Unit Hydrograph

S-Curve/S-Hydrograph Method

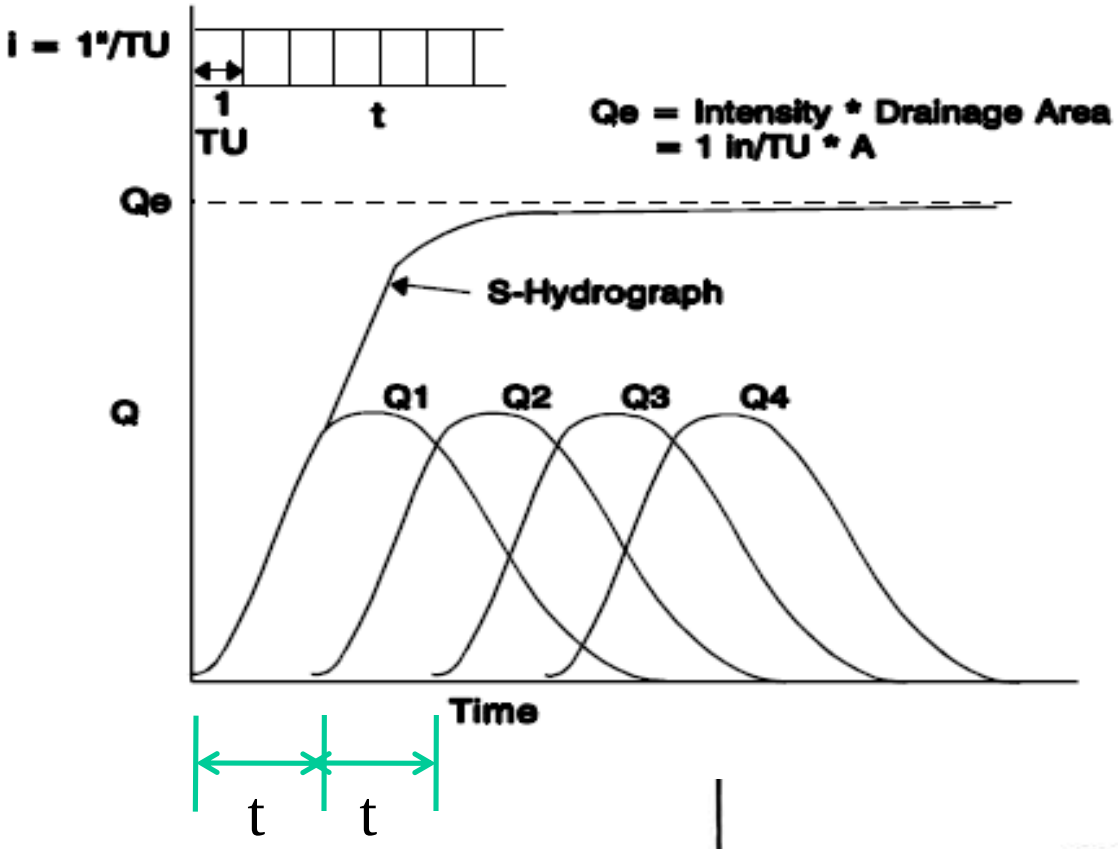
When the duration of effective or excess rainfall is infinitely long, and its intensity is one unit per unit storm duration (say, *D*-hour), the hydrograph so obtained is called S-hydrograph or summation hydrograph or S-curve.

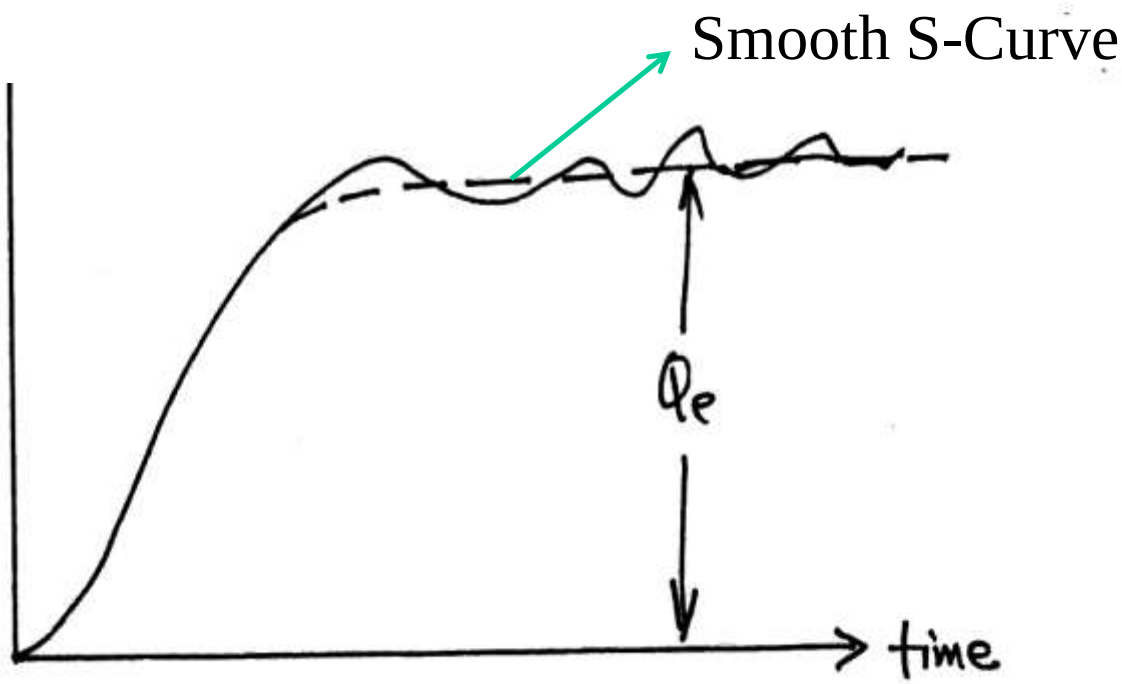
S-curve can be used to construct hydrographs from unit hydrographs with longer or shorter durations.

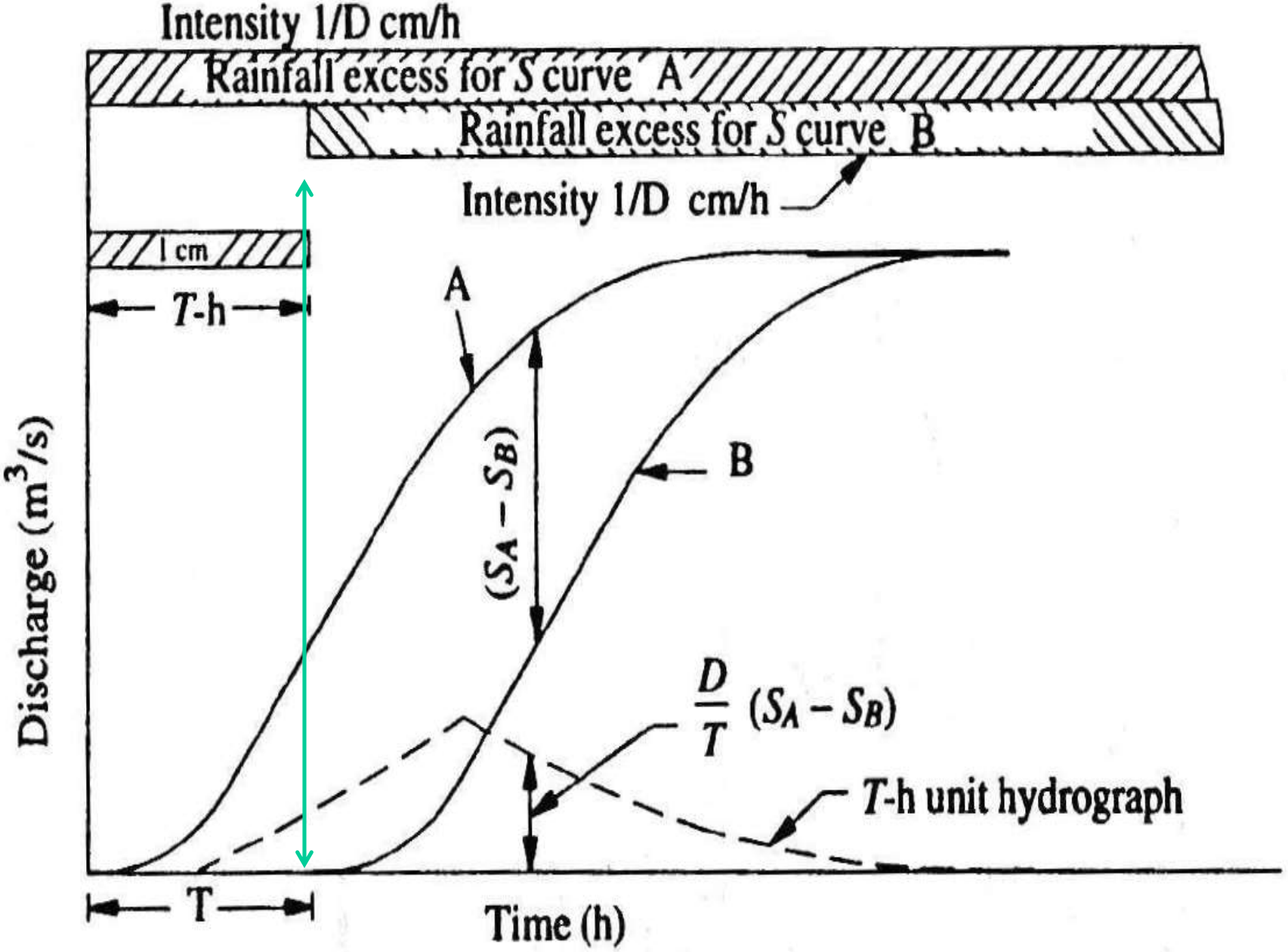
Develop a 2-hr unit hydrograph from a 6-hr unit hydrograph

The following steps are to be followed for developing a S-Curve:

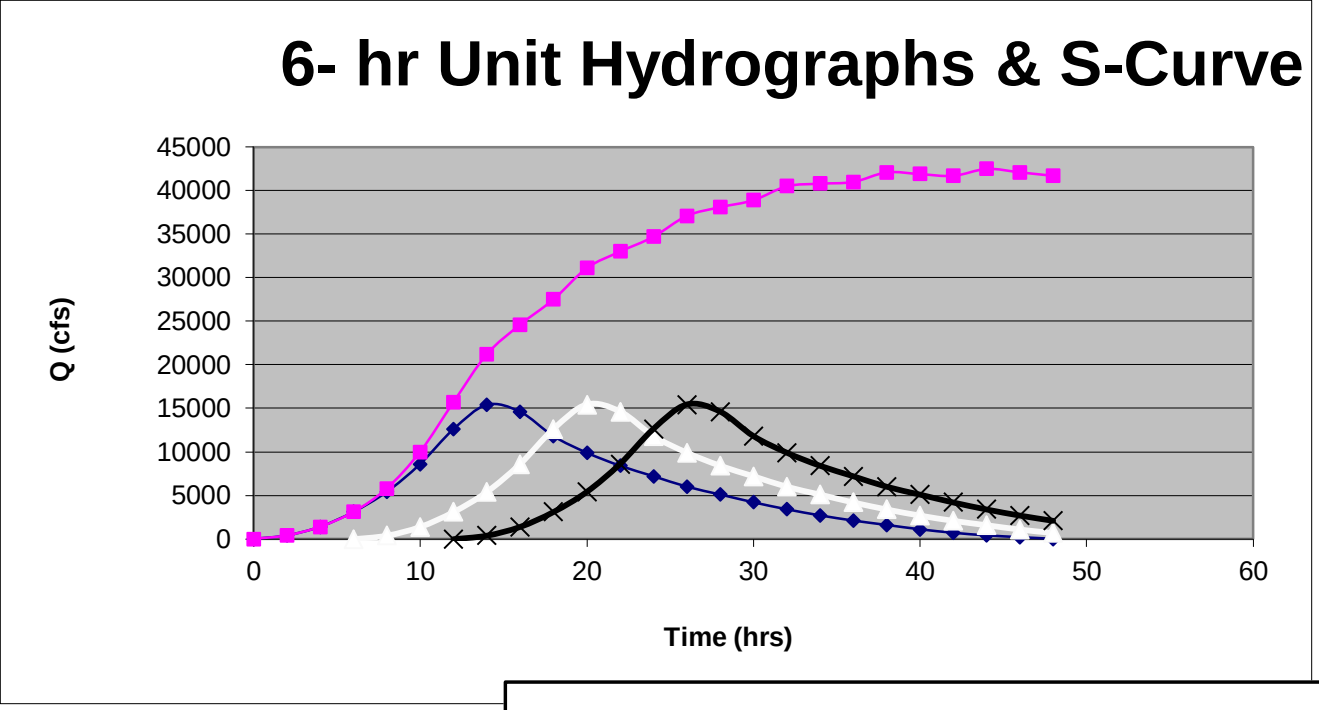
1. Add ordinates of many hydrographs lagged by 6-hr (eventually the curve levels off and remains constant: S-Curve)
2. Lag the S-curve by the proposed duration (2-hr)
3. Difference between S-curves is the hydrograph for storm duration (2-hr) with rainfall amount of (2/6) cm
4. Convert to unit hydrograph by multiplying ordinates by (6/2=3). A shorter duration would have higher ordinates and vice versa.
5. Smooth the data if necessary .

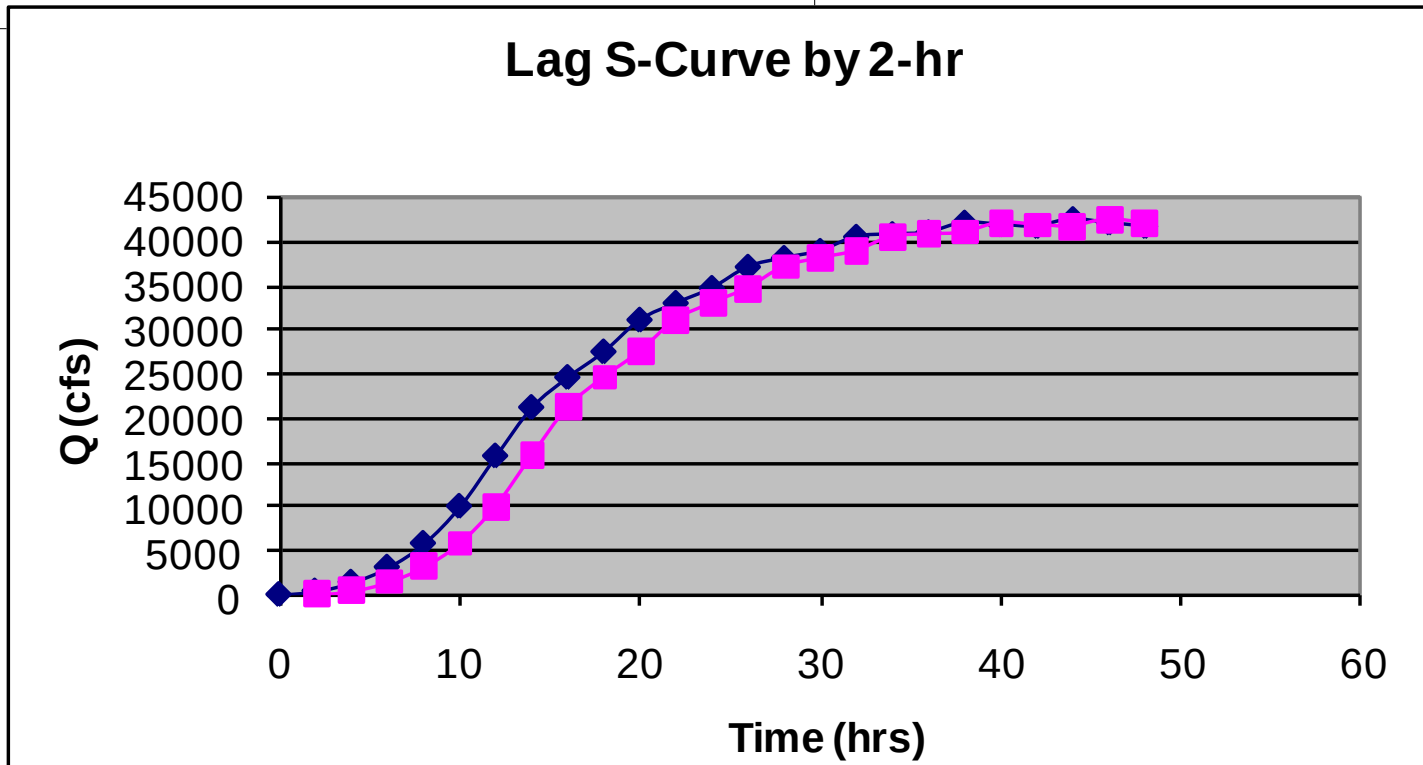

I = 1"/TU
1
TU
t
Qe = Intensity * Drainage Area
= 1 in/TU * A
Qe
S-Hydrograph
Q
Q1
Q2
Q3
Q4
Time
t
t

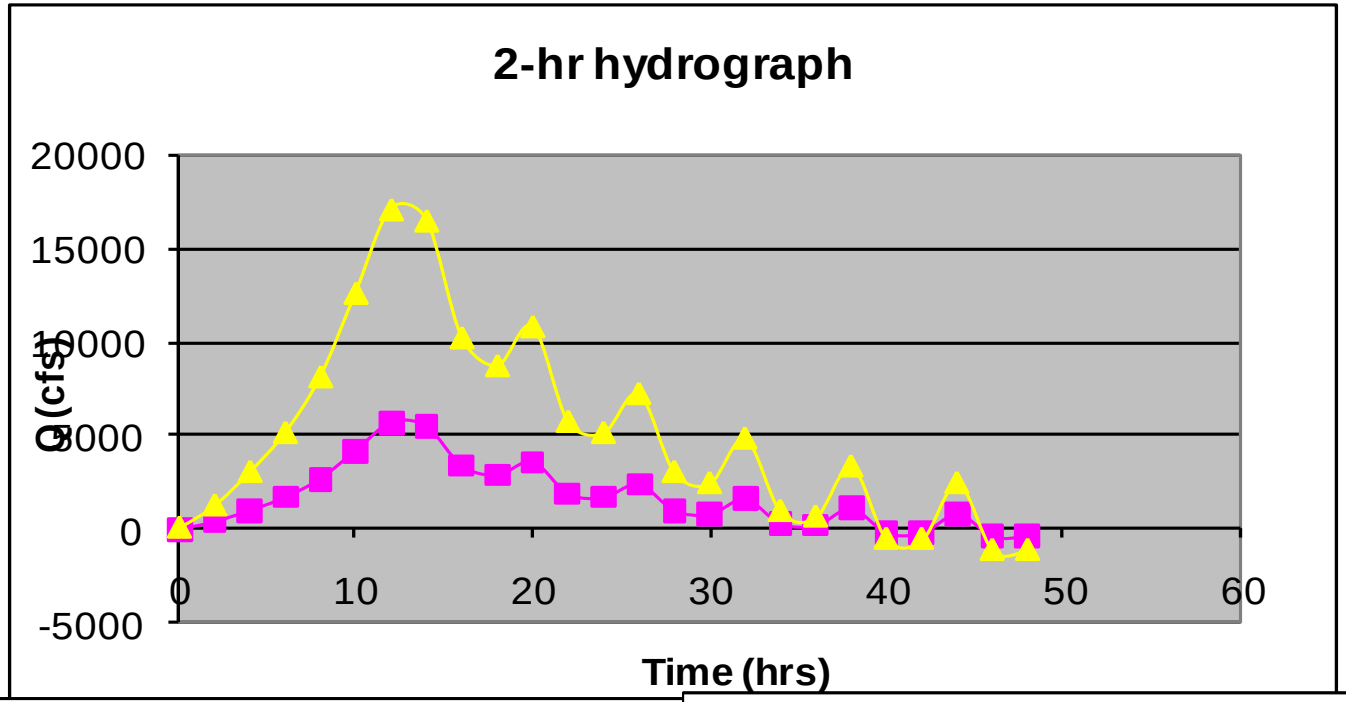

Smooth S-Curve
Qe
time

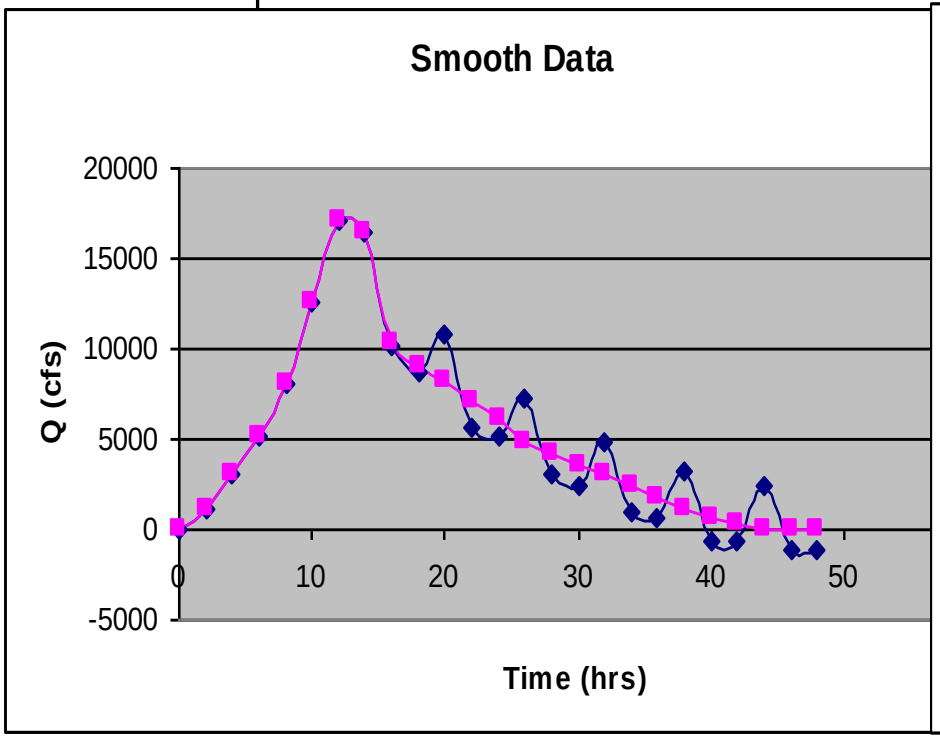

Intensity 1/D cm/h
Rainfall excess for S curve A
Rainfall excess for S curve B
Intensity 1/D cm/h
1 cm
T-h
A
B
$(S_A - S_B)$
$\frac{D}{T}(S_A - S_B)$
T-h unit hydrograph
Discharge (m^3/s)
T
Time (h)

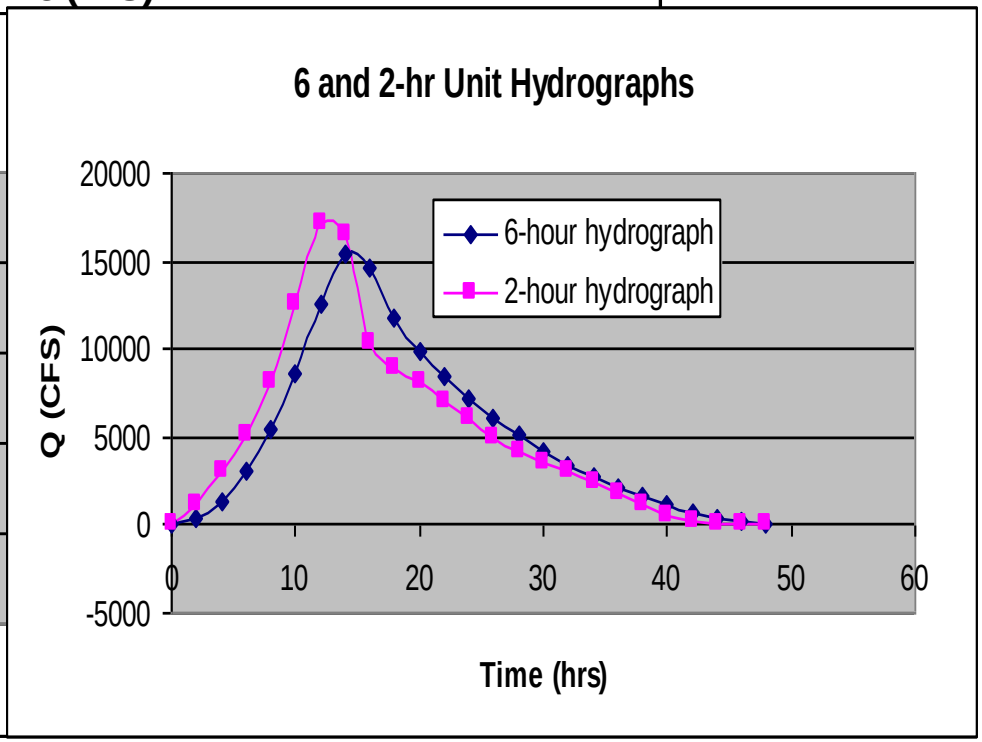
CTC 260-Hydrology							Tr	6	hours				
Synthetic Hydrograph: S-Curve Method							Tr'	2	hours				
Derive a 2-hr hydrograph from a 6-hour hydrograph							Tr/Tr'	3					
Time	Tr Unit								S-Curve	Lag by Tr'	Tr' Hydro	Tr' Unit	Smooth
(hrs)	Hydrograph											Hydrograph	Data
	(cfs)												
0	0								0		0	0	0
2	400								400	0	400	1200	1200
4	1400								1400	400	1000	3000	3000
6	3100	0							3100	1400	1700	5100	5100
8	5400	400							5800	3100	2700	8100	8100
10	8600	1400							10000	5800	4200	12600	12600
12	12600	3100	0						15700	10000	5700	17100	17100
14	15400	5400	400						21200	15700	5500	16500	16500
16	14600	8600	1400						24600	21200	3400	10200	10300
18	11800	12600	3100	0					27500	24600	2900	8700	9000
20	9900	15400	5400	400					31100	27500	3600	10800	8200
22	8400	14600	8600	1400					33000	31100	1900	5700	7100
24	7200	11800	12600	3100	0				34700	33000	1700	5100	6100
26	6000	9900	15400	5400	400				37100	34700	2400	7200	4900
28	5100	8400	14600	8600	1400				38100	37100	1000	3000	4200
30	4200	7200	11800	12600	3100	0			38900	38100	800	2400	3600
32	3400	6000	9900	15400	5400	400			40500	38900	1600	4800	3000
34	2700	5100	8400	14600	8600	1400			40800	40500	300	900	2400
36	2100	4200	7200	11800	12600	3100	0		41000	40800	200	600	1800
38	1600	3400	6000	9900	15400	5400	400		42100	41000	1100	3300	1200
40	1100	2700	5100	8400	14600	8600	1400		41900	42100	-200	-600	600
42	700	2100	4200	7200	11800	12600	3100	0	41700	41900	-200	-600	300
44	400	1600	3400	6000	9900	15400	5400	400	42500	41700	800	2400	0
46	200	1100	2700	5100	8400	14600	8600	1400	42100	42500	-400	-1200	0
48	0	700	2100	4200	7200	11800	12600	3100	41700	42100	-400	-1200	0
	126300										41700	125100	126300


6- hr Unit Hydrographs & S-Curve
Q (cfs)
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
0
10
20
30
40
50
60
Time (hrs)


Lag S-Curve by 2-hr
Q (cfs)
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
0
10
20
30
40
50
60
Time (hrs)


2-hr hydrograph
20000
15000
10000
5000
0
-5000
Q (cfs)
0
10
20
30
40
50
60
Time (hrs)


Smooth Data
20000
15000
10000
5000
0
-5000
Q (cfs)
0
10
20
30
40
50
Time (hrs)


6 and 2-hr Unit Hydrographs
20000
15000
10000
5000
0
-5000
Q (CFS)
0
10
20
30
40
50
60
6-hour hydrograph
2-hour hydrograph
Time (hrs)

- Convert the following 2-hr UH to a 3-hr UH

TIME (hr)	2-hr UH Ordinate (cfs)
0	0
1	75
2	250
3	300
4	275
5	200
6	100
7	75
8	50
9	25
10	0

TIME (hr)	2-HR UH	2-HR LAGGED UH'S				SUM
0	0					0
1	75					75
2	250	0				250
3	300	75				375
4	275	250	0			525
5	200	300	75			575
6	100	275	250	0		625
7	75	200	300	75		650
8	50	100	275	250	0	675
9	25	75	200	300	75	675
10	0	50	100	275	250	0	675
11		25	75	200	300	75	675

TIME (hr)	S-CURVE ORDINATE	S-CURVE LAGGED 3 HR	DIFFERENCE	3-HR UH ORDINATE
0	0		0	0
1	75		75	50.0
2	250		250	166.7
3	375	0	375	250.0
4	525	75	450	300.0
5	575	250	325	216.7
6	625	375	250	166.7
7	650	525	125	83.3
8	675	575	100	66.7
9	675	625	50	33.3
10	675	650	25	16.7
11	675	675	0	0

Unit Hydrograph

Synthetic Unit Hydrograph (Ungaged Basin)

In the absence of rainfall-runoff data, unit hydrographs can be derived by synthetic means. A synthetic unit hydrograph is a unit hydrograph derived following an established formula, without the need for rainfall-runoff data analysis.

The development of synthetic unit hydrograph requires relation between physical geometry of basin and resulting hydrographs.

Since peak flow and time of peak flow (lag time) are two of the most important parameters characterizing a unit hydrograph, the Snyder method (1938) employs factors defining these parameters, which are then used in the synthesis of the unit graph.

The parameters are C_p, the peak flow factor, and C_t, the lag factor.

The basic assumption in this method is that basins which have similar physiographic characteristics are located in the same area will have similar values of C_t and C_p.

The value of C_t varies between 0.3 to 6 and C_p between 0.3 to 6.5.

Therefore, for ungauged basins, it is preferred that the basin be near or similar to gauged basins for which these coefficients can be determined.

Unit Hydrograph

The basin lag t_p which is the time between center of mass of unit rainfall excess to the peak of the unit hydrograph is:

$$t_p = C_t (L . Lc)^{0.3} \quad (hrs)$$

Where, L = distance from gaging station (outlet) to catchment boundary (divide) along the main stream (km)

Lc = distance from gaging station to centroid of catchment area, measured along the main stream to the nearest point (km).

C_t = a coefficient depending on units and drainage characteristics (lag factor).

For any basin, the standard duration of rainfall excess t_r, (hrs) is:

$$t_r = t_p / 5.5 \quad (hrs)$$

The peak discharge Q_p (cumec) for the unit hydrograph is:

$$Q_p = 2.78 C_p A / t_p$$

Where, A = Catchment area (km^2), C_p = the peak flow factor and t_p = the basin lag (hrs).

Determination of C_p and C_t

3. The following are the ordinates of the 9-hour unit hydrograph for the entire catchment of the river Damodar up to Tenughat dam site: and the catchment characteristics are , A = 4480 km^2, L = 318 km, L_{ca} = 198 km.

Time (hr)	0	9	18	27	36	45	54	63	72	81	90
Flow (cumec)	0	69	1000	210	118	74	46	26	13	4	0

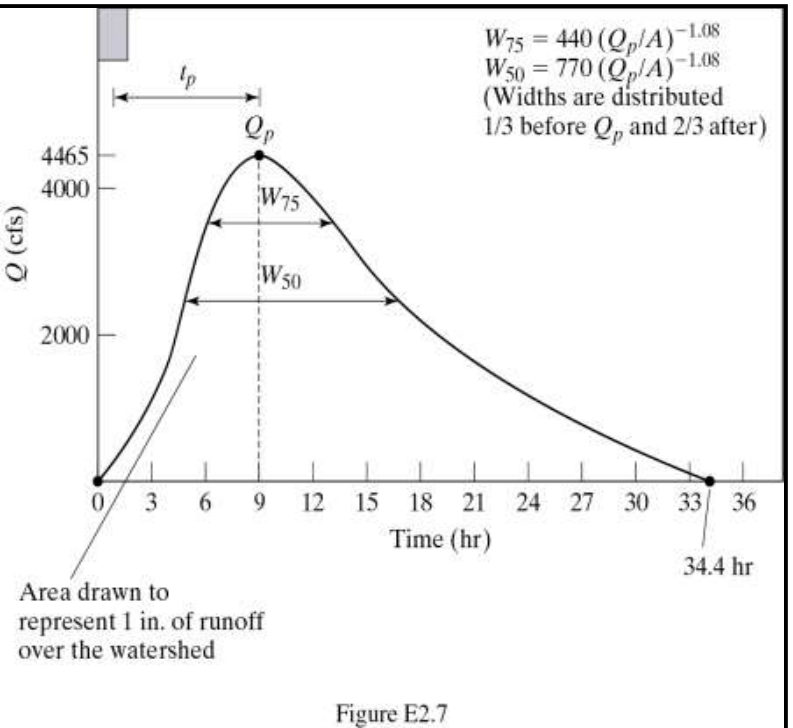


Figure E2.7

$Q_p = 2.78\, C_p\, A / t_p$

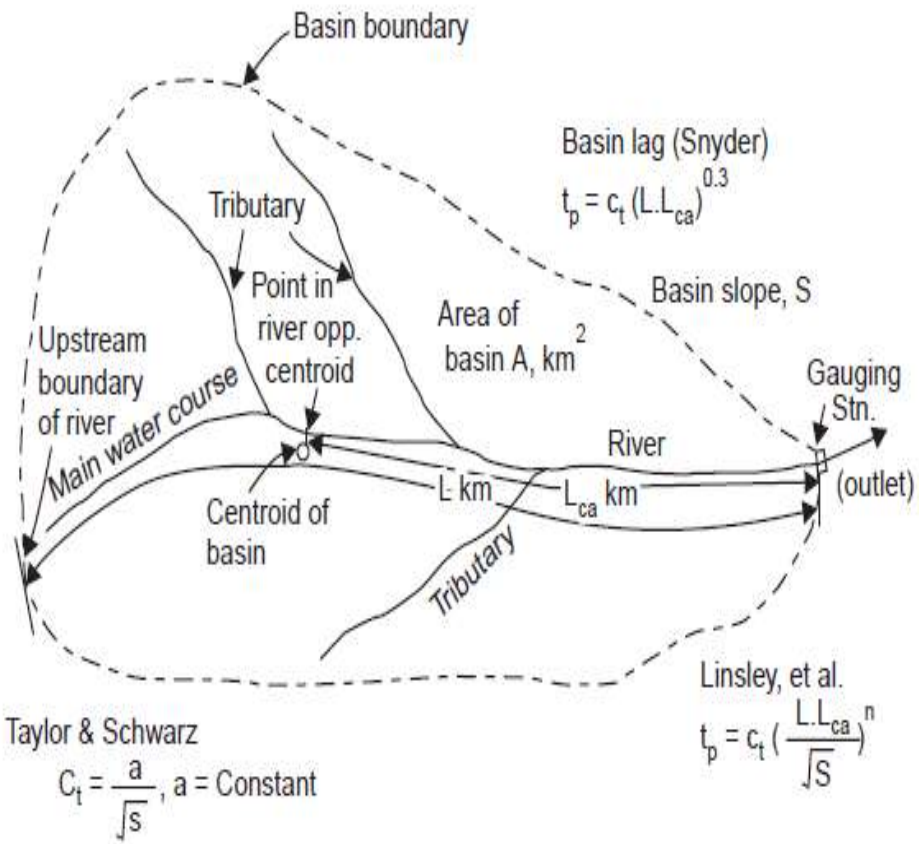
$1000 = 2.78\, C_p \times 4480 / (18 - 4.5)$

$C_p = 1.08$

$t_p = C_t\,(L \,.\, Lc)^{0.3}$

$13.5 = C_t\,(318 \times 198)^{0.3}$

$C_t = 13.5 / 27.5 = 0.49$


Basin boundary
Basin lag (Snyder)
$t_p = c_t (L.L_{ca})^{0.3}$
Tributary
Point in river opp. centroid
Basin slope, S
Area of basin A, km^2
Upstream boundary of river
Main water course
Gauging Stn.
River
L km
L_{ca} km
(outlet)
Centroid of basin
Tributary
Taylor & Schwarz
$C_t = \frac{a}{\sqrt{s}}$, a = Constant
Linsley, et al.
$t_p = c_t \left(\frac{L.L_{ca}}{\sqrt{S}}\right)^n$

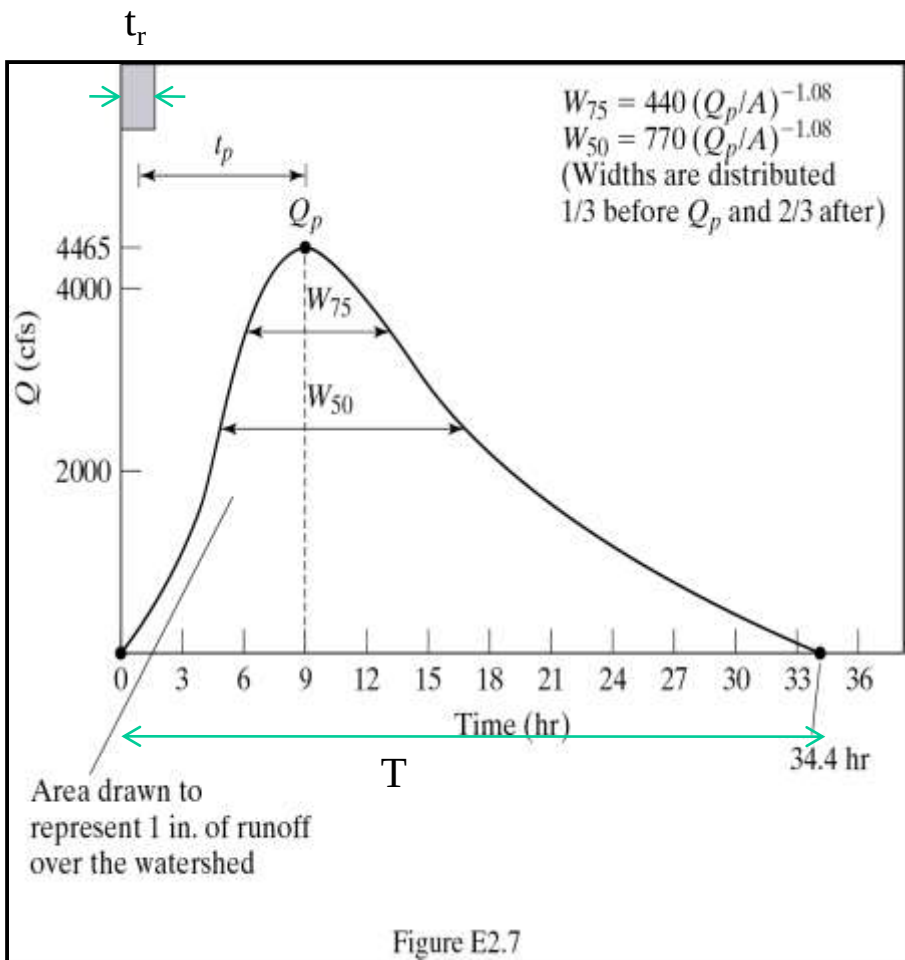

t_r
$W_{75} = 440\,(Q_p/A)^{-1.08}$
$W_{50} = 770\,(Q_p/A)^{-1.08}$
(Widths are distributed 1/3 before Q_p and 2/3 after)
t_p
Q_p
4465
4000
2000
W_{75}
W_{50}
Q (cfs)
0 3 6 9 12 15 18 21 24 27 30 33 36
Time (hr)
34.4 hr
T
Area drawn to represent 1 in. of runoff over the watershed

Figure E2.7

Unit Hydrograph

The duration of surface runoff, or the time base of unit hydrograph, T is:

$$T = 3 + t_p/8 \qquad \text{(days)}$$

For a basin if the unit hydrograph is required for any duration other than the standard duration t_r, (hrs), then a modified basin lag time t_p' is to be calculated.

For any other duration t_r', a modified basin lag time t_p' is:

$$t_p' = t_p + (t_r' - t_r) / 4$$

The peak discharge for duration t_r' is:

$$Q_p = 2.78\, C_p\, A / t_p'$$

The duration of surface runoff, or the time base of unit hydrograph, T is:

$$T = 3 + t_p' /8 \qquad \text{(days)}$$

While sketching the unit hydrograph, it is convenient to use the widths W_{50} and W_{75} of the hydrograph at 50 % and 75% of the height of the peak flow ordinate, respectively.

$$W_{50} = 5.9 / (Q_p /A)^{1.08} \quad \text{and}$$

$$W_{75} = W_{50} /1.75$$

Unit Hydrograph

Example

Derive a 3-hr unit hydrograph for an ungaged basin from the following data.

Length L =32 km; length Lc =25 km; Area of catchment =325 km^2

Assume C_t =0.9 and C_p =1.8

Solution

$$t_p = C_t (L . Lc)^{0.3} = 0.9 (32 \times 25)^{0.3} = 6.7 \text{ hrs}$$

$$t_r = t_p / 5.5 = 6.7 / 5.5 = 1.2 \text{ hrs}$$

As t_r is not equal to the desired unit duration t_r' (3-hr), we have to re-calculate the value of t_p'.

$$t_p' = t_p + (t_r' - t_r) / 4 = 6.7 + (3 – 1.2) / 4 = 7.2 \text{ hrs}$$

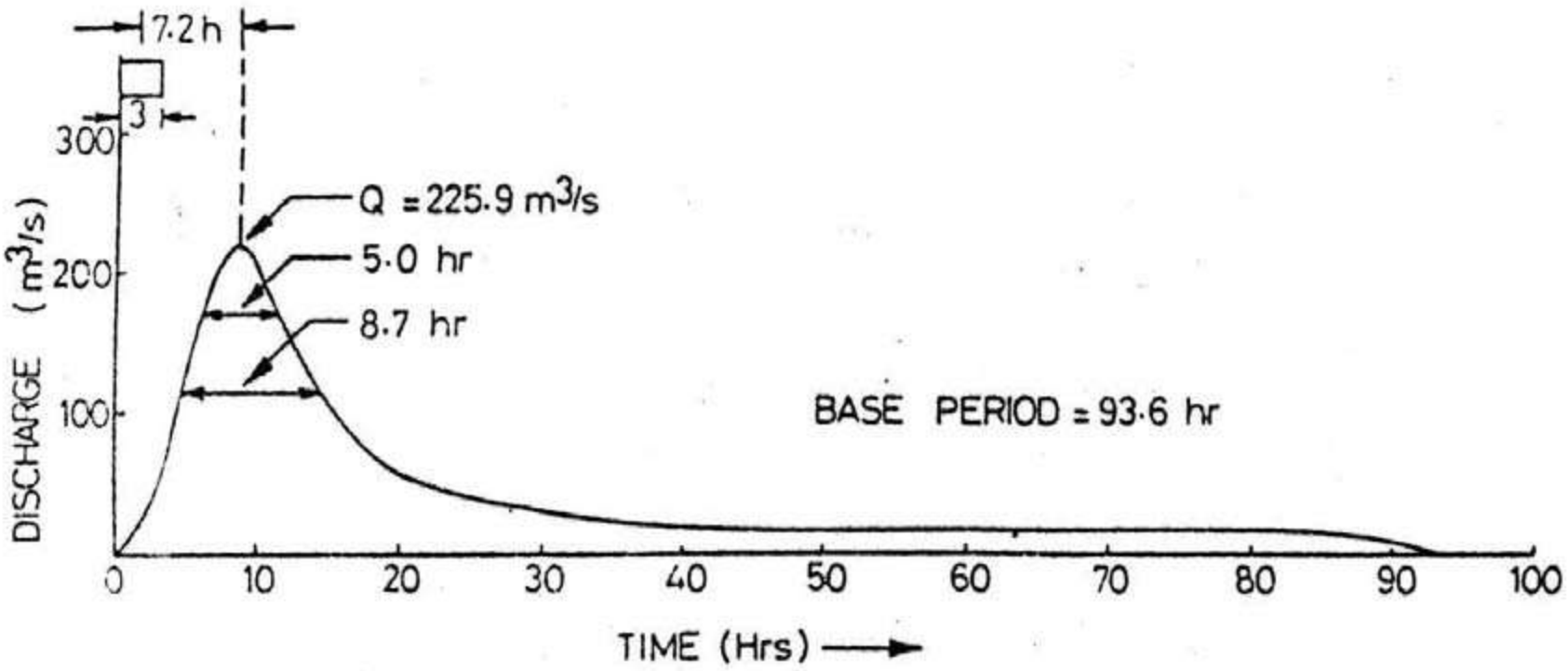
$$Q_p = 2.78 C_p A / t_p' = 2.78 \times 1.8 \times 325 / 7.2 = 225.9 \text{ cumec}$$

$$T = 3 + t_p' /8 = 3 + 7.2 / 8 = 3.9 \text{ days} = 93.6 \text{ hrs}$$

$$W_{50} = 5.9 / (Q_p /A)^{1.08} = 5.9 / (225.9 /325)^{1.08} = 8.7 \text{ hrs}$$

$$W_{75} = 8.7 / 1.75 = 5 \text{ hrs}$$

Synthetic Unit Hydrograph



Probability in Hydrology

The design of a water resources system is based on a 'design event' of rainfall, discharge, water level etc. The magnitude of the design event would vary from project to project, depending upon risk to life and property, cost, benefits etc.

No matter how large the 'design event' is used, there is always some chance (risk) that this 'design event' would be exceeded during the designed life of the project.

How large should be designed event? What is the risk of exceeding the designed event during the project life?

These and similar questions are answered using the concept of probability.

Probability is a measure of how likely an event will occur.

Probability is a number expressing the ratio of favorable outcome to the all possible outcomes. Thus:

- P (getting a head after flipping a coin) = ½
- P (getting a 3 after rolling a dice) = 1/6
- P (getting a club from a deck of playing cards) = 13/52 = 0.25

Probability in Hydrology

In Hydrology, probability is often expressed as **Recurrence Interval or Return Period (T)** which means the *average* number of years between occurrence of an event (e.g. flood, river level, rainfall etc.) of given magnitude to be equaled or exceeded.

For example, if the river level with a 50 year recurrence interval (return period) at a given location is 8 m, this is just another way of saying that a river level of 8 m, or greater, should occur at that location on the average only once every 50 years.

Probability of Occurrence (p) – is the probability that an event of the specified magnitude will be equaled or exceeded during a one year.

Frequency Analysis provides a systematic approach for using historical data to relate the magnitude of a naturally occurring event (e.g. rainfall, water level etc.) to the probability of its occurring in a given time period or to its return period (recurrence interval).

The objective of frequency analysis in a hydrologic context is to infer the probability that various size events will be exceeded or not exceeded from a given sample of recorded events.

Probability in Hydrology

There is a fundamental relationship between flood recurrence interval or return period (T) and probability of occurrence (p). These two variables are inversely related to each other. That is:

$$p = 1/T \quad \text{or} \quad T = 1/p.$$

For example, the probability (p) of a 50 year storm (return period) occurring in a one year period is 1/50 or 0.02.

The probability of occurrence (p) and probability of nonoccurrence (q) are related by the fact that something must either occur or not occur, so p + q = 1 and q = 1- p.

As the probability of occurrence or nonoccurrence of a storm is the same from year to year, the probability of a 50-year storm non-occurring in n successive years is: q^n or $q^n = (1 – p)^n$

Thus, the probability that the 50-year storm will occur at least once in n years is: p_n

Since $p_n = 1 – q^n = 1 - (1 – p)^n = 1 - (1 – 1/T)^n$

The probability of at least one event that exceeds design limits during the expected life of the structure is also called the risk. Thus, risk R is the same as p_n :

$$R = p_n = 1 – q^n = 1 - (1 – p)^n = 1 - (1 – 1/T)^n$$

The reliability of the designed event (Re) is defined as:

$$Re = 1 – R = (1 – p)^n = (1 – 1/T)^n$$

Probability in Hydrology

Example : What is the probability that a 50 year flood (river water level) will occur at least once within the next 10 years at any given location?

Solution: The answer to this can be found using the equation : $p_n = 1 - (1 - p)^n$

Since we are considering a 10 year period of time, n = 10. Also, for a 50 year river level, p = 1/50 = 0.02. Substituting into the equation: $P = 1 - (1 - 0.02)^{10} = 1 - 0.98^{10} = 0.183$. Thus, the **risk** of a 50 year river level occurring in 10 years is **18%**.

The reliability is: $Re = 1 - R = (1 - p)^n = (1 - 0.02)^{10} = 0.98^{10} = 0.817 =$ **82%**

The probability of k successes in n trials is:

$$P(\text{k successes in n trials}) = \frac{n!}{k!\,(n-k)!} P^k (1-P)^{n-k}$$

P (1 or more 50 year flood in n years) = 1 – P (no 50 year flood in n years)

$$= 1 - \frac{n!}{0!\,(n-0)!} P^0 (1-P)^{n-0}$$

$$= 1 - (1-P)^n = R \text{ (as derived earlier)}$$

Example: Compute the probability that one 10-year flood will occur in 10 years.

$$P(\text{One 10-year flood in 10 years}) = \frac{10!}{1!\,9!}(0.1)^1 (0.9)^9 \quad [P = 1/10 = 0.1]$$

$$= 10 \times 0.1 \times 0.387 = 0.387$$

Thus, the probability that one 10-year flood will occur in 10 years is about 39%.