Military Institute of Science and Technology

EWCE 261

Fluid Mechanics

3 Credit Hour

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Lecturer

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Syllabus:

1. Development and Scope of Fluid Mechanics
2. Fluid Properties
3. Fluid Statics
4. Similitude and Dimensional Analysis
5. Steady Incompressible Flow in Pressure Conduits: Laminar Flow and Turbulent Flow
6. General Equation for Fluid Friction
7. Empirical Equation for Pipe Flow
8. Minor Losses in Pipe Flow
9. Fluid Measurement: pitot tube, orifice, mouthpiece, nozzle, venturimeter, weir.
10. Pipe Flow Problems: Pipes in Series and Parallel, Branching Pipes, Pipe Networks

Syllabus: (To be Covered in Part A)

1. Kinematics of Fluid Flow
2. Fluid Flow Concepts and Basic Equations: Continuity Equation, Bernoulli's Equation, Energy Equation
3. Momentum Equation and Forces in Fluid Flow

Steady Incompressible Flow in Pressure Conduits

(Franzini, Chapter 8)

Flow Through Pipes

Turbulent Flow: Flow undergoes irregular fluctuations.

Reynolds Number:

- $\mathrm{Re} = \frac{Vd}{\upsilon}$
- Re < 2000 Laminar
- Re > 4000 Turbulant
- 2000 < Re < 4000 Transitional

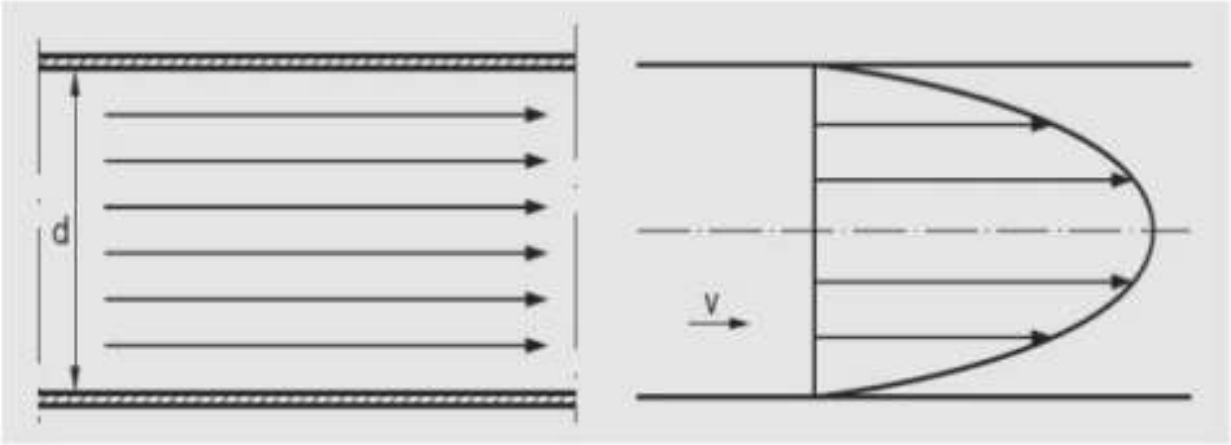


Fig. 1: Laminar Flow Velocity Profile

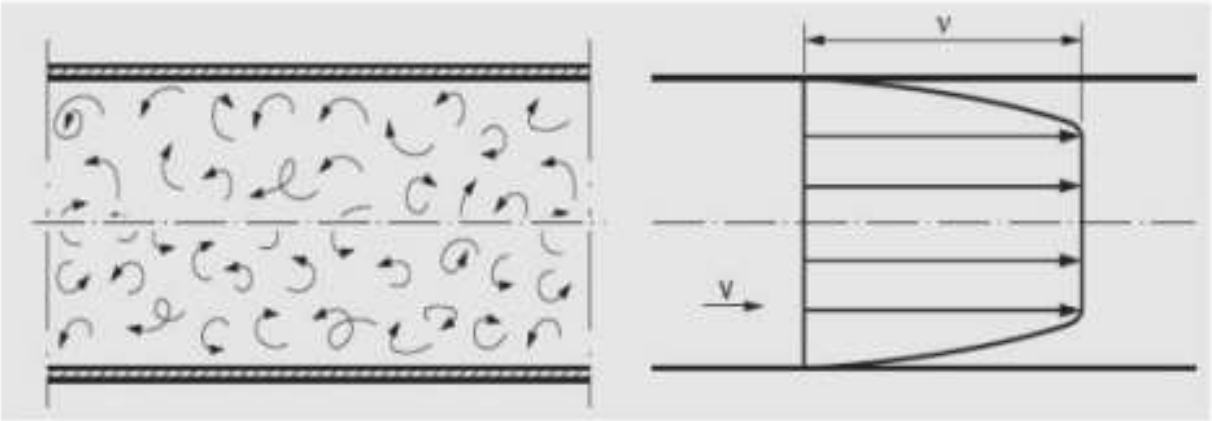


Fig. 2: Turbulent Flow Velocity Profile

Head loss is energy loss in fluid expressed in m – kg/kg or m- N/N. Loss of head occurs in all types of low in pipe. The head loss is caused by-

- Pipe friction.
- Change in velocity or direction in flow.
- Change in cross section of pipe.

Head loss due to pipe friction is termed as major loss.

Minor losses can be-

- Due to contraction or expansion of cross section.
- Due to obstructions like gates or valves etc.
- Due to bends or curves in pipes.

$H = h_f + h_c + h_e + h_g + h_b$

Frictional Loss With Laminar Flow: HAZEN- POISEUILLE EQUATION

Figure 8.4 represents a longitudinal section and a cross-section of a straight horizontal circular pipe of constant diameter d in which a fluid of uniform specific weight γ is moving from left to right with steady laminar motion. Consider a circular cylinder of fluid, abcd, of length L extending from section 1 where the pressure is $p_1 = \gamma h_1$ to section 2 where the pressure has decreased to $p_2 = \gamma h_2$. The difference in total pressure force on the two ends of the cylinder is thus

(8.1)

Total pressure force = pressure × area

$$= \gamma(h_1 - h_2)\pi y^2$$

$$= \gamma h_f \pi y^2 \quad \text{-----} (8.3)$$

It is considered that the cylinder is in equilibrium between this pressure difference and the shear resistance exerted by the surrounding fluid on the curved surface of the cylinder.

From the definition of viscosity, the unit shear stress on this surface is

$$\tau = \mu\left(\frac{-dv}{dy}\right)$$

since for each increment dy in distance from the pipe axis, there is a decrease dv in velocity. The total shear stress on the surface of the cylinder is thus

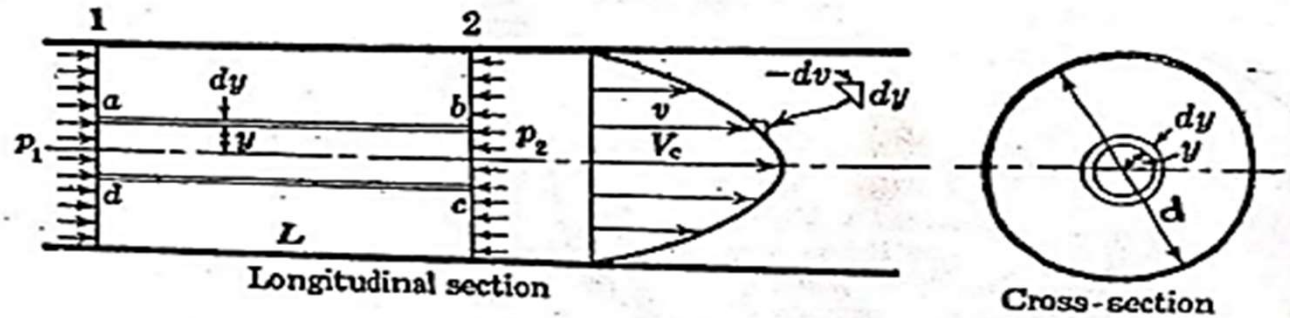


Fig. 8.4 Laminar flow in a pipe

Total shear force $= -2\pi y L \cdot \mu \left(\frac{dv}{dy}\right)$ ----(8·4)

Now, equating eqn (8·3) and (8·4)

$$\gamma h_f \pi y^2 = -2\pi y L \mu \frac{dv}{dy}$$

$$\Rightarrow dv = \frac{-\gamma h_f \pi y^2 \cdot dy}{2\pi y L \mu} \quad \therefore dv = \frac{-\gamma h_f y\, dy}{2L\mu} \quad ---(8·5)$$

Now, Integrating on both sides,

$$\int dv = \int \frac{-\gamma h_f y\, dy}{2L\mu}$$

$$\Rightarrow v = -\frac{\gamma h_f y^2}{2L\mu \cdot 2} + c_1$$

$$\therefore v = \frac{-\gamma h_f y^2}{4L\mu} + c_1 \quad ------(8·6)$$

when, $y = \frac{d}{2}$, $v = 0$, Therefore,

$$0 = -\frac{\gamma h_f \left(\frac{d}{2}\right)^2}{4L\mu} + C_1$$

$$\therefore C_1 = \frac{\gamma h_f d^2}{16 L \mu}$$

Substituting C_1 in eqn (8.6)

$$v = \frac{-\gamma h_f y^2}{4L\mu} + \frac{\gamma h_f d^2}{16 L \mu}$$

$$= \frac{\gamma h_f}{4L\mu}\left(\frac{d^2}{4} - y^2\right) \quad \text{---------- (8.7)}$$

This equation gives the velocity v at any distance y from the pipe axis. The discharge through the ring of width dy is

$$dQ = v \times 2\pi y dy$$

substituting the value of v from eqn (8.7),

$$dQ = \frac{\gamma h_f}{4L\mu}\left(\frac{d^2}{4} - y^2\right) \cdot 2\pi y \, dy$$

$$= 2\pi \frac{\gamma h_f}{4L\mu}\left(\frac{d^2}{4} y - y^3\right) dy$$

Now, integrating bet^{w} limits $y = 0$ & $y = \frac{d}{2}$,

$$\int dQ = 2\pi \frac{8\mu f}{4L\mu} \int \left(\frac{d^2}{4} y - y^3\right) dy$$

$$\Rightarrow \int dQ = 2\pi \frac{8\mu f}{4L\mu} \left[\int_0^{d/2} \frac{d^2}{4} y\, dy - \int_0^{d/2} y^3 dy\right]$$

$$\Rightarrow Q = 2\pi \frac{8\mu f}{4L\mu} \left[\frac{d^2}{4}\left(\frac{y^2}{2}\right)_0^{d/2} - \left(\frac{y^4}{4}\right)_0^{d/2}\right]$$

$$= 2\pi \frac{8\mu f}{4L\mu} \left[\frac{d^2}{4} \cdot \frac{d^2}{4\times 2} - \frac{d^4}{16\times 4}\right]$$

$$= 2\pi \frac{8\mu f}{4L\mu} \left[\frac{d^4}{32} - \frac{d^4}{64}\right]$$

$$= 2\pi \frac{8\mu f}{4L\mu} \left[\frac{2d^4 - d^4}{64}\right]$$

$$= 2\pi \frac{8\mu f}{4L\mu} \times \frac{d^4}{64}$$

$$\therefore Q = \frac{8\pi \rho f \cdot d^4}{128 L \mu}$$

$$\therefore h_f = \frac{128 L \mu Q}{\pi \rho g d^4} \quad \text{-----} \quad (8.10)$$

Now, $Q = AV = \frac{1}{4}\pi d^2 . V$

$$h_f = \frac{128 L \mu Q}{\pi \rho g d^4} = \left(\frac{128 L \mu}{\pi \rho g d^4}\right)\left(\frac{1}{4}\pi d^2 . V\right)$$

$$\Rightarrow h_f = \frac{32 L \mu . V}{\rho g d^2}$$

$$\therefore h_f = \frac{32 L \mu . V}{g d^2} \quad \text{---- (8.11)}$$

Equation (8.11) was first determined experimentally by Hagen in 1839 and simultaneously by Poiseuille in 1840 and it is usually known as the Hazen-Poiseuille equation. It indicates that in laminar flow the loss of head is proportional to the first power of the mean velocity. It was experimentally determined that Eq. (8.11) gives correct result. It is mostly used for the experimental determination of fluid viscosity by measuring the loss of head in a pipe of length L. Equation (8.11) can be written for the fluid viscosity as

Now,
$$\mu = \frac{\gamma h_f d^2}{32 L V} = \frac{\gamma (h_1 - h_2) d^2}{32 L V} = \frac{(P_1 - P_2) d^2}{32 L V} \quad \text{---- (8.12)}$$

Equation (8.7) shows that the velocity distribution along the diameter is parabolic, the maximum velocity being at the center of the pipe (y = 0) and having the value

$$V_c = \frac{h_f g d^2}{16 L \nu} \quad \cdots (8.13)$$

$$\left[\quad V = \frac{\gamma h_f}{4 L \mu} \left(\frac{d^2}{4} - y^2 \right) \right.$$

$$\Rightarrow V = \frac{\gamma h_f}{4 L \mu} \left(\frac{d^2}{4} - 0 \right)$$

$$= \frac{\gamma h_f d^2}{16 L \mu}$$

$$= \frac{\rho g h_f d^2}{16 L \mu}$$

$$\left. = \frac{h_f g d^2}{16 L \nu} \right]$$

$\nu = \frac{\mu}{\rho}$

Also, rearranging Eq (8.11)

$$V = \frac{h_f g d^2}{32 L \nu} \quad ---- (8.14)$$

from (8.13) & (8.14),

$$V_c = 2V$$

Frictional Loss With Turbulent Flow: Darcy-Weisbach Formula

Consider a straight pipe of internal diameter d in which a fluid flowing at a mean velocity V. Let the loss of head in length L be denoted by h_f. Certain general laws based upon observation and experiment appear to govern fluid friction in pipes and are expressed in all the generally accepted pipe formulas.

These laws briefly stated are:

- Frictional loss in turbulent flow generally increases with the roughness k of the pipe. When the flow is laminar the frictional loss is independent of the roughness.
- Frictional loss is directly proportional t the area of the wetted surface or to πdL.
- Frictional loss varies inversely as some power of the pipe diameter, or as $\frac{1}{d^x}$.
- Frictional loss varies as some power of the velocity, or as V^n.
- Frictional loss varies as some power of the ratio of viscosity to density of the fluid, or as $(\frac{\mu}{\rho})^r$.

Combining these factors, a rational equation for loss of head due to pipe friction for any fluid can be written in the form of:

$$h_f = K' \times k \times \pi\, d\, L \times \frac{1}{d^3} \times V^n \times \left(\frac{\mu}{\rho}\right)^r$$

$$= \left[K' k \pi \left(\frac{\mu}{\rho}\right)^r \right] \times \frac{L}{d^m} \times V^n$$

Where, K' is a constant of proportionality and $m = x - 1$.

The effect of viscosity and density of water on loss of head at usual flow velocities is so small that it can be neglected. If there is any little effect, it could be easily included in a general coefficient.

So, this equation becomes $h_f = K\frac{L}{d^m}V^n$.

A determination of K, m and n is necessary for practical application for using this equation. After some modification done by different scientists the equation becomes:

$$h_f = (K \times 2g) \times \frac{L}{d} \times \frac{V^2}{2g}$$

By substituting the friction factor f for (K×2g) we obtain

$$h_f = f\frac{LV^2}{d2g}$$

$$h_f = k \frac{L}{d^m} V^n$$

$$h_f = f \frac{L}{d} \frac{V^2}{2g}$$

$$h_f = 4f' \frac{L}{d} \frac{V^2}{2g} \longrightarrow \text{Fanning Equation}$$

Limitations of Darcy-Weisbach Formula

1. The loss of head with turbulent flow varies not only as the square of the mean velocity, but as some power varying from 1.7 to 2 or more depending on the roughness of the pipe. This discrepancy must be taken care of by varying the value of f.

2. Since $V = Q/A = Q/(\pi d^2/4)$, for a given Q, f and L, the loss of head by the Darcy-Weisbach formula varies inversely as the fifth power of the diameter. Tests have shown that the actual variation is closer to the 5.25 power and that the exponent of d in the formula should be close to 1.25. Again the discrepancy is taken care of by varying the value of f.

The friction factor must therefore be a function of velocity and diameter as well as of the pipe roughness and of the viscosity and density of the fluid.

Remarks

1. For a given velocity of flow, the value of f decreases as the diameter of pipe increases. This decrease of f is due to the decrease in relative roughness k/d of the material in the pipe wall.

2. Some kinds of pipe become rougher with age with resulting increase in f. This possibility is usually taken care in design by increasing the value of f for new pipe by a certain percentage. The increase in f for cast iron or steel pipe may be 50% to 100% after some years of service. However, wood pipe and asbestos cement pipe have shown little or no increase in f after many years of service.

Determination of Friction Factor, f

Laminar Flow

When the flow is laminar, the Hazen-Poiseuille equation, Eq. (8.11), applies. Equation (8.11) can be put in the Darcy-Weisbach form by multiplying numerator and denominator by 2V and replacing Vd/v by the Reynolds number Re. Thus.

$$h_f = \frac{64}{\mathrm{Re}} \frac{L}{d} \frac{V^2}{2g}$$

$$f \frac{L}{D} \frac{V^2}{2g} = 32\mu \frac{LV}{gD^2}$$

$$\Rightarrow f = \frac{32\mu LV \times D \times 2g}{gD^2 \times LV^2} \qquad (8.22)$$

$$= \frac{64\mu}{DV} = \frac{64}{\mathrm{Re}}$$

from which it is apparent that, for laminar flow

$$f = \frac{64}{\mathrm{Re}} \qquad (8.23)$$

Equation (8.23) indicates that when the flow is laminar, the loss of head depends on Re, but is independent of the roughness of the pipe.

Turbulent Flow

When the flow is turbulent, the value of f depends not only on the Reynolds number, but also on the relative roughness of the pipe. Blasius was the first to give an empirical equation for finding the friction factor f, as early as 1913, based on his experimental work on hydraulically smooth pipe in turbulent flow. The formula developed by Blasius that is valid up to about $\mathrm{Re} = 10^5$ is given by

$$f = \frac{0.3164}{\mathrm{Re}^{1/4}} \qquad (8.24)$$

For smooth pipes

$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{Re\sqrt{f}}{2.51}\right)$$

$\delta_v > k$ $\qquad \frac{ku_*}{\nu} < 5$

For rough pipes

$$\frac{1}{\sqrt{f}} = 2\log\left(3.7\frac{d}{k}\right)$$

$\delta_v < \frac{k}{14}$ $\qquad \frac{ku_*}{\nu} > 70$ (8.26)

These equations are regarded as great scientific achievement as they have been verified by experiment for pipes of all sizes and for different liquids. They show that for turbulent flow in pipes which are hydraulically smooth, f is independent of the relative roughness and is a function of the Reynolds number only. On the other hand, for turbulent flow in pipes which are hydraulically rough and turbulence is fully developed, f is independent of the Reynolds number and depends only on the relative roughness.

Between these two limiting conditions of flow, there is a transition region for which White and Colebrook developed the following formula for f for use with commercial pipes:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k}{3.7d} + \frac{2.51}{Re\sqrt{f}}\right)$$ → For any pipe (8.27)

The transition region merges at one end into the smooth pipe flow and at other end into the zone of fully developed turbulence.

$K > \delta_v > \frac{k}{14}$ $\qquad 5 \leq \frac{ku_*}{\nu} \leq 70$

Laminar Sublayer

Even in turbulent flow there exists next to the wall of the pipe a very thin layer in which the flow is laminar. This layer is known as the laminar or viscous sublayer. The thickness of this layer is given by

$$\partial_v = \frac{11.6\nu}{u^*} \qquad (8.21)$$

where $u^* = \sqrt{\tau/\rho}$ is the shear or friction velocity and τ is the shear stress on the pipe wall.

Hydraulically Smooth and Rough Boundaries

A pipe is said to be hydraulically smooth if the height of the roughness elements is less than the thickness of the laminar sublayer ($k < \partial_v$), i.e. the roughness elements are well-covered by the laminar sublayer. In such a case, variations in relative roughness k/d do not affect the value of the friction factor f. On the other hand, if the height of the roughness elements are greater than the thickness of the laminar sublayer ($k > \partial_v$), their presence affects the amount of turbulence and hence the value of f and the pipe is said to be hydraulically rough. As the height of the roughness elements k increases or the thickness of the laminar sublayer decreases with increasing Reynolds number, the turbulence increases to a maximum level at which it is said to be "fully developed".

Nikuradse Equivalent Sand Grain Roughness (k)

Nikuradse in 1933 conducted a series of experiments on flow through pipes which were artificially roughened by gluing sand grains of uniform diameter. He introduced the concept of equivalent sand grain roughness (k) as standard for all other types of roughness elements. The ratio k/d of the equivalent sand grain roughness to the pipe diameter is known as the *relative roughness*.

Table 8.1 gives the values of equivalent sand grain roughness for different pipe materials.

Table 8.1 Equivalent sand grain roughness (k) for various pipe materials

Sl. No.	Pipe material	k(mm)
1.	Glass	0.0003
2.	Wrought iron, steel	0.046
3.	Asphalted cast iron	0.12
4.	Galvanized iron	0.15
5.	Cast iron	0.26
6.	Concrete	0.30 – 3.0
7.	Riveted steel	0.90 – 9.0

The above values correspond to the material in new and clean condition. As the pipe becomes older, the roughness increases due to corrosion.

Example 8.1

An oil having kinematic viscosity of 21.4 stokes is flowing through a pipe of 300 mm diameter. Determine the type of flow, if the discharge through the pipe is 15 liters/sec.

Solution We have, $\nu = 21.4$ stokes $= 21.4\ cm^2/s = 21.4 \times 10^{-4}\ m^2/s$

Diameter of pipe, $d = 300\ mm = 0.30\ m$

$\therefore$ Area of pipe, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.30^2 = 0.0707\ m^2$

Discharge, $Q = 15$ liters/s $= 0.015\ m^3/s$

$\therefore$ Velocity of flow, $V = \frac{Q}{A} = \frac{0.015}{0.0707} = 0.212\ m/s$

The Reynolds number of the flow is given by

$$Re = \frac{Vd}{\nu} = \frac{0.212 \times 0.30}{21.4 \times 10^{-4}} = 29.75$$

As the Reynolds number is less than 2000, the flow is laminar.

Example 8.2

In a laboratory experiment, a crude oil is flowing through a pipe of 50 mm diameter with a velocity of 1.5 m/s. During this experiment, a pressure difference of 180 N/mm^2 was recorded from two pressure gages 8 m apart. Find the viscosity of the flowing oil.

Solution Given, diameter of pipe, d = 50 mm = 0.05 m

Velocity of oil, V = 1.5 m/s

Pressure difference, $p_1 - p_2 = \gamma h_f = 180$ N/mm^2 $= 180 \times 10^6$ N/m^2

Length of the pipe, L = 8 m

$$\therefore \ \mu = \frac{\gamma h_f d^2}{32LV} = \frac{180 \times 10^6 \times 0.05^2}{32 \times 8 \times 1.5} = 1171.88 \ \text{N.s/m}^2$$

Example 8.3

Find the loss of head due to friction in a pipe of 1 m diameter and 15 km long. The velocity of water in the pipe is 1 m/s. Take f = 0.020 and neglect minor losses. Use the Darcy-Weisbach formula.

Solution We have, d = 1 m, L = 15 km = 15000 m, V= 1 m/s, f = 0.020

$$\therefore h_f = f\frac{L}{d}\frac{V^2}{2g} = 0.020 \times \frac{15000}{1} \times \frac{1^2}{2 \times 9.81} = 15.29\ m \text{ of water}$$

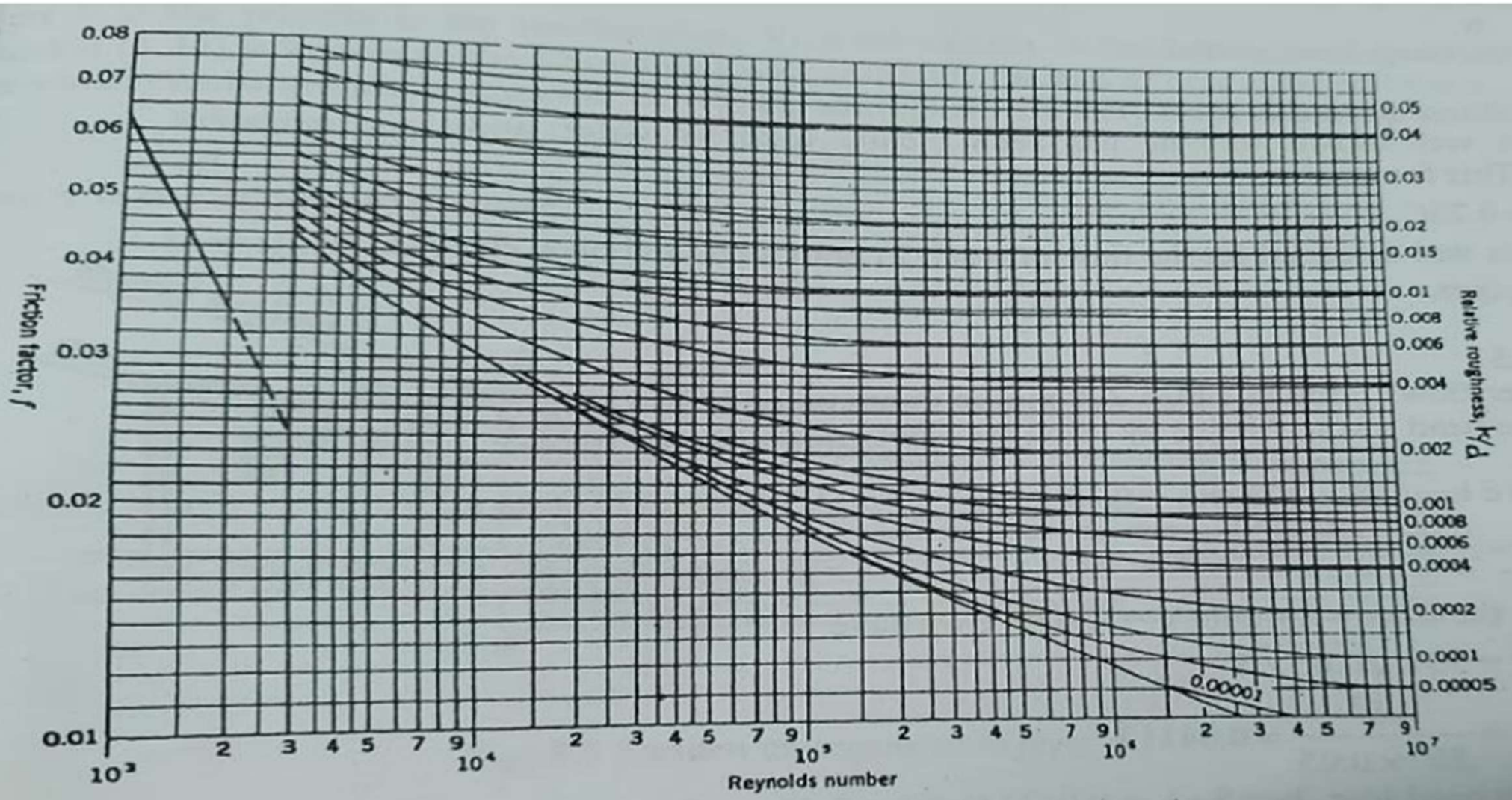


Fig. 8.6 Moody diagram

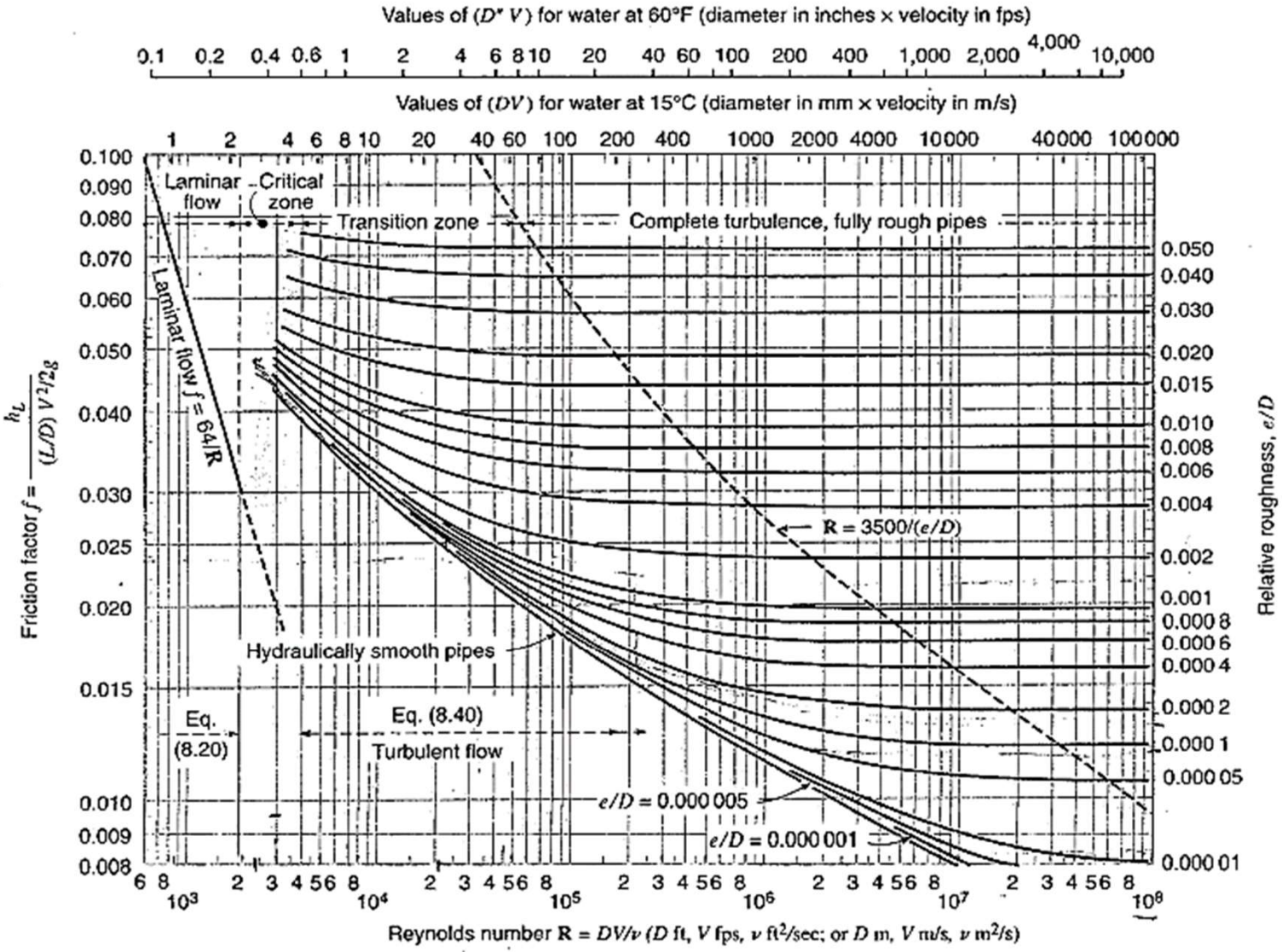


Figure 8.11
Moody chart for pipe friction factor (Stanton diagram).

Moody Diagram Math:

Math Example 1: Crude oil ($S=0.86$) at 20°c flows in a riveted steel pipe of 1m diameter. The mean velocity is 2m/s. What is the range of head loss expected in 1 km of pipeline. For riveted steel pipe, roughness ranges between 0.9 mm to 9mm. The dynamic viscosity of crude oil at 20°c is 0.00718 $N.s/m^2$.

=> Here, $D = 1m$
$v = 2m/s$
$h_L = ?$
$L = 1\ km = 1000\ m$
roughness = 0.9 mm to 9 mm
specific gravity, $S = 0.86$
at 20°c, $\rho_{water} = 998.23\ kg/m^3$ [from Table]

The table was given in fluid property slide

We know, $Re = \frac{VD}{\nu} = \frac{2 \times 1}{\frac{\mu}{\rho}} = \frac{2 \times 1}{\frac{0.00718}{0.86 \times 998.23}}$

$= 2.4 \times 10^5$

Now, $\frac{e_1}{D} = 9 \times 10^{-4}$ & $\frac{e_2}{D} = 9 \times 10^{-3}$

from moody diagram, $f = 0.021$ & $f = 0.0365$

$h_L = f \frac{L}{D} \frac{v^2}{2g}$

$= 0.021 \times \frac{1000}{1} \times \frac{(2)^2}{2 \times 9.81}$

$= 4.28\ m$

$h_L = f \frac{L}{D} \frac{v^2}{2g}$

$= 0.0365 \times \frac{1000}{1} \times \frac{2^2}{2 \times 9.81}$

$= 7.99\ m$

So, head loss ranges from 4.28m to 7.99 m

An example of how to read the value in moody diagram is in next page

Other Pipe Formulas

Chezy Formula

Using $h_f/L = S$ and $d = 4R$, where S is the slope of the energy grade line (Fig. 8.7) representing the loss of head per m of pipe and R (=A/P) is the hydraulic radius, the Darcy-Weisbach formula can be put in the form

$$V = \sqrt{\frac{2g}{f}} \times \sqrt{4R} \times \sqrt{S} = \sqrt{\frac{8g}{f}} \sqrt{RS} \qquad (8.28)$$

Substituting a coefficient C for $\sqrt{8g/f}$, we obtain

$$V = C\sqrt{RS} \qquad (8.29)$$

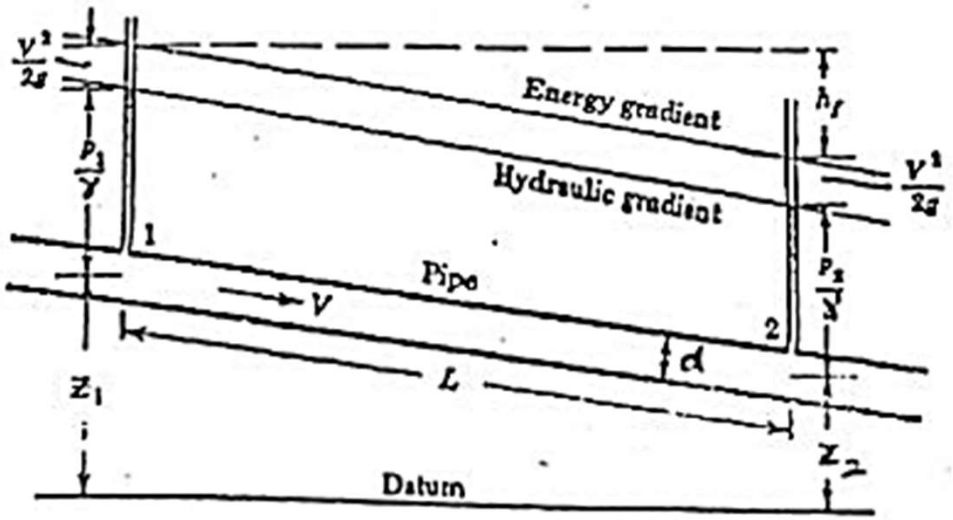


Fig. 8.7 Frictional loss in pipe

This formula for velocity of flow in terms of the hydraulic radius of a conduit and slope of the energy grade line is called the Chezy formula. The Chezy coefficient C is a function of the same variables as the Darcy-Weisbach friction factor f and the Chezy formula is therefore subject to same defects as noted in Art. 8.6 for the Darcy-Weisbach formula.

Manning Formula

The Manning formula is one of the best-known open channel formulas and is commonly used for pipes. In the form of Eq.(8.29), the Manning formula is

$$V = \frac{1}{n} R^{2/3} S^{1/2} \tag{8.30}$$

where n is the roughness coefficient.

Hazen-Williams Formula

This well-known formula has been mostly used by water works and irrigation engineers. This formula is

$$V = 0.25 C_1 R^{0.63} S^{0.54} \tag{8.31}$$

This formula was designed for the flow of water in both pipes and open channels, but is used mostly for pipes.

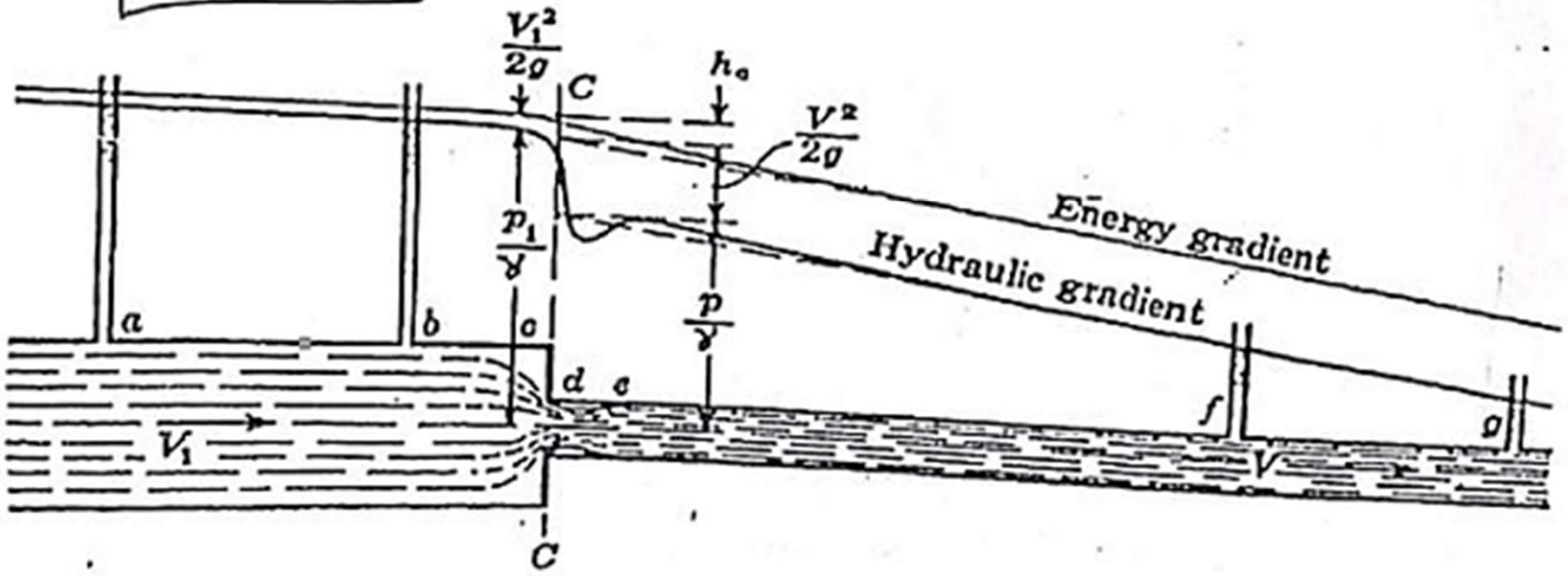
Minor Losses

Loss of Head due to Contraction: It is given by

$$h_c = K_c \frac{V^2}{2g} \qquad (8.32)$$

where K_c is an empirical coefficient and V is the velocity in the smaller pipe (Fig. 8.8). The value of K_c depends on V and the ratio of the smaller to the larger diameter. This loss is usually taken to be equal to

$$h_c = 0.5 \frac{V^2}{2g} \qquad (8.33)$$



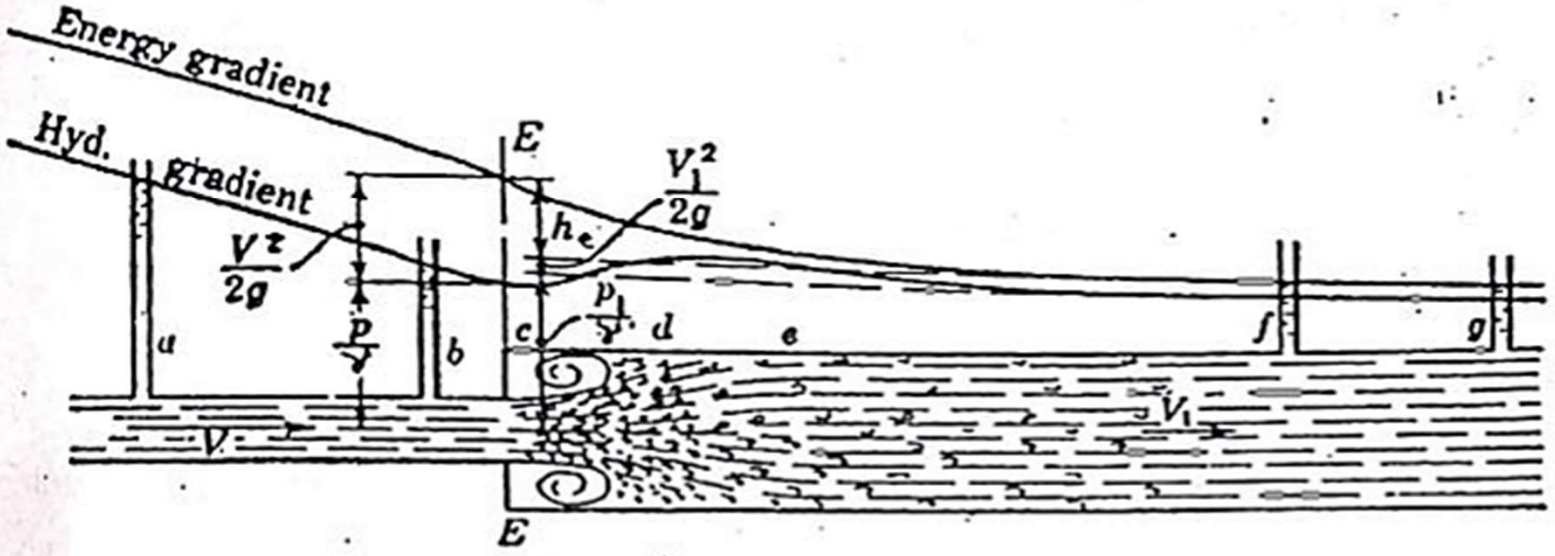
Loss of Head due to Enlargement: It is usually given by

$$h_e = \frac{(V - V_1)^2}{2g} \qquad (8.34)$$

where V is the velocity in the smaller pipe, V_1 is the velocity in the larger pipe (Fig. 8.9). Equation (8.34) is known as the Carnot or Borda equation. The loss of head at the outlet to a pipe which discharges into a reservoir or into the atmosphere is usually taken to be equal to

$$h_e = \frac{V^2}{2g} \qquad (8.35)$$

where V is the velocity in the pipe.



Loss of Head due to Obstructions: The loss of head in pipes due to gates, valves and other obstructions is given by

$$h_x = K_x \frac{V^2}{2g} \tag{8.36}$$

where V is the mean velocity in the pipe. The coefficient K_g depends on the size of obstruction compared to the diameter of the pipe.

Loss of Head due to Bends: This loss of head is usually expressed as a function of the velocity head in the pipe, i.e.

$$h_b = K_b \frac{V^2}{2g} \tag{8.37}$$

The value of the coefficient K_b varies with the ratio of the radius of curvature of the bend r to the pipe diameter d, the roughness of the surface in the bend and the Reynolds number. If the flow is turbulent, the effect of variation in Reynolds number is not of much practical importance and in that case K_b is a function of r/d and the roughness of the bend.

Math:

A pipe 60 m long and 15 cm in diameter is connected to a water tank at one end and flows freely into the atmosphere at the other end. The height of the water level in the tank 2.6 m above the center of the pipe. The pipe is horizontal and f = 0.040. Determine discharge through the pipe in liters/sec, if all the minor losses are to be considered.

Solution We have, L = 60 m, d = 15 cm = 0.15 m, H = 2.6 m, f = 0.040

Since the pipe is connected to a water tank and discharges freely into the atmosphere there are contraction loss at the entrance and a sudden enlargement loss at the outlet. The contraction loss is given by $h_c = 0.5V^2/2g$ and the sudden enlargement loss is given by $V^2/2g$, where V is the velocity of water through the pipe.

$$\therefore H = f\frac{L}{d}\frac{V^2}{2g} + 0.5\frac{V^2}{2g} + \frac{V^2}{2g} = \frac{V^2}{2g}\left(f\frac{L}{d} + 1.5\right)$$

or

$$2.6 = \frac{V^2}{2\times 9.81}\left(0.04\times\frac{60}{0.15} + 1.5\right)$$

$$\therefore V^2 = \frac{2.6\times 2\times 9.81}{17.5} = 2.915$$

$$\therefore V = 1.7073\ m/s$$

$$\therefore Q = AV = \frac{\pi}{4}\times 0.15^2\times 1.7073 = 0.0302\ m^3/s = 30.2\ \text{liters/sec}$$