COURSE OUTLINE

English
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- Development and Scope of Fluid Mechanics
- Fluid Properties
- Fluid Statics
- Kinematics of Fluid Flow
- Fluid Flow Concepts and Basic Equations: Continuity
- Equation, Bernoulli's Equation, Energy Equation
- Momentum Equation and Forces in Fluid Flow
- Similitude and Dimensional Analysis
- Steady Incompressible Flow in Pressure Conduits: Laminar
- Flow and Turbulent Flow
- General Equation for Fluid Friction
- Empirical Equation for Pipe Flow
- Minor Losses in Pipe Flow
- Fluid Measurement: pitot tube, orifice, mouthpiece, nozzle, venturimeter, weir.
- Pipe Flow Problems: Pipes in Series and Parallel, Branching Pipes, Pipe Networks.

Reference Books

1. Fluid Mechanics with Engineering Application by Franzini
2. Fluid Mechanics by Md. Abdul Halim

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Kinematics of Fluid Flow

Kinematics is defined as the bananch of science which deals with the motion of particles without considering the forces causing the motion.

Velocity at any point in a fluid flow is determined (consequently pressure distribution and hece forces acting on the fluid can be determined).

Here, velocity and acceleration will be determined.

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Method of Describing Fluid Motion

Fluid motion is a fascinating and complex phenomenon that plays a significant role in various scientific and engineering fields. Understanding and effectively describing fluid motion is crucial for gaining insights into natural processes, designing efficient systems, and predicting behaviors.

Two prominent methods: Lagrangian method and Eulerian method

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Lagrangian View of Fluid Motion

In this approach, a single fluid particle is followed during its motion through space and timeand its velocity, acceration, density etc. are described.

Instead of considering fixed points in a fluid flow, this approach focuses on tracking the particles' positions and observing their changes over time.

Application:

In environmental science, to track the movement of pollutants in water bodies.
In biomechanics, to study of blood flow and the transport of nutrients in biological systems.
Also to understanding and predicting the behavior of aerosols, emulsions, and colloidal suspensions.

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Eulerian View of Fluid Motion

Instead of tracking individual particles, this approach focuses on observing the change in fluid properties (velocity, pressure, density, and temperature etc.) at a point in space.

It provides valuable information about the overall behavior of the fluid, including its flow patterns, turbulence, and pressure distribution.

Application:

In meteorology, for the analysis of weather patterns and the prediction of atmospheric phenomena.

In oceanography, to understand ocean currents, wave dynamics, and the distribution of heat and salt in the oceans.

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Key differences between Lagrangian and Eulerian Views

While both the Lagrangian and Eulerian approaches provide valuable insights into fluid motion, they differ significantly in their methodologies and the information they provide.

The Lagrangian approach focuses on individual particles and tracks their trajectories, velocities, and accelerations. It **provides a microscopic view** of fluid motion, enabling the analysis of phenomena involving particle interactions and transports.

The Eulerian approach considers fixed points in space and tracks changes in fluid properties at those points. It **provides a macroscopic view** of fluid behavior and is suitable for analyzing overall flow patterns and properties.

Lagrangian descriptions are well-suited for studying particle dynamics, dispersion, and mixing processes.

Eulerian descriptions are effective in analyzing flow patterns, turbulence, and large-scale behaviors.

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Kinematics of Fluid Flow

Types of Fluid

1. Ideal Fluid:

Nonviscous or inviscid (devoid of viscosity)

No frictional effects between fluid layers or between fluid and boundary walls.

2. Real Fluid:

Possesses viscosity

Shear stresses developed between neighboring fluid particles when they move at differential velocities.

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Kinematics of Fluid Flow

Types of Fluid

Newtonian or non – Newtonian Fluid

Newton's Law of Viscosity $\tau = \mu \frac{dV}{dy}$

Where μ = viscosity of fluid

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Kinematics of Fluid Flow

Types of Fluid Flow

1. Laminar and Turbulent flows
2. Steady and Unsteady flows
3. Uniform and non-uniform flows
4. One-, Two- or Three dimensional flow
5. Compressible and incompressible flows
6. Rotational and irrotational flows

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Kinematics of Fluid Flow

Types of Flow: Laminar and Turbulent Flows

Laminar Flow:

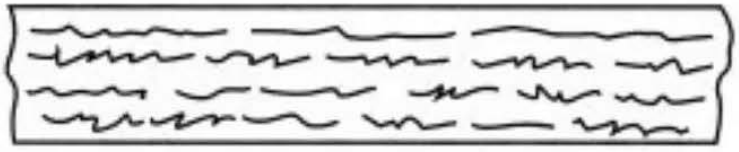
- **Fluid moves in layers or laminars**
- **One layer gliding smoothly over an adjacent layer**
- **Fluid particle moves in parallel paths.**

Turbulent Flow:

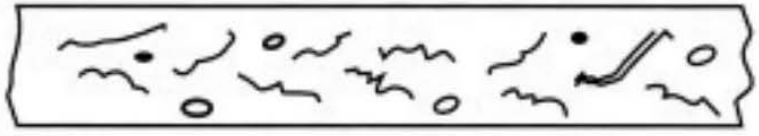
- **Fluid particles move in a erratic, random or haphazard manner**



Laminar



Critical Zone



Turbulent

- A flow of fluid where each particle of fluid follows a smooth path which never interfere with one another, and the velocity of the fluid is constant at any point is called laminar flow.
- Turbulent flow is an irregular flow where the velocity of a fluid is not constant at every point. In the case of turbulent flow liquid flowing in the pipe has no fixed speed or direction.

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Kinematics of Fluid Flow

Types of Flow: Laminar and Turbulent Flows

Reynold's Experment (1883), French Engineer Osborne Reynolds

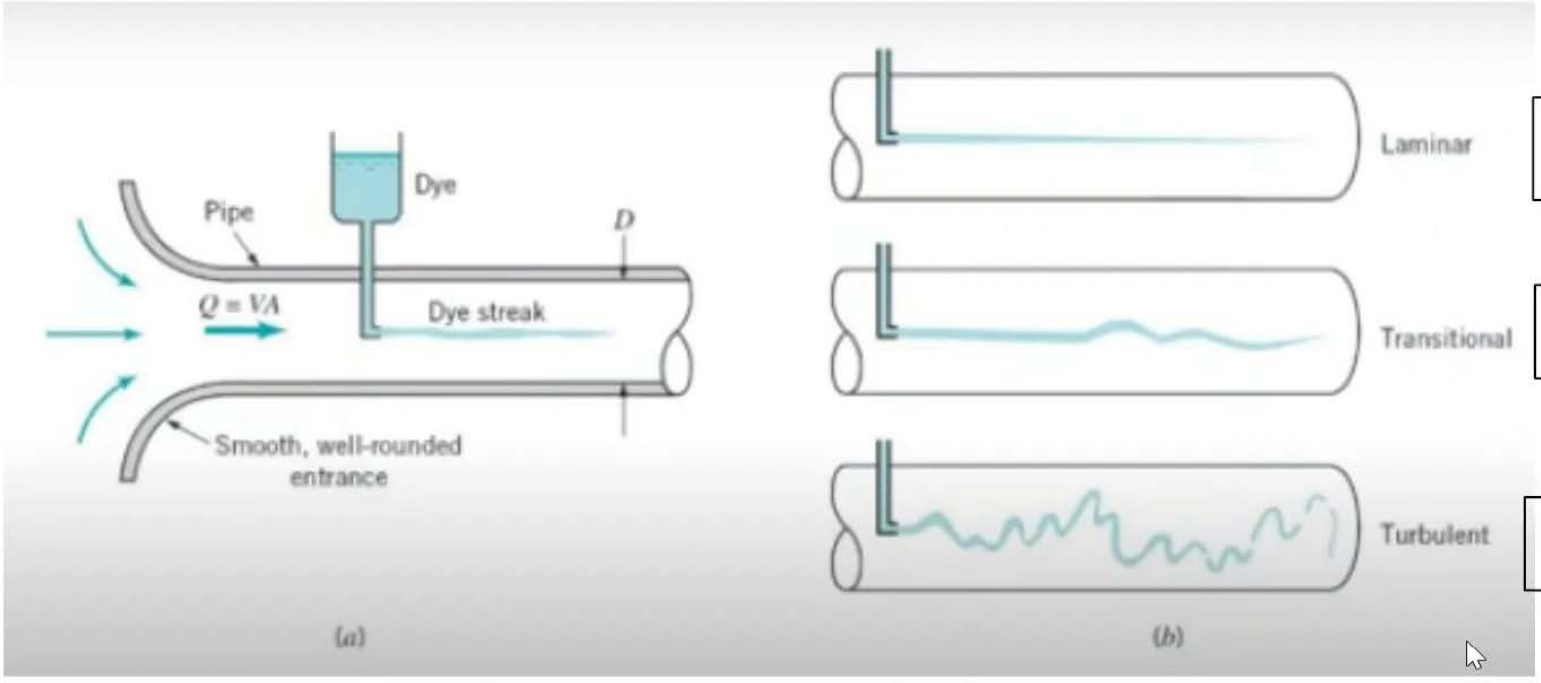
- **At low velocities (small flows), dye filament moved in a straight line** through the tube showing the flow to be **Laminar.**
- **At higher velocities (large flows), dye filament wavered and broke**, eventually diffused throughout the fluid in the tube showing the flow to be **turbulent.**
- There is also a **transitional stage between laminar and turbulent** flows, in which the dye stream will wander about and show intermittent bursts of mixing, followed by a more laminar behavior.
- **Reynolds found** that transition from laminar to turbulent flow in a pipe **depends on: Velocity (V), pipe diameter (D) and viscosity of fluid**.

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Kinematics of Fluid Flow

Types of Flow: Laminar and Turbulent Flows

Reynold's Experment (1883), French Engineer Osborne Reynolds



$Re < 2000$

$2000 < Re < 4000$

$Re > 4000$

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Kinematics of Fluid Flow

Types of Flow: Laminar and Turbulent Flows

Flow behavior in natural or artificial systems depends on which forces (inertia, viscous, gravity, surface tension, etc.) predominate.

In slow-moving laminar flows, **viscous forces are dominant**, and the fluid behaves as if the layers are sliding over each other.

In turbulent flows, the flow behavior is chaotic and changes dramatically, since the **inertial forces are more significant** than the viscous forces.

The Reynolds number (*Re*), provides a useful way of characterizing the flow. It is defined as: $R_e = \frac{vd}{\nu}$

where (ν) is the kinematic viscosity of the water, v is the mean flow velocity and d is the diameter of the pipe.

The Reynolds number is a dimensionless parameter that is the ratio of the inertial (destabilizing) force to the viscosity (stabilizing) force.

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Kinematics of Fluid Flow

Types of Flow: Laminar and Turbulent Flows

REYNOLD'S NUMBER, $R_e = \frac{Inertia\ force}{Viscous\ force} = \frac{m\,a}{\tau\,A}$

OR, $R_e = \frac{m\,a}{\tau\,A} = \frac{(\rho L^3)(LT^{-2})}{\left(\mu\frac{du}{dy}\right)(L^2)} = \frac{(\rho L^3)(LT^{-2})}{\left(\mu\frac{LT^{-1}}{L}\right)(L^2)} = \frac{(LT^{-1})L\rho}{\mu}$

$\therefore R_e = \frac{VL\rho}{\mu} = \frac{VL}{\mu/\rho} = \frac{VL}{\vartheta}$

WHERE L = CHARACTERISTIC LENGTH

μ = ABSOLUTE OR DYNAMIC VISCOSITY (N.S/M²)

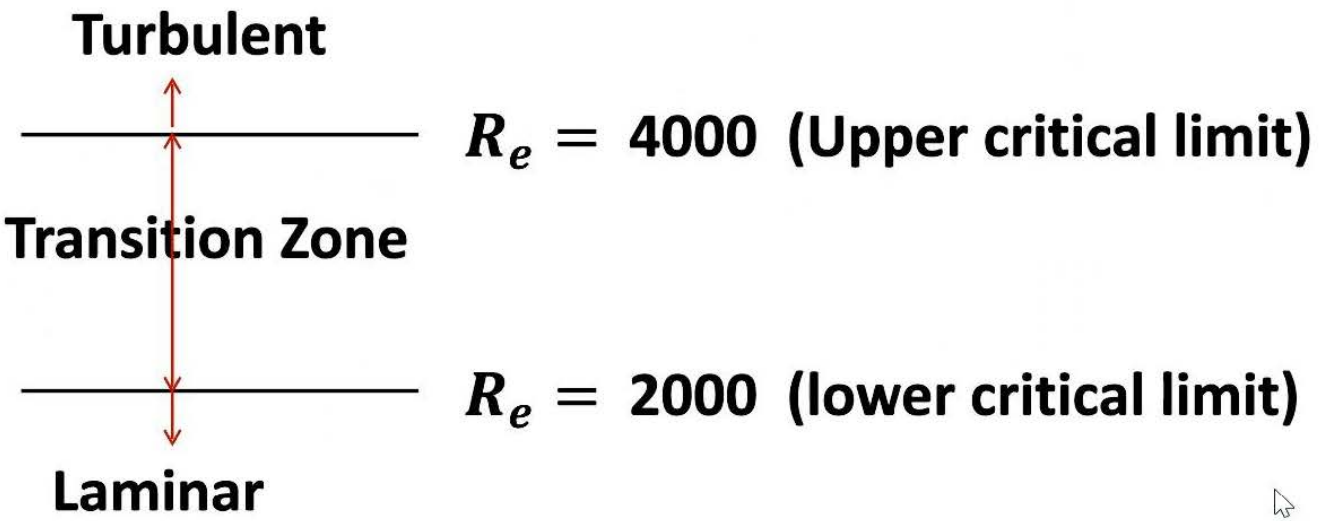
υ = KINEMATIC VISCOSITY (M²/S)

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Kinematics of Fluid Flow

Types of Flow: Laminar and Turbulent Flows

For practical purposes $R_e = \frac{VD\rho}{\mu}$ (D = diameter of the pipe)



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Kinematics of Fluid Flow

Types of Flow: Laminar and Turbulent Flows

REYNOLD'S NUMBER, $R_e = \frac{Inertia\ force}{Viscous\ force} = \frac{m\,a}{\tau\,A}$, $or, R_e = \frac{vd}{\nu}$

As R_e increases, the inertial force becomes relatively larger, and the flow destabilizes and becomes fully turbulent.

The critical Reynolds number for pipe flow at which laminar flow (*Re<2000*) becomes transitional (*2000<Re<4000*) and the transitional flow becomes turbulent (*Re>4000*).

The advantage of using a critical Reynolds number, instead of critical velocity, is that the results of the experiments are applicable to all Newtonian fluid flows in pipes with a circular cross-section.

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Kinematics of Fluid Flow

EXAMPLE 1. Water of kinematic viscosity $1.15 \times 10^{-6} m^2/s$ flows in a circular pipe of 30 mm diameter. Calculate the largest flow rate for which laminar flow can be expected. What s the equivalent flow rate of air? (υ_{AIR} = 1.37x 10^{-5} M^2/S)

Given, $\boldsymbol{\nu} = 1.15 \times 10^{-6} m^2/s$ and $\boldsymbol{D = 0.03\ m}$

$$\therefore A = \frac{\pi D^2}{4} = 0.0007065\ m^2$$

$$\boldsymbol{R_e = \frac{vd}{\nu}}, \mathbf{2000} = \frac{\boldsymbol{v \times 0.03}}{1.15 \times 10^{-6}}, \ \therefore v = 0.077\ m/s$$

Flow Rate, $Q = AV = 0.0007065 \times 0.077 = 5.4 \times 10^{-5} m^3/s$

Again, $\mathbf{2000} = \frac{\boldsymbol{v_{air} \times 0.03}}{1.37 \times 10^{-5}}, \ \therefore \boldsymbol{v_{air}} = 0.91\ m/s$

$\therefore \boldsymbol{Q_{air}} = 0.0007065 \times 0.91 = 6.4 \times 10^{-4} m^3/s$

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Kinematics of Fluid Flow

EXAMPLE 2. Oil with $\mu = 9.6 \times 10^{-3}\ N.s/m^2$, $s = 0.90$, flows at $4.2\ L/s$ througha pipe of 5 cm diameter. Calculate N_R and classify the flow.

Given, $\mu = 9.6 \times 10^{-3}\ N.s/m^2$, $Q = 4.21\ L/s = 4.21 \times 10^{-3} m^3/s$ and $D = 0.05\ m$,

$\therefore A = \frac{\pi D^2}{4} = 0.00196\ m^2$,

we know, $\rho = 900\ Kg/m^3$

$$\therefore v = \frac{Q}{A} = \frac{4.21 \times 10^{-3}}{0.00196} = 2.147\ m/s$$

$$N_R = \frac{2.147 \times 0.05 \times 900}{9.6 \times 10^{-3}} = 10068, \text{ Turbulent flow}$$

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Kinematics of Fluid Flow

EXAMPLE 3

GIVEN μ = 0.057 N.S/M^2 , s = 0.90, D = 75 MM

Q = 350 L/MIN.

IS THE FLOW LAMINAR OR TURBULENT?

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Kinematics of Fluid Flow

Types of Flow: Steady and Unsteady flows

Steady flow is defined as the type of flow in which **fluid characteristics like velocity, pressure, density, etc. at a point do not change with time.**
Unsteady flow is a type of flow in which the **velocity, pressure, and density at a point change with time.**

STEADY FLOW: occurs when the condition at any point in the fluid do not change with time.

$$\frac{\partial V}{\partial t} = 0, \qquad \frac{\partial \rho}{\partial t} = 0, \qquad \frac{\partial T}{\partial t} = 0, \qquad \frac{\partial P}{\partial t} = 0$$

UNSTEADY FLOW: condition at any point in the fluid change with time.

$$\frac{\partial V}{\partial t} \neq 0$$

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Kinematics of Fluid Flow

Types of flow: Uniform and Non-uniform flows

Uniform flow is defined as the type of flow in which the **velocity at any given time has the same magnitude and direction**.

Non-uniform flow is the type of flow in which **velocity changes from time to time** at any given instant of time.

UNIFORM FLOW: Velocity is same at every section at any instant of time.

$$\frac{\partial V}{\partial s} = 0, \text{ where, } s = \text{distance}$$

NON-UNIFORM FLOW: Velocity vector varies from place to place at any instant of time.

$$\frac{\partial V}{\partial s} \neq 0$$

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Kinematics of Fluid Flow

STEADY UNIFORM FLOW: FLOW THROUGH A LONG PIPE AT A CONSTANT RATE.

UNSTEADY UNIFORM FLOW: FLOW THROUGH A LONG PIPE AT A CHANGING RATE.

STEADY NON-UNIFORM FLOW: FLOW THROUGH A CONICAL PIPE AT A CONSTANT RATE.

UNSTEADY NON-UNIFORM FLOW: FLOW THROUGH A CONICAL PIPE AT A CHANGING RATE.

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Kinematics of Fluid Flow

1 – DIMENSIONAL FLOW: F = F(X.T)

2 – DIMENSIONAL FLOW: F = F(X,Y,T)

3 - DIMENSIONAL FLOW: F = F(X,Y,Z,T)

WHERE F =FLOW OR FLUID PROPERTY (VELOCITY, MASS, DENSITY, PRESSURE)

IN CARTESIAN CO-ORDINATE SYSTEM, $V = u\underline{i} + v\underline{j} + W\underline{k}$

ILLUSTRATION:

1 – D FLOW: FLOW THROUGH A STRAIGHT PIPE

2 – D FLOW: FLOW OVER A WEIR OR FLOW AROUND AN ISLAND

3 – D FLOW: FLOW OF COOL AIR FROM OUTLET OF AN AIR CONDITIONER INTO A ROOM.

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Kinematics of Fluid Flow

Types of flow: Compressible and Incompressible flows

- **Compressible flow** is that type of flow in which the density of the fluid changes from point to point i.e., the density is not constant for the fluid.

$\rho \neq constant$

- **INcompressible flow** is that type of flow in which the density of the fluid is constant. Liquid are generally incompressible while gasses are compressible.

$\rho = constant$

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Kinematics of Fluid Flow

Types of flow: Rotational and Irrotational Flows

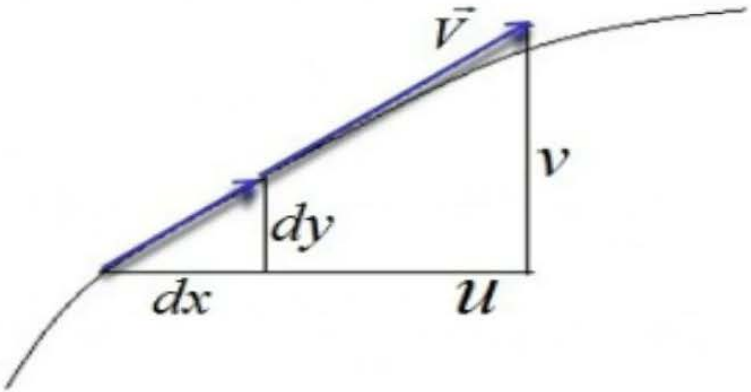
- A flow in which the fluid particles rotate about their own mass centers while flowing i.e., have same angular velocity normal to the plane of motion, is called a rotational flow.

- A flow in which the fluid particles do not rotate about their own mass centers while flowing and retain their original orientations is called an irrotational flow.

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Kinematics of Fluid Flow

Streamline

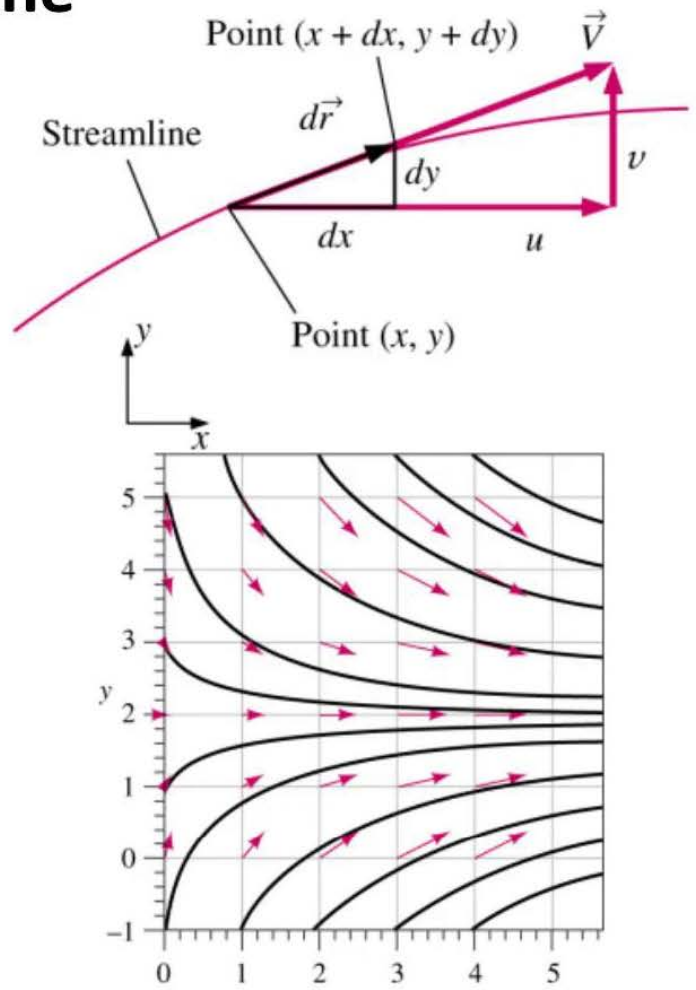


- A streamline is an imaginary line drawn through a flowing fluid in such a way that the tangent to it at any point gives the direction of the velocity of flow at that point.
- Since a fluid is composed of fluid particles, the pattern of flow of fluid may be represented by a series of streamlines.
- Since at any point the velocity is tangential to the stream line , the component of the velocity at right angle to the steam line is always zero. Thus there can be no flow occurring across the stream line
- A fluid particle always moves tangent to the streamline thus, for steady flow, the path lines and streamlines are identical.

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Kinematics of Fluid Flow

Streamline



- A **streamline** is a curve that is everywhere tangent to the *instantaneous* local velocity vector.
- Consider an infinitesimal arc length along a streamline:

$$d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$$

- By definition $d\vec{r}$ must be parallel to the local velocity vector

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

- Geometric arguments result in the equation for a streamline

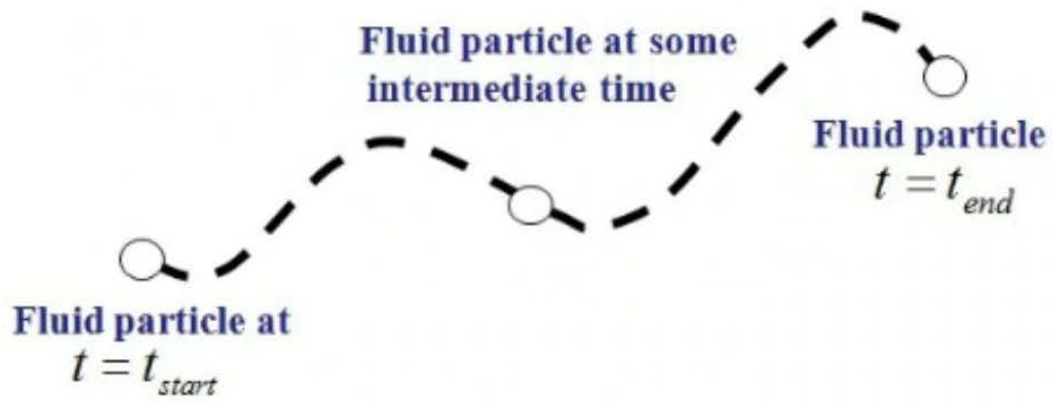
$$\frac{dr}{V} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

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Kinematics of Fluid Flow

Path Lines

- A path line is the line traced or actual path travelled by an individual fluid particle over a period of time.
- It indicates the direction of velocity of the same fluid particle at successive instants of time.



- Same as fluid particle's material position vector, $\{x_{particle}(t), y_{particle}(t), z_{particle}(t)\}$
- Particle location at time t: $\vec{x} = \vec{s}_{start} + \int_{start}^{t} \vec{V}.dt$
- PIV (particle Image Velocimetry is a modern experimental technique to measure velocity filed over a plane in the flow field.

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Path Line indicates the direction of velocity of the same fluid particle at successive instants of time.

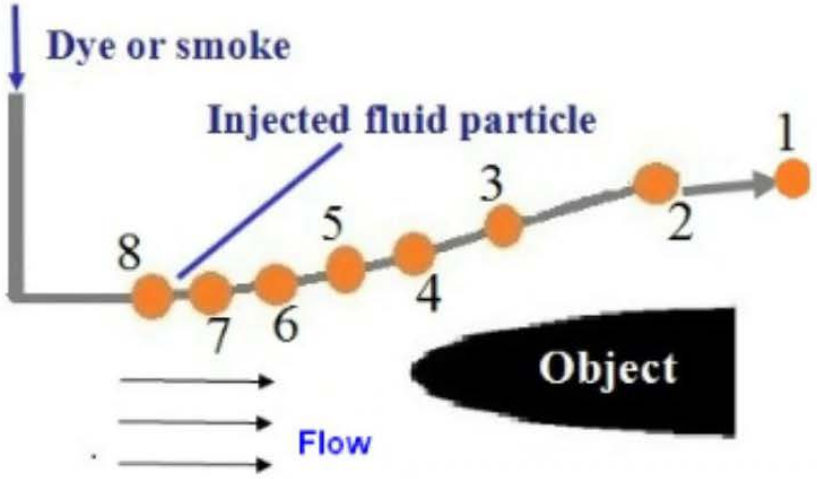
While a **streamline** shows the direction of velocity of a number of fluid particles at the same instant of time.

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Kinematics of Fluid Flow

Streak Line

- A streakline is the locus of fluid particles that have passed sequentially through a prescribed point in the flow.
- The instantaneous pictures of the position of all fluid particles, which have passed through a given point at some previous time are called streak lines or filament lines.



(c) Streakline

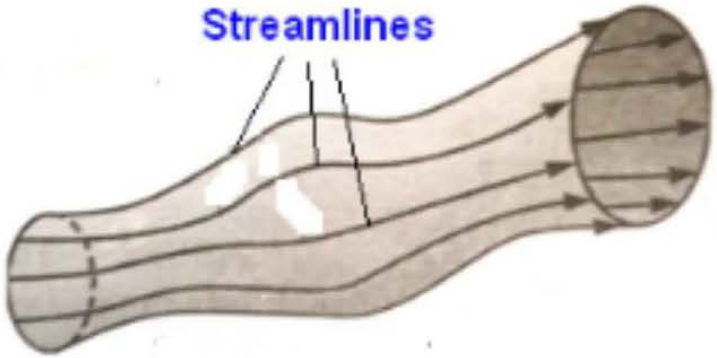
- In experimental work, often a colour or dye or some other substance is injected into the flowing fluid, in order to trace the motion of the fluid particles. The resulting trail of colour is known as streak line.
- In steady flow, since there is no change in the flow pattern, a streak line is same as a streamline and the path line of a particle.

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Kinematics of Fluid Flow

Stream tube

A stream tube is a tube imagined to be formed by a group of streamlines passing through a small closed curve, which may or may not be circular.



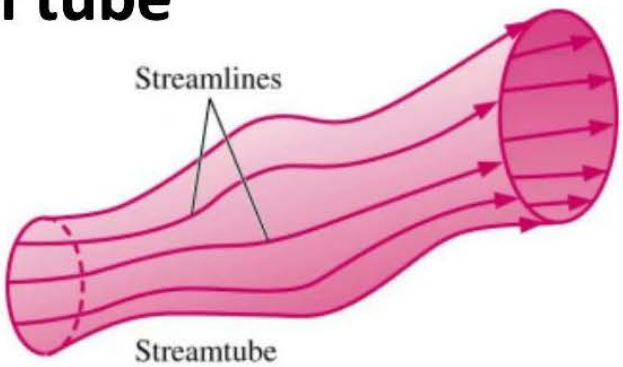
The stream surface thus functions as if it were a solid wall. The shape of stream tube changes from one instant to another because of change in the position of streamlines.

Flow in pipes and nozzles are the examples of stream tube. The concept of stream tube is quite useful in analyzing several fluid flow problems, since the entire flow field may be divided into a large number of stream tubes, thus yielding a clear picture of the actual pattern of flow.

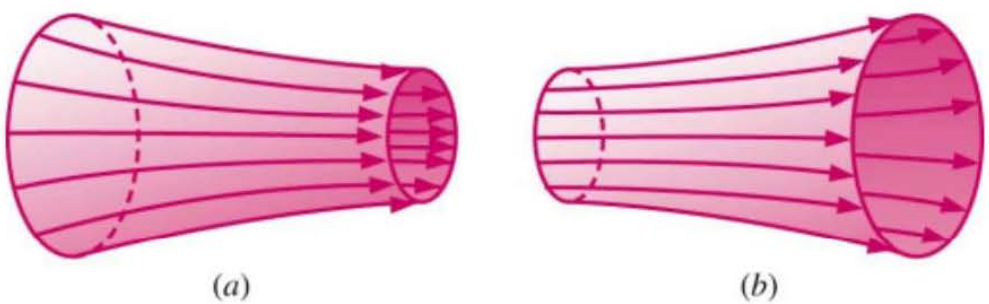
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Kinematics of Fluid Flow

Stream tube



A **streamtube** consists of a bundle of individual streamlines. Since fluid cannot cross a streamline (by definition), fluid within a streamtube must remain there. Streamtubes are, obviously, instantaneous quantities and they may change significantly with time.



In the converging portion of an incompressible flow field, the diameter of the streamtube must decrease as the velocity increases, so as to conserve mass.

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Control Volume (CV): A volume fixed in space through which matter, mass, energy etc., can flow.

Control Surface (CF): The boundary of control volume is control surface

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Continuity Equation

Since the stream tube is bounded on all sides by streamlines and since the velocity has no component normal to a streamline, no fluid can leave or enter the stream tube except at the ends.

The fixed volume between two fixed sections of the steam tube is known as the control volume (CV) and magnitude will be designated by volume.

According to Newtonian physics (disregarding the possibility of converting mass to energy), mass must be conserved.

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Continuity Equation

Based on the conservation of mass.

The quantity of fluid flowing per sec through all cross-section of a pipe is constant.

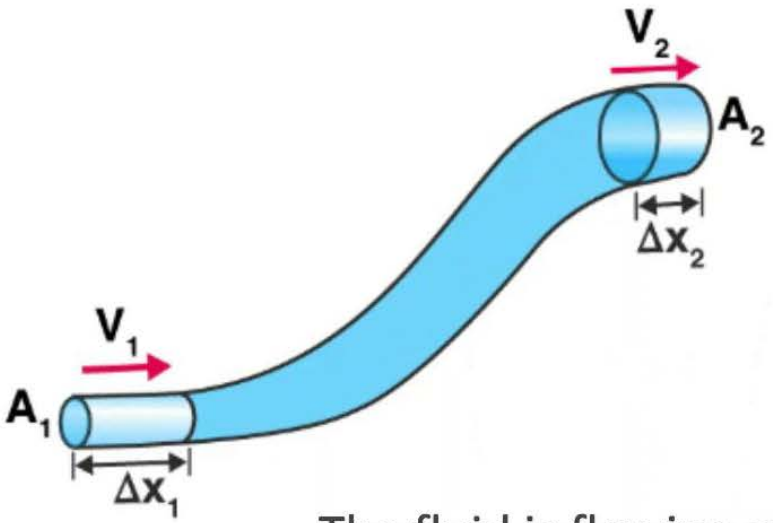
The cross sectional area of the pipe and the fluid speed at any point along the pipe is always constant.

This product is equal to the volume flow per second or simply the flow rate.

The continuity equation is given as: $\boldsymbol{Q = AV}$

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Continuity Equation



Assumption of Continuity Equation

- The tube is having a single entry and single exit
- The fluid flowing in the tube is non-viscous
- The flow is incompressible
- The fluid flow is steady

The fluid is flowing at speed v_1 through a short distance Δx_1 at short interval of time Δt. Then, distance covered, $\Delta x_1 = V_1 \Delta t$

Now, at the lower end of the pipe, the volume of the fluid that will be:

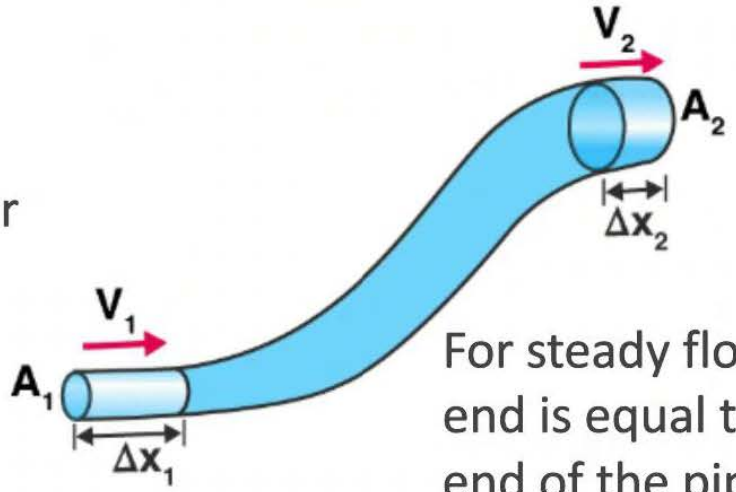
$V = A_1 \Delta x_1 = A_1 V_1 \Delta t$

and, ***mass*** will be: Δm_1***= Density × Volume***

$\therefore \Delta m_1 = \rho_1 A_1 V_1 \Delta t$

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Continuity Equation



Mass flow rate at the lower end , $\Delta m_1/\Delta t = \rho_1 A_1 v_1$

Mass flow rate at the upper end, $m_2/\Delta t = \rho_2 A_2 v_2$

For steady flow, the mass flux at the lower end is equal to the mass flux at the upper end of the pipe,

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

In a general form as: $\rho A V = constant$

The equation proves the law of conservation of mass in fluid dynamics.

As the fluid is incompressible, the density will remain constant for steady flow. So, $\rho_1 = \rho_2$.

Thus, the e***quation*** can be now written as: $A_1 V_1 = A_2 V_2$

In general form as: $A V = constant$

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Flow Rate and Mean velocity

The quantity of fluid flowing per unit time across any section is called Flow rate

Mass flow rate refers to the mass of a fluid passing through a specific cross-sectional area per unit time.

Expressed (kg/s) or (lb/min).

Essential chemical reactions, combustion processes, and mass balancing etc.

For compressible fluid, expressed in weight of fluid flowing across a section

Volumetric flow rate measures the volume of fluid passing through a given cross-sectional area per unit time.

Expressed in (m^3/s) or GPM or L/s.

Essential for sizing equipment, monitoring fluid flow, and assessing system performance.

Mass flow rate is affected by the fluid's density, while volumetric flow rate is influenced by the fluid's compressibility and changes in temperature and pressure.

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Flow Rate and Mean velocity

Mass flow rate can be calculated by multiplying the volume flow rate by the mass density of the fluid, ρ.

Mass Flow Rate = Volumetric Flow Rate × Density

The volume flow rate is calculated by multiplying the flow velocity of the mass elements, v, by the cross-sectional vector area, A.

Volumetric Flow Rate = Flow velocity × Density

$$A_1V_1 = A_2V_2 = \ldots\ldots\ldots\ldots = Q \qquad \textbf{Volume flow rate}$$

$$\rho_1 A_1V_1 = \rho_2 A_2V_2 = \ldots\ldots\ldots\ldots = M \qquad \textbf{Mass flow rate}$$

$$\gamma_1 A_1V_1 = \gamma_2 A_2V_2 = \ldots\ldots\ldots\ldots = G \qquad \textbf{Weight flow rate}$$

$$\textbf{Mean velocity, } V = \frac{Q}{A} = \frac{G}{A} = \frac{M}{A}$$

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Problem 5.1 *The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 m/s. Determine also the velocity at section 2.*

At section 1, $D_1 = 10 \text{ cm} = 0.1 \text{ m}$

$$A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(.1)^2 = 0.007854 \text{ m}^2$$

$V_1 = 5$ m/s.

At section 2, $D_2 = 15 \text{ cm} = 0.15 \text{ m}$

$$A_2 = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$$

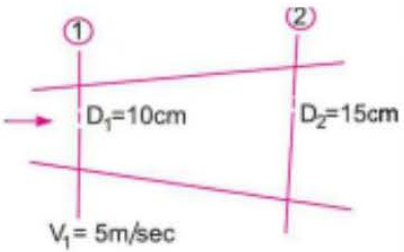


Fig. 5.2

(*i*) Discharge through pipe is given by equation (5.1)

or $Q = A_1 \times V_1$

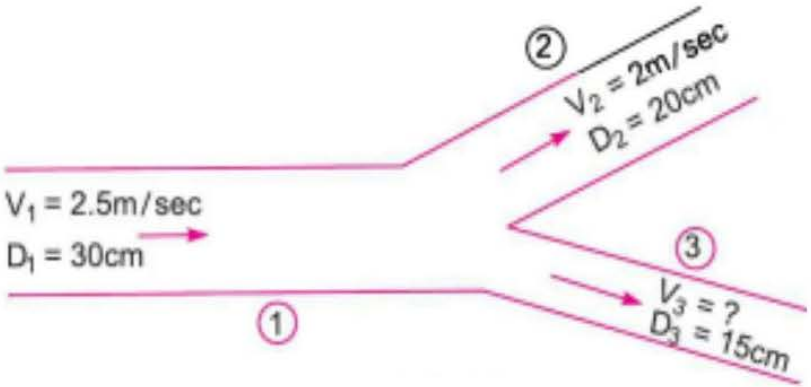
$= 0.007854 \times 5 = \mathbf{0.03927\ m^3/s.}$ **Ans.**

Using equation (5.3), we have $A_1V_1 = A_2V_2$

(*ii*) $\therefore$ $$V_2 = \frac{A_1V_1}{A_2} = \frac{0.007854}{0.01767} \times 5.0 = \mathbf{2.22\ m/s.}$$ **Ans.**

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Problem 5.2 *A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.*



$$D_1 = 30 \text{ cm} = 0.30 \text{ m}$$

$$\therefore \quad A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .3^2 = 0.07068 \text{ m}^2$$

$$V_1 = 2.5 \text{ m/s}$$

$$D_2 = 20 \text{ cm} = 0.20 \text{ m}$$

$$\therefore \quad A_2 = \frac{\pi}{4} (.2)^2 = \frac{\pi}{4} \times .4 = 0.0314 \text{ m}^2,$$

$$V_2 = 2 \text{ m/s}$$

$$D_3 = 15 \text{ cm} = 0.15 \text{ m}$$

$$\therefore \quad A_3 = \frac{\pi}{4} (.15)^2 = \frac{\pi}{4} \times 0.225 = 0.01767 \text{ m}^2$$

Find (*i*) Discharge in pipe 1 or Q_1

(*ii*) Velocity in pipe of dia. 15 cm or V_3

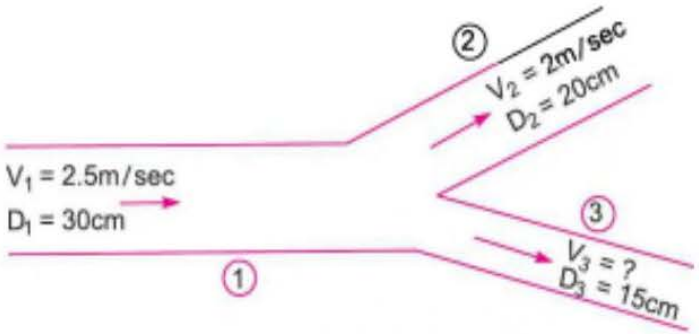
Let Q_1, Q_2 and Q_3 are discharges in pipe 1, 2 and 3 respectively.

Then according to continuity equation

$$Q_1 = Q_2 + Q_3$$

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Problem 5.2 *A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.*



(*ii*) Velocity in pipe of dia. 15 cm or V_3

Let Q_1, Q_2 and Q_3 are discharges in pipe 1, 2 and 3 respectively.

Then according to continuity equation

$$Q_1 = Q_2 + Q_3$$

(*i*) **The discharge Q_1** in pipe 1 is given by

$$Q_1 = A_1V_1 = 0.07068 \times 2.5 \text{ m}^3/\text{s} = \mathbf{0.1767\ m^3/s.\ Ans.}$$

(*ii*) **Value of V_3**

$$Q_2 = A_2V_2 = 0.0314 \times 2.0 = 0.0628 \text{ m}^3/\text{s}$$

Substituting the values of Q_1 and Q_2 in equation (1)

$$0.1767 = 0.0628 + Q_3$$

$\therefore$ $$Q_3 = 0.1767 - 0.0628 = 0.1139 \text{ m}^3/\text{s}$$

But $$Q_3 = A_3 \times V_3 = 0.01767 \times V_3 \quad \text{or} \quad 0.1139 = 0.01767 \times V_3$$

$\therefore$ $$V_3 = \frac{0.1139}{0.01767} = \mathbf{6.44\ m/s.\ Ans.}$$

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A pipeline carries oil (relative density 0.86) at v = 2 m/s through 200mm cross-section. Find

- Mass flow rate
- Weight flow rate
- Volume flow rate
- Velocity in 60mm cross-section.

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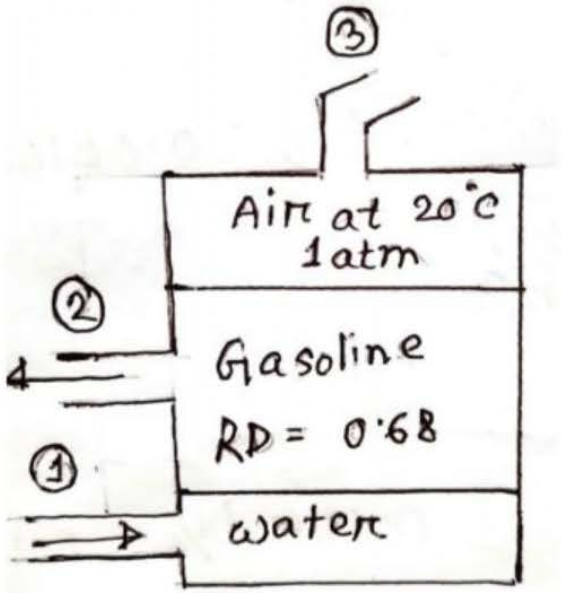
In steady viscous flow through a circular pipe the velocity profile is approximated by,

$$u = Um[1 - (\frac{r}{r_0})^2],$$

Compute the average velocity if the density is constant.

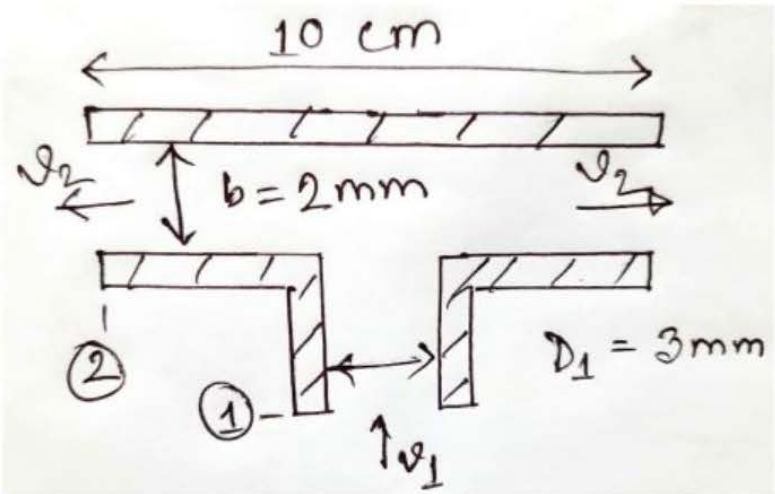
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The tank in the figure admitting water at 90 N/s and ejecting gasoline (RD = 0.68) at 50 N/s. If all the fluids are incompressible, how much air is passing through the vent and in what direction? $\gamma_{air} = 0.0118\ KN/m3$



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Oil enters at section 1 at 0.06N/s to lubricate the thrust bearing. The 10cm diameter bearing plates are 2mm apart. Assume steady flow compute the inlet velocity V1 and outlet velocity V2.



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Velocity and acceleration in steady flow

$$u_{st} = u(x, y, z)$$

$$v_{st} = v(x, y, z)$$

$$w_{st} = w(x, y, z)$$

$$\mathbf{a}_{st} = \frac{d}{dt}\mathbf{V}(x, y, z) = \frac{\partial \mathbf{V}}{\partial x}\frac{dx}{dt} + \frac{\partial \mathbf{V}}{\partial y}\frac{dy}{dt} + \frac{\partial \mathbf{V}}{\partial z}\frac{dz}{dt}$$

$$|\mathbf{V}| = (u^2 + v^2 + w^2)^{1/2}$$

$$\mathbf{a}_{st} = u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}$$

$$(a_x)_{st} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$(a_y)_{st} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$(a_z)_{st} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

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Velocity and acceleration in unsteady flow

$$a_x = \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) + \frac{\partial u}{\partial t}$$

$$a_y = \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) + \frac{\partial v}{\partial t}$$

$$a_z = \left(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) + \frac{\partial w}{\partial t}$$

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Acceleration of Fluid Particle

The acceleration of a fluid particle is the time rate of change of its velocity vector. Let a be the acceleration at point P(x,y,z) and a_x, a_y and a_z denote the components of the acceleration in x, y and z directions, respectively. Then,

$$a_x = \frac{du}{dt}, \qquad a_y = \frac{dv}{dt}, \qquad a_z = \frac{dw}{dt} \tag{5.26}$$

and

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} \tag{5.27}$$

If $\overline{V}(x, y, z, t)$ is the velocity vector and $\overline{a}$ is the acceleration vector, then

$$\overline{a} = \frac{d\overline{V}}{dt}(x, y, z, t) \tag{5.28}$$

Now,

$$d\overline{V} = \frac{\partial \overline{V}}{\partial t}dt + \frac{\partial \overline{V}}{\partial x}dx + \frac{\partial \overline{V}}{\partial y}dy + \frac{\partial \overline{V}}{\partial z}dz \tag{5.29}$$

$$\therefore \frac{d\overline{V}}{dt} = \frac{\partial \overline{V}}{\partial t} + \frac{\partial \overline{V}}{\partial x}\frac{dx}{dt} + \frac{\partial \overline{V}}{\partial y}\frac{dy}{dt} + \frac{\partial \overline{V}}{\partial z}\frac{dz}{dt} \tag{5.30}$$

$$\therefore \overline{a} = \frac{\partial \overline{V}}{\partial t} + \frac{\partial \overline{V}}{\partial x}\frac{dx}{dt} + \frac{\partial \overline{V}}{\partial y}\frac{dy}{dt} + \frac{\partial \overline{V}}{\partial z}\frac{dz}{dt} = \frac{\partial \overline{V}}{\partial t} + u\frac{\partial \overline{V}}{\partial x} + v\frac{\partial \overline{V}}{\partial y} + w\frac{\partial \overline{V}}{\partial z} \tag{5.31}$$

In Cartesian coordinates

$$a_x = \frac{\partial u}{\partial t} + (u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}) \tag{5.32a}$$

$$a_y = \frac{\partial v}{\partial t} + (u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}) \tag{5.32b}$$

$$a_z = \frac{\partial w}{\partial t} + (u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}) \tag{5.32c}$$

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A two dimensional flow field is given by $u = 2y$, $v = x$. Derive a general expression for the velocity and acceleration. Find the acceleration in the flow field at point A ($x = 3.5$, $y = 1.2$)

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Problem 5.6 *The velocity vector in a fluid flow is given*

$$V = 4x^3 i - 10x^2 yj + 2tk.$$

Find the velocity and acceleration of a fluid particle at (2, 1, 3) at time t = 1.

Solution. The velocity components u, v and w are $u = 4x^3$, $v = -10x^2 y$, $w = 2t$

For the point (2, 1, 3), we have $x = 2$, $y = 1$ and $z = 3$ at time $t = 1$.

Hence velocity components at (2, 1, 3) are

$$u = 4 \times (2)^3 = 32 \text{ units}$$

$$v = -10(2)^2(1) = -40 \text{ units}$$

$$w = 2 \times 1 = 2 \text{ units}$$

$\therefore$ Velocity vector V at (2, 1, 3) $= 32i - 40j + 2k$

or Resultant velocity $= \sqrt{u^2 + v^2 + w^2}$

$$= \sqrt{32^2 + (-40)^2 + 2^2} = \sqrt{1024 + 1600 + 4} = \mathbf{51.26 \text{ units. Ans.}}$$

Acceleration is given by equation (5.6)

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

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$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Now from velocity components, we have

$$\frac{\partial u}{\partial x} = 12x^2,\ \frac{\partial u}{\partial y} = 0,\ \frac{\partial u}{\partial z} = 0 \text{ and } \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial v}{\partial x} = -20xy,\ \frac{\partial v}{\partial y} = -10x^2,\ \frac{\partial v}{\partial z} = 0\ \frac{\partial v}{\partial t} = 0$$

$$\frac{\partial w}{\partial x} = 0,\ \frac{\partial w}{\partial y} = 0,\ \frac{\partial w}{\partial z} = 0 \text{ and } \frac{\partial w}{\partial t} = 2.1$$

Substituting the values, the acceleration components at (2, 1, 3) at time $t = 1$ are

$$a_x = 4x^3(12x^2) + (-10x^2y)(0) + 2t \times (0) + 0$$
$$= 48x^5 = 48 \times (2)^5 = 48 \times 32 = 1536 \text{ units}$$
$$a_y = 4x^3(-20xy) + (-10x^2y)(-10x^2) + 2t(0) + 0$$
$$= -80x^4y + 100x^4y$$
$$= -80(2)^4(1) + 100(2)^4 \times 1 = -1280 + 1600 = 320 \text{ units.}$$
$$a_z = 4x^3(0) + (-10x^2y)(0) + (2t)(0) + 2.1 = 2.0 \text{ units}$$

$\therefore$ Acceleration is $A = a_x i + a_y j + a_z k = \mathbf{1536i + 320j + 2k.\ Ans.}$

or Resultant $A = \sqrt{(1536)^2 + (320)^2 + (2)^2}$ units

$= \sqrt{2359296 + 102400 + 4} = \mathbf{1568.9\ units.\ Ans.}$

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Stream Function

In fluid dynamics, the stream function is a mathematical concept used to describe the flow field of an incompressible fluid. It is a scalar field that represents the streamlines of the fluid flow.

The stream function is defined such that its partial derivatives give the components of the velocity vector in the perpendicular direction to the streamlines. In other words, if u and v are the x and y components of the velocity vector, respectively, then the stream function, denoted by Ψ, can be expressed as:

$$\frac{\delta\psi}{\delta x} = v; \frac{\delta\psi}{\delta y} = -u$$

Here, Ψ is a scalar function that depends on the spatial coordinates (x, y).

The stream function is particularly useful in two-dimensional flow analysis, where the fluid motion occurs in a plane. By solving the continuity equation and using the definition of the stream function, the velocity components can be determined.

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Stream Function

The discharge per unit width between two stream lines will be the difference in the values of the two stream functions, i.e.

$$q = \psi_2 - \psi_1 \qquad (5.21)$$

In a two-dimensional flow, the resultant velocity V at any point P(x,y) is given by the relation

$$V = \sqrt{u^2 + v^2} \qquad (5.22)$$

Example 5.2

For a two-dimensional flow, the stream function is given by $\psi = 2xy$. Calculate the velocity at the point (3, 6).

Solution If u and v are the velocity components in the x and y directions, respectively, then

$$u = \frac{\delta\psi}{\delta y} = \frac{\delta}{\delta y}(2xy) = 2x = 2 \times 3 = 6 \text{ m/s}$$

$$v = -\frac{\delta\psi}{\delta x} = -\frac{\delta}{\delta y}(2xy) = -2y = -2 \times 6 = -12 \text{ m/s}$$

$\therefore$ The resultant velocity at the point (3,6) is obtained as

$$V = \sqrt{u^2 + v^2} = \sqrt{6^2 + (-12)^2} = 13.42 \text{ m/s}$$

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Flow Net

- A grid obtained by drawing a series of streamlines ψ and equipotential lines Φ is known as a flow net.

- Flow net provides a simple graphical technique for studying tow-dimensional irrotational flows, when the mathematical calculation is difficult and cumbersome.

- The streamlines ψ and equipotential lines Φ are mutually perpendicular to each other.

- The stream lines are so spaced that rate of flow Q is same between each successive pair of lines.

- The stream lines in flow net show the direction of flow and the equinoctial lines join the points the equal velocity potential Φ.

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Flow Net

Characteristics of Flow Nets

1.Flow lines or stream lines represent flow paths of particles of water.
2.Flow lines and equipotential line are orthogonal to each other.
3.The area between two flow lines is called a flow channel.
4.The rate of flow in a flow channel is constant.
5.Flow cannot occur across flow lines.
6.An equipotential line is a line joining points with the same head.
7.The velocity of flow is normal to the equipotential line.
8.The difference in head between two equipotential lines is called the potential drop or head loss (*dh*).
9.A flow line cannot intersect another flow line.
10.An equipotential line cannot intersect another equipotential line.

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5.11 FLOW NET

It is a graphical representation of stream lines and potential lines, which are orthogonal. The stream lines show the direction of flow. The potential lines show the lines joining the points of equal velocity potential. The flow net can be drawn only for irrotational flow. It helps in depicting and analyzing the behavior of irrotational flow. Certain flow phenomena which cannot be easily analyzed by mathematical means, may be analyzed and studied by drawing flow nets, e.g. flow through a Francis water turbine runner.

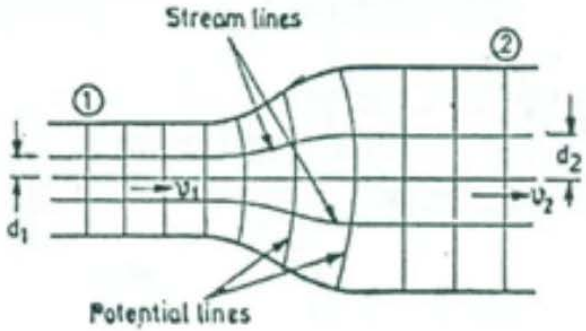


Fig. 5.8 Flow net

Consider a flow net where the stream lines are diverging as shown in Fig. 5.8. Consider two sections of the flow net 1 and 2. Let d_1 be the spacing between two stream lines at section 1, d_2 be the spacing between two stream lines at section 2, v_1 be the velocity of fluid particles at section 1 and v_2 be the velocity of fluid particles at section 2. Since there can be no flow across the stream lines, therefore the discharge per unit width between two consecutive stream lines will be equal, i.e.

$$q = d_1 v_1 = d_2 v_2 \qquad (5.33)$$

It is clear that the velocity of fluid particles varies inversely with the spacing between the stream lines, i.e. velocity decreases with increase in stream line spacing and vice versa.

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Example 5.4

From a flow net diagram, it was found that the distance between two consecutive stream lines at two successive sections are 10 mm and 6 mm, respectively. If the velocity at the first section is 1 m/s, find the velocity at the other section. Also, find the discharge between the two stream lines.

Solution We have, $d_1 = 10$ mm $= 0.01$ m, $d_2 = 6$ mm $= 0.006$ m and $v_1 = 1$ m/s

$\therefore$ Using the relation, $d_1 v_1 = d_2 v_2$, we have

$$0.01 \times 1 = 0.006 \times v_2$$

or

$$v_2 = \frac{0.01}{0.006} = 1.67 \text{ m/s}$$

The discharge between the streamlines is given by

$$q = d_1 v_1 = 0.01 \times 1 = 0.01 \text{ m}^2/\text{s}$$

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Potential Function

5.9 POTENTIAL FUNCTION

Potential function ϕ is a function of x, y and z such that its partial derivative with respect to displacement in any direction is equal to the velocity component in that direction. Mathematically,

$$u = \frac{\partial \phi}{\partial x}, \; v = \frac{\partial \phi}{\partial y}, \; w = \frac{\partial \phi}{\partial z} \tag{5.23}$$

where u, v and w are the velocity components in the x, y and z directions, respectively.

The relationship between the stream function and the potential function for two-dimensional steady, irrotational and incompressible flow is obtained from Eqs.(5.16), (5.17) and (5.23) as follows:

$$u = \frac{\partial \phi}{\partial x} = \frac{\delta \psi}{\delta y} \tag{5.24}$$

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Potential Function

$$v = \frac{\partial \phi}{\partial y} = -\frac{\delta \psi}{\delta x} \qquad (5.25)$$

Potential line is a line along which the potential function ϕ is constant, i.e. $d\phi = 0$. The potential lines are orthogonal to the stream lines as is shown below.

Since $\phi = f(x,y)$ for steady flow in two dimensions, so

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

But along a potential line, $d\phi = 0$.

$$\therefore \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = 0$$

or

$$\frac{dy}{dx} = -\frac{\partial \phi}{\partial x} / \frac{\partial \phi}{\partial y} = -\frac{u}{v}$$

$\therefore$ Slope of the potential line = - u/v.

Again, $\psi = f(x, y)$ for steady flow in two dimensions. So

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

But along a stream line, $d\psi = 0$.

$$\therefore \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = 0$$

or

$$\frac{dy}{dx} = -\frac{\partial \psi}{\partial x} / \frac{\partial \psi}{\partial y} = \frac{v}{u}$$

$\therefore$ Slope of the stream line = v/u.

Thus, slope of the potential line × slope of the stream line = $-\frac{u}{v} \times \frac{v}{u} = -1$, which shows that the potential lines and the stream lines are orthogonal, i.e. perpendicular to one another.

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