Chapter 9

PIPE FLOW PROBLEMS

9.1 INTRODUCTION

A pipe line is made of small lengths of pipes of uniform or different diameters, joined together with the help of pipe fittings such as couplings, flanges, reducers, bends, elbows and pipe junctions. Pipe fittings like valves and sluices are employed for the regulation of flow. Long pipes are used to convey different kinds of fluids such as water, oil, gas, etc. For water supply installations, pipe line is meant to distribute water from one reservoir to one or more reservoirs.

9.2 PIPES IN SERIES

Figure 9.1 represents a system of pipes of different lengths and diameters conveying liquid from one reservoir to another. Such a pipe line is called a compound pipe or pipes in series. As the pipes are in series, the discharge through them will be continuous. Assuming the liquid at rest in both reservoirs, the difference H in elevation of free surfaces is the total head producing discharge. The losses of head, as indicated by the drops in the energy gradient are successively: h_{c1}, due to contraction at entrance at A; h_{f1}, due to friction in pipe 1; h_{c2}, due to contraction to smaller pipe at B; h_{f2}, due to friction in pipe 2; h_{e1}, due to enlargement to larger pipe at C; h_{f3}, due to friction in pipe 3; and h_{e2}, due to enlargement at outlet at D. Therefore,

$$H = h_{c1} + h_{f1} + h_{c2} + h_{f2} + h_{e1} + h_{f3} + h_{e2} \qquad (9.1)$$

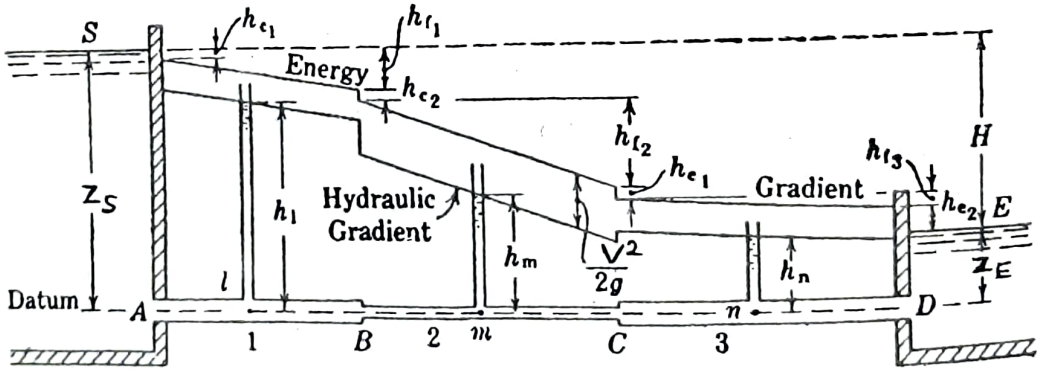


Fig. 9.1 System of pipes connecting two reservoirs

The hydraulic gradient is at a distance $V^2/2g$ below the energy gradient at all points in the three pipes. Since pipe 2 is smaller than pipe 1, the velocity head is greater and the hydraulic gradient is farther below the energy gradient. With the enlargement at C, however, the velocity head becomes less, resulting in a rise in the hydraulic gradient at that point.

In most hydraulic problems the major pipe friction losses h_{f1}, h_{f2} and h_{f3} in Eq.(9.1) constitute most of the total head H, and the minor losses are mostly so small as to be negligible. If the pipe length is about 500 pipe diameters, the error resulting from neglecting minor losses will ordinarily not exceed 5 per cent, and if the pipe length is 1000 diameters or more, the effect of minor losses can usually be considered negligible. However, if it is desired to include these losses, a solution may be obtained first neglecting them and then correcting the results by including them.

Figure 9.2 shows a simplified diagram of flow through a pipe line of different diameters in series connecting two reservoirs. Minor losses are negligible and only the hydraulic gradient is shown. The flow is assumed to be continuous and steady. Two common problems which arise are:

1. Sizes and lengths of pipe and Q given, to find the total loss of head.

This problem can be solved as follows. The total loss of head is given by

$$H = h_{f1} + h_{f2} + h_{f3}$$

$$= f_1 \frac{L_1}{d_1}\frac{V_1^2}{2g} + f_2 \frac{L_2}{d_2}\frac{V_2^2}{2g} + f_3 \frac{L_3}{d_3}\frac{V_3^2}{2g} \qquad (9.2)$$

Since $f\dfrac{L}{d}\dfrac{V^2}{2g} = f\dfrac{L}{d}\dfrac{Q^2}{2gA^2} = f\dfrac{L}{d}\dfrac{Q^2}{2g}\dfrac{1}{(\pi d^2/4)^2} = \dfrac{8Q^2}{\pi^2 g}\dfrac{fL}{d^5}$, therefore

$$H = \frac{8Q^2}{\pi^2 g}\left(\frac{f_1 L_1}{d_1^5} + \frac{f_2 L_2}{d_2^5} + \frac{f_3 L_3}{d_3^5}\right) \qquad (9.3)$$

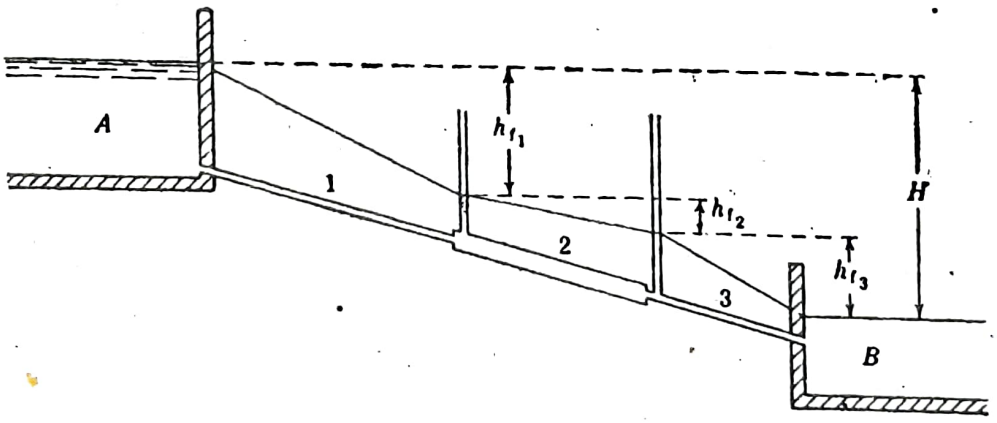


Fig. 9.2 Pipe lines of different diameters in series

Now, if the coefficient of friction is the same for all the pipes, then Eqs.(9.2) and (9.3) become

$$H = \frac{f}{2g}\left(\frac{L_1 V_1^2}{d_1} + \frac{L_2 V_2^2}{d_2} + \frac{L_3 V_3^2}{d_3}\right), \qquad (9.4)$$

and

$$H = \frac{8Q^2 f}{\pi^2 g}\left(\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right) \qquad (9.5)$$

respectively. The minor losses can be computed and included if considerable.

Example 9.1

A 5 cm pipe takes off abruptly from a large tank, runs 8 m, and then expands abruptly to 10 cm diameter, again runs 45 m, and then discharges directly into the open air with a velocity of 1.5 m/s. Compute the necessary height of water surface above the point of discharge. Take $f_1 = 0.024$ and $f_2 = 0.026$. Include the minor losses.

Solution We have, $d_1 = 5$ cm $= 0.05$ m, $L_1 = 8$ m, $d_2 = 10$ cm $= 0.10$ m, $L_2 = 45$ m, $V_2 = 1.5$ m/s, $f_1 = 0.024$, $f_2 = 0.026$

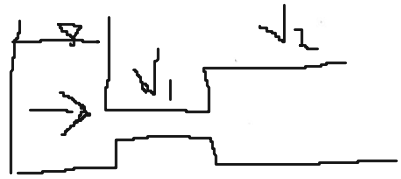
Now, height of water surface above the point of discharge = total head loss

$$\therefore H = 0.5\frac{V_1^2}{2g} + f_1\frac{L_1}{d_1}\frac{V_1^2}{2g} + \frac{(V_1 - V_2)^2}{2g} + f_2\frac{L_2}{d_2}\frac{V_2^2}{2g} + \frac{V_2^2}{2g} \qquad \text{(i)}$$

Since the flow is continuous, $Q = A_1V_1 = A_2V_2$

$$\therefore \frac{\pi}{4}d_1^2V_1 = \frac{\pi}{4}d_2^2V_2$$

$$\therefore V_1 = V_2\left(\frac{d_2}{d_1}\right)^2 = 1.5 \times \left(\frac{0.10}{0.05}\right)^2 = 6\ m/s$$



$\therefore$ From (i), we have

$$H = 0.5 \times \frac{6^2}{2\times 9.81} + 0.024 \times \frac{8}{0.05} \times \frac{6^2}{2\times 9.81} + \frac{(6-1.5)^2}{2\times 9.81} + 0.026 \times \frac{45}{0.10} \times \frac{1.5^2}{2\times 9.81} + \frac{1.5^2}{2\times 9.81}$$

$$= 0.917 + 7.045 + 1.032 + 1.341 + 0.115 = 10.45 \text{ m of water}$$

2. Allowable losses of head and lengths and sizes of pipes given, to find Q.

Four different methods of solution of this problem using Eq. (9.4) or (9.5) are as follows.

i) Trial solution: Assume a Q. Compute loss of head in each pipe by formula or diagram and add the losses. Compare with the allowable loss and revise Q in proper direction. Repeat until satisfactory check is obtained.

ii) Algebraic solution: Write $f\frac{L}{d}\frac{V^2}{2g}$ for each pipe, assuming values of f and equating sum of the terms to the allowable head loss. Express all velocity heads in terms of velocity head in one of the given sizes of pipes. Solve for the velocity head and velocity. Compute the velocities in the other pipes. Look up proper f's and check total loss of head. Revise solution if necessary.

This method is adapted to the condition in which minor losses are appreciable since they can also be expressed in terms of the velocity head and included in the equation. The minor loss coefficients selected for the first solution may also need revision.

iii) Equivalent-length solution: Reduce the overall length of compound pipe to an equivalent length of some selected diameter. With this selected diameter and computed equivalent length, determine Q for the given loss of head. Using this Q, check summation of losses in the line. A pipe diagram is of much assistance in the application of this method.

iv) Equivalent-diameter solution: Reduce the different sizes of pipe in series to an equivalent diameter of the given overall length. With this diameter and length, determine Q from the given loss of head. Using this Q, check summation of losses in the line. A pipe diagram is of much assistance in the application of the method.

Example 9.2

A pipeline 40 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 15 cm in diameter and its diameter suddenly enlarges to 30 cm. The height of water level in the tank is 8 m above the center of the pipe. Considering all losses of head which occur, determine the rate of flow. Assume f = 0.020 for both the pipes.

Solution We have, L = 40 m, L_1 = 25 m, d_1 = 15 cm = 0.15 m, L_2 = 40 – 25 = 15 m, d_2 = 30 cm = 0.30 m, H = 8 m, f = 0.020

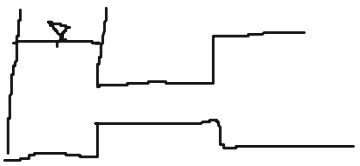
The total head loss is given by

$$\therefore H = 0.5\frac{V_1^2}{2g} + f_1\frac{L_1}{d_1}\frac{V_1^2}{2g} + \frac{(V_1 - V_2)^2}{2g} + f_2\frac{L_2}{d_2}\frac{V_2^2}{2g} + \frac{V_2^2}{2g} \qquad \text{(i)}$$

Since the flow is continuous, $Q = A_1V_1 = A_2V_2$

$$\therefore \frac{\pi}{4}d_1^2V_1 = \frac{\pi}{4}d_2^2V_2$$

$$\therefore V_1 = V_2\left(\frac{d_2}{d_1}\right)^2 = V_2 \times \left(\frac{0.30}{0.15}\right)^2 = 4V_2$$



$\therefore$ From (i), we have

$$8 = 0.5 \times \frac{(4V_2)^2}{2g} + 0.020 \times \frac{25}{0.15} \times \frac{(4V_2)^2}{2g} + \frac{(4V_2 - V_2)^2}{2g} + 0.020 \times \frac{15}{0.30} \times \frac{V_2^2}{2g} + \frac{V_2^2}{2g}$$

$$= \frac{8V_2^2}{2g} + \frac{53.33V_2^2}{2g} + \frac{9V_2^2}{2g} + \frac{V_2^2}{2g} + \frac{V_2^2}{2g} = \frac{72.33V_2^2}{2g}$$

$$\therefore V_2 = \sqrt{\frac{8 \times 2 \times 9.81}{72.33}} = 1.473\ m/s$$

$$\therefore Q = A_2V_2 = \frac{\pi}{4} \times 0.30^2 \times 1.473 = 0.104\ m^3/s$$

9.3 EQUIVALENT PIPES

Two pipe systems are said to be equivalent when the same head loss produces the same discharge in both systems. We have, for the first pipe

$$h_{f1} = f_1 \frac{L_1}{d_1}\frac{V_1^2}{2g} = f_1 \frac{L_1}{d_1}\frac{Q^2}{2g(\pi d_1^2/4)^2} = \frac{8Q^2 f_1 L_1}{\pi^2 g d_1^5} \tag{9.6}$$

and for the second pipe

$$h_{f2} = \frac{8Q^2 f_2 L_2}{\pi^2 g d_2^5} \tag{9.7}$$

For the two pipes to be equivalent, $h_{f1} = h_{f2}$ for the same discharge Q. So, equating Eqs.(9.6) and (9.7), we have after simplifying

$$\frac{f_1 L_1}{d_1^5} = \frac{f_2 L_2}{d_2^5} \tag{9.8}$$

$$\therefore L_2 = L_1 \frac{f_1}{f_2}\left(\frac{d_2}{d_1}\right)^5 \tag{9.9}$$

which determines the length of the second pipe to be equivalent to the first pipe.

Example 9.3

Replace 300 m of 250 mm diameter pipe (f = 0.020) with an equivalent length of 150 mm diameter pipe (f = 0.018).

Solution We have, $L_1 = 300$ m, $d_1 = 250$ mm $= 0.25$ m, $f_1 = 0.020$, $d_2 = 150$ mm $= 0.15$ m, $f_2 = 0.018$

From Eq. (9.9), we have

$$L_2 = L_1 \frac{f_1}{f_2}\left(\frac{d_2}{d_1}\right)^5 = 300 \times \frac{0.020}{0.018} \times \left(\frac{0.15}{0.25}\right)^5 = 25.9\ m$$

Therefore, 25.9 m of 150 mm diameter pipe is equivalent to 300 m of 250 mm diameter pipe.

Obviously, a compound pipe may be replaced by a pipe of uniform diameter and of the same length as that of a compound pipe, such that the loss of head and discharge are the same in both the cases. The new pipe of uniform diameter is called *equivalent pipe* and its diameter is called *equivalent size of the pipe.*

Let L_1, L_2 and L_3 be the lengths and d_1, d_2 and d_3 be the diameters of the three pipes of a compound pipe. Assume that the three pipes have the same friction factor f as that of the compound pipe. Then, the total head loss in the compound pipe is given by

$$H = f\frac{L_1}{d_1}\frac{V_1^2}{2g} + f\frac{L_2}{d_2}\frac{V_2^2}{2g} + f\frac{L_3}{d_3}\frac{V_3^2}{2g} = \frac{8Q^2 f}{\pi^2 g}\left(\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right) \tag{9.10}$$

If L is the length and d is the diameter of the equivalent pipe that would give the same discharge for the same head loss due to friction, then total head loss in the equivalent pipe

$$H = \frac{8Q^2 fL}{\pi^2 g d^5} \qquad (9.11)$$

where $L = L_1 + L_2 + L_3$ = length of the equivalent pipe. Equating Eqs.(9.10) and (9.11) and simplifying, we have

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5} \qquad (9.12)$$

Equation (9.12) gives the uniform diameter of an equivalent pipe which has the same length, same head loss and same discharge as the compound pipe.

It is also sometimes required to determine the length L of a pipe of uniform diameter d which is available for use to replace the compound pipe. To have the same discharge and same loss of head due to friction, the length of such a pipe would be

$$L = d^5\left(\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right) \qquad (9.13)$$

Remember that in this case, $L \neq L_1 + L_2 + L_3$.

Example 9.4

A compound pipe line is made up of pipes 45 cm in diameter for 900 m, 37.5 cm in diameter for 450 m and 30 cm in diameter for 300 m. (a) It is required to be replaced by a pipe of uniform diameter. Find the diameter of the new pipe, assuming the length to remain the same. (b) If we want to replace the compound pipe with a pipe of 40 cm diameter, what would be the length of this pipe?

Solution We have, $d_1 = 45$ cm $= 0.45$ m, $L_1 = 900$ m, $d_2 = 37.5$ cm $= 0.375$ m, $L_2 = 450$ m, $d_3 = 30$ cm $= 0.30$ m, $L_3 = 300$ m

(a) $L = L_1 + L_2 + L_3 = 900 + 450 + 300 = 1650$ m

Using the equation

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

we obtain

$$\frac{1650}{d^5} = \frac{900}{0.45^5} + \frac{450}{0.375^5} + \frac{300}{0.30^5} = 232911$$

$$\therefore d = \left(\frac{1650}{232911}\right)^{1/5} = 0.3716\ m = 37.16\ cm$$

(b) d = 40 cm

$$\therefore L = d^5\left(\frac{L_1}{d_1^5}+\frac{L_2}{d_2^5}+\frac{L_3}{d_3^5}\right)$$

$$= 0.40^5\left(\frac{900}{0.45^5}+\frac{450}{0.375^5}+\frac{300}{0.30^5}\right) = 0.01024 \times 232911 = 2385.01 \text{ m}$$

9.4 PIPES IN PARALLEL

A combination of two or more pipes connected as in Fig. 9.3, so that the flow divided among the pipes and then is joined again, is a parallel-pipe system. **In series pipes, the discharge is same in all the pipes and head losses are cumulative. But in parallel pipes, the head losses are the same and discharges are cumulative.**

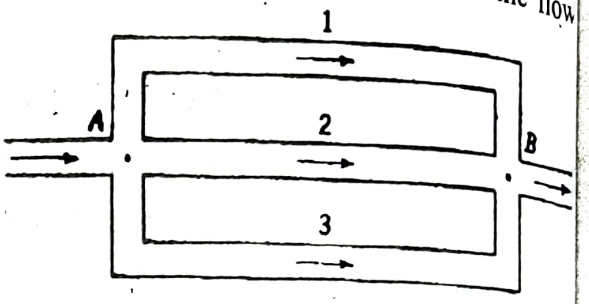


Fig. 9.3 Parallel pipe system

To solve a problem involving parallel-pipe system, the following two conditions must be satisfied:

1. The head loss in each parallel pipe is the same, i.e. $h_{f1} = h_{f2} = h_{f3}$.
2. The discharge in the main line is equal to the sum of the discharges in the parallel lines, i.e. $Q = Q_1 + Q_2 + Q_3$.

A mathematical determination of the division of flow can be made by use of the Darcy-Weisbach formula

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = \frac{8Q^2 fL}{\pi^2 g d^5} \tag{9.14}$$

so that

$$Q = \pi\sqrt{g}\,\frac{\sqrt{h_f}\,d^{5/2}}{\sqrt{f}\sqrt{L}} \times \frac{1}{\sqrt{8}} \tag{9.15}$$

If only two branches 1 and 2 are in parallel, then for a given h_f

$$\frac{Q_1}{Q_2} = \left(\frac{d_1}{d_2}\right)^{5/2} \times \sqrt{\frac{f_2}{f_1}} \times \sqrt{\frac{L_2}{L_1}} \tag{9.16}$$

If the diameters and the lengths of the two branches are known and the friction factors are known or assumed, then Eq. (9.16) can be expressed as

$$Q_1 = FQ_2 \tag{9.17}$$

where F is a known quantity. Moreover,

$$Q = Q_1 + Q_2 \tag{9.18}$$

where Q is the discharge through the approach or the exit pipe. With Q known or assumed, simultaneous solution of Eqs.(9.17) and (9.18) gives Q_1 and Q_2. Then, using these discharges, head losses in pipes 1 and 2 can be computed. These must be equal. If the computations do not show them equal, the discharges should be adjusted by trial until reasonable agreement is obtained.

This method can be extended to any number of branches in parallel. For example, with three branches in parallel as in Fig. 9.3, it is possible to develop the relations

$$Q_2 = F'Q_1 \quad \text{and} \quad Q_3 = F''Q_1 \tag{9.19}$$

which can be combined with the equation

$$Q = Q_1 + Q_2 + Q_3 \tag{9.20}$$

to obtain the flow in each of the three parallel branches.

Two types of problems with parallel-pipe system generally arise:

1. Having given the lengths and the diameters of all the pipes and the allowable head loss, to determine Q.

This type is, in effect, the solution of simple pipe problems for discharge, since the head loss is known. These discharges are added to determine the total discharge.

Example 9.5

In Fig. 9.3, L_1 = 900 m, d_1 = 300 mm, f_1 = 0.021, L_2 = 600 m, d_2 = 200 mm, f_2 = 0.018, L_3 = 1200 m, d_3 = 400 mm, f_3 = 0.019 and the head loss between A and B is 6.62 m. Determine the discharge.

Solution We have, $h_{f1} = h_{f2} = h_{f3} = 6.62$ m. From Eq.(9.6), we obtain for pipe 1

$$h_{f1} = f_1 \frac{L_1}{d_1}\frac{V_1^2}{2g} = f_1 \frac{L_1}{d_1}\frac{Q_1^2}{2g(\pi d_1^2/4)^2} = \frac{8Q_1^2 f_1 L_1}{\pi^2 g d_1^5}$$

or

$$Q_1 = \sqrt{\frac{h_{f1}\times\pi^2\times g\times d_1^5}{8 f_1 L_1}} = \sqrt{\frac{6.62\times\pi^2\times 9.81\times 0.30^5}{8\times 0.021\times 900}} = 0.1015\ m^3/s$$

Similarly,

$$Q_2 = \sqrt{\frac{h_{f2}\times\pi^2\times g\times d_2^5}{8 f_2 L_2}} = \sqrt{\frac{6.62\times\pi^2\times 9.81\times 0.20^5}{8\times 0.018\times 600}} = 0.0487\ m^3/s$$

$$Q_3 = \sqrt{\frac{h_{f3}\times\pi^2\times g\times d_3^5}{8 f_3 L_3}} = \sqrt{\frac{6.62\times\pi^2\times 9.81\times 0.40^5}{8\times 0.019\times 1200}} = 0.1897\ m^3/s$$

$$\therefore Q = Q_1 + Q_2 + Q_3 = 0.1015 + 0.0487 + 0.1897 = 0.3399\ m^3/s$$

2. Having given the total discharge Q and the diameters and the lengths of th pipes, to find the distribution of flow and the head loss.

This type of problem is more complex, as neither the head loss nor the discharge f any pipe is known. Two methods of solution of this problem are as follows.

Method 1 Algebraic solution

1. Determine the division of flow in the branches in parallel. To do so, express t discharges Q_2 and Q_3 in terms of Q_1. Then, using Eq. (9.20), determine Q_1. From Q_1, Q_2 a Q_3 can be obtained.

2. Compute the head losses h_{f1}, h_{f2} and h_{f3}. The loss of head in the parallel branch should be equal.

Method 2 Trial solution

1. Assume a discharge Q_1' through pipe 1.

2. Determine h_{f1}' using the assumed discharge.

3. Using h_{f1}', determine Q_2' and Q_3'.

4. The given total discharge Q is split up among the three pipes in the same proportio as Q_1', Q_2' and Q_3', i.e.

$$Q_1 = \frac{Q_1'}{\sum Q'} Q, \quad Q_2 = \frac{Q_2'}{\sum Q'} Q, \quad Q_3 = \frac{Q_3'}{\sum Q'} Q \tag{9.21}$$

where $\sum Q' = Q_1' + Q_2' + Q_3'$.

5. Check the correctness of these discharges by computing h_{f1}, h_{f2} and h_{f3}.

Example 9.6

In Fig. 9.3, L_1 = 900 m, d_1 = 300 mm, f_1 = 0.021, L_2 = 600 m, d_2 = 200 mm, f_2 = 0.018, L_3 = 1200 m, d_3 = 400 mm and f_3 = 0.019. For a discharge of 0.34 m^3/s, determine th flow through each pipe and the loss of head between A and B.

Solution by Method 1 We have from Eq.(9.16)

$$\frac{Q_2}{Q_1} = \left(\frac{d_2}{d_1}\right)^{5/2} \times \sqrt{\frac{f_1}{f_2}} \times \sqrt{\frac{L_1}{L_2}} = \left(\frac{0.20}{0.30}\right)^{5/2} \times \sqrt{\frac{0.021}{0.018}} \times \sqrt{\frac{900}{600}} = 0.480$$

$$\therefore Q_2 = 0.480 Q_1 \tag{i}$$

Similarly

$$\frac{Q_3}{Q_1} = \left(\frac{d_3}{d_1}\right)^{5/2} \times \sqrt{\frac{f_1}{f_3}} \times \sqrt{\frac{L_1}{L_3}} = \left(\frac{0.40}{0.30}\right)^{5/2} \times \sqrt{\frac{0.021}{0.019}} \times \sqrt{\frac{900}{1200}} = 1.869$$

$$\therefore Q_3 = 1.869 Q_1 \tag{ii}$$

Since $Q = Q_1 + Q_2 + Q_3$, we have

$$0.34 = Q_1 + 0.480Q_1 + 1.869Q_1 = 3.349Q_1$$

$$\therefore Q_1 = \frac{0.34}{3.349} = 0.1015\ m^3/s$$

Then

$$Q_2 = 0.480 \times 0.1015 = 0.0487\ m^3/s$$

and

$$Q_3 = 1.869 \times 0.1015 = 0.1897\ m^3/s$$

Using Eq.(9.6), we have

$$h_{f1} = \frac{8Q_1^2 f_1 L_1}{\pi^2 g d_1^5} = \frac{8 \times 0.1015^2 \times 0.021 \times 900}{\pi^2 \times 9.81 \times 0.30^5} = 6.62\ m$$

So, the head loss between A and B is 6.62 m.

Solution by Method 2 Assume $Q_1' = 0.085$ m^3/s. Then, using Eq.(9.6)

$$h_{f1}' = \frac{8Q_1'^2 f_1 L_1}{\pi^2 g d_1^5} = \frac{8 \times 0.085^2 \times 0.021 \times 900}{\pi^2 \times 9.81 \times 0.30^5} = 4.643\ m$$

$\therefore$ For pipe 2, we get

$$Q_2' = \sqrt{\frac{h_{f1}' \times \pi^2 \times g \times d_2^5}{8 f_2 L_2}} = \sqrt{\frac{4.643 \times \pi^2 \times 9.81 \times 0.20^5}{8 \times 0.018 \times 600}} = 0.0408\ m^3/s$$

and for pipe 3, we obtain

$$Q_3' = \sqrt{\frac{h_{f1}' \times \pi^2 \times g \times d_3^5}{8 f_3 L_3}} = \sqrt{\frac{4.643 \times \pi^2 \times 9.81 \times 0.40^5}{8 \times 0.019 \times 1200}} = 0.1589\ m^3/s$$

The total discharge for the assumed condition is

$$Q_1' + Q_2' + Q_3' = 0.085 + 0.0408 + 0.1589 = 0.2847\ m^3/s$$

But the total discharge is 0.34 m^3/s. So correcting the discharges in proportion, we obtain

$$Q_1 = \frac{0.085}{0.2847} \times 0.34 = 0.1015\ m^3/s$$

$$Q_2 = \frac{0.0408}{0.2847} \times 0.34 = 0.0487\ m^3/s$$

$$Q_3 = \frac{0.1589}{0.2847} \times 0.34 = 0.1898\ m^3/s$$

Now, check the values of h_{f1}, h_{f2} and h_{f3} using Eq.(9.6).

$$h_{f1} = \frac{8Q_1^2 f_1 L_1}{\pi^2 g d_1^5} = \frac{8 \times 0.1015^2 \times 0.021 \times 900}{\pi^2 \times 9.81 \times 0.30^5} = 6.62\,m$$

$$h_{f2} = \frac{8Q_2^2 f_2 L_2}{\pi^2 g d_2^5} = \frac{8 \times 0.0487^2 \times 0.018 \times 600}{\pi^2 \times 9.81 \times 0.20^5} = 6.61\ m$$

$$h_{f3} = \frac{8Q_3^2 f_3 L_3}{\pi^2 g d_3^5} = \frac{8 \times 0.1898^2 \times 0.019 \times 1200}{\pi^2 \times 9.81 \times 0.40^5} = 6.63\ m$$

It is seen that the loss of head in the three branches is almost the same.

Note: Method 1 seems to be better than method 2.