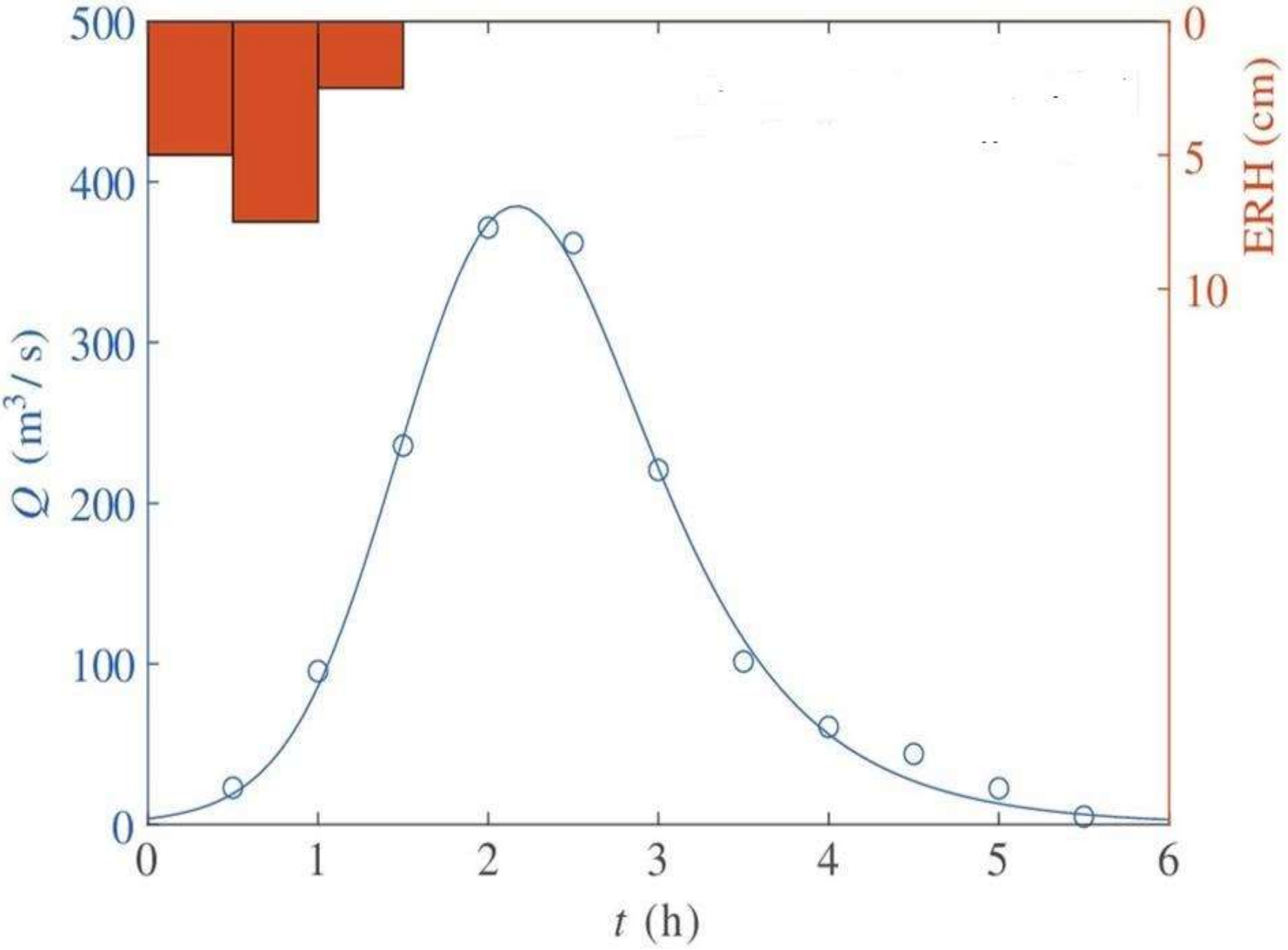
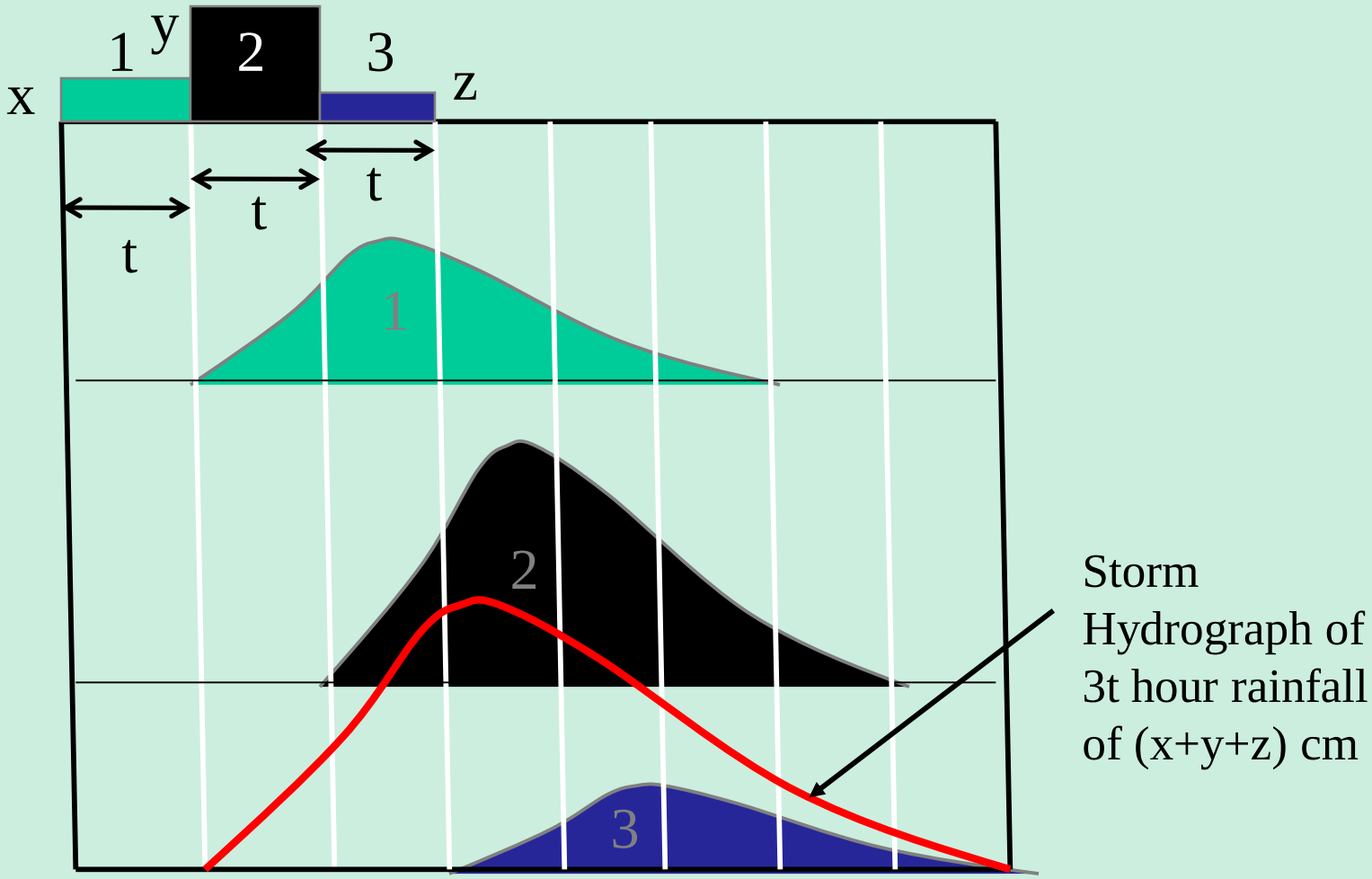
Hydrograph for complex storm from unit hydrograph



Hydrograph Convolution

Add and Lag Method



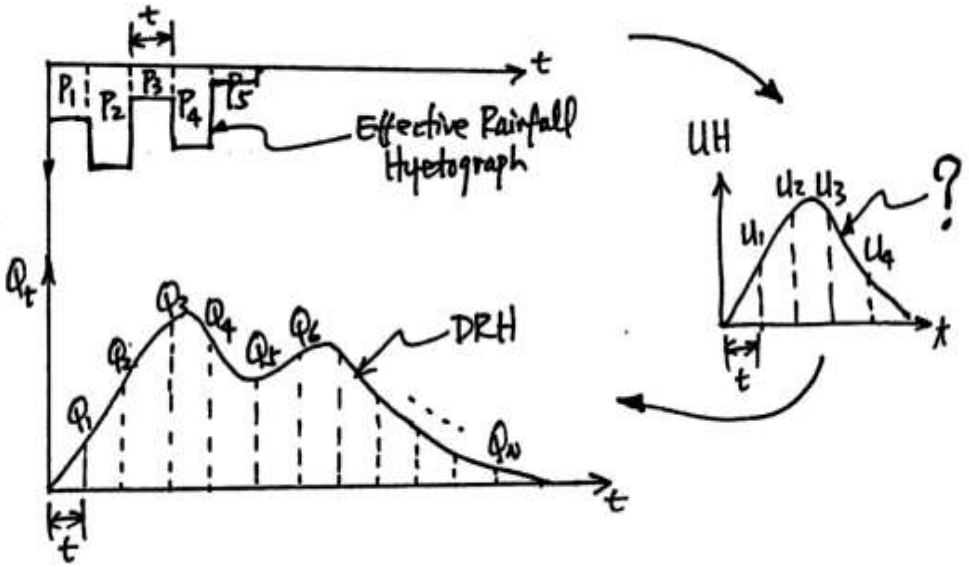
Add up the ordinates of all three to produce storm hydrograph. The durations of the rainfalls must be same. (Note: break in rainfall)

Hydrograph Convolution

Now that we have developed the unit hydrograph, we can use it to determine the direct runoff hydrograph for any rainfall amount with any time distribution. The process is called **convolution**.

Example

Determine the DRO hydrograph from two 1-hr rainfall in a catchment as shown in the table and also calculate the peak flow.

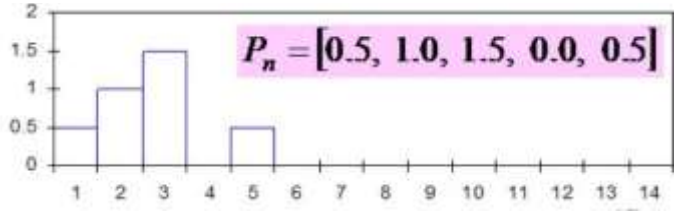


Time Interval (1 hr)	Rainfall (cm)	Unit Hydrograph (cumec)
1	1.3	10
2	0.5	46
3		51.6
4		199.4
5		1060.7
6		299.4
7		128.2
8		117.4
9		37.5
10		14.7
11		5.2
12		0

Time Interval (1 hr)	Rainfall (cm)	Unit Hydrograph (cumec)	UH x 1.3	UH x 0.5	Direct Runoff (cumec)
1	1.3	10	13	0	**13**
2	0.5	46	59.8	5	**64.8**
3		51.6	67.08	23	**90.08**
4		199.4	259.33	25.8	**285.13**
5		1060.7	1378.91	99.7	**1478.61**
6		299.4	389.22	530.35	**919.57**
7		128.2	166.66	149.75	**316.41**
8		117.4	152.62	64.1	**216.72**
9		37.5	48.75	58.7	**107.45**
10		14.7	19.11	18.75	**37.86**
11		5.2	6.76	7.35	**14.11**
12		0	0	2.6	**2.6**

Peak discharge





Hydrograph from a complex storm

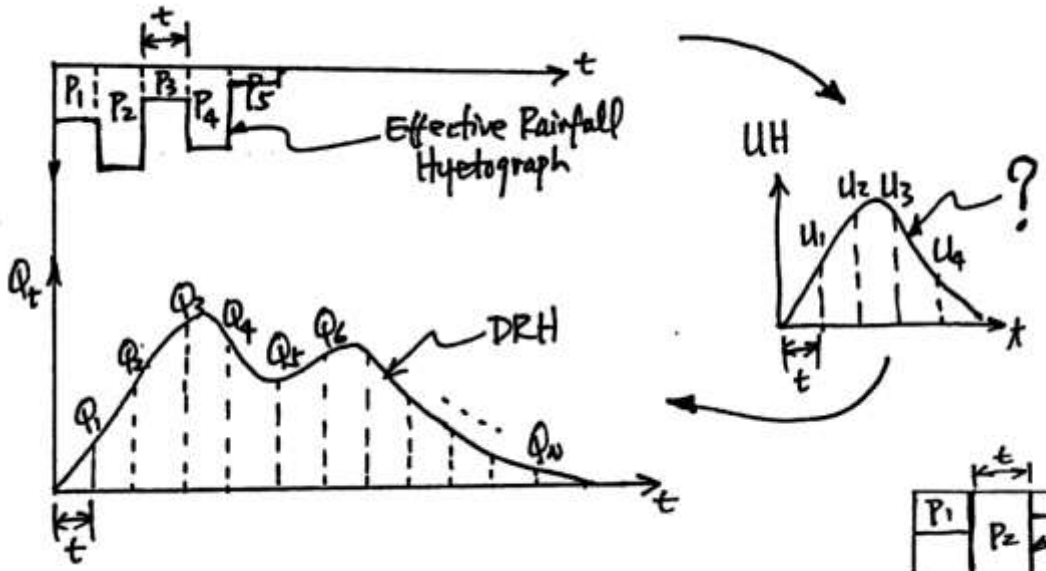
Rainfall

Hydrograph

	(1) Time (n hr)	(2) P_1U_n (cfs)	(3) P_2U_n (cfs)	(4) P_3U_n (cfs)	(5) P_4U_n (cfs)	(6) P_5U_n (cfs)	(7) Q_n (cfs)
0	0	0					0
100	1	50	0				50
320	2	160	100	0			260
450	3	225	320	150	0		695
370	4	185	450	480	0	0	1115
250	5	125	370	675	0	50	1220
160	6	80	250	555	0	160	1045
90	7	45	160	375	0	225	805
40	8	20	90	240	0	185	535
0	9	0	40	135	0	125	300
	10		0	60	0	80	140
	11			0	0	45	45
	12				0	20	20
	13					0	0

Unit Hydrograph

Unit Hydrograph from complex storm (Deconvolution)

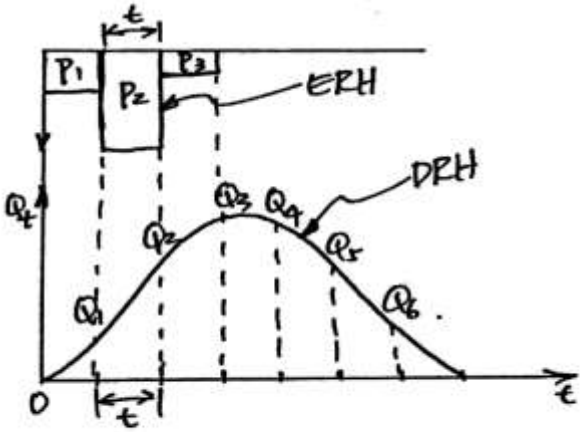


The number of unit hydrograph ordinates (J) is given by:

$J = N - M + 1$ where:

N = ordinates of given hydrograph with same interval as rainfall (t); and

M = number of rainfalls with interval t.



M=3, N=6,

J=N-M+1=6-3+1=4

$Q_1 = P_1 u_1$

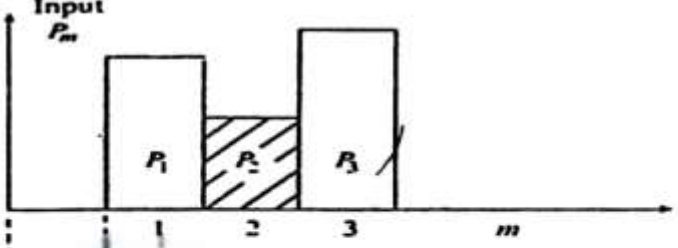
$Q_2 = P_2 u_1 + P_1 u_2$

$Q_3 = P_3 u_1 + P_2 u_2 + P_1 u_3$

$Q_4 = P_4 u_1 + P_3 u_2 + P_2 u_3 + P_1 u_4$

$Q_5 = P_5 u_1 + P_4 u_2 + P_3 u_3 + P_2 u_4 + P_1 u_5$

$Q_6 = P_6 u_1 + P_5 u_2 + P_4 u_3 + P_3 u_4 + P_2 u_5 + P_1 u_6$

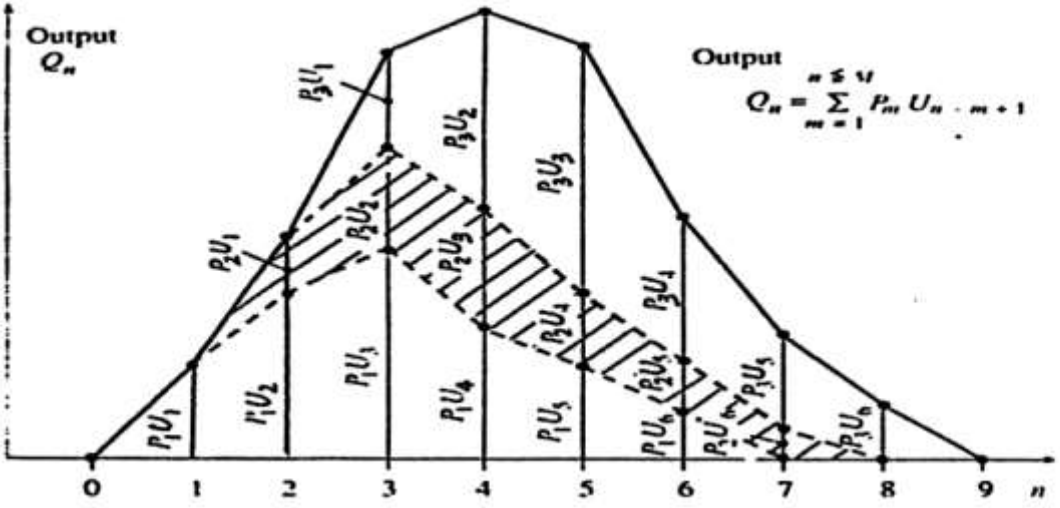


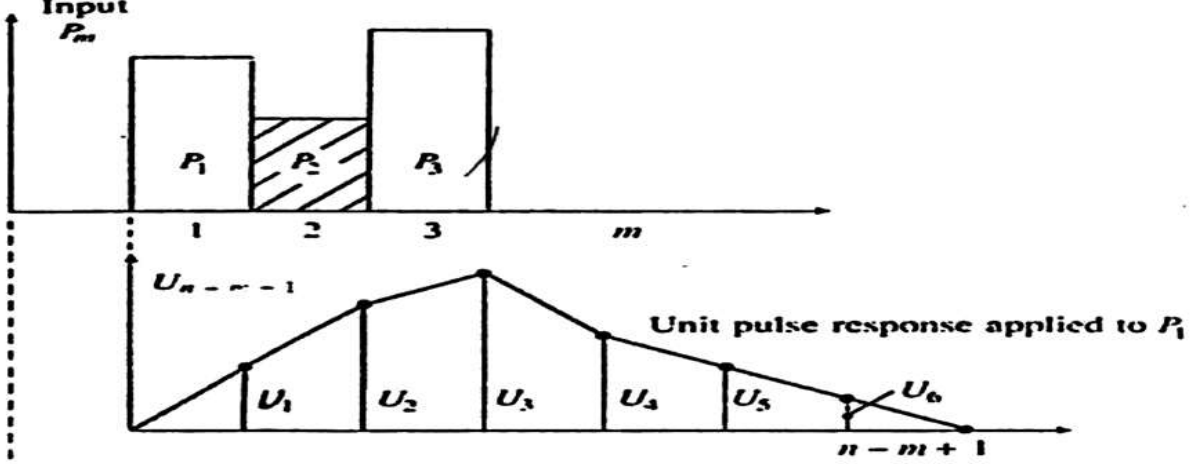
$$Q_1 = P_1U_1$$

$$Q_2 = P_2U_1 + P_1U_2$$

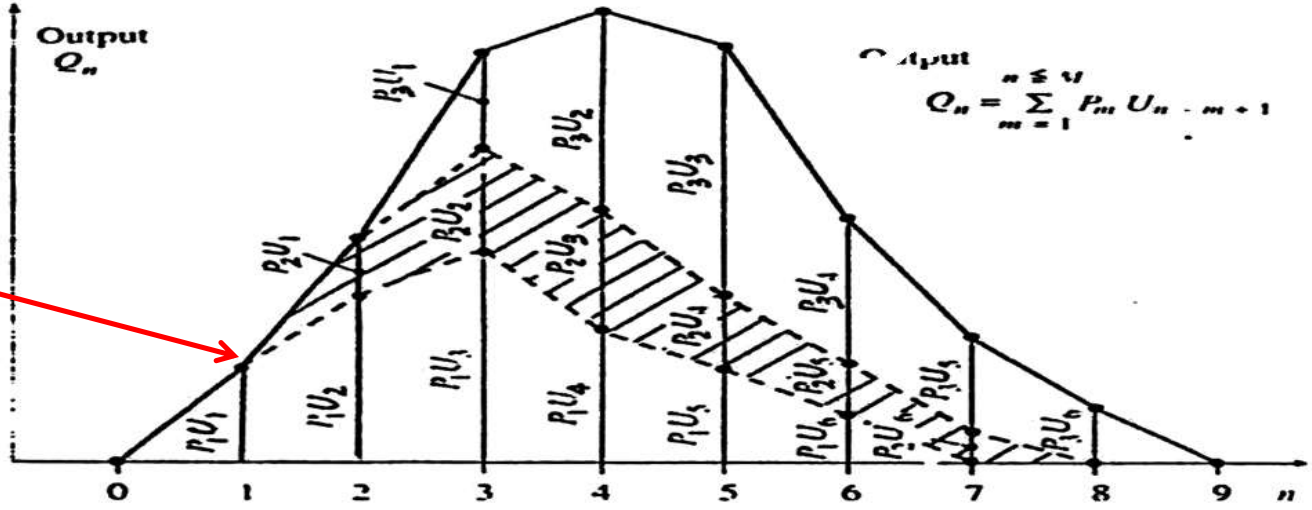
$$Q_3 = P_3U_1 + P_2U_2 + P_1U_3$$

$$Q_M = P_MU_1 + P_{M-1}U_2 + \ldots\ldots + P_1U_M$$

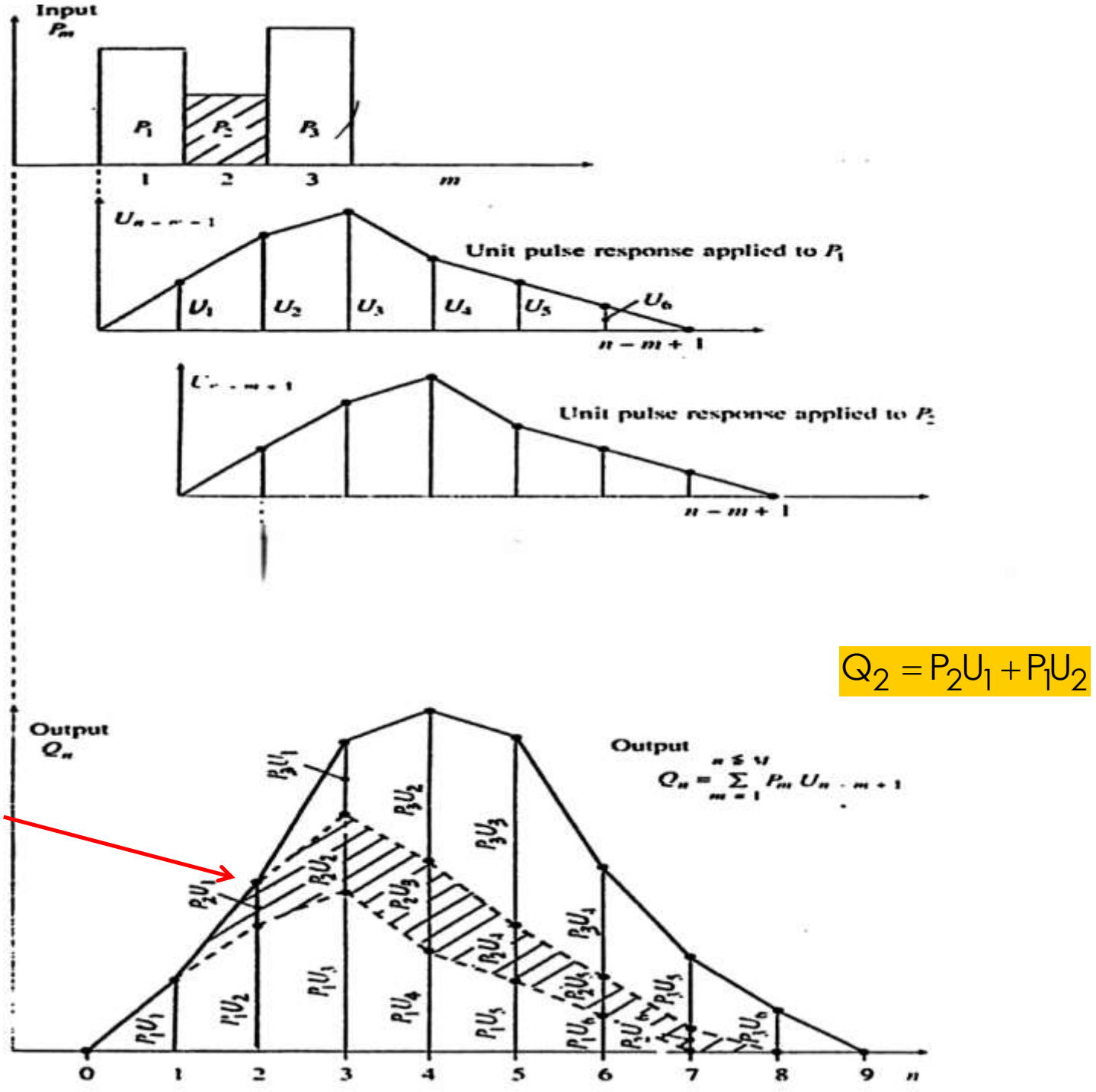



Input
P_m
P_1
P_2
P_3
1
2
3
m
U_{n-m+1}
Unit pulse response applied to P_1
U_1
U_2
U_3
U_4
U_5
U_6
$n-m+1$

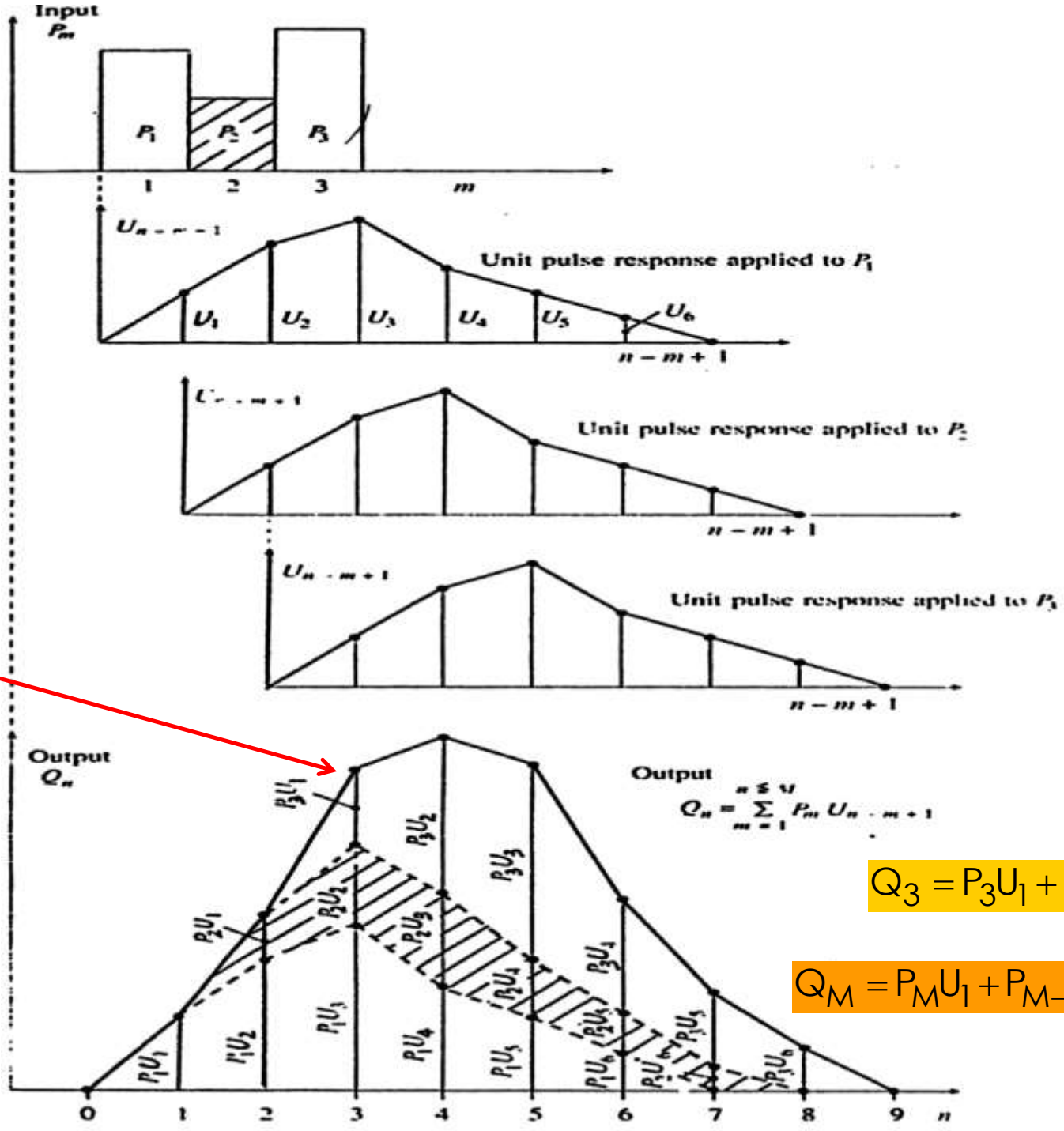
$Q_1 = P_1U_1$


Output
Q_n
$Q_n = \sum_{m=1}^{n \le 9} P_m U_{n-m+1}$
P_1U_1
P_1U_2
P_1U_3
P_1U_4
P_1U_5
P_1U_6
P_2U_1
P_2U_2
P_2U_3
P_2U_4
P_2U_5
P_3U_1
P_3U_2
P_3U_3
P_3U_4
P_3U_5
P_3U_6
0
1
2
3
4
5
6
7
8
9
n

Q_1


Input
P_m
P_1
P_2
P_3
1
2
3
m
Unit pulse response applied to P_1
U_1
U_2
U_3
U_4
U_5
U_6
$n-m+1$
Unit pulse response applied to P_2
$n-m+1$
Output
Q_n
Output
$Q_n = \sum_{m=1}^{n \le N} P_m U_{n-m+1}$
Q_2
P_1U_1
P_1U_2
P_1U_3
P_1U_4
P_1U_5
P_1U_6
P_2U_1
P_2U_2
P_2U_3
P_2U_4
P_2U_5
P_3U_1
P_3U_2
P_3U_3
P_3U_4
P_3U_5
P_3U_6
0
1
2
3
4
5
6
7
8
9
n

$$Q_2 = P_2U_1 + P_1U_2$$



Q_3

$$Q_3 = P_3U_1 + P_2U_2 + P_1U_3$$

$$Q_M = P_MU_1 + P_{M-1}U_2 + \ldots\ldots + P_1U_M$$

Example

There are 3 pulses of rainfall so M = 3. There are 13 pulses of observed direct runoff so N = 13. The total number of unit hydrograph ordinates are N – M +1 == 13 – 3 + 1 = 11 ordinates. So we have to solve 11 linear equations.

Hour	Excess Rainfall (inches)	Observed Direct Discharge (ft^3/s)	Unit Hydro-graph (ft^3/s)
1	0.5	5	10
2	1.2	35	46
3	0.9	90	51.6
4		203	199.4
5		816	1060.7
6		1602	299.4
7		1378	128.2
8		482	117.4
9		275	37.5
10		158	14.7
11		54	5.2
12		47	
13		12	

EQ 1. $5 = 0.5\ U_1$ $\boldsymbol{U_1}$ = 5/0.5 = **10 cfs direct runoff / inch excess rainfall**

EQ 2. $35 = 1.2\ U_1 + 0.5\ U_2$

$\boldsymbol{U_2} = (35 – 1.2\ U_1)/0.5 = (35 – 1.2(10))/0.5 =$ **46**

EQ 3. $90 = 0.9\ U_1 + 1.2\ U_2 + 0.5\ U_3$

$\mathbf{U_3} = (90 – 0.9U_1 – 1.2U_2)/0.5 = (90 – 0.9(10) – 1.2(46))/0.5 =$ **51.6**

EQ 4. $203 = 0.9U_2 + 1.2U_3 + 0.5U_4$

$\mathbf{U_4} = (203 - 0.9U_2 - 1.2U_3)/0.5 = (203 – 0.9(46) – 1.2(51.6))/0.5 =$ **199.4**

EQ 5. $816 = 0.9U_3 + 1.2U_4 + 0.5U_5$

$\mathbf{U_5} = (816 -0.9U_3 - 1.2U_4)/0.5 = (816 – 0.9(51.6) – 1.2(199.4))/0.5 =$ **1060.7**

EQ 6. $1602 = 0.9U_4 + 1.2U_5 + 0.5U_6$

$\mathbf{U_6} = (1602 - 0.9U_4 - 1.2U_5)/0.5 = (1602 - 0.9(199.4) – 1.2(1060.7)/0.5 =$ **299.4**

EQ 7. $1378 = 0.9U_5 + 1.2U_6 + 0.5U_7$

$\mathbf{U_7} = (1378 - 0.9U_5 - 1.2U_6)/0.5 = (1378 – 0.9(1060.7) – 1.2(299.4)/0.5 =$ **128.2**

EQ 8. $482 = 0.9U_6 + 1.2U_7 + 0.5U_8$

$\mathbf{U_8} = (482 - 0.9U_6 - 1.2U_7)/0.5 = (482 – 0.9(299.4) – 1.2(128.2)/0.5 =$ **117.4**

EQ 9. $275 = 0.9U_7 + 1.2U_8 + 0.5U_9$

$\mathbf{U_9} = (275 - 0.9U_7 - 1.2U_8)/0.5 = (275 – 0.9(128.2) – 1.2(117.4)/0.5 =$ **37.5**

EQ 10 $158 = 0.9U_8 + 1.2U_9 + 0.5U_{10}$

$\mathbf{U_{10}} = (158 - 0.9U_8 - 1.2U_9)/0.5 = (158 – 0.9(117.4) – 1.2(37.5)/0.5 =$ **14.7**

EQ 11 $54 = 0.9U_9 + 1.2U_{10} + 0.5U_{11}$

$\mathbf{U_{11}} = (54 - 0.9U_9 - 1.2U_{10})/0.5 = (54 – 0.9(37.5) – 1.2(14.7) / 0.5 =$ **5.2**

Unit Hydrograph

UH of Various Durations

There are two methods to **change the duration of unit hydrograph:**

(1) the superposition method and

(2) the S-hydrograph (S-Curve) method.

The superposition method converts an X-hour unit hydrograph into an nX – hour unit hydrograph, in which "n" is an integer.

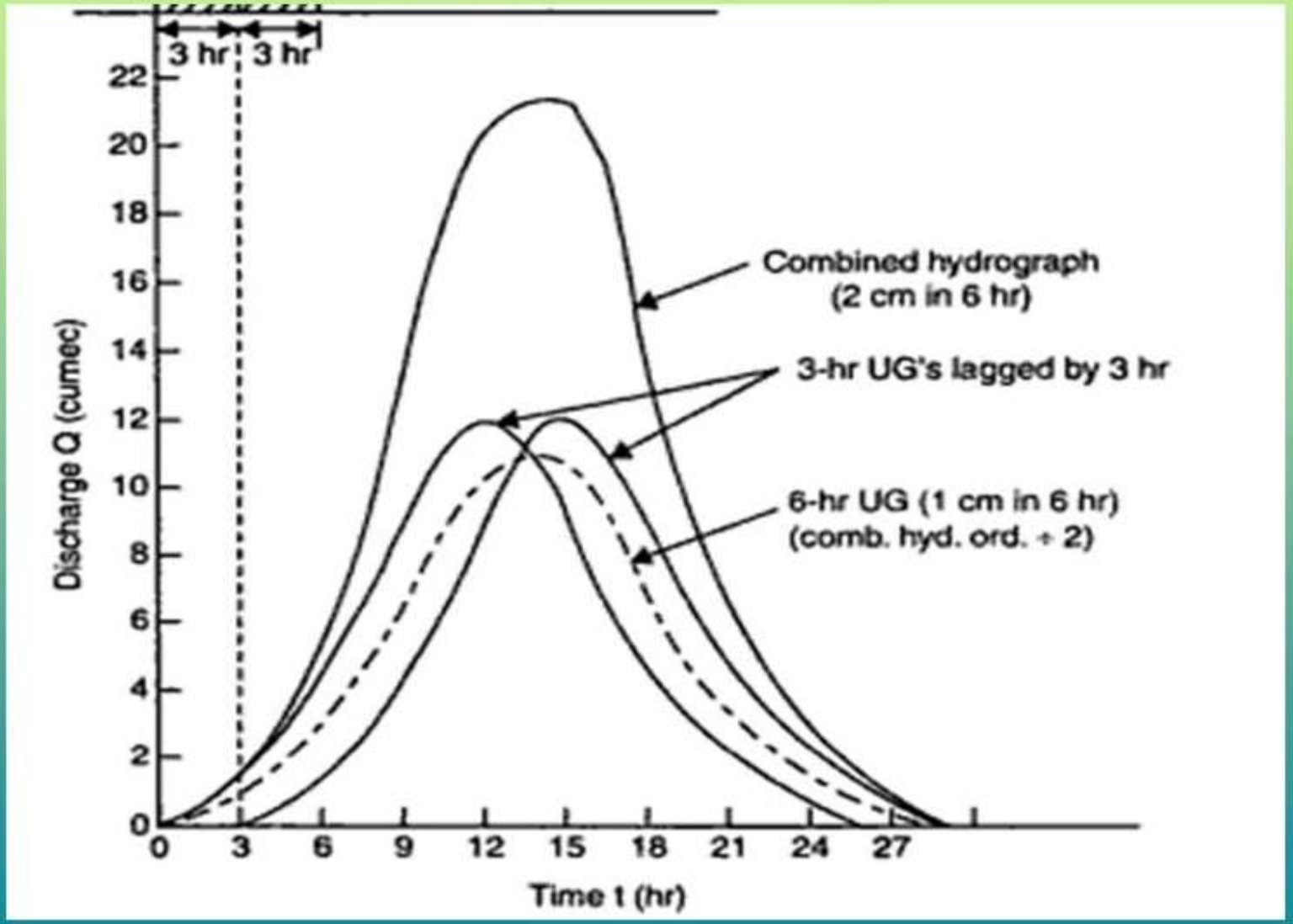
The S-hydrograph method converts an X-hour unit hydrograph into a Y-hour unit hydrograph, regardless of the ratio between X and Y.

Superposition Method

The procedure consists of lagging X-hour unit hydrographs in time, each for an interval equal to X-hours, summing the ordinates of all "n" hydrographs, and dividing the summed ordinates by n to obtain the nx-hour unit hydrograph.

The volume under X-hour and nX-hour unit hydrograph is the same. If Tb is the time base of the X-hour hydrograph, the time base of the nX-hour hydrograph is equal to Tb + (n – 1) X.

Convert the duration of unit hydrograph from short to long (superposition)



Time (hrs)	6-hr Unit Hydrograph (cms)
0	0
2	8
4	18
6	23
8	21
10	15
12	13
14	10
16	8
18	6
20	5
22	4
24	3
26	2
28	1
30	1
32	0
34	
36	
38	
SUM	138

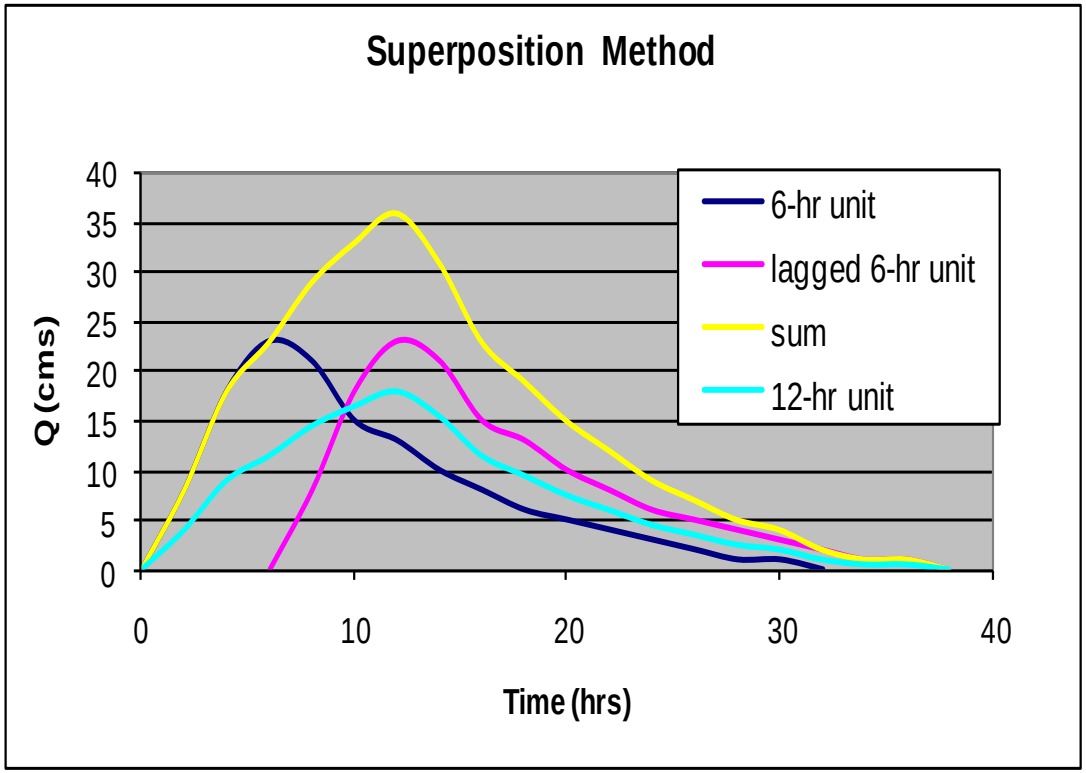
Example of Superposition Method

Develop a 12-hr UH from a 6-hr UH

Example of Superposition Method

Develop a 12-hr UH from a 6-hr UH

Synthetic Hydrograph: Lagging Storm Method				
Derive a 12-hr unit hydrograph from a 6-hr unit hydrograph				
Time (hrs)	6-hr Unit Hydrograph (cms)	Lag by 6 hrs	Sum	12-hr Unit Hydrograph (cms)
0	0		0	0
2	8		8	4
4	18		18	9
6	23	0	23	11.5
8	21	8	29	14.5
10	15	18	33	16.5
12	13	23	36	18
14	10	21	31	15.5
16	8	15	23	11.5
18	6	13	19	9.5
20	5	10	15	7.5
22	4	8	12	6
24	3	6	9	4.5
26	2	5	7	3.5
28	1	4	5	2.5
30	1	3	4	2
32	0	2	2	1
34		1	1	0.5
36		1	1	0.5
38		0	0	0
SUM	138	138	276	138



Example: Convert the 2 hr UHG in col. 2 to a 6 hr UHG

Time (hrs)	2hr UHG	2hr UHG lagged 2 hrs	2 hr UHG lagged 4 hrs	6 hr 30 mm hydrograph	6 hr UHG
0	0	-	-	0	0
1	0	-	-	0	0
2	2.87	0	-	2.87	0.96
3	6.44	0	-	6.44	2.15
4	10.14	2.87	0	13.01	4.34
5	13.15	6.44	0	19.59	6.53
6	8.22	10.14	2.87	21.23	7.08
7	6.33	13.15	6.44	25.94	8.65
8	4.66	8.22	10.14	23.02	7.67
9	2.77	6.33	13.15	22.25	7.42
10	1.10	4.66	8.22	13.98	4.66
11	0	2.70	6.33	9.03	3.01
12	-	1.10	4.66	5.76	1.92
13	-	0	2.70	2.70	0.70
14	-	-	1.10	1.10	0.37
15	-	-	0	0	0

.....