0.$ **10**

.c. Define pole of a circle .Find the pole of the straight line $2x+y+12=0$ with respect to the circle $x^2+y^2-4x+3y-1=0$. **10**

Question 7

a. Find the shortest distance between the lines $\frac{x-6}{1}=\frac{y-2}{-2}=\frac{z-2}{2}$ and $\frac{x+4}{3}=\frac{y}{-2}=\frac{z+1}{-2}.$ Also find the equation of the shortest distance. **14**

b. Find the distance of the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z-5=0$ from the point $(-1,-5,-10).$ **10**

c. Find the angle between the planes $2x-3y+4z=5$ and $3x+5y-2z=7.$ **06**

Question 8

a. Identify the nature of the following conic. Reduce the conic to its standard form. Find its all properties and sketch it. $x^2+2xy+y^2-6x-2y+4=0.$ **16**

b. Show that in polar coordinates, the equation $x^4+x^2y^2-(x+y)^2=0$ takes the form $r=\pm(1+tan\theta).$ **07**

c. Are the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ coplanar? Justify your answer. **07**