Class-01

23.04.24 (TUESDAY)

Co-ordinate → Sec B

Point

Straight line, Circle, Conics → 2D

3D Co-ordinate system.

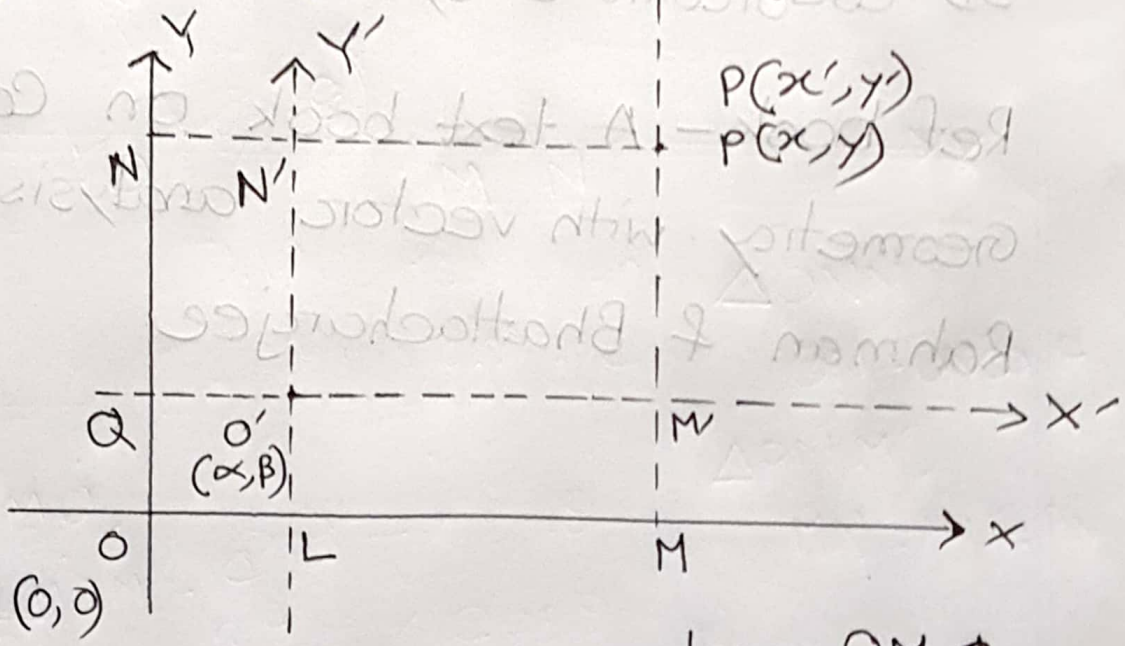
Ref book-. A text book on Co-ordinate Geometry with vector analysis by Rahman & Bhattacharjee

Class-2

24.04.24 (WEDNESDAY)

Change of Axis:

1. Change of origin: or shifting



$x' = x - \alpha$ $\quad x = x' + \alpha$

$y' = y - \beta$ $\quad y = y' + \beta$

$x = OM$

$= OL + LM$

$= \alpha + x'$

$y = ON$

$= OQ + QN$

$= \beta + y'$

Math: Examine the eqn of the curve $2x^2 + 4y^2 - 6x + 8y - 7 = 0$ when the origin is shifted to a point $(2, -1)$.

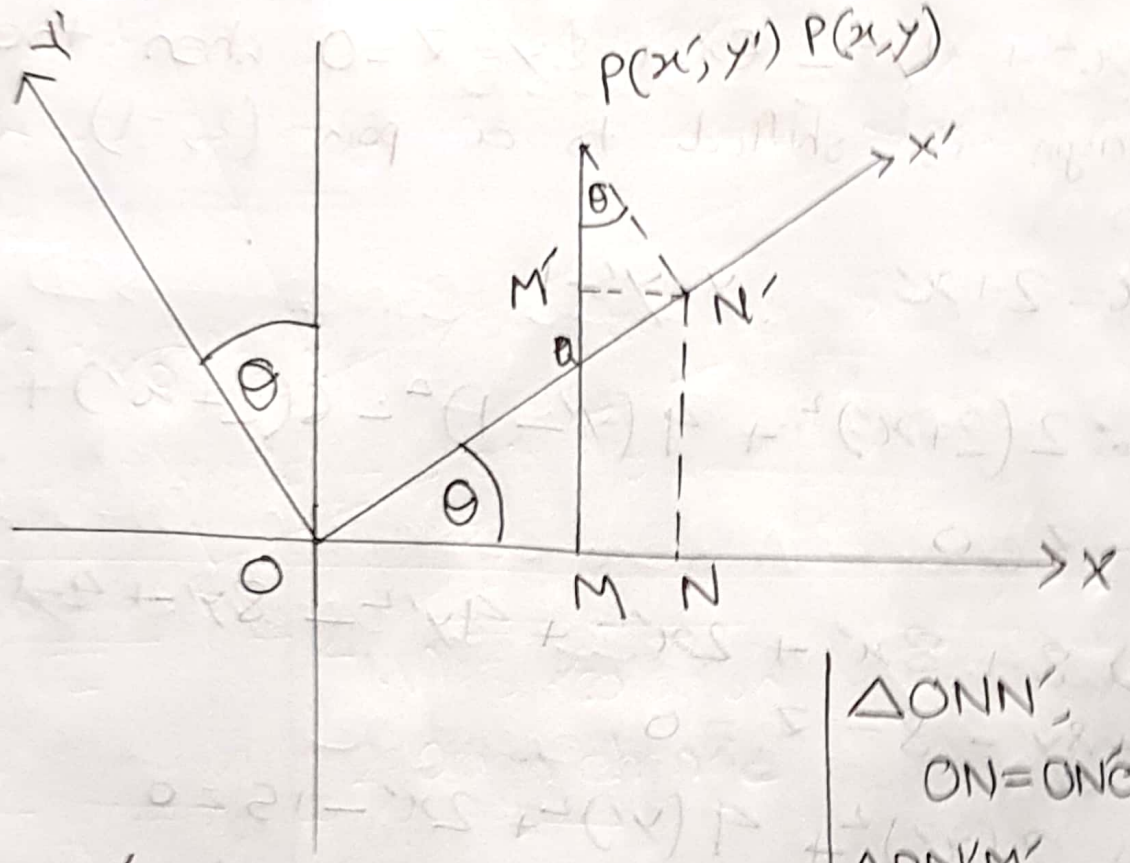
$x = 2 + x'$ $\quad y = y' - 1$

$\therefore 2(2+x')^2 + 4(y'-1)^2 - 6(2+x') + 8(y'-1) - 7 = 0$

$\Rightarrow 8 + 8x' + 2x'^2 + 4y'^2 - 8y' + 4 - 12 - 6x' + 8y' - 8 - 7 = 0$

$\Rightarrow 2(x')^2 + 4(y')^2 + 2x' - 15 = 0$

2. Rotation of axis :



$\triangle ONN'$,

$ON = ON' \cos\theta$

$\triangle PN'M'$,

$MN = MN' = PN' \sin\theta$

$\triangle M'MO$,

$\angle M'OM = \theta$

$\therefore \angle MM'O = 90^\circ - \theta$

$\angle MM'O = \angle M'QN' = 90^\circ - \theta$ [

$\therefore \angle M'N'Q = 90^\circ - \angle M'QN' = \theta$

$\therefore \angle PN'M' = 90^\circ - \angle M'N'Q = 90^\circ - \theta$

$\therefore \angle N'PM' = 90^\circ - \angle PN'M' = \theta^\circ$

$x = OM$

$= ON - MN$

$= ON' \cos\theta - PN' \sin\theta$

$\Rightarrow x = x' \cos\theta - y' \sin\theta$

$y = PM = PM' + MM'$

$\Rightarrow y = y' \cos\theta + x' \sin\theta$

$\Delta PM'N'$,

$PM' = PN' \cos\theta$

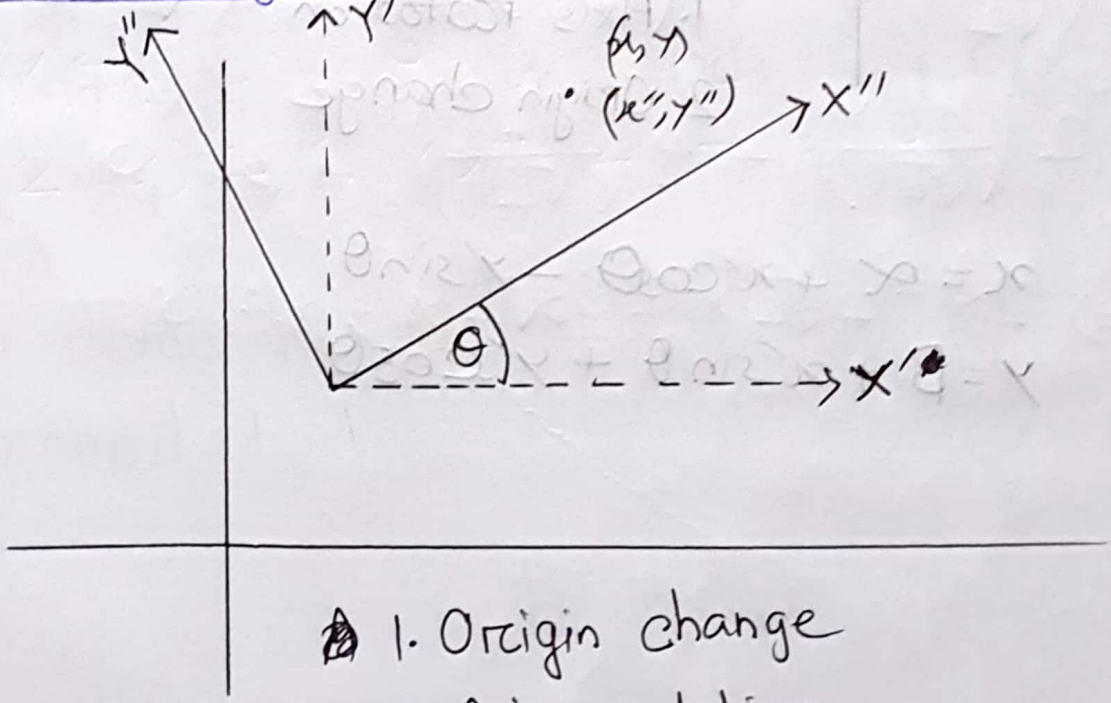
$= y' \cos\theta$

$\Delta ONN'$,

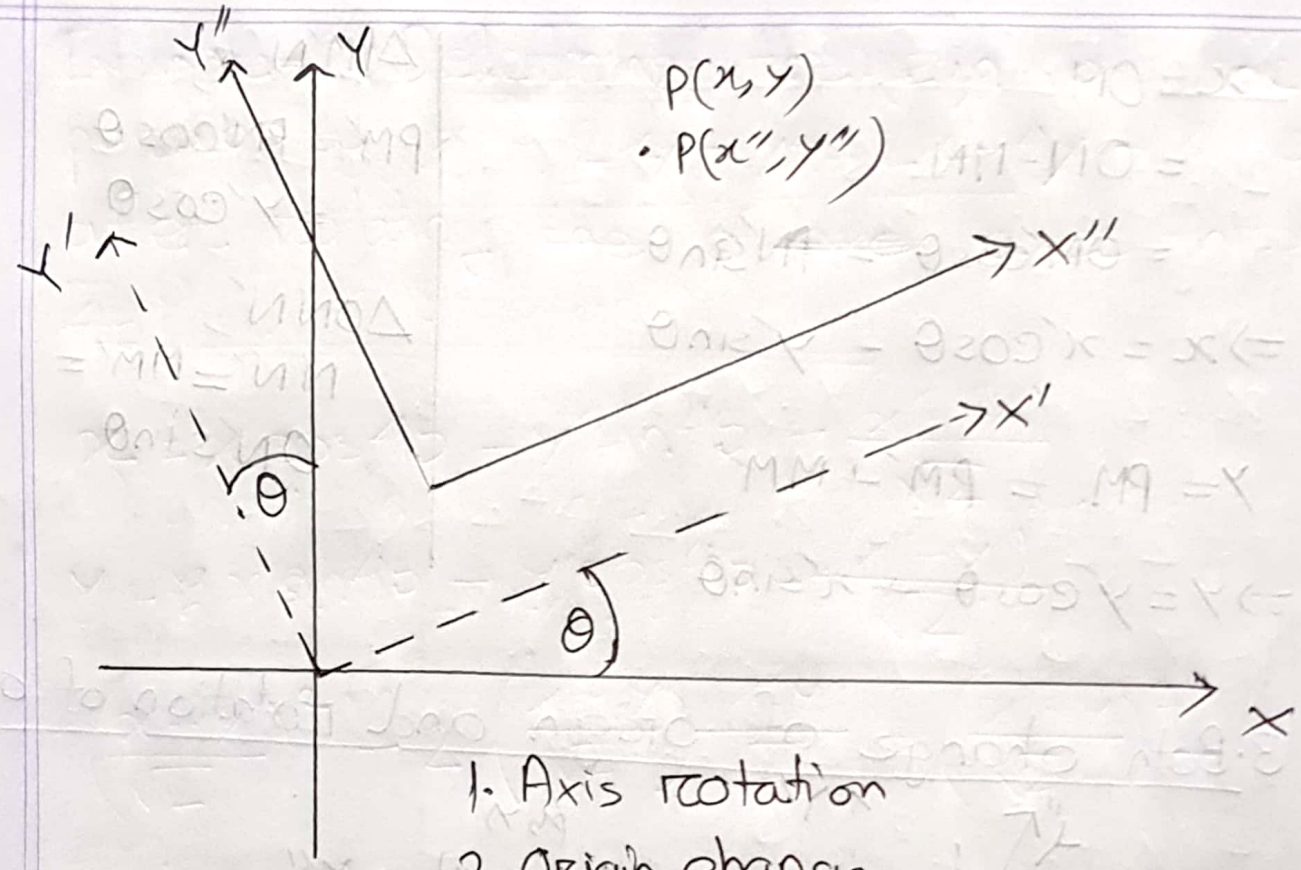
$NN' = MM' =$

$ON' \sin\theta$

3. Both change of origin and rotation of axis:



1. Origin change
2. Axis rotation



1. Axis rotation
2. Origin change

$$x = \alpha + x'\cos\theta - y'\sin\theta$$

$$y = \beta + x'\sin\theta + y'\cos\theta$$

expression $ax^2 + 2hyx + by^2$ vanishes.

Ex. 1. Determine the equation of the curve $2x^2 + 3y^2 - 8x + 6y - 7 = 0$ when then origin is transferred to the point (2, −1).

when then origin is transferred to the point (2, −1) Put $x = x' + 2$, $y = y' - 1$ in the above equation. Then $2(x' + 2)^2 + 3(y' - 1)^2 - 8(x' + 2) + 6(y' - 1) - 7 = 0$

or, $2x'^2 + 3y'^2 = 18$

Now removing suffixes the equation referred to new axes is $2x^2 + 3y^2 = 18$

Ex. 2. Determine the equation of the parabola

$x^2 - 2xy + y^2 + 2x - 4y + 3 = 0$ ... ... (i)

after rotating of axes through 45°

When the axes have been roated through an angle 45°, then

$x = x' \cos 45° - y' \sin 45° = (x' - y')/\sqrt{2}$

$y = x' \sin 45° + y' \cos 45° = (x' + y)/\sqrt{2}$

Put them in (1), then $\left(\frac{x' - y'}{\sqrt{2}}\right)^2 - 2\left(\frac{x' - y'}{\sqrt{2}}\right)\frac{x' + y'}{\sqrt{2}} + \left(\frac{x' + y'}{\sqrt{2}}\right)^2 + 2\left(\frac{x' - y'}{\sqrt{2}}\right) - 4\left(\frac{x' + y'}{\sqrt{2}}\right) + 3 = 0$

On simplification the equation reduces to

$2y'^2 - \sqrt{2}x' - 3\sqrt{2}y' + 3 = 0$

Now dropping the suffixes, the equation is

$2y^2 - \sqrt{2}x - 3\sqrt{2}y + 3 = 0$ Ans.

Ex. 3. Remove the first degree terms in

$3x^2 + 4y^2 - 12x + 4y + 13 = 0$

Complete the square,

or, $3(x^2 - 4x + 4)\left(y^2 + y + \frac{1}{4}\right) = 0$

or, $3(x - 2)^2 + 4\left(y + \frac{1}{2}\right)^2 = 0$ ... ... (ii)

Put $x-2=x'$ and $y+\frac{1}{2}=y'$ in (ii)

$\therefore \quad 3x'^2+4y'^2=0$

This is satisfied only by $x'=0, y'=0$, which is the new origin.

By shifting origin at $(2, 1/2)$, the first degree terms can be removed.

Ex. 4. By transforming to parallel axes through a properly chosen point (h, k), prove that the equation.

$12x^2-10xy+2y^2+11x-5y+2=0$

can be reduced to one containing only the terms of the 2nd degree.

Transforming to parallel axes through (h, k) we have

$12(x'+h)^2-10(x'+h)(y'+k)+2(y'+k)^2+11(x'+h)-5(y'+k)+2=0$

or, $12x'^2-10x'y'+2y'^2+(24h-10k+11)x'+(-10h+4k-5)y'+(12h^2-10hk+2k^2+11h-5k+2)=0$ ... ... (i)

Now equate the co-efficients of x, y to zero then

$24h-10k+11=0$ also $-10h+4k-5=0$

and $2h^2-10hk+2k^2+11h-5k+2=0$

Solve the first two equations $h=-\frac{3}{2}, k=-\frac{5}{2}$ and it is clear that these values of h, k satisfy the equation (ii). Hence the equation. (i) becomes $12x'^2-10x'y'+2y'^2=0$

Class-03

30.04.24 (TUESDAY)

Pair of Straight lines:

① Show that, the 2nd degree homogenous eqn $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines.

Soln:

Given eqn,

$$ax^2 + 2hxy + by^2 = 0$$

$$\Rightarrow \frac{a}{b} + \frac{2h}{b}\frac{y}{x} + \left(\frac{y}{x}\right)^2 = 0 \quad \text{——} (1)$$

Let m_1 and m_2 be the soln of eqn ①.

$$\therefore m_1 + m_2 = -\frac{2h}{b}$$

$$m_1 m_2 = \frac{a}{b}$$

$\therefore$ ① implies,

$$\left(\frac{y}{x}\right)^2 - (m_1+m_2)\frac{y}{x} + m_1 m_2 = 0$$

$$\Rightarrow \left(\frac{y}{x} - m_1\right)\left(\frac{y}{x} - m_2\right) = 0$$

$$\Rightarrow \frac{y}{x} - m_1 = 0 \quad \text{or} \quad \frac{y}{x} - m_2 = 0$$

$$\Rightarrow y = m_1 x \qquad y = m_2 x$$

$\therefore$ The eqn $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines.

② Find the eqn of pair of straight lines perpendicular to the lines represented by the eqn $ax^2 + 2hxy + by^2 = 0$

Soln: We know that the eqn $ax^2 + 2hxy + by^2 = 0$ — ①

represents a pair of straight lines.

The lines are,

$$\left.\begin{aligned} y &= m_1 x \\ y &= m_2 x \end{aligned}\right\} \text{— ②}$$

The lines which are perpendicular to the lines ② are,

$$y = \frac{1}{m_1}x \quad \text{and}$$

$$y = -\frac{1}{m_2}x$$

$$\Rightarrow x + m_1 y = 0$$

$$x + m_2 y = 0$$

Now the required eqn is

$$(x + m_1 y)(x + m_2 y) = 0$$

$$\Rightarrow x^2 + (m_1 + m_2)xy + m_1 m_2 y^2 = 0$$

$$\Rightarrow x^2 - \frac{2h}{b}xy + \frac{a}{b}y^2 = 0$$

$$\Rightarrow bx^2 - 2hxy + ay^2 = 0$$

Math: Determine the eqn of pair of straight lines which are perpendicular to the lines $5x^2 - 7xy - 3y^2 = 0$

③ Find the eqn of bisectors of the lines represented by

$$ax^2 + 2hxy + by^2 = 0$$

$$\left.\begin{aligned} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{aligned}\right\}$$

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{|a_2x + b_2y + c_2|}{\sqrt{a_2^2 + b_2^2}}$$

Or,

$$ax^2 + 2hxy + by^2 = 0$$

$y - mx = 0$

$y - m_1x = 0$ $\qquad$ $y - m_2x = 0$

eqn of bisector,

$$\frac{|y - m_1 x|}{\sqrt{1^2 + (-m_1)^2}} = \pm \frac{|y - m_2 x|}{\sqrt{1^2 + (-m_2)^2}}$$

$$\Rightarrow \frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

5

PAIR OF STRAIGHT LINES

Art. 36. Homogeneous Quadratic Equations.

Let us consider the equation $ax^2 + 2hxy + by^2 = 0$ ... ... (1)

On multiplying it by a, it may written in the form

$a^2x^2 + 2ahxy + aby^2 = 0$, if $a \neq 0$

or, $a^2x^2 + 2ahxy + h^2y^2 - (h^2 - ab)\, y^2 = 0$

or, $(ax + hy)^2 - \{y\sqrt{h^2 - ah}\}^2 = 0$

or, $(ax + hy) + \sqrt{(h^2 - ab)}\, y\}\, \{ax + hy - (h^2 - ab)y\} = 0$

The equation (1) therefore, represents the two straight lines whose equations are

$ax + hy + y\sqrt{(h^2 - ab)} = 0$ ... ... (2)

and $ax + hy - y\sqrt{(h^2 - ab)} = 0$ ... ... (3)

each of which passes through the origin. The two lines (2), (3) form the locus of the equation (1) for (1) is satisfied by the co-ordinates of all points which satisfy (2) and (3).

These two straight lines are real and different if $h^2 > ab$, real and coincident if $h^2 = ab$, and imaginary if $h^2 < ab$.

In the case when $h^2 = ab$, the straight lines though themselves imaginary intersect in a real point. For the origin lies on the locus given by (1) since the equation (1) is always satisfied by the values $x = 0$ and $y = 0$.

Alternative method

If $b \pm 0$ dividing both sides of the equation (1) by $x^2\, b$ we have

$$\left(\frac{y}{x}\right)^2 + 2\frac{h}{b}\frac{y}{x} + \frac{a}{b} = 0 \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

Let m_1, m_2, be the roots of this quadratic equation in x/y.

Sum of the roots = $m_2 + m_2 = -2\, h/b$

and product of the roots = $m_1m_2 = a/b$ ... ... (5)

The Eq. (4) must be equivalent to

$$\left(\frac{y}{x} - m_1\right)\left(\frac{y}{x} - m_2\right) = 0 \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

Ex. 9. The axes being rectangular, find the equation to the pair of sraight lines meeting at the origin which are perpendicular to the pair given by the equation $ax^2 + 2hxy + by^2 = 0$

If the lines represented by $ax^2 + 2hxy + by^2 = 0$ be $y - m_1x = 0$ and $y - m_2x = 0$, then we have $m_1 + m_2 = 2\frac{h}{b}$ and $m_1m_2 = \frac{a}{b}$ ... ... (1)

Now the equations of the lines perpendicular to the above lines and passing through the origin are $m_1y + x = 0$ and $m_2y + x = 0$

Their joint equation is

$(m_1y + x)(m_2y + x) = 0$, or $m_1m_2y^2 + (m_1 + m_2)xy + x^2 = 0$

or, $(\frac{a}{b})y^2 - (\frac{2h}{b})xy + x^2 = 0$ by (1)

or, $bx^2 - 2hxy + ay^2 = 0$ Ans.

of the lines pass through the origin, these are

Cor. 3. Two lines represented by $ax^2 + 2hxy + by^2 = 0$ will be real if $h^2 > ab$

Cor. 4. Two lines of above equation will be imaginary if $h^2 < ab$.

Art. 39. The bisectors of the angles between the lines represented by $ax^2 + 2hxy + by^2 = 0$

Let the lines $y - m_1x = 0$ and $y - m_2x = 0$ represented by

$ax^2 + 2hxy + by^2 = 0$ ... ... (1) have slopes m_1 and m_2 and make angles θ_1 and θ_2 with OX. Let one of the bisectors make an angle θ with OX. from the diagram we have

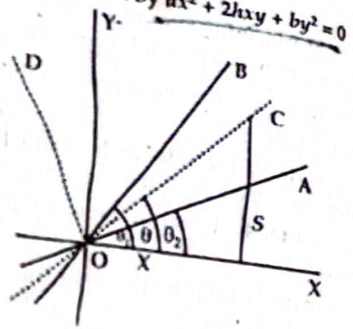


Fig. 12

$\theta = \theta_1 + \frac{1}{2}(\theta_2 - \theta)$ or, $\frac{1}{2}\pi + \theta_1 + \frac{1}{2}(\theta_2 - \theta_1) = \frac{1}{2}(\theta_1 + \theta_2)$

or, $\frac{1}{2}\pi + \frac{1}{2}(\theta_1 + \theta_2)$

$\therefore 2\theta = \theta_1 + \theta_2$

or, $\pi + (\theta_1 + \theta_2)$

... ... (2)

Note. 1. In (2), first value θ is the angle made by the internal bisector OC with OX and the second value of θ is the angle made by the external bisector OD with OX.

Alternative Method :

Let the given equation $ax^2 + 2hxy + by^2 = 0$ represent the line

$y - m_1x = 0$ and $y - m_2x = 0$,

$\therefore m_1 + m_2 = -2\frac{h}{b}$, $m_1m_2 = \frac{a}{b}$ ... ... (1)

The equations of the required bisectors are

$$\frac{y - m_1x}{\sqrt{(1 + m_1^2)}} = \pm\frac{y - m_2x}{\sqrt{(1 + m_2)}} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Squaring, we have

$(y - m_1x)^2 (1 + m_2^2) = (y - m_2x)^2 (1 + m_1^2)$

or, $(m_1^2 - m_2^2)(x^2 - y^2) = 2xy\ [(m_1 - m_2) - m_1m_2 (m_1 - m_2)]$

or, $(m_1 - m_2)(m_1 - m_2)(x^2 - y^2) = 2xy\ (m_1 - m_2)(1 - m_1m_2)$

or, $(m_1 + m_2)(x^2 - y^2) = 2xy(1 - m_1m_2)$

or, $(-2\frac{h}{b}(x^2 - y^2) = 2xy\frac{(b - a)}{b}$

or, $h(x^2 - y^2) = xy(a - b)$

or, $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$

4] Find an equation for the bisectors of the angles between the lines represented by $ax^2+2hxy+by^2=0$.

Soln: The given eqn is,

$$ax^2+2hxy+by^2=0 \quad \text{—①}$$

We know the lines represented by ① are,

$$y-m_1x=0 \text{ and } y-m_2x=0$$

then; $m_1+m_2=-\frac{2h}{b}$; $m_1m_2=\frac{a}{b}$

Thus, the equation of the required bisectors are, $\frac{y-m_1x}{\sqrt{1^2+(-m_1)^2}} = \pm \frac{y-m_2x}{\sqrt{1^2+(-m_2)^2}}$

$$\Rightarrow \frac{(y-m_1x)^2}{1+m_1^2} = \frac{(y-m_2x)^2}{1+m_2^2}$$

$$\Rightarrow (1+m_2^2)(y-m_1x)^2 = (1+m_1^2)(y-m_2x)^2$$

$$\Rightarrow (1+m_2^2)(y^2-2xym_1+m_1^2x^2) = (1+m_1^2)(y^2-2xym_2+m_2^2x^2)$$

$$\Rightarrow \{(1+m_2^2)-(1+m_1^2)\}y^2-2\{m_1(1+m_2^2)-m_2(1+m_1^2)\}xy + \{m_1^2(1+m_2^2)-m_2^2(1+m_1^2)\}x^2=0$$

$$\Rightarrow (m_2^2-m_1^2)y^2-2\{(m_1-m_2)+m_1m_2(m_2-m_1)\}xy + (m_1^2-m_2^2)x^2=0$$

$$\Rightarrow (x^2-y^2)(m_1^2-m_2^2) - 2\{(m_1-m_2) - m_1m_2(m_1-m_2)\}xy = 0$$

$$\Rightarrow (x^2-y^2)(m_1^2-m_2^2) - 2(m_1-m_2)(1-m_1m_2)xy = 0$$

$$\Rightarrow (x^2-y^2)(m_1+m_2) - 2(1-m_1m_2)xy = 0$$

$$\Rightarrow (x^2-y^2)\left(-\frac{2h}{b}\right) - 2\left(1-\frac{a}{b}\right)xy = 0$$

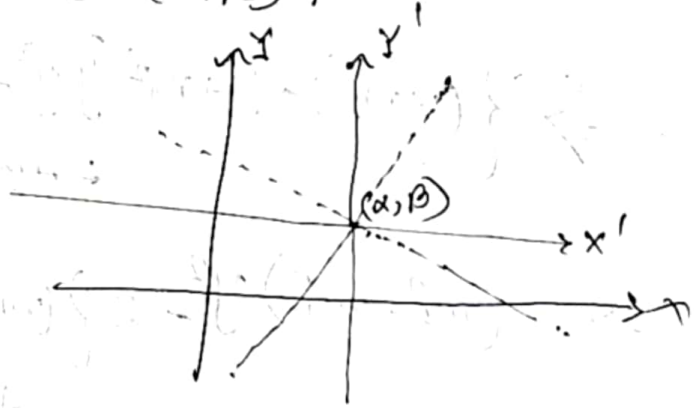
$$\Rightarrow \frac{x^2-y^2}{a-b} = \frac{xy}{h}$$

Which is the required equation of bisectors.

* Find the condition that the general eqn of second degree may represent a pair of straight line.

<u>Soln:</u> Let us consider the general eqn of second degree, $ax^2+2hxy+by^2+2gx+2fy+c=0$ —— (1) represents a pair of straight lines. Let, their point of intersection be (α,β).

By transforming the origin to (α,β) keeping the direction of the axes unaltered.



Let, (x,y) and (x',y') reffered to the same point with original coordinate and transformed coordinate system. then,

$$x = x' + \alpha$$
$$y = y' + \beta$$

then from (1) we get,

$$a(x'+\alpha)^2 + 2h(x'+\alpha)(y'+\beta) + b(y'+\beta)^2 + 2g(x'+\alpha) + 2f(y'+\beta) + c = 0$$

$$\Rightarrow a(x'^2 + 2x'\alpha + \alpha^2) + 2h(x'y' + y'\alpha + x'\beta + \alpha\beta) + b(y'^2 + 2\beta y' + \beta^2) + 2gx' + 2g\alpha + 2fy' + 2f\beta + c = 0$$

$$\Rightarrow ax'^2 + 2hx'y' + by'^2 + 2(a\alpha + h\beta + g)x' + 2(h\alpha + b\beta + f)y' + a\alpha^2 + 2h\alpha\beta + b\beta^2 + 2g\alpha + 2f\beta + c = 0 \quad \text{---(3)}$$

Since eqn (3) represents a pair of straight lines through the new origin, it must be homogeneous in x' and y'.

Therefore the first order in x', y' and the constant term must be disappeared and leave only,

$$ax'^2 + 2hx'y' + by'^2 = 0 \quad \text{---(4)}$$

Then from eqn ③ we get,

$$a\alpha + h\beta + g = 0 \quad \text{—⑤}$$
$$h\alpha + b\beta + f = 0 \quad \text{—⑥}$$

and $a\alpha^2 + 2h\alpha\beta + b\beta^2 + 2g\alpha + 2f\beta + c = 0$

$\Rightarrow \alpha(a\alpha + h\beta + g) + \beta(h\alpha + b\beta + f) + g\alpha + f\beta + c = 0$

$\Rightarrow \alpha \cdot 0 + \beta \cdot 0 + g\alpha + f\beta + c = 0$ [using ⑤, ⑥]

$\Rightarrow g\alpha + f\beta + c = 0 \quad \text{—⑦}$

Now eliminating α, β from eqn ⑤, ⑥, ⑦ the required condition is,

$$\Delta \equiv \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

Or, $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

Which is the required condition.

Note: $ax + by = 0 \quad \text{—①}$
$cx + dy = 0 \quad \text{—②}$

Now eliminating x, y from ① and ②

① imply $x = -\frac{b}{a}y$

② imply, $c(-\frac{b}{a})y + dy = 0$

$\Rightarrow (-bc + ad)y = 0$

since, $y \neq 0$

$\therefore ad - bc = 0$

$\Rightarrow \begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0$

Problem 01. Show that, $3x^2 - 14xy - 5y^2 - 54x - 2y + 51 = 0$ represents a pair of straight lines.

Soln: Given,

$$3x^2 - 14xy - 5y^2 - 54x - 2y + 51 = 0 \quad \text{—(1)}$$

Comparing (1) with general eqn of second degree we can write,

$$a = 3,\ h = -7,\ b = -5,\ g = -27,\ f = -1,$$
$$c = 51$$

Now, (1) will represent a pair of straight lines if

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

Now,

$$\Delta = \begin{vmatrix} 3 & -7 & -27 \\ -7 & -5 & -1 \\ -27 & -1 & 51 \end{vmatrix}$$

$$= 3(-255-1) - (-7)(-357-27) + (-27)(7-135)$$

$$= -768 - 2688 + 3456$$

$$= 0$$

Since $\Delta = 0$

$\therefore$ eqn (1) represents a pair of straight line.

Problem 02: Find the value of λ so that the equation $\lambda x^2+4xy+y^2-4x-2y-3=0$ may represent a pair of straight line.

Soln: Given,

the eqn is,

$$\lambda x^2+4xy+y^2-4x-2y-3=0 \quad \text{— (1)}$$

Comparing (1) with general eqn of second degree we can write,

$$a=\lambda,\ h=2,\ b=1,\ g=-2,\ f=-1,\ c=-3$$

Since, (1) represents a pair of straight lines,

$$\therefore \Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \lambda & 2 & -2 \\ 2 & 1 & -1 \\ -2 & -1 & -3 \end{vmatrix} = 0$$

$$\Rightarrow \lambda(-3-1)-2(-6-2)-2(-2+2)=0$$

$$\Rightarrow 4\lambda = 16$$

$$\Rightarrow \lambda = 4 \quad \text{(Ans)}$$

Problem-03: Show that the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents two parallel lines if $a:h = h:b = g:f$. Also show that the distance between them is $2\sqrt{\frac{g^2-ac}{a(a+b)}}$

Soln: We know ~~that~~ the condition that the general equation represents two straight lines if,

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

$\Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ ——— (1)

since the lines are parallel, so, $ab = h^2$

from (1) we can write,

$$h^2c + 2fg\sqrt{ab} - af^2 - bg^2 - ch^2 = 0$$

$\Rightarrow af^2 - 2fg\sqrt{ab} + bg^2 = 0$

$\Rightarrow (\sqrt{a}f)^2 - 2\sqrt{a}f\sqrt{b}g + (\sqrt{b}g)^2 = 0$

$\Rightarrow (\sqrt{a}f - \sqrt{b}g)^2 = 0$

$\Rightarrow \sqrt{a}f = \sqrt{b}g$

$\Rightarrow \frac{g}{f} = \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$

$\Rightarrow \frac{g}{f} = \frac{h}{b} \Rightarrow g:f = h:b$ ——— (2)

Again,

$$ab = h^2$$

$$\Rightarrow \frac{a}{h} = \frac{h}{b}$$

$$\Rightarrow a : h = h : b \quad \text{— (3)}$$

Combining (2) and (3) we can write,

$$a : h = h : b = g : f$$

2nd part: Let the two ~~lines~~ parallel lines be,

$$lx + my + n = 0 \text{ and } lx + my + n_1 = 0$$

$$\therefore ax^2 + 2hxy + by^2 + 2gx + 2fy + c = (lx + my + n)(lx + my + n_1)$$

$$= l^2x^2 + m^2y^2 + (lm + ml)xy + (ln + ln_1)x + (mn + mn_1)y + nn_1 = 0$$

Equating like terms we can write,

$$l^2 = a,\ m^2 = b,\ nn_1 = c,\ l(n_1 + n) = 2g$$

Now, the distance between two lines ~~are~~ is:

$$\frac{n_1 - n}{\sqrt{l^2 + m^2}} = \frac{\sqrt{(n_1 + n)^2 - 4nn_1}}{\sqrt{l^2 + m^2}} = \frac{\sqrt{\left(\frac{2g}{l}\right)^2 - 4c}}{\sqrt{a + b}}$$

$$= \sqrt{\frac{4g^2 - 4cl^2}{l^2(a + b)}} \quad \cancel{= \frac{2\sqrt{g^2 - ac}}{\ldots}}$$

$$= 2\sqrt{\frac{g^2 - ac}{a(a + b)}} \quad \text{[showed]}$$