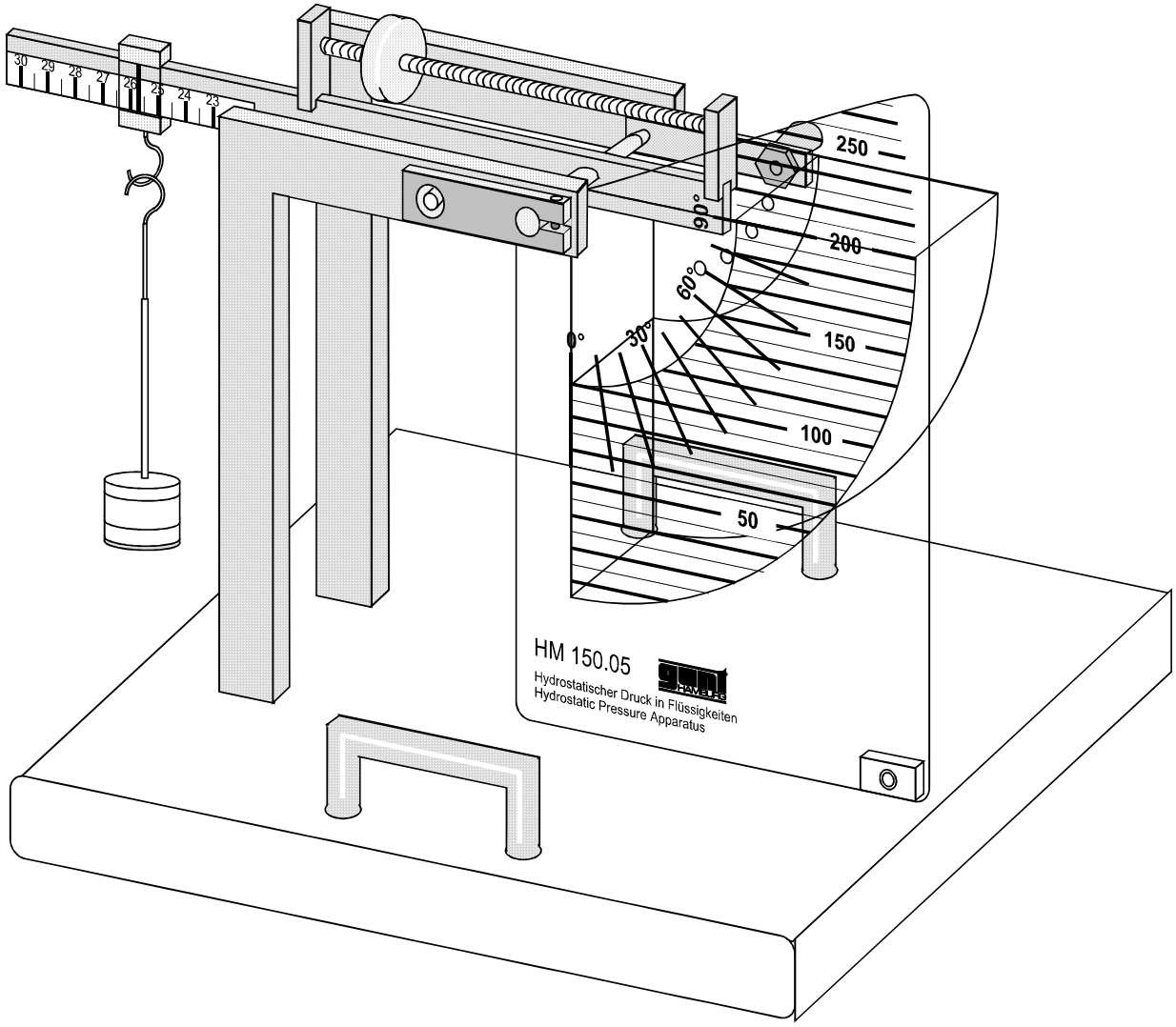
Equipment for Engineering Education

Instruction Manual

HM 150.05 Hydrostatic Pressure Apparatus

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HM 150.05 Hydrostatic Pressure Apparatus



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Instruction Manual

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HM 150.05 Hydrostatic Pressure Apparatus

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HM 150.05 Hydrostatic Pressure Apparatus

1 Introduction

The effect of hydrostatic pressure is of major significance in many areas of engineering, such as shipbuilding, the construction of dykes, weirs and locks, and in sanitary and building services engineering.

With the **HM 150.05 Hydrostatic Pressure Apparatus** the following key topics can be investigated by experimentation:

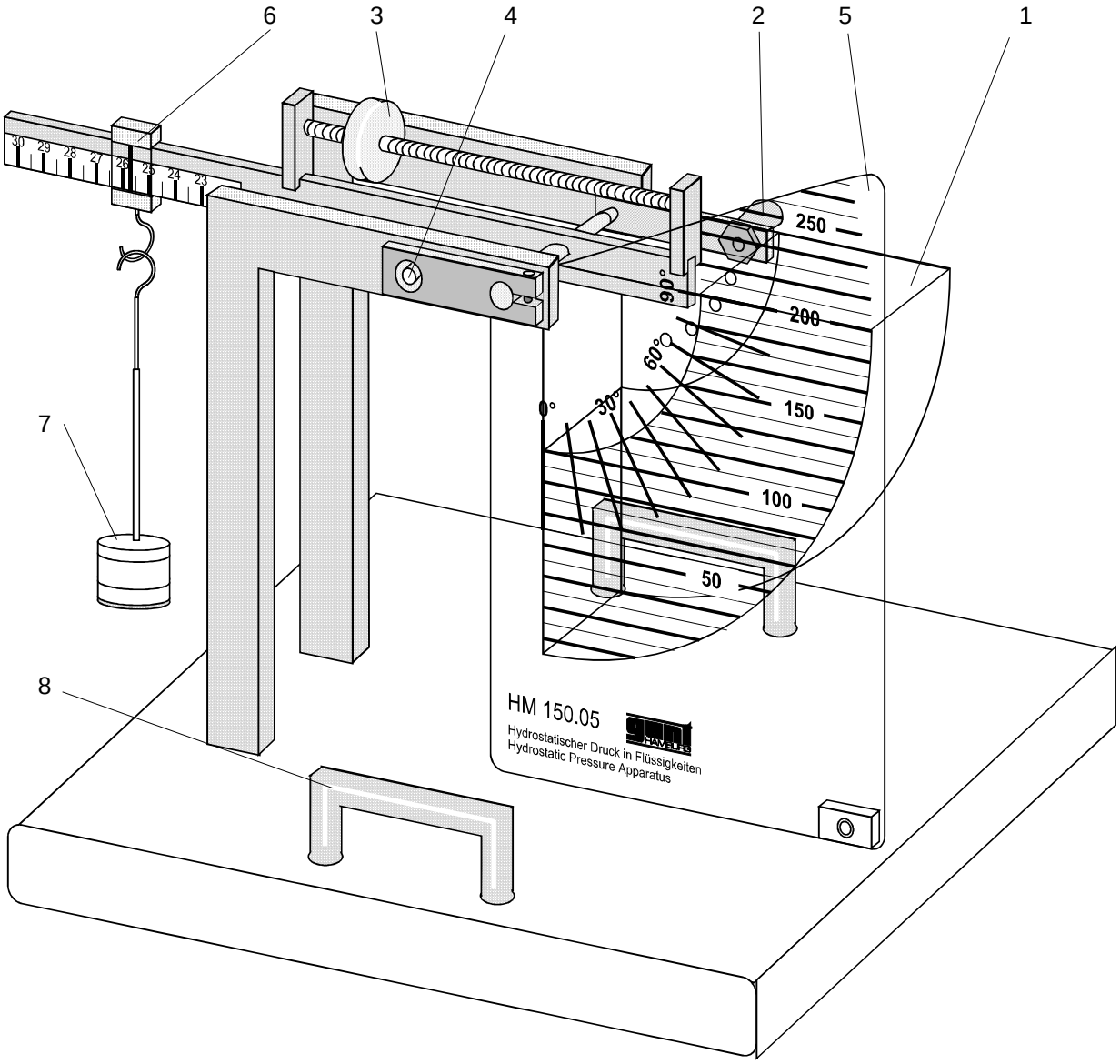
- Pressure distribution in a liquid taking into account gravity
- "Lateral force" of the hydrostatic pressure
- Centre of pressure of the lateral force

T he appendix to this Test Instruction contains prepared worksheets which facilitate methodical evaluation of the results of the experiments.

The apparatus is designed for use in the fields of education.

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2 Unit Description



1. Water vessel
2. Detent
3. Slider
4. Stop pin
5. Water level scale
6. Rider
7. Weights
8. Handles

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With **HM 150.05 "Hydrostatic Pressure Apparatus "**the correlation between the water level and the dependent side pressure can be investigated.

The unit is of robust construction and can be set up quickly. It is therefore highly suitable for everyday use in schools and universities.

The unit can be easily transported by means of two handles.

A transparent measuring vessel with mm scale and a scale with mm increments permits precise water level and lever arm readings.

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2.1 Commissioning

The **"Hydrostatic Pressure Apparatus"** should be placed on a horizontal, waterproof surface . An additional vessel for filling and emptying the water vessel should be provided.

2.2 Care and Maintenance

The unit and its accessories require practically no maintenance. To prevent liming, empty the unit after use and dry it with a soft cloth.

3 Theory on the Centre of Pressure

The hydrostatic pressure of liquids is the "gravitational pressure" p_{hyd}. It rises due to the intrinsic weight as the depth t increases, and is calculated from:

$$p_{hyd} = \rho \cdot g \cdot t. \qquad (1)$$

ρ - Density of water
g - Acceleration due to gravity (g=9,81 m/s^2)
t - Distance from liquid surface

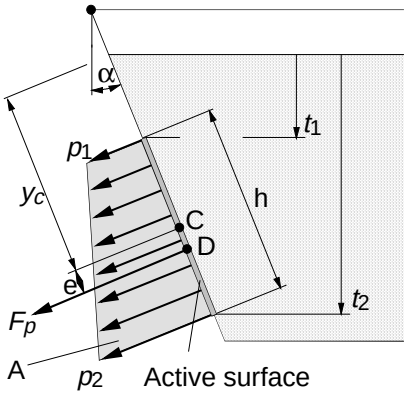
To calculate forces acting on masonry dams or ships' hulls, for example, from the hydrostatic pressure, two steps are required:

- Reduce the pressure load on an active surface down to a resultant force F_p, which is applied at a point of application of force, the "centre of pressure", vertical to the active surface.
- Determine the position of this centre of pressure by determining a planar centre of force on the active surface.

It is first demonstrated how the centre of pressure can be determined. The resultant force F_p is then calculated.

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3.1 Determining the Centre of Pressure

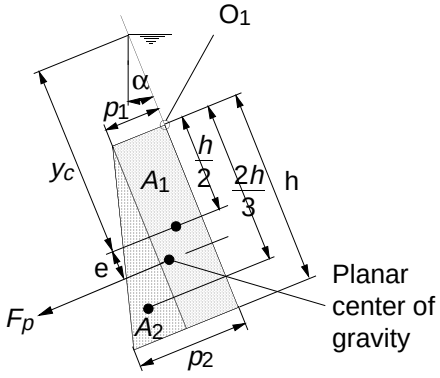


A linear pressure profile is acting on the active surface shown, because the hydrostatic pressure rises proportional to the depth t.

The resultant force F_p is therefore **not** applied at the centre of force C of the active surface, but always slightly below it, at the so-called centre of pressure D! To determine the distance e of the centre of pressure from the planar centre of force, the following model demonstration is used:

Imagine an area A in front of the active surface, formed by the height h and the pressure profile of the hydrostatic pressure p_1-p_2. This area is in the form of a trapezium.

The centre of pressure D lies on the extension of the **planarcentre of force** of this area A. A can be broken down into partial areas A_1 and A_2. The respective planar centres of force are identified by black dots.



A **balance of moments between the areas** is then established around the point O_1 in order to find the common planar centre of force (dynamic effect in direction F_p):

$$\Sigma M^{(O_1)}=0:\ A\cdot\left(\frac{h}{2}+e\right)=A_1\cdot\frac{h}{2}+A_2\cdot\frac{2h}{3} \qquad (2)$$

Where

$$A_1 = p_1\cdot h, \qquad (3)$$

$$A_2=\frac{p_2-p_1}{2}\cdot h \text{ and} \qquad (4)$$

$$A = A_1+A_2 \qquad (5)$$

the result is

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$$e = \frac{1}{6} h \cdot \frac{p_2 - p_1}{p_2 + p_1}. \quad (6)$$

With the hydrostatic pressure

$$p_2 = \rho g \cos\alpha \cdot (y_c + \frac{h}{2}) \text{ and} \quad (7)$$

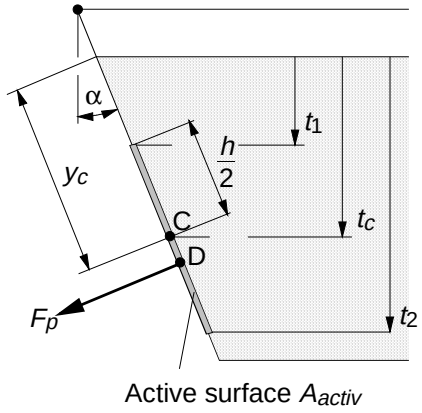
$$p_1 = \rho g \cos\alpha \cdot (y_c - \frac{h}{2}) \text{ the result is} \quad (8)$$

$$e = \frac{1}{12} \cdot \frac{h^2}{y_c}. \quad (9)$$

e is the distance of the centre of pressure from the planar centre of force of the active surface which we are looking for.

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3.2 Determining the Resultant Force



Active surface A_{activ}

The hydrostatic pressure acting on the active surface can be represented as resultant force F_p, of which the line of application leads through the centre of pressure D. The size of this resultant force corresponds to the hydrostatic pressure at the **planar centre of force C of the active surface**:

$$p_c = \rho \cdot g \cdot t_c \quad (10)$$

p_c - Hydrostatic pressure at the planar centre of force of the active surface

t_c - Vertical distance of the **planar centre of force** from the surface of the liquid

In visual terms, the pressure at the planar centre of force corresponds to precisely the mean value between the highest and lowest pressure, becau-

se of the linear pressure distribution.
If the wall is tilted by an angle α:

$$p_c = \rho \cdot g \cdot \cos\alpha \cdot y_c. \quad (11)$$

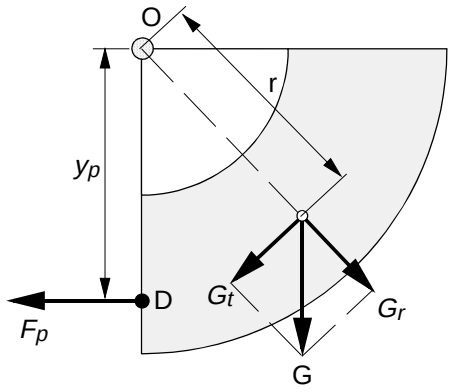
The resultant force F_p can now be calculated:

$$F_p = p_c \cdot A_{active} \quad (12)$$

Important! To calculate the resultant force the planar centre of force of the active surface is applied, but the line of application of the resultant force F_p runs through the centre of pressure (see section 3.1).

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3.3 Mode of Functioning of the HM 150.05 unit



The unit's water vessel is designed as a ring segment with constant cross-section. The force due to weight G of the water always produces the same moment of momentum referred to the centre of motion O as the resultant force F_p of the active surface running through the centre of pressure D. Consequently, this apparatus can be used to determine the force of pressure F_p and the centre of pressure.
To illustrate the point, imagine the ring segment completely filled. The force due to weight G applied to the centre of volume of the water can be broken down into two components:

- A radially applied component **G_r** running precisely through the centre of motion, and
- a tangential component **G_t** with a lever arm r acting on the centre of motion O.

The radial component G_r exerts no momentum on centre of motion O, because its lever arm is zero. Now, regardless of the water level:

$$F_p \cdot y_p = G_t \cdot r \qquad (13)$$

That is to say, **the force due to weight G of the water volume always exerts the same moment of momentum as the force F_p at the centre of pressure D**.

The derivation of (13) leads via determination of the centre of force of a ring segment and its volume.

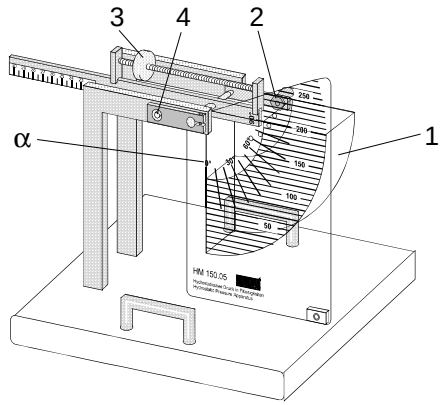
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4 Experiments Relating to the Centre of Pressure

4.1 Centre of Pressure with Vertical Positioning of the Water Vessel

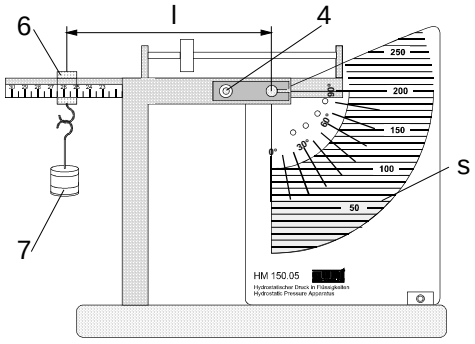
4.1.1 Performing the Experiment

4.1.1.1 Counterbalancing the Water Vessel



- Set the water vessel (1) to an angle of $\alpha = 0°$ using the detent (2) as shown
- Counterbalance the unit with a rotating slider (3): The stop pin (4) must be precisely in the middle of the hole for this

4.1.1.2 Performing the Measurement



- Mount the rider (6), set the lever arm on the scale (e.g. l=150 mm)
- Top up with water until the unit is balanced (stop pin (4) at centre of hole)
- Read off water level s and enter it in the prepared worksheet (see Appendix)
- Increase the appended weights (7) in increments of 0.5 - 1 N and repeat the measurement

4.1.2 Evaluating the experiment

Measured values:

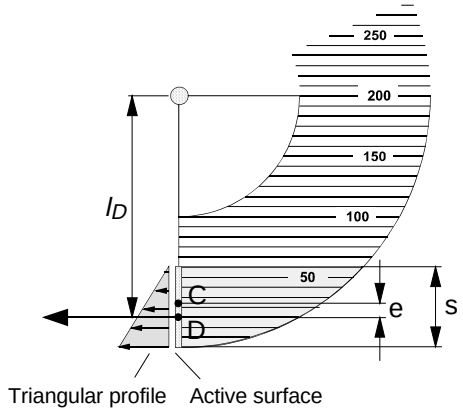
s - Water level reading

l - Lever arm of the force due to weight

F_G - Force due to weight of the appended weights

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4.1.2.1 Determining the Centre of Pressure



At a water level s, below the 100 mm mark, the height of the active surface changes with the water level. If the water level is above that mark, the height of the active surface is always 100 mm.

Meaning:

s - Water level

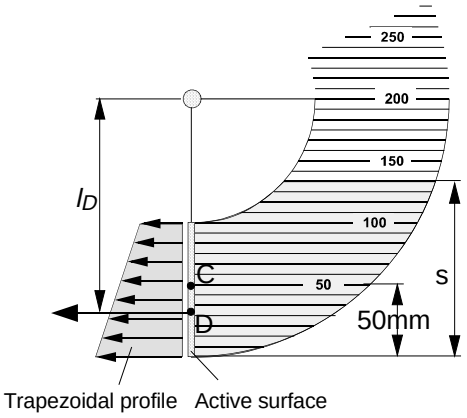
e - Distance of centre of pressure D from planar centre of force C of the active surface

l_D - Distance to centre of motion of the unit:

For a water level s < 100 mm:
(pressure has a triangular profile)

$$e = \frac{1}{6} \cdot s \qquad (1)$$

$$l_D = 200mm - \frac{1}{3} \cdot s \qquad (2)$$



For a water level s > 100 mm:
(pressure has a trapezoidal profile)

$$e = \frac{1}{12} \cdot \frac{(100mm)^2}{s - 50mm} \qquad (3)$$

$$l_D = 150mm + e \qquad (4)$$

4.1.2.2 Determining the Resultant Force

The resultant force corresponds to the hydrostatic pressure at the planar centre of force C of the active surface. Thus, the height of water level s must again be differentiated:

Meaning:

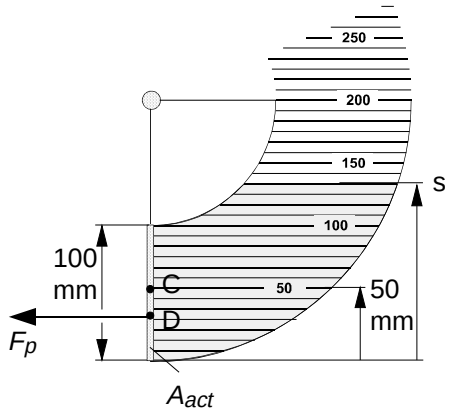
A_{act} - Superficial content of active surface

b=75 mm - Width of liquid vessel

p_C - Hydrostat. pressure at planar centre of force

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F_p - Resultant force for hydrostat. pressure on active surface:



For s < 100 mm:
(triangular profile)

$$p_c = \rho \cdot g \cdot \frac{s}{2} \text{ and } A_{act} = s \cdot b \qquad (5)$$

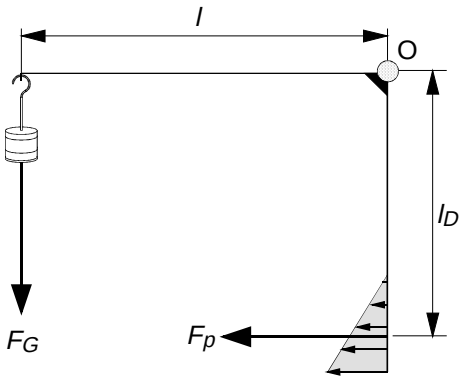
For s>100 mm:
(trapezoidal profile)

$$p_c = \rho \cdot g \cdot (s - 50mm) \text{ and } A_{act} = 100mm \cdot b \qquad (6)$$

The resultant force is produced as

$$F_p = p_c \cdot A_{act} . \qquad (7)$$

4.1.2.3 Balance of Moments



Calculated variables:

F_G - Appended weight

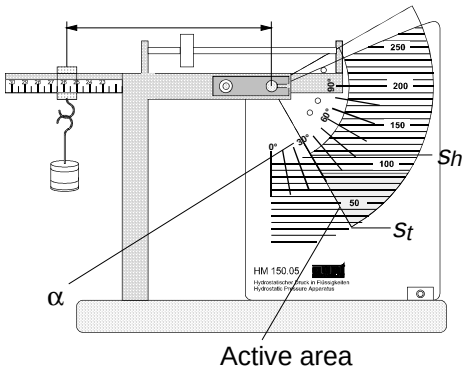
l - Lever arm of appended weight referred to centre of motion O

To check the theory, a balance of moments around the centre of motion O can be established and checked:

$$\Sigma M^{(O)}=0: F_G \cdot l = F_p \cdot l_D \qquad (8)$$

4.2 Centre of Pressure with Water Vessel Tilted

4.2.1 Performing the Experiment



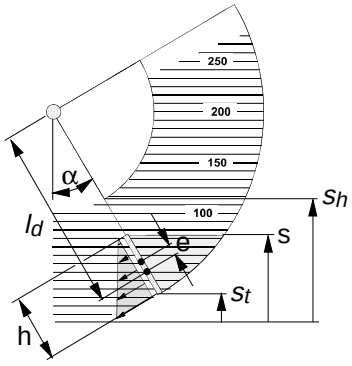
- Set an angle α and counterbalance the water vessel as described under 4.1.1.1.
- Enter the characteristic values in the prepared worksheet: Lowest water level s_t and highest water level s_h of the active surface
- Perform the measurement as described under 4.1.1.2.

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4.2.2 Evaluating the experiment

The difference between evaluation of the tilted vessel and that of the vertical vessel lies in the translation of the water levels onto the tilted active surface: A factor cosα must be taken into account here.

4.2.2.1 Determining the centre of pressure



When the water vessel is at a tilt, too, a triangular pressure profile is produced when the water level is below s_h; above that level a trapezoidal profile is produced.

Measured values:

s - Water level reading

α - Tilt angle of vessel

Meaning:

s_t - Water level at lowest point of vessel

s_h - Water level of active surface at rim

e - Position of centre of pressure

h - Height of active surface

l_D - Distance between centre of pressure/centre of motion

For a water level $s < s_h$ a triangular profile as follows applies:

$$h = \frac{s - s_t}{\cos\alpha} \qquad (9)$$

$$e = \frac{1}{6} \cdot h \qquad (10)$$

$$l_D = 200mm - \frac{1}{3} \cdot h \qquad (11)$$

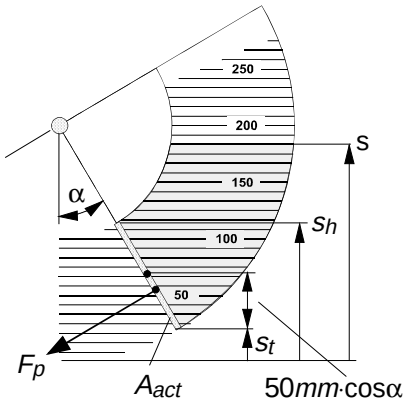
For a water level $s > s_h$ a trapezoidal profile as follows applies:

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$$e = \frac{1}{12} \cdot \frac{(100mm)^2}{\frac{s - s_t}{\cos\alpha} - 50mm} \qquad (12)$$

$$l_D = 150mm + e \qquad (13)$$

4.2.2.2 Determining the resultant force



Meaning:

A_{act} - Superficial content of active surface

b=75 mm - Width of liquid vessel

p_c - Hydrostat. pressure at planar centre of force of active surface

For $s<s_h$ with h from section 4.2.2.1:

$$p_c = \rho \cdot g \cdot \frac{s - s_t}{2} \text{ and } A_{act} = h \cdot b \qquad (14)$$

For $s>s_h$ the trapezoidal profile as follows applies:

$$p_c = \rho \cdot g \cdot (s - s_t - 50mm \cdot \cos\alpha)\ , \qquad (15)$$

$$A_{act} = 100mm \cdot b \qquad (16)$$

The resultant force is produced as

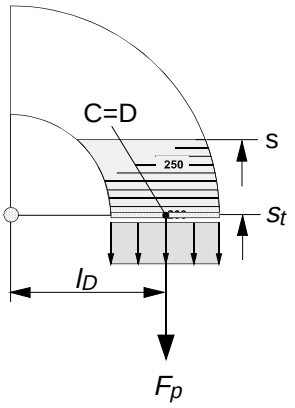
$$F_p = p_c \cdot A_{act}\ . \qquad (17)$$

4.2.2.3 Balance of moments

The results can be checked with the balance of moments, as described in section 4.1.2.3.

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4.3 Centre of pressure with 90° positioning of the water vessel



The angle α=90° represents a special case. The resultant pressure profile has the form of a triangle, because the hydrostatic pressure **is equal** at every point on the active surface.

For this reason, the centre of pressure C lies precisely at the planar centre of force D of the active surface

$$e = 0 \tag{18}$$

and has the lever arm

$$l_D = 150mm. \tag{19}$$

The resultant force is produced as

$$F_p = \rho \cdot g \cdot (s - s_t) \cdot (100mm \cdot b) \tag{20}$$

The result can be checked with the balance of moments from section 4.1.2.3.

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5 Appendix

5.1 Worksheet for Centre of Pressure

Angle α [°]	Lowest water level s_t [mmWC]	Highest water level s_h [mmWC]

Lever arm l [mm]	Appended weight F_G [N]	Water level reading s [mm]	Calculated lever arm l_D [mm]	Resultant force F_p [N]

Angle α [°]	Lowest water level s_t [mmHOW]	Highest water level s_h [mmHOW]

Lever arm l [mm]	Appended weight F_G [N]	Water level reading s [mm]	Calculated lever arm l_D [mm]	Resultant force F_p [N]

Angle α [°]	Lowest water level s_t [mmWC]	Highest water level s_h [mmWC]

Lever arm l [mm]	Appended weight F_G [N]	Water level reading s [mm]	Calculated lever arm l_D [mm]	Resultant force F_p [N]

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5.2 Symbols and Units

ρ	Density of liquids	g/cm^3
p_{hyd}	Hydrostatic pressure	mmHOW
p_c	Hydrostatic pressure in planar centre of gravity of active surface	mmHOW
F_p	Resultant force in centre of pressure	N
F_G	Force due to weight of appended weights	N
G	Force due to weight of water in measuring vessel	N
h	Height of active surface	mm
b	Width of liquid container	mm
s	Water level reading/scale	mm
s_h	Water level at highest point of active surface	mm
s_t	Water level at lowest point of measuring vessel	mm
e	Distance between centre of pressure and planar centre of gravity	mm
l_D	Lever arm of resultant force F_p	mm
l	Lever arm of force due to weight	mm
y_p	Distance between centre of pressure and liquid level along the active surface	mm
y_c	Distance between planar centre of gravity and liquid level along the active surface	mm
A_{act}	Superficial content of active surface	mm^2

5.3 Technical data

Dimensions: LxWxH	400x500x360	mm^3
Capacity of measuring vessel:	approx. 1.8	l
Weight:	approx. 10	kg

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