Class of 8.5.2024

Ex-3: If the pair of straight lines $x^2-2axy-y^2=0$ and $x^2-2bxy-y^2=0$ be such that each pair bisects the angle between the other pair, prove that $ab=-1$.

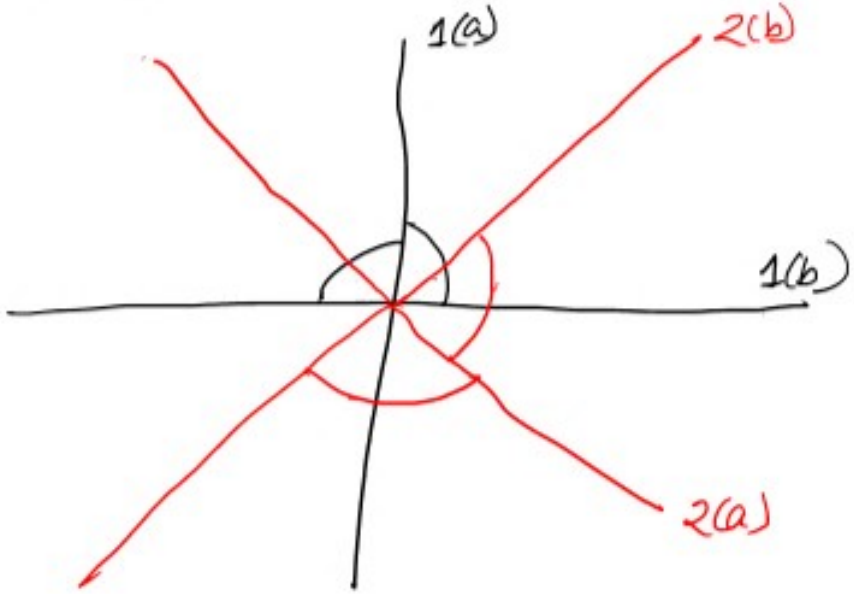
Sol^n: The given two lines are,

$$x^2-2axy-y^2=0 \quad \text{—} (1)$$

$$x^2-2bxy-y^2=0 \quad \text{—} (2)$$

Comparing eq^n (1) with $ax^2+2hxy+by^2=0$

we get, $a=1,\ h=-a,\ b=-1$



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We know that,
the eqn of bisectors is, $\boxed{\frac{x^2-y^2}{a-b} = \frac{xy}{h}}$ ✓

∴ the eqn of bisectors of the angle represents by eqn (1) is, $\frac{x^2-y^2}{1-(-1)} = \frac{xy}{-a}$ ✓

$$\Rightarrow \frac{x^2-y^2}{xy} = \frac{2}{-a} \quad \text{—— (3)}$$

By question eqn (2) itself, the eqn of bisectors of (1), so from eqn (2) we can write, $x^2-y^2 = 2bxy \Rightarrow \frac{x^2-y^2}{xy} = 2b$ — (4)

comparing eqns (3) and (4) we can write, $\frac{2}{-a} = 2b$

$\Rightarrow ab = -1$.

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Prove that, the straight lines represented by the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ will be equidistant from the origin, if $f^4-g^4=c(bf^2-ag^2)$.

Soln: The given equation is,

$$ax^2+2hxy+by^2+2gx+2fy+c=0 \quad \text{---------} (1)$$

Let, the straight lines represented by eqn (1) are,

$$lx+my+n=0 \quad \text{---------} (2) \text{ and}$$

$$l_1x+m_1y+n_1=0 \quad \text{---------} (3)$$

Since, they are equidistance from the origin $(0,0)$,

$\therefore$ We can write

$$\frac{n}{\sqrt{l^2+m^2}} = \frac{n_1}{\sqrt{l_1^2+m_1^2}}$$

$$\Rightarrow \frac{n^2}{l^2+m^2} = \frac{n_1^2}{l_1^2+m_1^2}$$

$$\Rightarrow n^2(l_1^2+m_1^2) = n_1^2(l^2+m^2)$$

$$\Rightarrow n^2l_1^2 - n_1^2l^2 = n_1^2m^2 - n^2m_1^2$$

$$\Rightarrow (nl_1 + n_1l)(nl_1 - n_1l) = (n_1m + nm_1)(n_1m - nm_1)$$

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$$\Rightarrow (nl_1 + n_1 l)\sqrt{(nl_1+n_1 l)^2 - 4nn_1 ll_1} = (nm_1 + mn_1)\sqrt{(nm_1+mn_1)^2 - 4nn_1 mm_1} \quad \text{---(3)}$$

Let, $ax^2+2hxy+by^2+2gx+2fy+c = (lx+my+n)(l_1x+m_1y+n_1)$

$$= ll_1x^2 + mm_1y^2 + (lm_1+ml_1)xy + (ln_1+nl_1)x + (nm_1+n_1m)y + nn_1 \quad \text{---(4)}$$

equating coefficents from bothsides of eqn (4) we can write, $ll_1 = a$, $mm_1 = b$, $nn_1 = c$, $lm_1+ml_1 = 2h$, $(ln_1+nl_1) = 2g$, $(nm_1+n_1m) = 2f$

Substituting the above values in eqn (3) we can write,

$$2g\sqrt{(2g)^2-4ac} = 2f\sqrt{(2f)^2-4bc}$$

$$\Rightarrow 4g^2(4g^2-4ac) = 4f^2(4f^2-4bc)$$

$$\Rightarrow g^4 - g^2ac = f^4 - f^2bc$$

$$\Rightarrow f^4 - g^4 = c(bf^2 - ag^2)$$ [Proved]