9.5 PIPE NETWORKS

City water supply distribution systems are constructed in the form of many loops, branches and junctions, more or less complicated in arrangement. Such a system is called a network. The methods we have so far considered are not suitable for solving complicated pipe network problems. An important advance in the solution of pipe network problems was made by Hardy Cross, who developed a method of successive approximations by which the distribution of flow in a pipe network can be determined.

Consider an elementary loop A in a general network of pipes, as illustrated in Fig. 9.4, The arrowheads indicate direction of flow. The following two conditions must be satisfied for the solution of pipe network problems.

1. At any junction the total inflow must be equal to the total outflow.

2. The loss of head due to flow in a clockwise direction around a loop must be equal to the loss of head due to flow in a counterclockwise direction. Thus in Fig. 9.4, the loss of head in pipes ab and bc must be equal to the loss of head in pipes ad, de and ec.

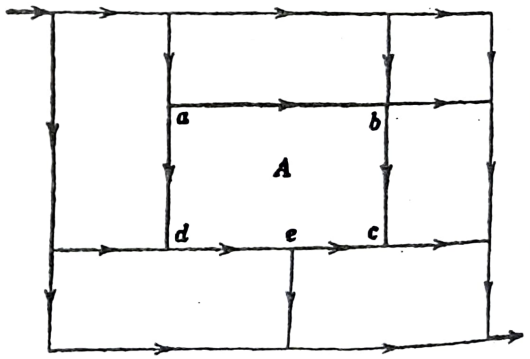
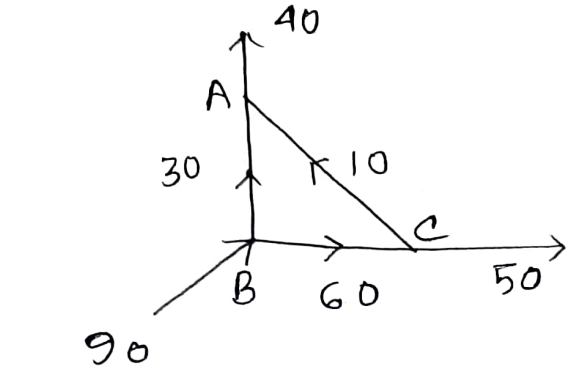


Fig. 9.4 Pipe network

Hardy Cross Method:



$\Sigma h_f = h_{fAB} - h_{fBC} - h_{fAC}$

$= 0$

clockwise(+), anticlockwise(−)



Darcy weisbach formula,

$$h_f = f \cdot \frac{L}{d} \cdot \frac{v^2}{2g}$$

$$\Rightarrow h_f = f \cdot \frac{L}{d} \cdot \frac{Q^2}{A^2 \cdot 2g}$$

$$\Rightarrow h_f = f \cdot \frac{L}{d} \cdot \frac{Q^2}{\left(\frac{1}{4}\pi d^2\right)^2 \cdot 2g}$$

$$\Rightarrow h_f = f \cdot \frac{L}{d} \cdot \frac{Q^2}{\frac{1}{16}\pi^2 d^4 \cdot 2g}$$

$$\Rightarrow h_f = f \cdot \frac{L}{d} \cdot \left(\frac{Q^2}{1} \times \frac{16}{\pi^2 d^4 \cdot 2g}\right)$$

$$\Rightarrow h_f = f \cdot \frac{L}{d} \cdot \frac{16Q^2}{2g\pi^2 d^4}$$

$$\Rightarrow h_f = f \cdot \frac{L}{d^5} \cdot \frac{16Q^2}{2g\pi^2}$$

$$\Rightarrow h_f = \frac{8fLQ^2}{g\pi^2 d^5}$$

$$\Rightarrow h_f = KQ^2 \quad \left[K = \frac{8fL}{g\pi^2 d^5}\right]$$

$$\therefore h_f = KQ^n$$

for single pipe, $h_f = KQ^n$

for more pipes,

$$\Sigma h_f = \Sigma KQ^n$$

$Q = Q_0 + \Delta Q$

Q = actual discharge

Q_0 = assumed "

ΔQ = changed "

$$\Sigma h_f = \Sigma KQ^n$$

$$\Rightarrow \Sigma h_f = \Sigma K(Q_0 + \Delta Q)^n$$

$$\Rightarrow \Sigma h_f = \Sigma K\left(Q_0^n + nQ_0^{n-1}\Delta Q + \cdots\right)$$

ΔQ is small compared to Q_0

$$\therefore \Sigma h_f = \Sigma KQ_0^n + \Delta Q \Sigma KnQ_0^{n-1} = 0$$

[when, $\Delta Q \approx 0$, $\Sigma h_f \approx 0$]

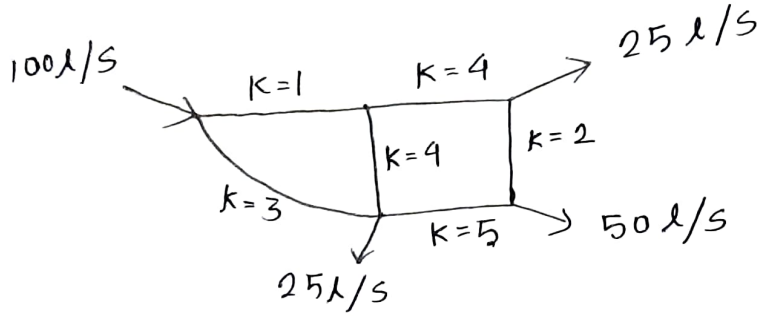
$$\Rightarrow -\Sigma KQ_0^n = \Sigma KnQ_0^{n-1}\Delta Q$$

$$\therefore \Delta Q = \frac{-\Sigma KQ_0^n}{\Sigma\left|KnQ_0^{n-1}\right|}$$

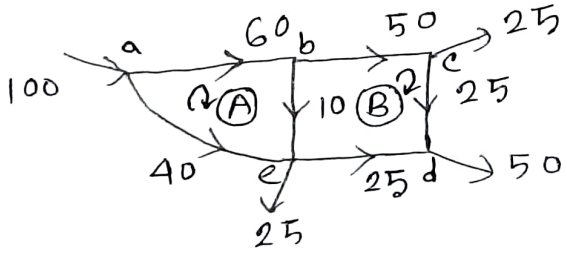
Hardy Cross Method;

Math Example

If the flows into and out of a two loop pipe system are as shown, determine the flow in each pipe. The K values for each pipe were calculated from the pipe and minor loss characteristics and from an assumed value of f. Show two trials.



Soln: first approximation,



ΔQ = (+), clockwise (+) anticlockwise (−)

ΔQ = (−), clockwise (−) anticlockwise (+)

Loop A

pipe	K	Assumed Q	$\Sigma K Q_0^n$	$\Sigma \lvert K n Q_0^{n-1} \rvert$	
ab	1	60	$1\times60^{\downarrow}\times2 = 3600$	$1\times2\times60 = 120$	(+)
be	4	10	$4\times10^{\downarrow}\times2 = 400$	$4\times2\times10 = 80$	
ae	3	40	$3\times40^{\uparrow} = 4800$	$3\times2\times40 = 240$	
			$(3600+400)-4800 = -800$	440	

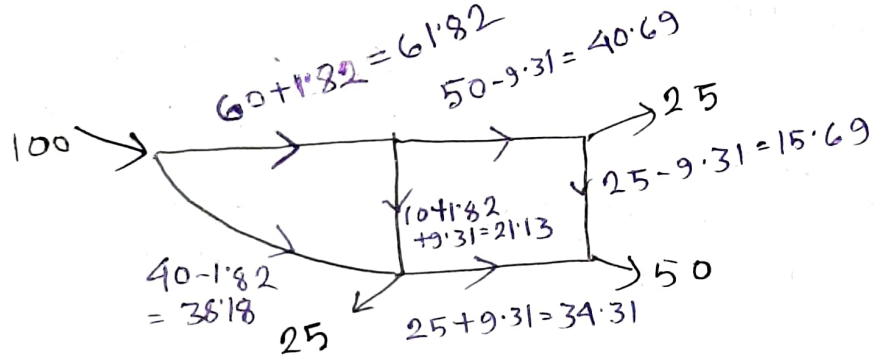
$$\therefore \Delta Q_1 = \frac{-\Sigma K Q_0^n}{\Sigma \lvert K n Q_0^{n-1} \rvert} = \frac{-(-800)}{440} = 1.82$$

Loop B

pipe	K	Assumed Q	$\sum kQ_0^n$	$\sum \lvert knQ_0^{n-1} \rvert$
bc	4	50	$4\times50^2 = 10000$ ↓	$4\times2\times50 = 400$
cd	2	25	$2\times25^2 = 1250$ ↓	$2\times2\times25 = 100$
de	5	25	$5\times25^2 = 3125$ ↑	$5\times2\times25 = 250$ (+)
be/ed	4	10	$4\times10^2 = 400$ ↑	$4\times2\times10 = 80$
			$(10000+1250)-(3125+400)$	830
			$= 7725$	

$$\therefore \Delta Q_1 = \frac{-7725}{830} = -9.31$$

After first Correction,



Loop A

pipe	K	Assumed Q	$\sum kQ_0^n$	$\sum \lvert knQ_0^{n-1} \rvert$
ab	1	61.82	$1\times(61.82)^2$ ↓ $= 3821.71$	$1\times2\times61.82 = 123.64$
be	4	21.13	$4\times(21.13)^2$ ↓ $= 1785.91$	$4\times2\times21.13 = 169.09$ (+)
ae	3	38.18	$3\times(38.18)^2$ ↑ $= 4373.14$	$3\times2\times38.18 = 229.08$
			$(3821.71+1785.91)-4373.19$	571.76
			$= 1234.98$	

$$\therefore \Delta Q_2 = \frac{-1234.98}{571.76} = -2.16$$

Loop B

pipe	K	Assumed Q	$\Sigma K Q_0^n$	$\Sigma \lvert kn Q_0^{n-1} \rvert$
bc	4	40.69	$4 \times (40.69)^2 = 6622.70$ ↓	$4 \times 2 \times 40.69 = 325.52$
cd	2	15.69	$2 \times (15.69)^2 = 492.35$ ↓	$2 \times 2 \times 15.69 = 62.76$
de	5	34.31	$5 \times (34.31)^2 = 5885.88$ ↑	$5 \times 2 \times 34.31 = 343.1$
ed	4	21.13	$4 \times (21.13)^2 = 1785.91$ ↑	$4 \times 2 \times 21.13 = 169.09$
			$(6622.70 + 492.35) - (5885.88 + 1785.91) = -556.79$	900.42

$$\Delta Q_2 = \frac{-(-556.79)}{900.42} = 0.62$$

After second correction,

ΔQ = (+), clockwise (+)
anticlockwise (−)

ΔQ = (−)
clockwise (−)
anticlockwise (+)

