Military Institute of Science and Technology

EWCE 261

Fluid Mechanics

3 Credit Hour

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Lecturer

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Syllabus:

1. Development and Scope of Fluid Mechanics
2. Fluid Properties
3. Fluid Statics
4. Similitude and Dimensional Analysis
5. Steady Incompressible Flow in Pressure Conduits: Laminar Flow and Turbulent Flow
6. General Equation for Fluid Friction
7. Empirical Equation for Pipe Flow
8. Minor Losses in Pipe Flow
9. Fluid Measurement: pitot tube, orifice, mouthpiece, nozzle, venturimeter, weir.
10. Pipe Flow Problems: Pipes in Series and Parallel, Branching Pipes, Pipe Networks

Syllabus: (To be Covered in Part A)

1. Kinematics of Fluid Flow
2. Fluid Flow Concepts and Basic Equations: Continuity Equation, Bernoulli's Equation, Energy Equation
3. Momentum Equation and Forces in Fluid Flow

Pipe Network
(Hardy Cross Method)

9.5 PIPE NETWORKS

City water supply distribution systems are constructed in the form of many loops, branches and junctions, more or less complicated in arrangement. Such a system is called a network. The methods we have so far considered are not suitable for solving complicated pipe network problems. An important advance in the solution of pipe network problems was made by Hardy Cross, who developed a method of successive approximations by which the distribution of flow in a pipe network can be determined.

Consider an elementary loop A in a general network of pipes, as illustrated in Fig. 9.4, The arrowheads indicate direction of flow. The following two conditions must be satisfied for the solution of pipe network problems.

1. At any junction the total inflow must be equal to the total outflow.

2. The loss of head due to flow in a clockwise direction around a loop must be equal to the loss of head due to flow in a counterclockwise direction. Thus in Fig. 9.4, the loss of head in pipes ab and bc must be equal to the loss of head in pipes ad, de and ec.

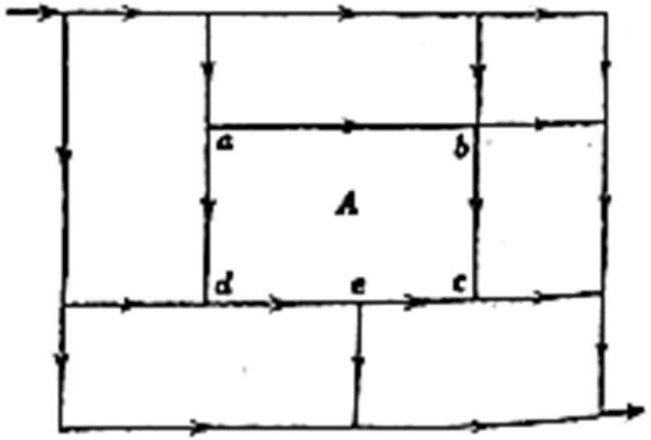
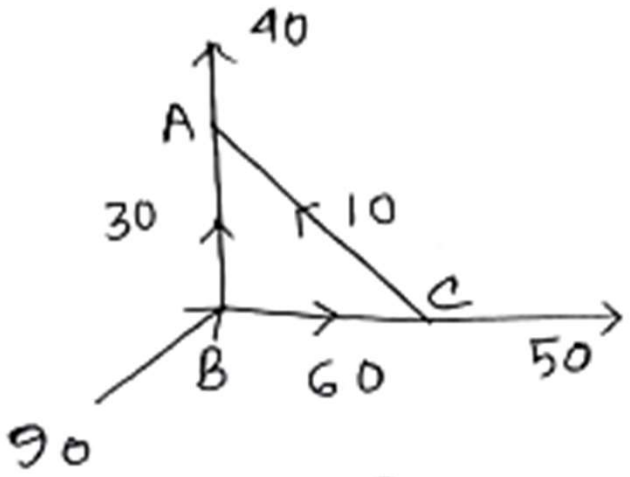


Fig. 9.4 Pipe network

Hardy Cross Method:



$\Sigma h_f = h_{fAB} - h_{fBC} - h_{fAC}$

$= 0$

clockwise (+), anticlockwise (−)



Darcy weisbach formula,

$$h_f = f \cdot \frac{L}{d} \cdot \frac{v^2}{2g}$$

$$\Rightarrow h_f = f \cdot \frac{L}{d} \cdot \frac{Q^2}{A^2 \cdot 2g}$$

$$\Rightarrow h_f = f \cdot \frac{L}{d} \cdot \frac{Q^2}{\left(\frac{1}{4}\pi d^2\right)^2 \cdot 2g}$$

$$\Rightarrow h_f = f \cdot \frac{L}{d} \cdot \frac{Q^2}{\frac{1}{16}\pi^2 d^4 \cdot 2g}$$

$$\Rightarrow h_f = f \cdot \frac{L}{d} \cdot \left(\frac{Q^2}{1} \times \frac{16}{\pi^2 d^4 \cdot 2g}\right)$$

$$\Rightarrow h_f = f \cdot \frac{L}{d} \cdot \frac{16 Q^2}{2g\pi^2 d^4}$$

$$\Rightarrow h_f = f \cdot \frac{L}{d^5} \cdot \frac{16 Q^2}{2g\pi^2}$$

$$\Rightarrow h_f = \frac{8 f L Q^2}{g\pi^2 d^5}$$

$$\Rightarrow h_f = K Q^2 \quad \left[K = \frac{8fL}{g\pi^2 d^5}\right]$$

$$\therefore h_f = K Q^n$$

for single pipe, $h_f = K Q^n$

for more pipes,

$$\Sigma h_f = \Sigma K Q^n$$

$Q = Q_0 + \Delta Q$

Q = actual discharge

Q_0 = assumed "

ΔQ = changed "

$$\Sigma h_f = \Sigma K Q^n$$

$$= \Sigma h_f = \Sigma K (Q_0 + \Delta Q)^n$$

$$\Rightarrow \Sigma h_f = \Sigma K \left(Q_0^n + n Q_0^{n-1} \Delta Q + \cdots\right)$$

ΔQ is small compared to Q_0

$$\therefore \Sigma h_f = \Sigma K Q_0^n + \Delta Q \Sigma K n Q_0^{n-1} = 0$$

[when, $\Delta Q \neq 0$, $\Sigma h_f = 0$]

$$\Rightarrow -\Sigma K Q_0^n = \Sigma K n Q_0^{n-1} \Delta Q$$

$$\therefore \Delta Q = \frac{-\Sigma K Q_0^n}{\Sigma \left| K n Q_0^{n-1} \right|}$$

Problem 11.56 *Calculate the discharge in each pipe of the network shown in Fig. 11.34. The pipe network consists of 5 pipes. The head loss h_f in a pipe is given by $h_f = rQ^2$. The values of r for various pipes and also the inflow or outflows at nodes are shown in the figure.*

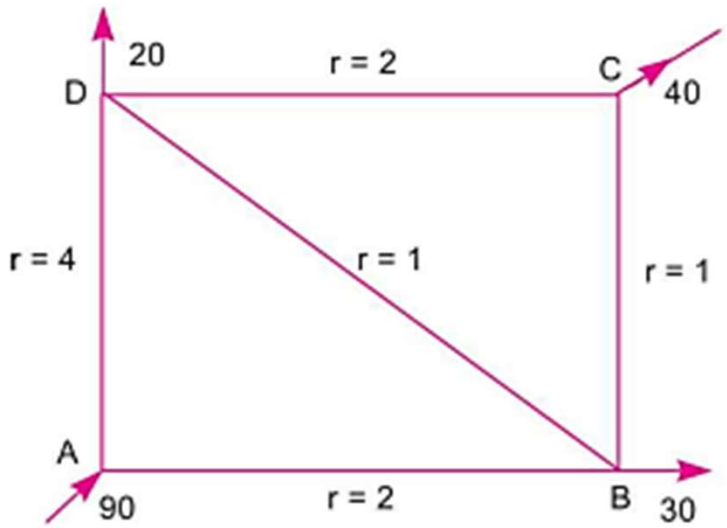


Fig. 11.34

Solution. Given :

Inflow at node $A = 90$, outflow at $B = 30$, at $C = 40$ and at $D = 20$.

Values of r for $AB = 2$, for $BC = 1$, for $CD = 2$, for $AD = 4$ and for $BD = 1$.

For the first trial, the discharges are assumed as shown in Fig. 11.34 (a) so that continuity is satisfied at each node (*i.e.*, flow into a node = flow out of the node). For this distribution of discharge, the corrections ΔQ for the loops ABD and BCD are calculated.

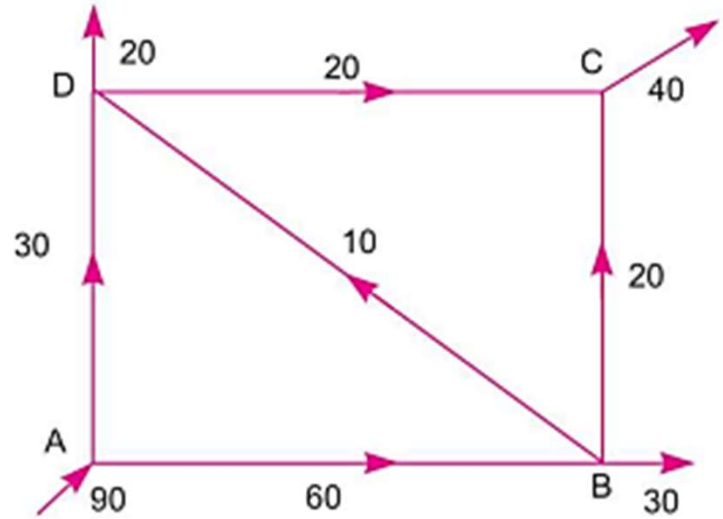


Fig. 11.34(a)

First Trial

Loop ADB					Loop DCB				
Pipe	*r*	Q_0	$h_f = rQ_0^2$	$2rQ_0$	*Pipe*	*r*	Q_0	$h_f = rQ_0^2$	$2rQ_0$
AD	4	30	$4 \times 30^2 = 3600$	$2 \times 4 \times 30 = 240$	*DC*	2	20	$2 \times 20^2 = 800$	$2 \times 2 \times 20 = 80$
DB	1	10	$-1 \times 10^2 = -100$	$2 \times 1 \times 10 = 20$	*CB*	1	20	$-1 \times 20^2 = -400$	$2 \times 1 \times 20 = 40$
AB	2	60	$-2 \times 60^2 = -7200$	$2 \times 2 \times 60 = 240$	*BD*	1	10	$1 \times 10^2 = 100$	$2 \times 1 \times 10 = 20.$
			$\Sigma rQ_0^2 = -3700,$	$\Sigma 2rQ_0 = 500,$				$\Sigma 2\, rQ_0^2 = 500$	$\Sigma 2rQ_0 = 140$

$$\therefore \quad \Delta Q = \frac{-\Sigma\, r\, Q_0^2}{\Sigma\, 2r\, Q_0} = \frac{-(-3700)}{500} = 7.4$$

$$\therefore \quad \Delta Q = \frac{-\Sigma\, r\, Q_0^2}{\Sigma\, 2r\, Q_0} \frac{-500}{140} = -3.57 \simeq -3.6.$$

In the loop *ADB*, the head loss h_f is negative in pipes *DB* and *AB* as the direction of discharges in these pipes is anticlockwise.

As ΔQ is positive for loop *ADB*, hence it should be added to the flow in the clockwise direction and subtractedfrom the flow in the anticlockwise direction. Hence the corrected flow for second trial for loop *ADB* will be as follows :

Pipe *AD* = 30 + 7.4 = 37.4 (flow is clockwise)
Pipe *AB* = 60 – 7.4 = 52.6 (flow is anticlockwise)
Pipe *BD* = 10 – 7.4 = 2. 6 (flow is anticlockwise)

The head loss in pipe *BC* for loop *DCB* is negative as the direction of discharge in pipe *BC* in anticlockwise.

As ΔQ is negative for loop *DCB*, hence it should be subtracted from the flow in the clockwise direction and added to the flow in the anticlockwise direction. Hence corrected flow for second trial for loop *DCB* will be as follows :

Pipe *DC* = 20 – 3.6 = 16.4
Pipe *BC* = 20 + 3.6 = 23.6
Pipe *BD** = 2.6 – 3.6 = – 1

Note. The pipe *BD* is common to two loops (*i.e.*, loop *ADB* and loop *DCB*). Hence this pipe will get two corrections. After the two corrections, the resultant flow in pipe *BD* is negative in loop *DCB*. Hence the direction of flow will be anticlockwise in pipe *BD* for loop *DCB*.

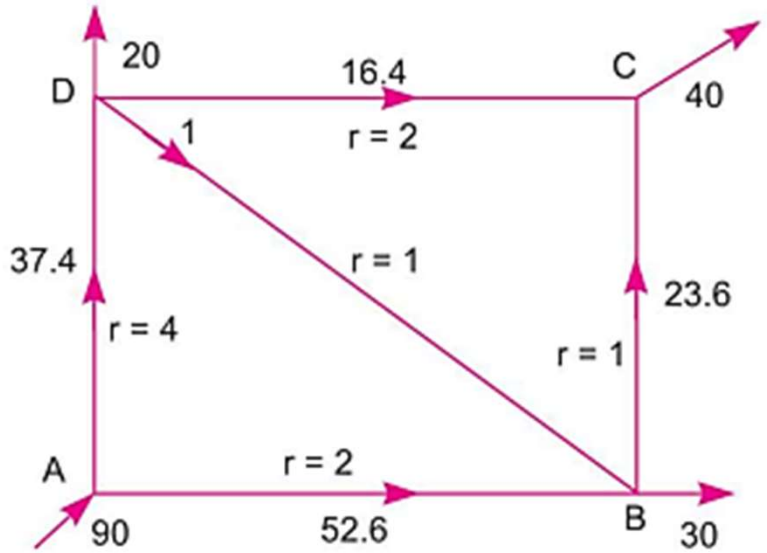


Fig. 11.34 (*b*)

The distribution of discharges in various pipes for second trial is shown in Fig. 11.34 (*b*). For second trial the correction ΔQ for loops *ADB* and *DCB* are calculated as follows :

Loop ADB

Pipe	r	Q_0	$h_f = rQ_0^2$	$2rQ_0$
AD	4	37.4	$4 \times 37.4^2 = 5595$	$2 \times 4 \times 37.4 = 299.2$
DB	1	1	$1 \times 1^2 = 1$	$2 \times 1 \times 1 = 2$
AB	2	52.6	$-2 \times 52.6^2 = -5533.5$	$2 \times 2 \times 52.6 = 210.4$

$\Sigma rQ_0^2 = 62.54, \quad \Sigma 2rQ_0 = 511.6$

$$\therefore \Delta Q = \frac{-\sum r Q_0^2}{\sum 2r Q_0} = \frac{62.54}{-511.6}$$

$$= -0.122 \simeq -0.1$$

As ΔQ is negative, hence it should be subtracted from the flow in the clockwise direction and added to the flow in the anticlockwise direction

As the correction (ΔQ) is small (*i.e.*, $\Delta Q = -0.1$), this correction is applied and further trials are discontinued.

Hence corrected flow for loop *ADB* will be as follows :

For pipe *AD*, $Q_0 = 37.4 - 0.1 = 37.3$ (as flow is clockwise)

For pipe *DB*, $Q_0 = 1 - 0.1 = 0.9$ (as flow is clockwise)

For pipe *AB*, $Q_0 = 52.6 + 0.1 = 52.7$ (as flow is anti-clockwise)

Loop DCB

Pipe	r	Q_0	$h_f = rQ_0^2$	$2rQ_0$
DC	2	16.4	$2 \times 16.4^2 = 537.9$	$2 \times 2 \times 16.4 = 65.6$
CB	1	23.6	$-1 \times 23.6^2 - 556.9$	$2 \times 1 \times 23.6 = 47.2$
BD	1	1	$-1 \times 1^2 = -1$	$2 \times 1 \times 1 = 2$

$\Sigma rQ_0^2 = -20, \quad \Sigma 2rQ_0 = 114.8$

$$\therefore \Delta Q = \frac{-\sum r Q_0^2}{\sum 2r Q_0} = \frac{-(-20)}{114.8}$$

$$= \frac{20}{114.8} = 0.174$$

$$\simeq 0.2$$

As ΔQ is positive, hence it should be added to the flow in the clockwise direction and subtracted from the flow in the anticlockwise direction.

As the correction (ΔQ) is small (*i.e.*, $\Delta Q = 0.2$), this correction is applied and further trials are discontinued.

Hence corrected flow for loop *DCB* will be as follows :

For pipe *DC*, $Q_0 = 16.4 + 0.2 = 16.6$ (clockwise flow)

For pipe *CB*, $Q_0 = 23.6 - 0.2 = 23.4$ (anticlockwise flow)

For pipe *BD*, $Q_0 = 0.9 - 0.2 = 0.7$ (anticlockwise flow)

The final distribution of discharges in each pipe is as follows :

Discharge in pipe $AD = 37.3$ from A to D

$AB = 52.7$ from A to B

$DB = 0.7$ from D to B

$DC = 16.6$. from D to C

$BC = 23.4$ from B to C

The final discharge in each *pipe is shown* in Fig. 11.34 (*c*)

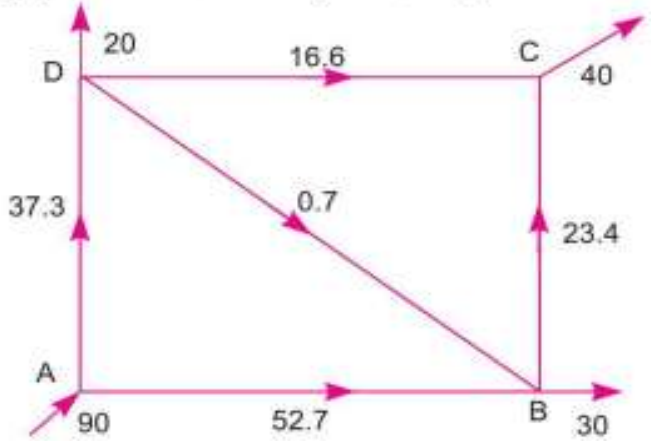


Fig. 11.34 (*c*)

Note. The pipe *DB* is common to two loop (*i.e.*, loops *ADB* and loop *DBC*). Hence this pipe will get two corrections. For loop *ADB*, the correction $\Delta Q = -0.1$ and hence the corrected flow in pipe *DB* is $1 - 0.1 = 0.9$. Now again, the correction is applied to pipe *DB* when we consider loop *DBC*. For loop *DBC*, the correction $\Delta Q = 0.2$ but flow is anticlockwise and hence the final correct flow in pipe *DB* will be $0.9 - 0.2 = 0.7$.

9.1 INTRODUCTION

A pipe line is made of small lengths of pipes of uniform or different diameters, joined together with the help of pipe fittings such as couplings, flanges, reducers, bends, elbows and pipe junctions. Pipe fittings like valves and sluices are employed for the regulation of flow. Long pipes are used to convey different kinds of fluids such as water, oil, gas, etc. For water supply installations, pipe line is meant to distribute water from one reservoir to one or more reservoirs.

9.2 PIPES IN SERIES

Figure 9.1 represents a system of pipes of different lengths and diameters conveying liquid from one reservoir to another. Such a pipe line is called a compound pipe or pipes in series. As the pipes are in series, the discharge through them will be continuous. Assuming the liquid at rest in both reservoirs, the difference H in elevation of free surfaces is the total head producing discharge. The losses of head, as indicated by the drops in the energy gradient are successively: h_{c1}, due to contraction at entrance at A; h_{f1}, due to friction in pipe 1; h_{c2}, due to contraction to smaller pipe at B; h_{f2}, due to friction in pipe 2; h_{e1}, due to enlargement to larger pipe at C; h_{f3}, due to friction in pipe 3; and h_{e2}, due to enlargement at outlet at D. Therefore,

$$H = h_{c1} + h_{f1} + h_{c2} + h_{f2} + h_{e1} + h_{f3} + h_{e2} \qquad (9.1)$$

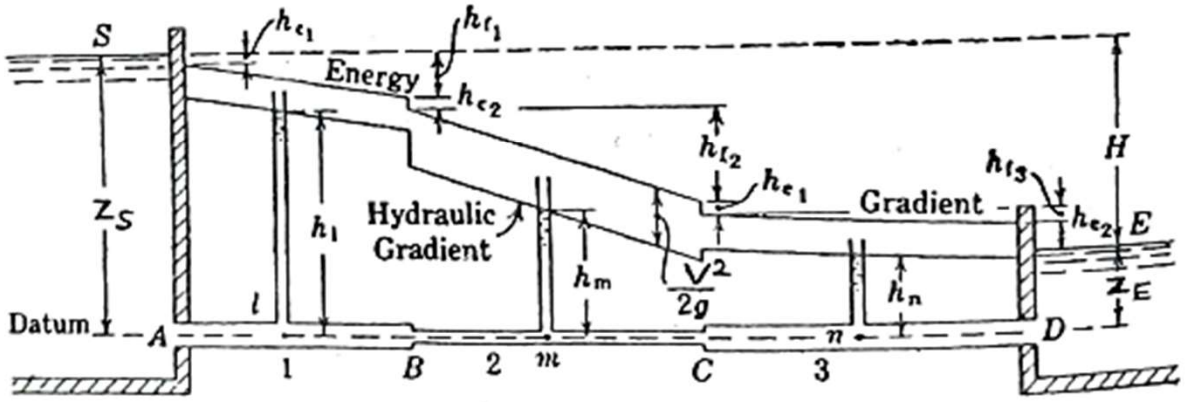


Fig. 9.1 System of pipes connecting two reservoirs