Military Institute of Science and Technology

EWCE 261

Fluid Mechanics

3 Credit Hour

Md. Atikuzzaman

Lecturer

Department of Environmental, Water Resources and Coastal Engineering

Syllabus:

1. Development and Scope of Fluid Mechanics
2. Fluid Properties
3. Fluid Statics
4. Similitude and Dimensional Analysis
5. Steady Incompressible Flow in Pressure Conduits: Laminar Flow and Turbulent Flow
6. General Equation for Fluid Friction
7. Empirical Equation for Pipe Flow
8. Minor Losses in Pipe Flow
9. Fluid Measurement: pitot tube, orifice, mouthpiece, nozzle, venturimeter, weir.
10. Pipe Flow Problems: Pipes in Series and Parallel, Branching Pipes, Pipe Networks

Syllabus: (To be Covered in Part A)

1. Kinematics of Fluid Flow
2. Fluid Flow Concepts and Basic Equations: Continuity Equation, Bernoulli's Equation, Energy Equation
3. Momentum Equation and Forces in Fluid Flow

Fluid Statics

Total Pressure: It is defined as the force exerted by a static fluid on a surface (either plane or curved) when the fluid comes in contact with the surface. This force is always normal to the surface.

Center of Pressure: The center of pressure is defined as the point of application of the resultant pressure on the surface

2.2 Pascal's Law

Pascal states that when a fluid at rest, the pressure at any point is equal in all direction. This law can be proved as follows:

Let us consider a small wedge shaped free body of liquid. Let us assume that the length perpendicular to paper is unity.

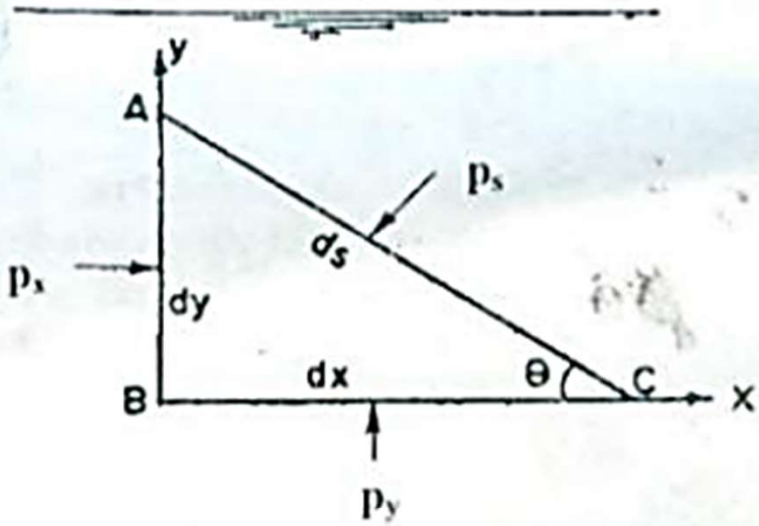


Fig. 2.1: Proof of Pascal's law

$\frac{1}{2} \times dx dy \times 1 \times \gamma$

$V \times \gamma = W$

Since there can be no shear forces, the only force is the pressure force acting normal to the three faces and the gravity force or the weight of the element acting vertically downward. For the equilibrium of the element, we have in the x and y directions, the force equation:

$$\sum F_x = p_x dy - p_s ds \sin\theta = 0 \quad \text{.....(2.3)}$$

$$\sum F_y = p_y dx - p_s ds \cos\theta - \gamma \frac{dxdy}{2} = 0 \quad \text{.....(2.4)}$$

In which, p_x, p_y and p_s are the average pressure on the three faces, γ is the specific weight and ρ is the density of the fluid. From the geometry we have from figure 2.1:

$dy = ds\sin\theta$, and $dx = ds\cos\theta$.

So from equation 2.3 and equation 2.4 we have

neglecting this term as dx and dy are so small

0

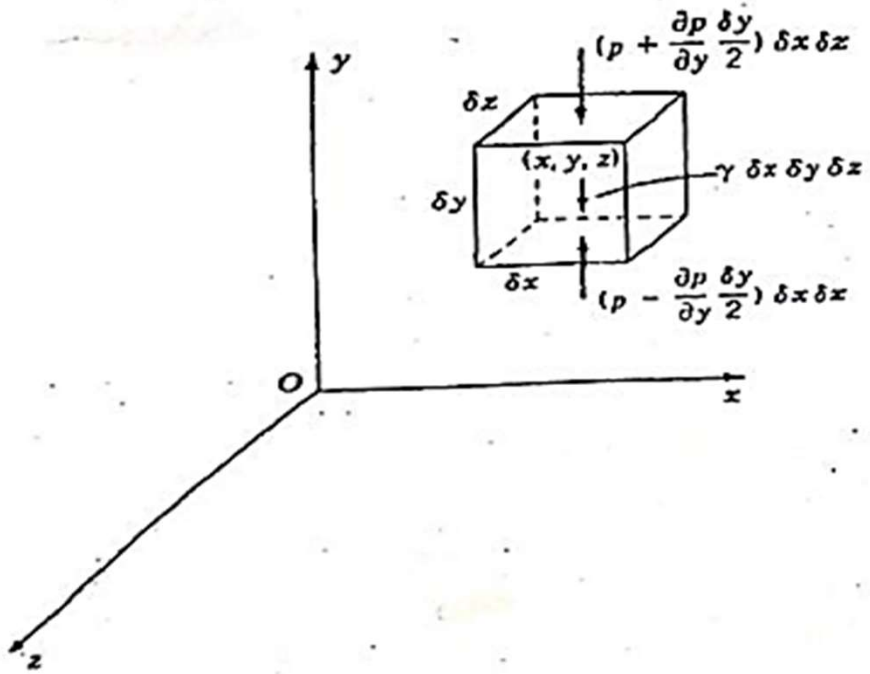
$p_x dy - p_s dy = 0$ and $p_y dx - p_s dx - \gamma (dxdy/2) = 0$

here, $p_x = p_y$ and $p_y = p_s$

so, we get $p_x = p_y = p_z$

2.3 PRESSURE VARIATION IN A STATIC FLUID

Consider the forces acting on an element of fluid at rest (Fig. 2.2). The element has sides δx, δy and δz in form of a parallelepiped with pressure p at its center (x, y, z). Let the pressure increases @ $\partial p/\partial x$, $\partial p/\partial y$ and $\partial p/\partial z$ along the positive x, y and z directions, respectively. The forces acting on the element consist of surface forces and body forces. The forces which are distributed over the entire mass of the fluid are called body forces. For the element considered above, the body force is the gravity force or weight of the fluid element $\gamma\delta x\delta y\delta z$ acting vertically downward from the center of the element. Surface forces are those which act normal or tangential to a surface. In a static fluid, there is no stress (e.g. shear stress) tangential to a surface and the only force is the pressure force normal to a surface.



Taking the y axis vertically upward, the force exerted on the bottom surface normal to the y axis is

$$(p-\frac{\partial p}{\partial y}\frac{\delta y}{2})\delta x\delta z$$

and the force exerted on the opposite face is

$$(p+\frac{\partial p}{\partial y}\frac{\delta y}{2})\delta x\delta z$$

where $\delta y/2$ is the distance from the center to a face normal to y. Summing the forces acting on the element in the y direction gives the resulting force in the y direction as

$$F_y=(p-\frac{\partial p}{\partial y}\frac{\delta y}{2})\delta x\delta z-(p+\frac{\partial p}{\partial y}\frac{\delta y}{2})\delta x\delta z-\gamma\delta x\delta y\delta z$$

$$=-\frac{\partial p}{\partial y}\delta x\delta y\delta z-\gamma\delta x\delta y\delta z$$

For the x and z directions, since no body forces act, we will have

$$F_x=-\frac{\partial p}{\partial x}\delta x\delta y\delta z \qquad F_z=-\frac{\partial p}{\partial z}\delta x\delta y\delta z$$

Since the fluid element is in static equilibrium, the resultant force in any direction must be zero. So, setting $F_x = F_y = F_z = 0$ since $\delta x \delta y \delta z$, which is the volume of the parallelepiped is not zero, we obtain

$$\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial p}{\partial y} = -\gamma \qquad \frac{\partial p}{\partial z} = 0 \tag{2.4}$$

Equation (2.4) indicates that the pressure is independent of x and z and it depends on y alone and we can replace $\partial p/\partial y$ by dp/dy and so

$$\frac{dp}{dy} = -\gamma \qquad \text{or,} \quad dp = -\gamma dy \tag{2.5}$$

Equation (2.5) is known as the fundamental equation of fluid statics. As the orientation of the element is chosen arbitrarily, it may be concluded that

i) pressure is same at every point in a horizontal plane (Pascal's law), and

ii) pressure varies in the y or vertical direction only and it decreases as y increases in the positive y direction.

For fluids which may be considered homogeneous and incompressible, γ is constant and Eq. (2.5) when integrated gives

$$p - p_1 = -\gamma(y - y_1) \tag{2.6}$$

where p is the pressure at an elevation y. For the case of a liquid at rest, it is convenient to measure distances vertically downward from the free liquid surface. If h is the distance or depth below the free liquid surface and if the pressure of air and vapor on the surface is arbitrarily taken to be zero, Eq. (2.6) can be written as

$$p = \gamma h \tag{2.7}$$

or

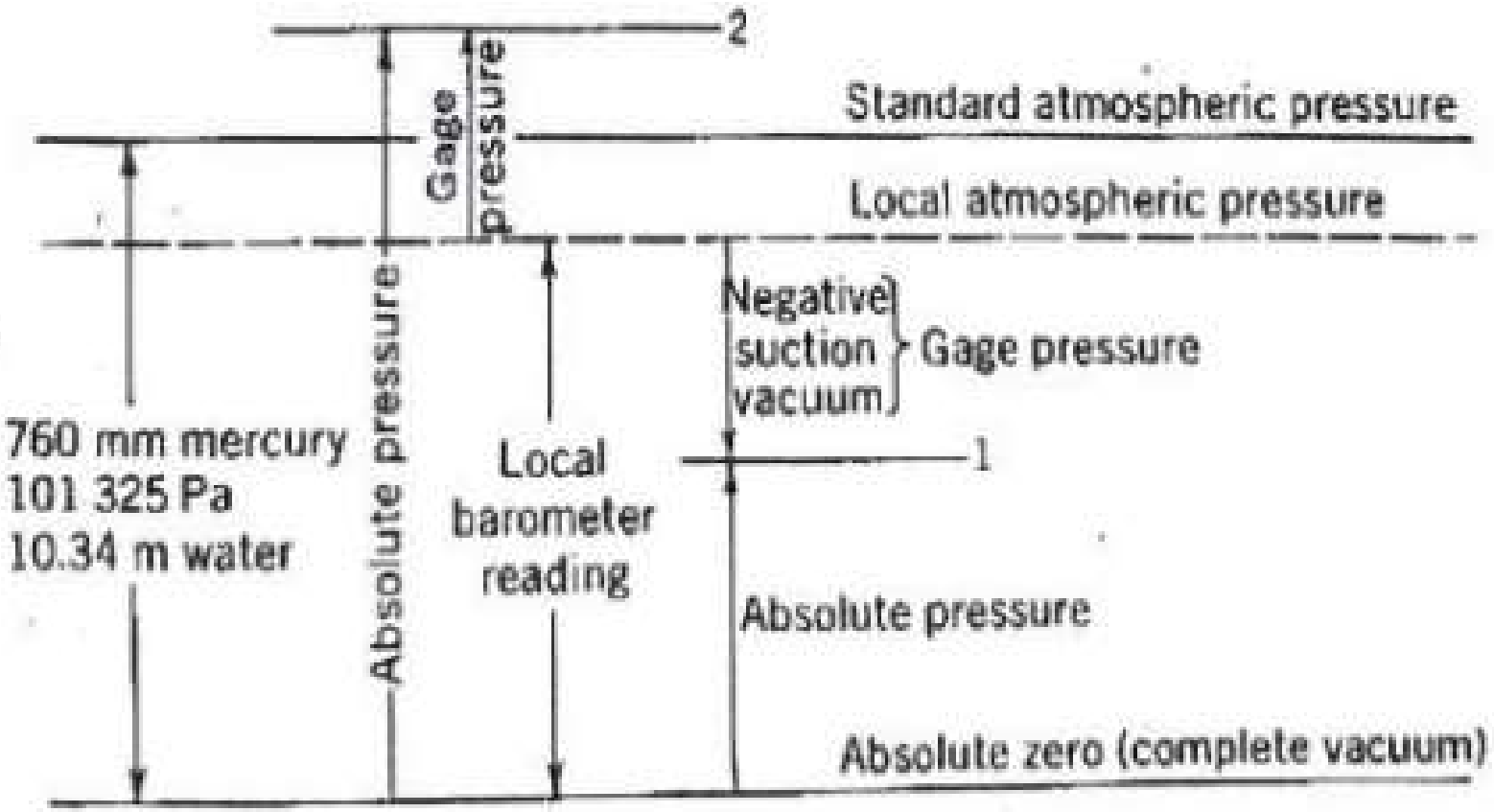
$$\frac{p}{\gamma} = h \tag{2.8}$$

which is the hydrostatic law of variation of pressure, p is the increase in pressure from that at the free surface and h is the hydrostatic pressure head measured vertically downward from the free liquid surface. Equation (2.7) or (2.8) demonstrates that the hydrostatic pressure p varies with h.

It is obvious that the pressure can be expressed either as a force per unit area p, given by Eq.(2.7), or as a height of the equivalent liquid column h, given by Eq. (2.8).

As there must always be some pressure on the surface of any liquid, the total pressure at any depth h is given by Eq. (2.7) plus the pressure on the surface. In many situations this surface pressure may be disregarded, as pointed out in the following section.

From Eq. (2.7) it may be seen that all points in a connected body of constant density fluid at rest are under the same pressure if they are at the same depth below the free liquid surface (Pascal's law). This indicates that a surface of equal pressure for a liquid at rest is a horizontal plane. Strictly speaking, it is a surface everywhere normal to the direction of gravity and is approximately a spherical surface concentric with the earth. For practical purposes, a limited portion of this surface may be considered a plane area.


2
Gage pressure
Standard atmospheric pressure
Local atmospheric pressure
Negative suction vacuum
Gage pressure
760 mm mercury
101 325 Pa
10.34 m water
Absolute pressure
Local barometer reading
1
Absolute pressure
Absolute zero (complete vacuum)

Manometer

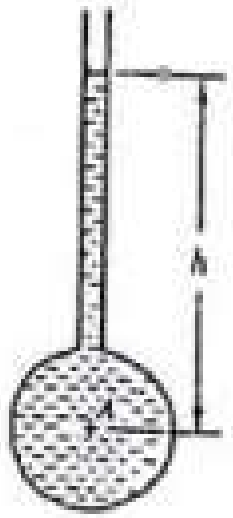
Manometers are devices that employ liquid columns to determine pressure or the difference in pressure.

Manometers are of two types:

Simple manometer, usually called a piezometer, measures the pressure in a pipe or vessel full of liquid

Differential manometer measures the difference of pressure between any two points on a pipeline or on two different pipes full of liquid

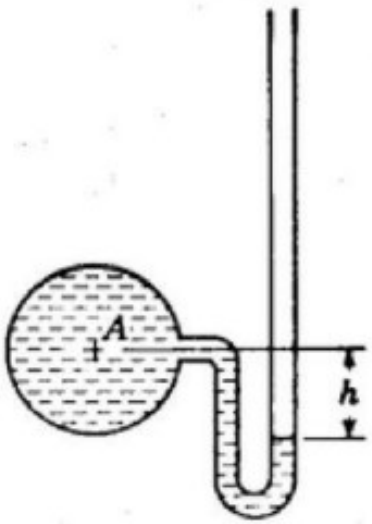
The most elementary manometer, called a piezometer, is a vertical glass tube one end of which is open to the atmosphere and the other end it connected to the side of the container containing the fluid whose pressure is to be determined.



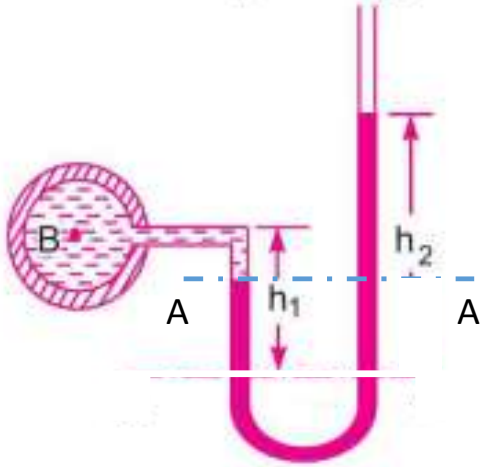
$$p_A = \gamma h$$

The diameter of the tube should not be less than 10 mm to avoid error due to capillary rise.

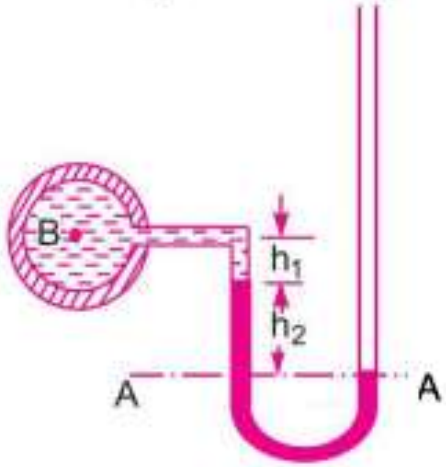
Piezometer does not work for negative gage pressure and also it is suitable measuring large pressure. For measurement of small negative or positive gage pressures, in a liquid a double column or U tube manometer may be conveniently used When the gage pressure is negative, the meniscus comes to rest below A as in figure.



2.6.2 U-tube Manometer. It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in Fig. 2.9. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.



(a) For gauge pressure



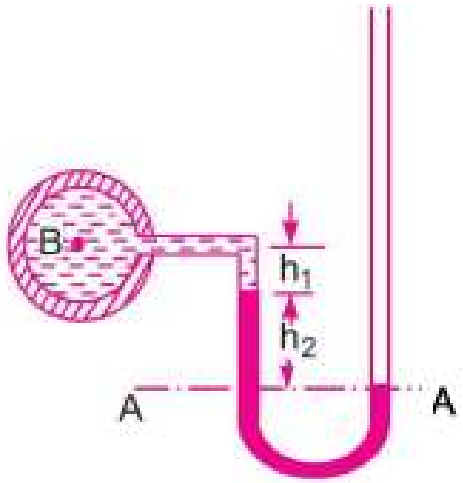
(b) For vacuum pressure

Pressure above A-A in the left column $= p + \rho_1 \times g \times h_1$

Pressure above A-A in the right column $= \rho_2 \times g \times h_2$

Hence equating the two pressures $p + \rho_1 g h_1 = \rho_2 g h_2$

$\therefore$ $p = (\rho_2 g h_2 - \rho_1 \times g \times h_1)$.



(b) For vacuum pressure

(*b*) **For Vacuum Pressure.** For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in Fig. 2.9 (*b*). Then

Pressure above *A-A* in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above *A-A* $= 0$

$\therefore \quad \rho_2 g h_2 + \rho_1 g h_1 + p = 0$

$\therefore \quad p = -(\rho_2 g h_2 + \rho_1 g h_1).$ ...(2.8)

Problem 2.9 *The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.*

Solution. Given :

Sp. gr. of fluid, $S_1 = 0.9$

$\therefore$ Density of fluid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Sp. gr. of mercury, $S_2 = 13.6$

$\therefore$ Density of mercury, $\rho_2 = 13.6 \times 1000 \text{ kg/m}^3$

Difference of mercury level, $h_2 = 20 \text{ cm} = 0.2 \text{ m}$

Height of fluid from A-A, $h_1 = 20 - 12 = 8 \text{ cm} = 0.08 \text{ m}$

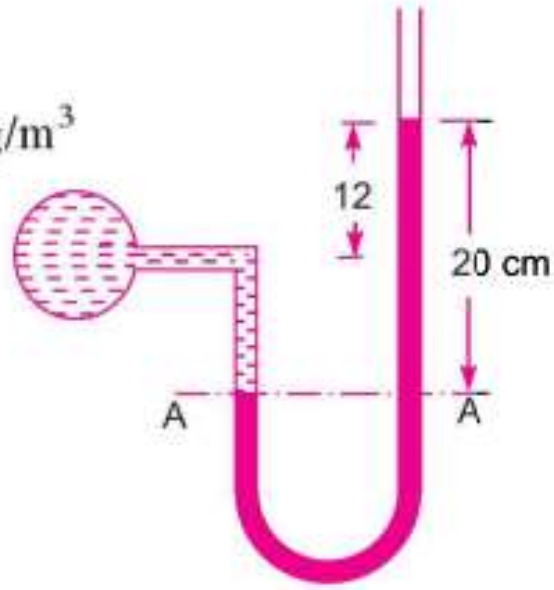


Fig. 2.10

Let p = Pressure of fluid in pipe

Equating the pressure above A-A, we get

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

or $$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times .2$$

$$p = 13.6 \times 1000 \times 9.81 \times .2 - 900 \times 9.81 \times 0.08$$

$$= 26683 - 706 = 25977 \text{ N/m}^2 = \mathbf{2.597 \text{ N/cm}^2}. \textbf{ Ans.}$$

Problem 2.10 *A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp. gr. 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the centre of pipe is 15 cm below.*

Solution. Given :

Sp. gr. of fluid, $S_1 = 0.8$

Sp. gr. of mercury, $S_2 = 13.6$

Density of fluid, $\rho_1 = 800$

Density of mercury, $\rho_2 = 13.6 \times 1000$

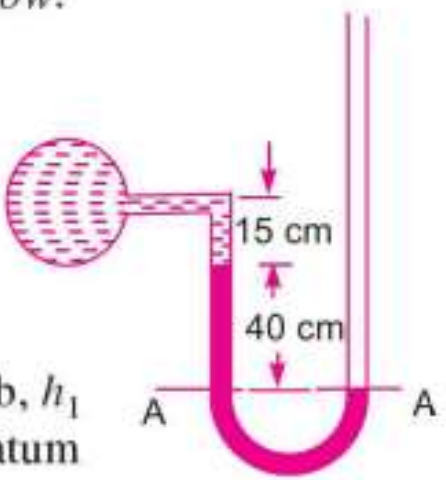


Fig. 2.11

Difference of mercury level, $h_2 = 40$ cm $= 0.4$ m. Height of liquid in left limb, $h_1 = 15$ cm $= 0.15$ m. Let the pressure in pipe $= p$. Equating pressure above datum line A-A, we get

$$\rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$p = -[\rho_2 g h_2 + \rho_1 g h_1]$$

$$= -[13.6 \times 1000 \times 9.81 \times 0.4 + 800 \times 9.81 \times 0.15]$$

$$= -[53366.4 + 1177.2] = -54543.6 \text{ N/m}^2 = \mathbf{-5.454 \text{ N/cm}^2. \text{ Ans.}}$$

Differential Manometer: A differential manometer (Fig. 2.5) is used to determine the difference of pressure at two points A and B when the actual pressure at any point in the system can not be determined. The two points A and B may be in the same pipeline or in two different pipes.

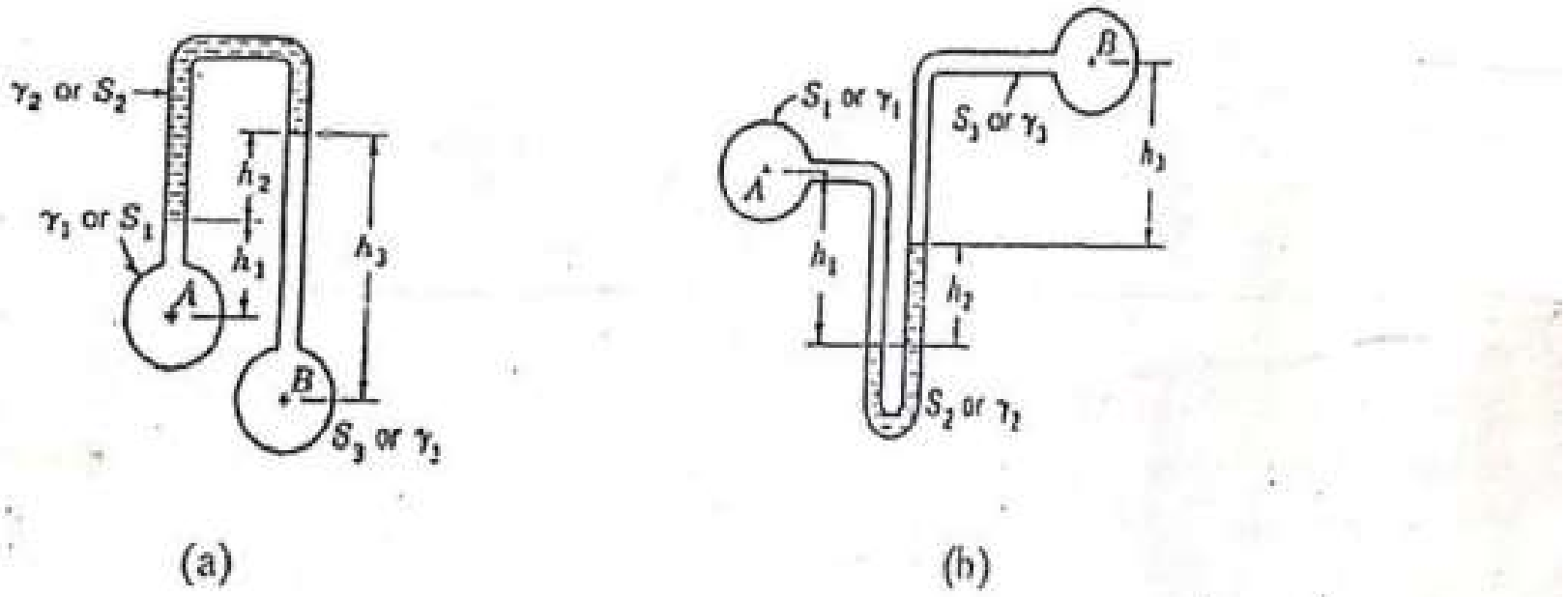


Fig. 2.5 Differential manometer

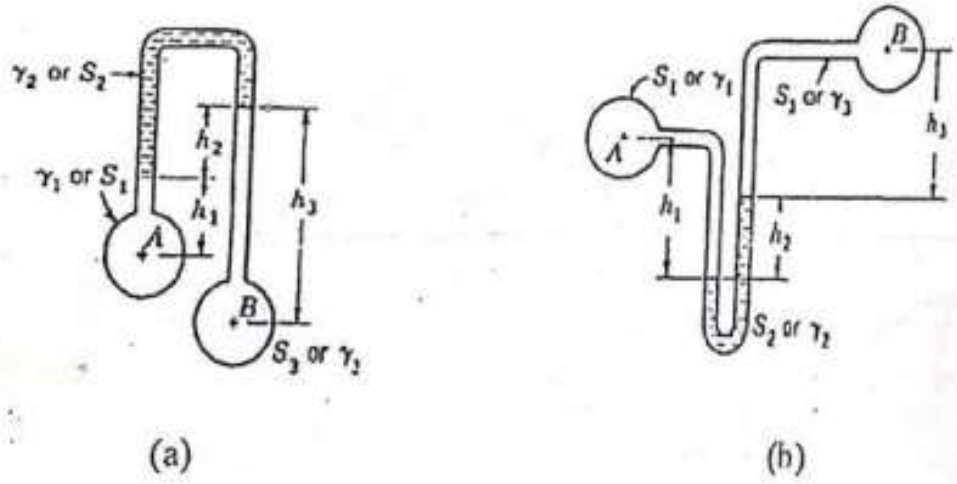


Fig. 2.5 Differential manometer

Application of the procedure outlined above to Fig. 2.5(a) produces

$$p_A - h_1\gamma_1 - h_2\gamma_2 + h_3\gamma_3 = p_B \tag{2.15}$$

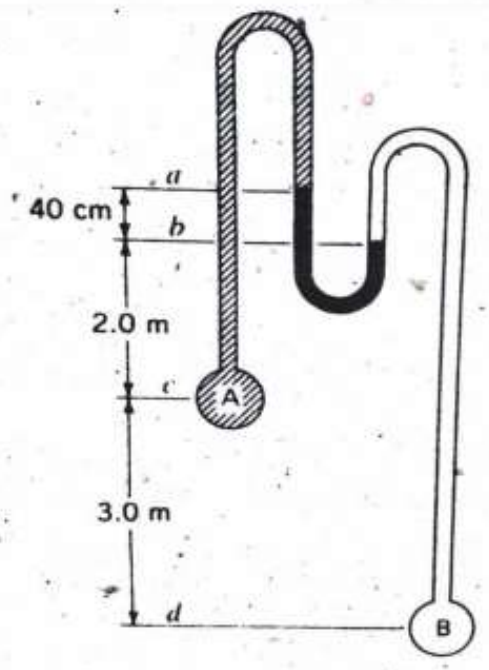
or

$$p_A - p_B = h_1\gamma_1 + h_2\gamma_2 - h_3\gamma_3 \tag{2.16}$$

Similarly, for Fig. 2.5(b),

$$p_A + h_1\gamma_1 - h_2\gamma_2 - h_3\gamma_3 = p_B \quad \text{or} \quad p_A - p_B = -h_1\gamma_1 + h_2\gamma_2 + h_3\gamma_3 \tag{2.17}$$

Math: Liquid A weighs $8.9\ kN/m^3$. Liquid B weighs $12.3\ kN/m^3$. Manometer liquid M is mercury. If the pressure at B is $200\ kN/m^2$, find the pressure at A.



Soln:

$$P_A - (\gamma_A h_A) + (\gamma_m h_B) + (\gamma_B h_c) = P_B$$

$$\Rightarrow P_A = P_B + (\gamma_A h_A) - (\gamma_m h_B) - (\gamma_B h_c)$$

$$= 200 + (8.9 \times 2.9) - (13.6 \times 9.81 \times 0.9) - (12.3 \times 5)$$

$$= 105.3\ kPa$$

Total Pressure and Centre of Pressure

- **Total Pressure**: It is defined as the force exerted by a static fluid on a surface (either plane or curved) when the fluid comes in contact with the surface. This force is always normal to the surface.
- **Center of Pressure:** The center of pressure is defined as the point of application of the resultant pressure on the surface.

2.6 HYDROSTATIC PRESSURE FORCE ON SUBMERSED PLANE SURFACES

Total Pressure Force on Submerged Plane Surfaces

Consider a plane surface MN acted upon by a liquid of unit weight γ. The surface makes an angle θ with the horizontal (Fig. 2.6). Consider the surface MN to be made up of an infinite number of horizontal strips each having an area dA and a width dy so small that the unit (or intensity of) pressure on a strip may be considered constant. The unit pressure on a strip at depth h below the free surface and at distance y from the line S-S is

$$p = \gamma h = \gamma y \sin\theta \qquad (2.20)$$

The total pressure force on the strip is

$$dP = \gamma y \sin\theta \, dA \qquad (2.21)$$

and the total pressure force on MN is

$$P = \gamma \sin\theta \int_0^A y \, dA \qquad (2.22)$$

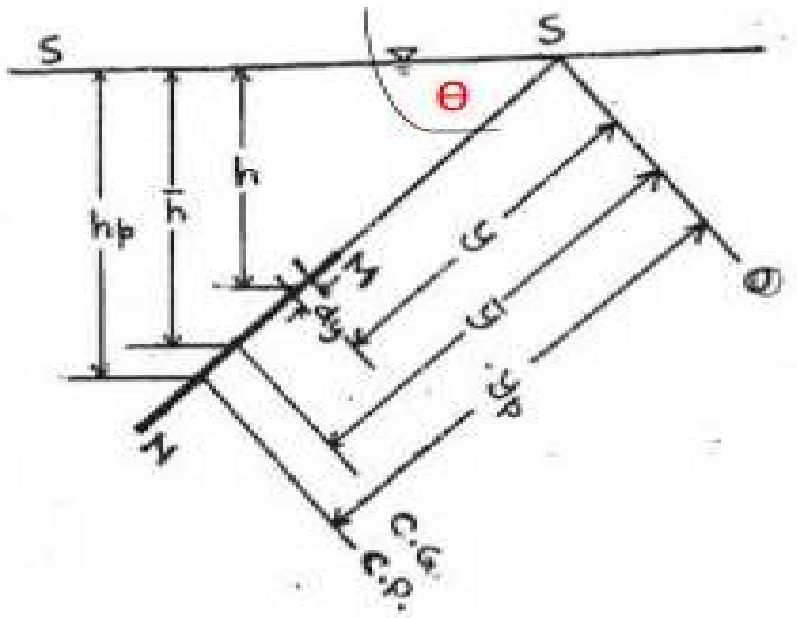


Fig. 2.6 Pressure on a plane surface

From the definition of center of gravity

$$\int_0^A y dA = A\bar{y} \tag{2.23}$$

where $\bar{y}$ is the distance of the center of gravity of A from the line S-S. Hence,

$$P = \gamma \sin\theta \, A \bar{y} \tag{2.24}$$

Since the vertical depth of the center of gravity below the surface is

$$\bar{h} = \bar{y} \sin\theta \tag{2.25}$$

it follows that

$$P = \gamma \bar{h} A \tag{2.26}$$

where $\gamma \bar{h}$ represents the unit (or intensity of) pressure at the center of gravity of A.

Thus, **the total hydrostatic pressure force on any plane surface is equal to the product of the area of the surface and the unit pressure at its center of gravity.**

Center of Pressure of Submerged Plane Surfaces

The intensity of pressure on an immerged surface is not uniform, but increases with depth. As the pressure is greater over the lower part of the surface, the resultant pressure force will act at some point which is below the center of gravity of the immerged surface and towards the lower edge of the surface. The point through which the resultant pressure force acts is known as the *center of pressure* and is always expressed in terms of depth from the free liquid surface.

The position of the center of pressure of a plane surface subjected to hydrostatic pressure may be determined by taking moments of all the forces acting on the surface about some horizontal axis in its plane. In Fig. 2.6, the line S-S may be taken as the axis of moments for the surface MN. Designating by y_p the distance to the center of pressure from the axis of moments, it follows from the definition of the center of pressure that

$$P.y_p = \int_0^A y dP \tag{2.29}$$

or

$$y_p = \frac{\int_0^A y dP}{P} \tag{2.30}$$

But, as in the previous article

$$dP = \gamma\, y \sin\theta\, dA \tag{2.31}$$

and

$$P = \gamma \sin\theta\, A\, \bar{y} \tag{2.32}$$

Substituting Eq.(2.32) in Eq.(2.30), we obtain

$$y_p = \frac{\gamma \sin\theta \int_0^A y^2 dA}{\gamma \sin\theta A \bar{y}} = \frac{\int_0^A y^2 dA}{A\bar{y}} = \frac{I_s}{S_s} \qquad (2.33)$$

where I_S is the moment of inertia or second moment of area of MN about the axis S-S from which y is measured and S_S is the static moment or first moment of area of MN about the same axis.

We know from the theory of parallel axis that

$$I_s = I_g + A\bar{y}^2 = I_g + \frac{A\bar{h}^2}{\sin^2\theta} \qquad (2.34)$$

where I_g is the moment of inertia about an axis passing through its center of gravity and parallel to OS (Fig. 2.6), $\bar{y}$ = distance between the liquid surface and the center of gravity of the area along MN and $\bar{h}$ is the vertical depth of the center of gravity of the area from the free liquid surface. Obviously,

$$\bar{y} = \frac{\bar{h}}{\sin\theta} \qquad (2.35)$$

If h_p is the vertical depth of the center of pressure of the area from the free liquid surface, then

$$h_p = y_p \sin\theta = \frac{I_s \sin\theta}{S_s} = \left(\frac{I_g + A\bar{y}^2}{A\bar{y}}\right)\sin\theta = \bar{y}\sin\theta + \frac{I_g}{A\bar{y}}\sin\theta \qquad (2.36)$$

Since $\bar{h} = \bar{y}\sin\theta$, we obtain

$$h_p = \bar{h} + \frac{I_g}{A\bar{h}}\sin^2\theta \qquad (2.37)$$

The above relationships for h_p do not contain any term for specific weight of liquid. Therefore, the center of pressure will remain the same for all liquids.

In Eq.(2.37), if $\theta = 0$, then it becomes the case of horizontal plan surface and the center of gravity and the center of pressure coincide.

For vertical plane surfaces, $\theta = 90^0$, and Eq.(2.37) gives

$$h_p = \bar{h} + \frac{I_g}{A\bar{h}} \qquad (2.38)$$

Curved Immersed Surface: The total pressure force on the curved surface, is given by

The direction of the resultant force on the curved surface with the horizontal is given by

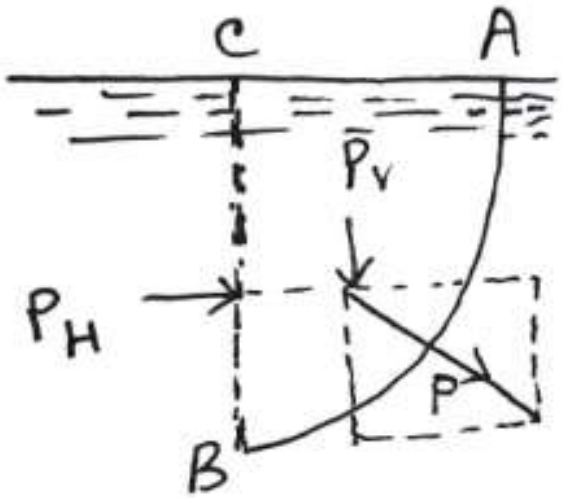
P_H = Horizontal force on the curved surface and is equal to the total pressure on the projected area of the curved surface on the vertical plane, and

P_V = Vertical force on the curved surface and is equal to the weight of the liquid supported by the curved surface upto the liquid surface.

$$P = \sqrt{P_H^2 + P_V^2}$$

The inclination of the resultant force with the horizontal α is given by

$$\alpha = \tan^{-1} \frac{P_V}{P_H}$$



2.7 FORCES ON SUBMERGED CURVED SURFACES

In case of a submerged plane surface, the forces acting perpendicularly on all the strips have the same directions and form a system of parallel forces. But in case of a submerged curved surface, all the strips do not lie in the same plane and the forces, though perpendicular to their respective strips, do not form a system of parallel forces. Due to this reason, the magnitude of the total force and its point of application (i.e. center of pressure) cannot be determined easily by the methods explained earlier. However, the same can be conveniently obtained by calculating the horizontal and vertical components of the resultant or total pressure force, which are then combined together to give the total force on the curved surface.

Consider a curved surface AB immersed in a liquid. Let BC be the vertical projection and AC be the horizontal projection of the curved surface (Fig. 2.9). The horizontal force P_H will be the total horizontal pressure force on the projection BC of the curved surface and will act through the center of pressure of the surface. The vertical force P_V will be the total weight of the liquid in the portion ABC and will act through the center of gravity of the volume ABC.

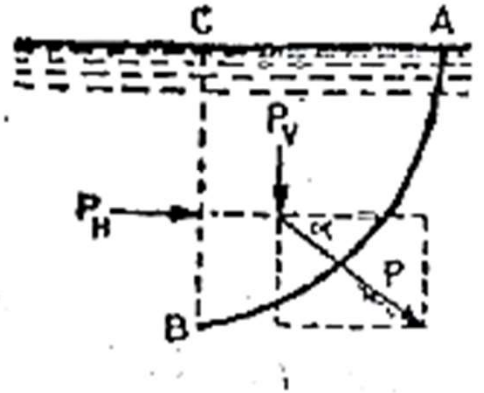
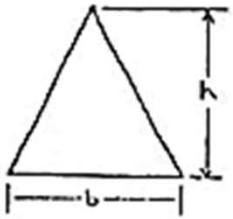
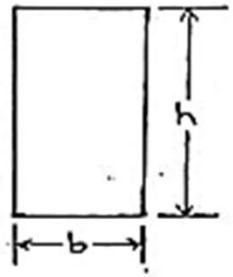
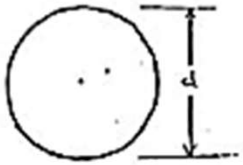


Fig. 2.9 Resultant pressure on a curved surface

Table 2.1 The center of gravity (C.G.) and the moment of inertia (I) of some surfaces

Sl. No.	Surface	C.G. from base	I_g	I from base
1.Triangle		$\frac{h}{3}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
2.Rectangle		$\frac{h}{2}$	$\frac{bh^3}{12}$	$\frac{bh^3}{3}$
3.Circle		$\frac{d}{2}$	$\frac{\pi d^4}{64}$	

Example 2.1

Determine the intensity of pressure of oil of specific gravity 0.75 at a depth of 4 m below the free surface.

Solution We have, specific weight of oil, $\gamma = 0.75 \times 1000 \times 9.81 = 7357.5\ N/m^3$, $h = 4\ m$

$\therefore$ Intensity of pressure, $p = \gamma h = 7357.5 \times 4 = 29430\ N/m^2$

Example 2.2

Determine the height of a water column equivalent to a pressure of 4 kN/m^2.

Solution We have, $p = 4\ kN/m^2 = 4000\ N/m^2$, $\gamma = 9810\ N/m^3$

$$\therefore h = \frac{p}{\gamma} = \frac{4000}{9810} = 0.41\ m$$

Example 2.7:

A rectangular tank having a base 5 m long and 2 m wide contains water up to a depth of 2.5 m. Calculate the total pressure force on the base of the tank.

Solution:

Area of the base of the tank, $A = 5 \times 2 = 10\ m^2$

Depth of the base from the water surface, $h_c = 2.5\ m$

Specific weight for water, $\gamma = 9.81\ kN/m^3$

Hence the total pressure force on the base

$F = h_c \gamma A = 9.81 \times 2.5 \times 10 = 245.25\ kN.$

Example 2.8:

A circular door of 1 m diameter closes an opening in the vertical side of a bulkhead, which retains sea water. The center of the opening is at a depth of 2 m from the water level. Determine the total pressure force on the door. Take the specific gravity of sea water is 1.03.

Solution:

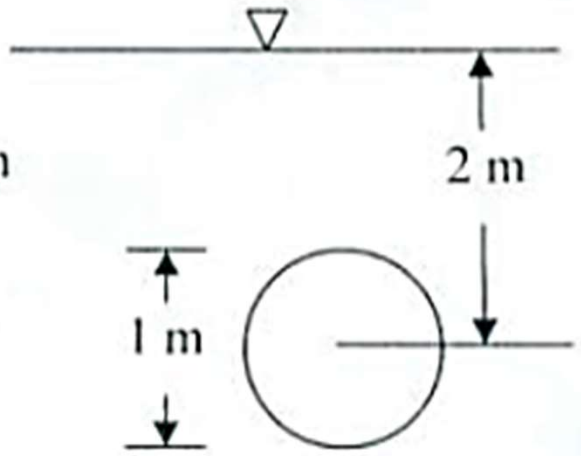
Area of the door, $A = \pi d^2/4 = \pi \times 1^2/4 = 0.785\ m^2$

Depth at the center of the door from the water surface, $h_c = 2\,m$

Specific weight for sea water, $\gamma = 9.81 \times 1.03 = 10.10\ kN/m^3$

Hence the total pressure force on the door

$P = h_c \gamma A = 10.10 \times 2 \times 0.785 = 15.865\ kN.$



Example 2.9:

A rectangular gate is 2 m wide and 1.5 m high is in a vertical plane. The water surface coincides with the top of the gate. (a) Find the force exerted by the water on the gate and the position of the center of pressure. (b) If the same area is immerged vertically downwards, the top side being at a depth of 3 m below the free surface find the force exerted by the water on the gate and the position of the center of pressure.

Solution:

(a) Total pressure force, $F = \gamma h_c A$

Here, $\gamma = 9.81\ \text{kN/m}^3$, $h_c = \frac{1.5}{2} = 0.75m$, $A = 2 \times 1.5 = 3\ \text{m}^2$

$\therefore F = 9.81 \times 0.75 \times 3 = 22.07\ kN$

Depth of center of pressure below the free water surface

$$h_p = h_c + \frac{I_c}{A h_c} = 0.75 + \frac{bh^3/12}{A h_c} = 0.75 + \frac{bh^3/12}{3 \times 0.75} = 0.75 + 0.25 = 1.0\ m$$

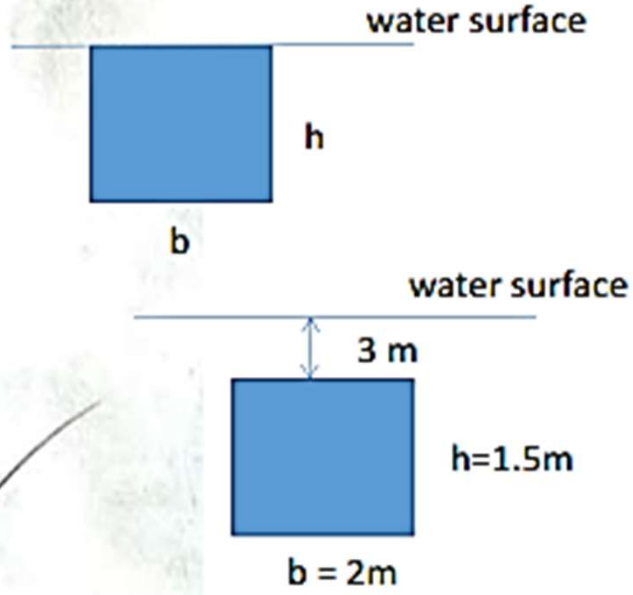
(b) Total pressure force

$F = \gamma h_c A = 9.81 \times (3 + 0.75) \times 3 = 110.36\ \text{kN}$

And the position of the center of pressure below the free water surface is

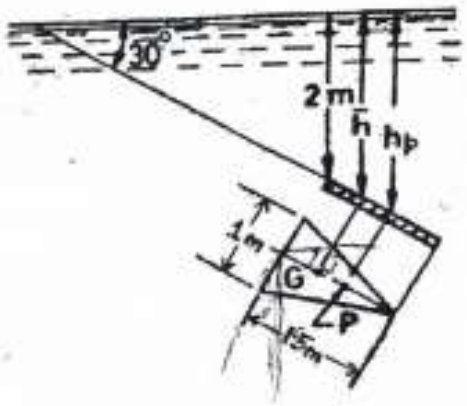
$$h_p = h_c + \frac{I_c}{A h_c} = 3.75 + \frac{bh^3/12}{A h_c} = 3.75 + \frac{2 \times 1.5^3}{12 \times 3 \times 3.75} = 3.75 + 0.05 = 3.80\ m$$

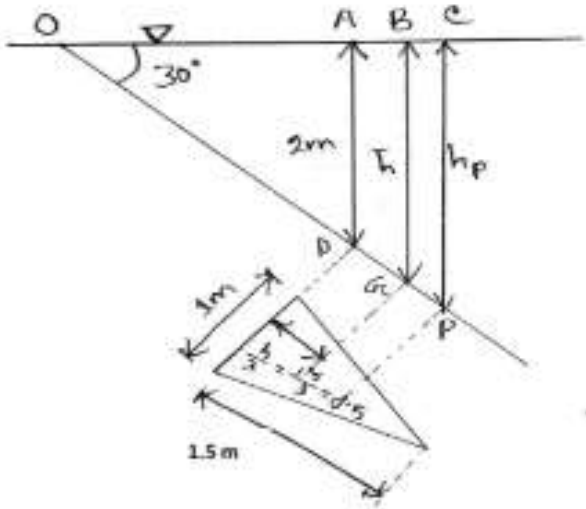
$$\frac{bh^3}{12} = \frac{2x(1.5)^3}{12}$$



Example 2.11

A triangular plate of 1. m base and 1.5 m altitude is immersed in water as shown in the figure. The plane of the plate is inclined at 30^0 with the free surface of water and the base is parallel to and at a depth of 2 m from the free surface. Find the total pressure force on the plate and the position of the center of pressure.





Solution Total pressure force, $P = \gamma \bar{h} A$

Here, $\gamma = 9.81\ \text{kN/m}^3$

$$\bar{h} = 2 + \frac{1.5}{3}\sin 30^0 = 2 + 0.5 \times \frac{1}{2} = 2.25\ m$$

$$A = \frac{1}{2} \times 1 \times 1.5 = 0.75\ m^2$$

$$\therefore P = \gamma \bar{h} A = 9.81 \times 2.25 \times 0.75 = 16.55\ kN$$

The position of the center of pressure below the free water surface is given by

$$h_p = \bar{h} + \frac{I_g}{A\bar{h}}\sin^2\theta = \bar{h} + \frac{bh^3/36}{A\bar{h}}\sin^2\theta = 2.25 + \frac{1 \times 1.5^3}{36 \times 0.75 \times 2.25} \times \sin^2 30^0$$

$$= 2.25 + 0.014 = 2.264\ m$$

from, $\triangle OAD$

$\sin 30^\circ = \frac{AD}{OD}$

$\Rightarrow OD = \frac{2}{\sin 30^\circ}$

$= 4$

$\therefore OD + DG = 4 + 0.5$

$= 4.5$

$= OG$

from, $\triangle OBG$,

$\sin 30^\circ = \frac{BG}{OG}$

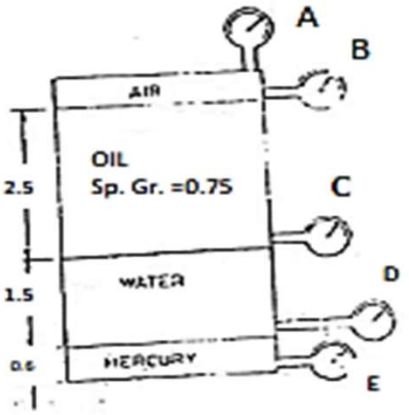
$\Rightarrow BG = OG \sin 30^\circ$

$= 4.5 \sin 30^\circ$

$= 2.25 = \bar{h}$

Example 2.6

In the figure shown, a closed tank contains 0.6 m of Hg, 1.5 m of water, 2.5 m of oil of specific gravity 0.75 and air space above the oil. If the pressure at the bottom of the tank is 275 kN/m^2 gauge, what would be the reading of the gauge A at the top of the tank?



Solution Pressure at E (at bottom of Hg)

$$= \frac{275}{9.81} = 28.03 \text{ m of water}$$

Pressure at D (at bottom of water)

= Pressure at E – pressure of 0.6 m of Hg

$= 28.03 - 0.6 \times 13.6 = 19.87$ m of water

Pressure at C (at bottom of oil)

= Pressure at D – pressure of 1.5 m of water

$= 19.87 - 1.5 = 18.37$ m of water

Pressure at B (at bottom of air)

= Pressure at C – pressure of 2.5 m of oil

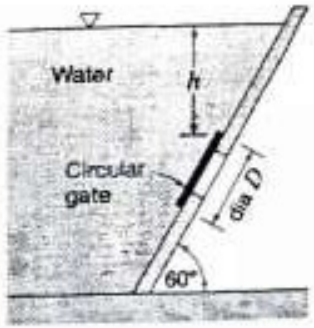
$= 18.37 - 0.75 \times 2.5 = 16.495$ m of water

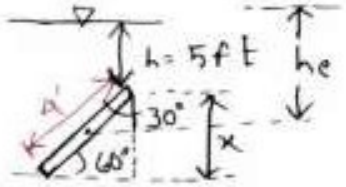
Now, pressure at A = Pressure at B ($\because$ the space is full of air)

$\therefore$ Reading of the pressure gage A at the top of the tank

$= 16.495$ m of water $= 16.495 \times 9.81 = 161.82$ kN/m^2

Ques:Find the magnitude and depth of the point of application of the force on the circular gate shown in fig if h= 5 ft and D = 4 ft dia





$\sin 60^\circ = \frac{x}{4}$

$\therefore x = 4\sin 60^\circ$

So, $\frac{x}{2} = 2\sin 60^\circ$

$\therefore h_e = 5 + 2\sin 60^\circ$ (from water surface)

h_c calculation

Solve:

$F = \gamma h_c A = 62.4 \times (5 + 2\sin 60^\circ) \times \pi (2)^2$

$= 5280\ lb$

$y_p = y_c + \frac{I_c}{y_c A}$

$= \left(\frac{5}{\cos 30^\circ} + 2\right) + \frac{\frac{\pi (4)^4}{64}}{\left(\frac{5}{\cos 30^\circ} + 2\right)(\pi \times 2^2)}$

$= 7.9\ ft$

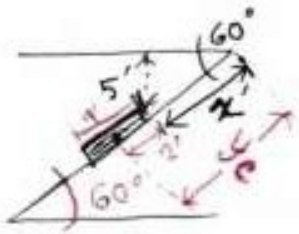
$\therefore h_p = y_p \cos 30^\circ$

$= 7.9 \times \cos 30^\circ$

$= 6.84\ ft$

$\sin 60^\circ = \frac{5}{x'}$

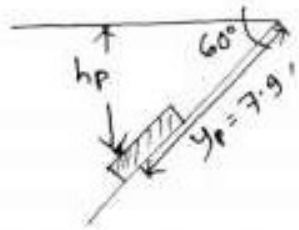
$\therefore x' = \frac{5}{\sin 60^\circ}$



So, $y_c = \frac{5}{\sin 60^\circ} + 2$

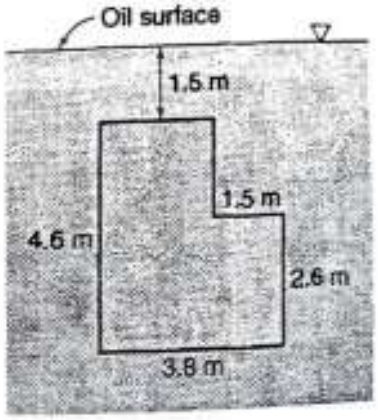
$= \frac{5}{\cos 30^\circ} + 2$

y_c calculation

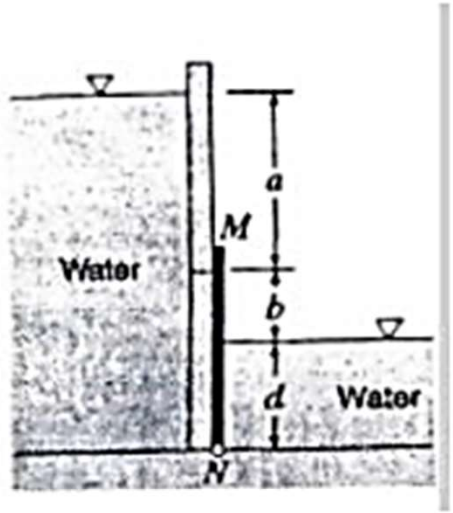


h_p (centre of pressure) calculation

Ques: The plate shown in figure is submerged in oil (s=0.94) and lies in a vertical plane. Find the magnitude and location of the hydrostatic force acting on one side of the plate.



Ques: The gate MN in fig rotates about an axis through N. If a= 3.3 ft, b = 1.3 ft, d=2 ft and the width perpendicular to the plane of the fig is 3 ft, what torque applied to the shaft through N is required to hold the gate closed?



Solve

For the left side,

$h_{cg_1} = 3.3 + \frac{1.3+2}{2} = 4.95\ ft$

$A_1 = 3 \times (2+1.3) = 9.9\ ft^2$

$\gamma = 62.4\ lb/ft^3$

$F_1 = \gamma h_{cg_1} A_1 = (62.4 \times 4.95 \times 9.9)$

$= 3058\ lb$

For the right side,

$h_{cg_2} = \frac{2}{2} = 1\ ft$

$A_2 = (3 \times 2) = 6\ ft^2$

$\gamma = 62.4\ lb/ft^3$

$F_2 = \gamma h_{cg_2} A_2 = (62.4 \times 1 \times 6) = 374.4\ lb$

$h_{cp_1} = h_{cg_1} + \frac{I_{cg_1}}{A_1 h_{cg_1}}$

$= 4.95 + \frac{\frac{1}{12} \times 3 \times (3.3)^3}{9.9 \times 4.95}$

$= 5.1333\ ft$

for left side,

$h_{cg_1} = 4.95'$

$l_1 = (3.3 + 1.3 + 2) - 5.1333 = 1.467$ ft

for right side,

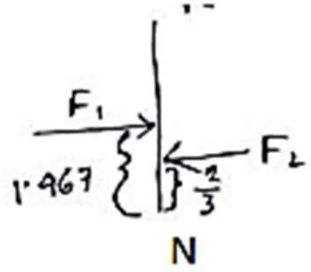
$h_{cg_1} = 1$ ft

$\therefore h_{cp_2} = h_{cg_1} + \dfrac{I_{cg_2}}{A_2 h_{cg_2}}$

$= 1 + \dfrac{\frac{1}{12} \times 3 \times 2^3}{6 \times 1} = \dfrac{4}{3}$ ft

$\therefore l_2 = \left(2 - \dfrac{4}{3}\right) = \dfrac{2}{3}$ ft

Now,

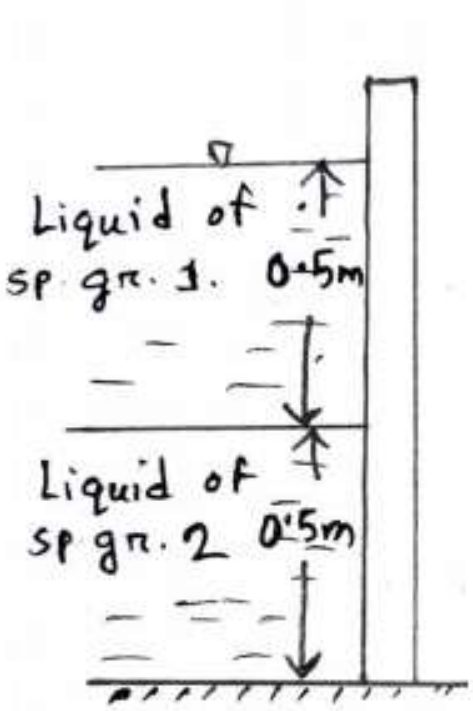


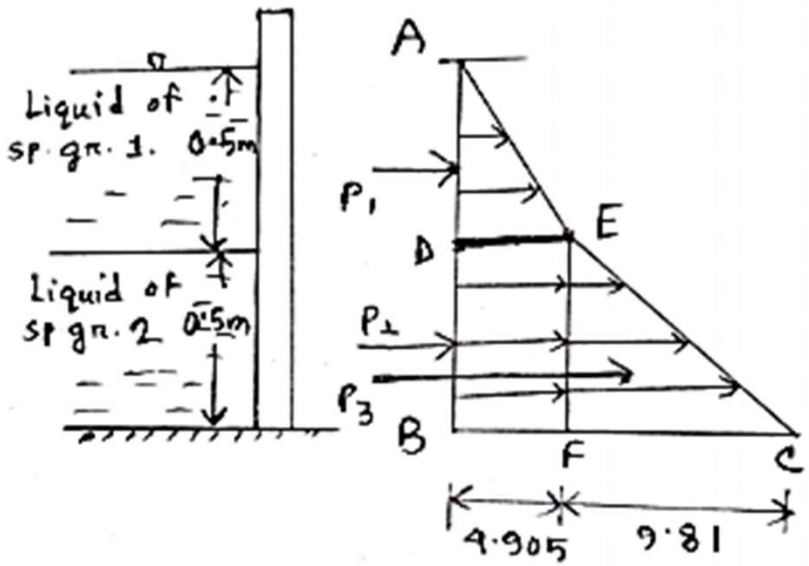
$\sum M_N^{\rightarrow}$ -ve=0,

$(3058 \times 1.467) - (374.4 \times \frac{2}{3}) - T = 0$

$\therefore T = 4236.5$ lb-ft

Ques: Find the magnitude and line of action of pressure force exerted on the side of a tank, which is 1.5 m square and 1 meter deep. The tank is filled half full with a liquid having specific gravity of 2, while the remainder is filled with a liquid having specific gravity 1.





Intensity of pressure at D (or B) due to liquid of specific gravity 1,

$$DE = BF = \gamma_1 H_1 = (1 \times 9.81) \times 0.5 = 4.905 \text{ KN/m}^2$$

Total pressure force,

P_1 = area of triangle ADE X length of tank wall

= (1/2 X 4.905 X 0.5) x 1.5 =1.84 KN

Total pressure force,

P_2 = area of rectangle BDFE X length of tank wall

= (4.905 x 0.5) x 1.5 =3.68 KN

Similarly, intensity of pressure at B due to liquid of sp gr. 2,

$$FC = \gamma_2 H_2 = (2 \times 9.81) \times 0.5 = 9.81 \text{ KN/m}^2$$

Total pressure force ,

P_3 = area of triangle EFC X length of tank wall

=(1/2 x 9.81 x 0.5) x 1.5 = 3.68 KN

Magnitude of the pressure force exerted on one side of tank

$P = P_1 + P_2 + P_3$

= (1.84 + 3.68 + 3.68)

=9.2 KN

Now,

Taking moments of all the pressures about A and equating the same,

$$P \times \bar{h} = \left[P_1 \times \frac{2 \times 0.5}{3}\right] + \left[P_2 \times \left(0.5 + \frac{0.5}{2}\right)\right] + \left[P_3 \times \left(0.5 + \frac{2 \times 0.5}{3}\right)\right]$$

$$\Rightarrow 9.2 \times \bar{h} = \left[1.84 \times \frac{1}{3}\right] + \left[3.68 \times \frac{3}{4}\right] + \left[3.68 \times \frac{5}{6}\right]$$

$$= 0.613 + 2.76 + 3.067$$

$$= 6.44$$

$$\therefore \bar{h} = \frac{6.44}{9.2} = 0.7 \text{ m}$$ ($\bar{h}$ is from point A)

Example 2.10:

Find the total pressure force acting on the gate per m length, which is quadrant of a circle of radius 2 m. At what angle will it be acting to the horizontal? Prove that resultant force passes through the hinge C.

Solution:

$$P_H = \gamma h_c A = (9.81 \times \frac{2}{2} \times 2 \times 1) = 19.62\ kN$$

$$h_p = h_c + \frac{I_c}{Ah_c} = h_c + \frac{bh^3/12}{Ah_c}$$

$$= 1 + \frac{\frac{1}{12} \times 1 \times 2^3}{(2 \times 1) \times 1}$$

$$= 1.33\ m \text{ (from the surface)}$$

$$P_V = \gamma V = 9.81 \times \left(\frac{\pi \times 2^2}{4} \times 1\right) = 30.82\ kN$$

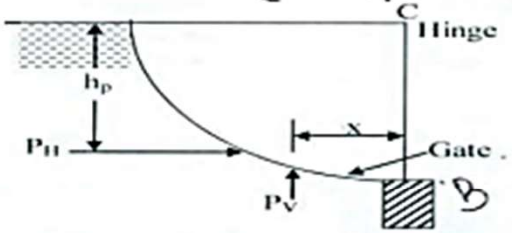
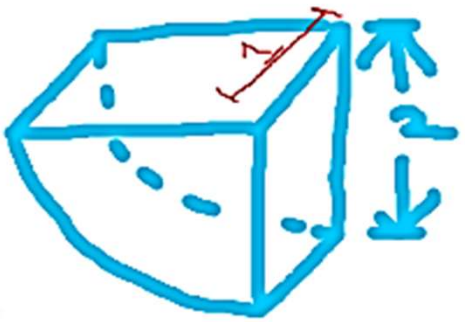


Fig. 2.11: Resultant Force on a curved surface



$$x = \frac{4R}{3\pi} = \frac{4 \times 2}{3\pi} = 0.849\ m$$

$$P = \sqrt{P_H^2 + P_V^2} = \sqrt{(19.62)^2 + (30.82)^2}$$

$$= 36.54\ kN$$

$$\alpha = \tan^{-1}\left(\frac{P_V}{P_H}\right) = \tan^{-1}\left(\frac{30.82}{19.62}\right)$$

$$= 57.5^\circ \text{ (with the horizontal)}$$

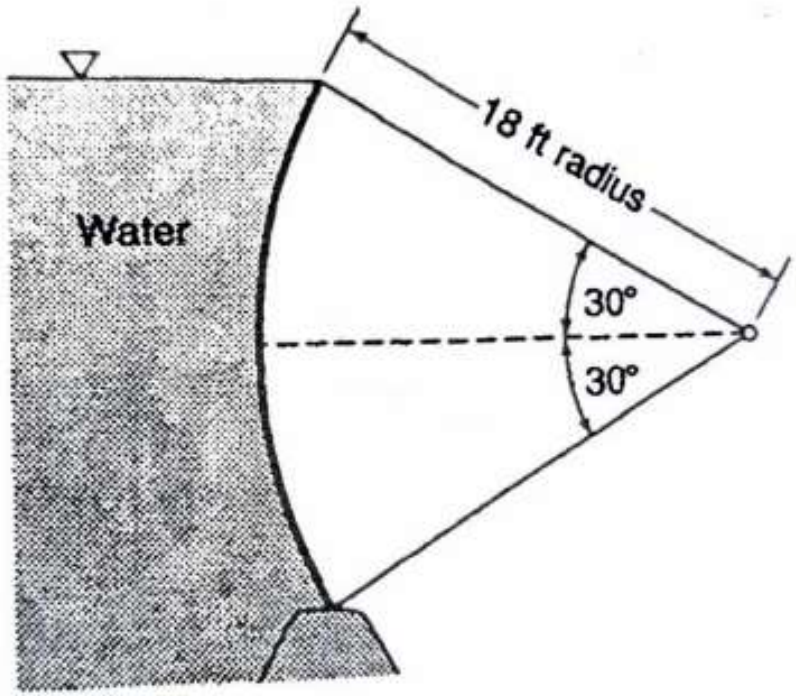
The sum of moments of the forces P_H and P_V about C is,

$$(P_H \times h_p) - (P_V \times x)$$

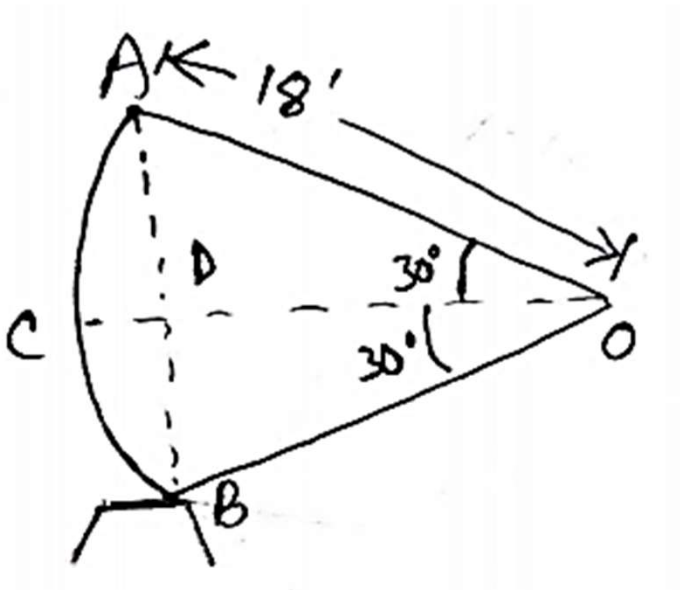
$$= (19.62 \times 1.33) - (30.82 \times 0.849)$$

$$= 0$$

Ques: Find the horizontal and vertical forces per foot of width acting on the tainter gate shown in figure



solve



$\sin 30^\circ = \frac{AD}{AO}$

$\Rightarrow AD = \sin 30^\circ \times AO = \frac{1}{2} \times 18 = 9'$

$\therefore AB = 2 \times AD = (2 \times 9) = 18'$

$\text{area} = AB \times 1 = (18 \times 1) = 18\ ft^2$

$h_c = \frac{AB}{2} = \frac{18}{2} = 9'$

$F_H = \gamma h_c A = (62.4) \times 9 \times 18 = 10108.8\ lb/ft$

$\cos 30^\circ = \frac{OD}{AO}$

$\Rightarrow OD = AO \cos 30^\circ$

$= 18 \times \frac{\sqrt{3}}{2} = 9\sqrt{3}$

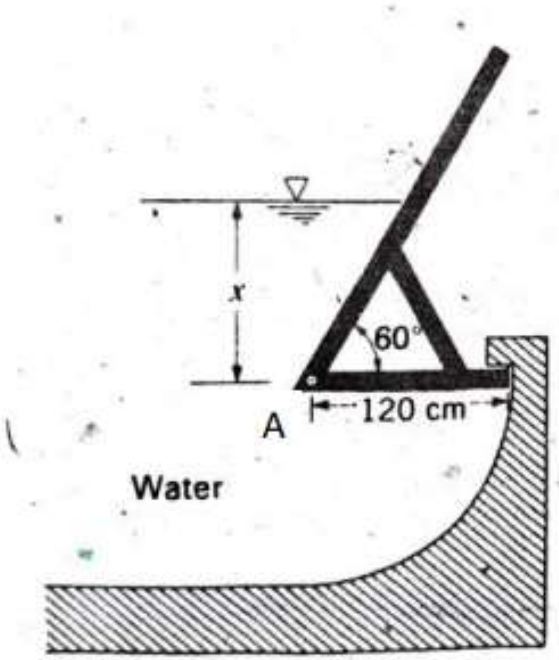
Volm (ADBC) = volume ACBO - Volume ABO

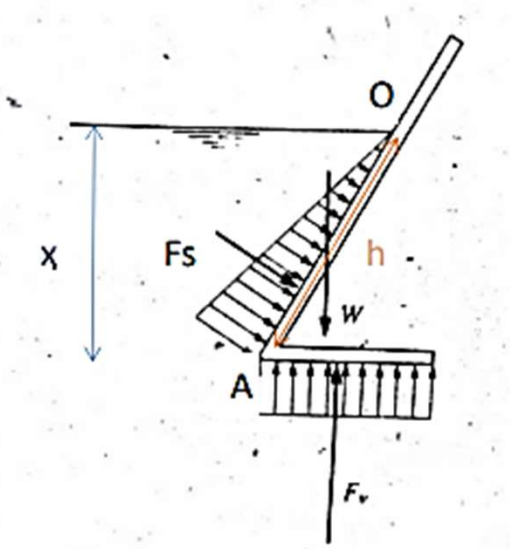
$= \left[\frac{\pi (18)^2}{6} \times 1 - \left(\frac{1}{2} \times 18 \times 9\sqrt{3} \times 1 \right) \right]$

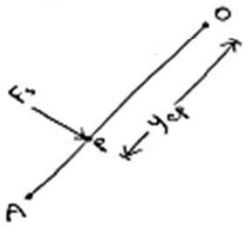
$= 29.35\ ft^3$

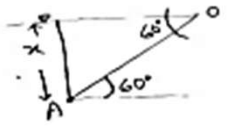
$F_V = \gamma V = (62.4 \times 29.35) = 1831.46\ lb/ft$

Ques: This gate is 60 cm wide perpendicular to the sketch. It is pivoted at A. The gate weighs 2224 N. Its center of gravity is 36 cm to the right of and 27 cm above A. For what values of water depth x above A will the gate remain closed? Neglect friction at the pivot and neglect thickness of the gate.









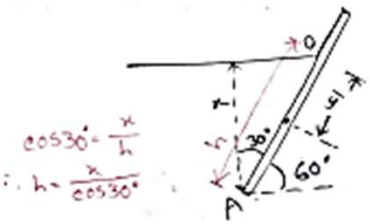
$\sin 60° = \frac{x}{OA}$

$\Rightarrow OA = \frac{x}{\sin 60°} = \frac{x}{\cos 30°}$

$F_v = \gamma h_c A = \gamma \times x \times (0.6 \times 1.2) = 0.72\gamma x$

$F_s = \gamma h_c A = \gamma \times \left(\frac{x}{2}\right) \times \left(\frac{x}{\cos 30°} \times 0.6\right)$

$= 0.465\gamma x^2$



$\cos 30° = \frac{x}{h}$

$\therefore h = \frac{x}{\cos 30°}$

$\bar{y} = \frac{h}{2}$

$= 0.5h = 0.5 \times \frac{x}{\cos 30°}$

$y_{cp} = \bar{y} + \frac{I_{cg}}{A\bar{y}}$

$= 0.5 \frac{x}{\cos 30°} + \frac{\frac{1}{12} \times 0.6 \times \left(\frac{x}{\cos 30°}\right)^3}{\left(0.5 \times \frac{x}{\cos 30°}\right) \times \left(\frac{x}{\cos 30°} \times 0.6\right)}$

$= 0.770x$

$PA = AO - 0.770x$

$= \left(\frac{x}{\cos 30°} - 0.770x\right) = 0.385x$

9.81 kN/m^3

$\Sigma M_{about\ A} = 0,$

$F_s(0.385x) - (F_v \times 0.6) + (w \times 0.36) = 0$

$\Rightarrow 0.465\gamma x^2(0.385x) - 0.72\gamma x \times 0.6 + (2.224 \times 0.36) = 0$

$\Rightarrow 1.7592x^3 - 4.237x + 0.80064 = 0$

By solving, $x = -1.639\ m, 1.45\ m, 0.192 m$.

So, $0.192 < x < 1.45\ m$.

The gate will remain closed when 0.192 m < x < 1.45 m