Probability in Hydrology

Similarly: Compute the probability that at least one 10-year flood will occur in 10 years.

P (at least one 10-year flood in 10 years) = 1- P(no 10-year flood in 10 years)

$$= 1 - \frac{10!}{0!\ 10!}(0.1)^0\ (0.9)^{10}$$

$$= 1 - 0.349 = 0.651$$

Thus, the probability that at least one 10-year flood will occur in 10 years (risk) is 65%.

Similarly: P (at least one 10-year flood in 10 years) = 1- P(no 10-year flood in 10 years)

$$= 1 - q^n = 1 - (1 - p)^n = 1 - (1 - 0.1)^{10} = 1 - 0.349 = 0.651$$

Probability (%) of at least one X-yr flood in Y period

Number of years in period (Y)	***N = Average return period T_r (X - years)***				
	5	***10***	***20***	***50***	***100***
1	20	10	5	2	1
5	67	41	23	10	5
10	89	65	40	18	10
20	99	88	64	33	18
60	—	99.8	95	70	43
100	—	—	99.4	87	63

Probability in Hydrology

Frequency Analysis

The objective of frequency analysis of hydrologic data is to relate the magnitude of extreme events to their frequency of occurrence through the use of probability distributions.

The two methods of frequency analysis are:

1. Plotting position technique
2. Frequency distribution

The cumulative distribution function (derived from Plotting position technique) provides a rapid means of determining the probability of an event equal to or less than some specified quantity.

The frequency distribution technique is based on the distributional assumptions that is made of the mean, variance and for some distribution coefficient.

Plotting Position Method

The frequency of an even can be obtained by use of Weibull's equation as:

$$p = 100 \times \frac{m}{N+1}$$

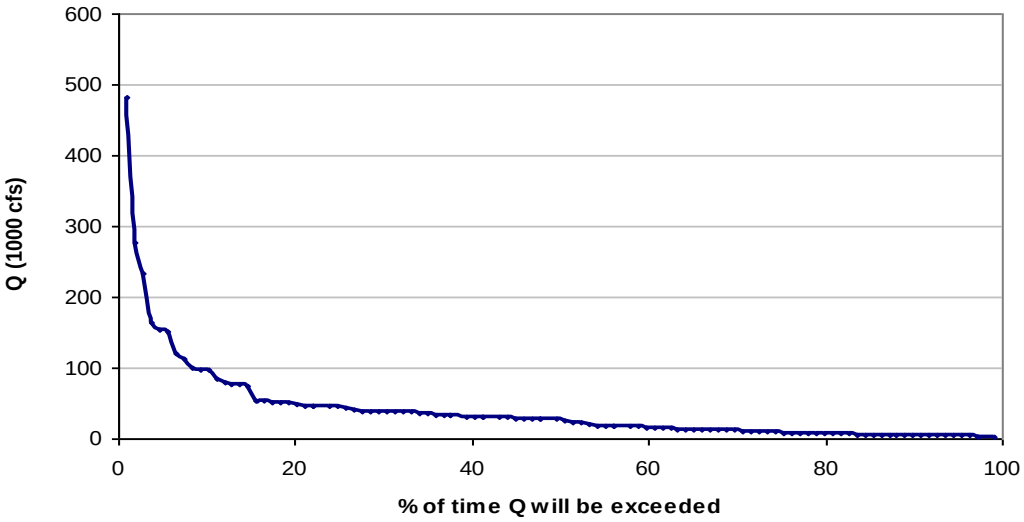
Where, p = the probability of occurrence

N = the number of values, and

m = the rank of descending values with largest equal to 1

Probability in Hydrology

- **Steps**
  - Find the number of records (N)
  - Sort the data from highest to lowest
  - Rank the data (m=1 for the highest value and m=N for the lowest value)
  - Compute exceedance probability for each value using the Weibull's method.
  - Plot p on x axis and the data (sorted) on y axis in normal paper.
  - If p and data are plotted in probability paper (p in probability scale and data in normal scale) then the data fits into straight line (extrapolation).
  - Example: 108 years of discharge data (N). For m = 1, p = 100 (1/109) = 0.92 etc. (0.92%) chance of exceedance in a given year.

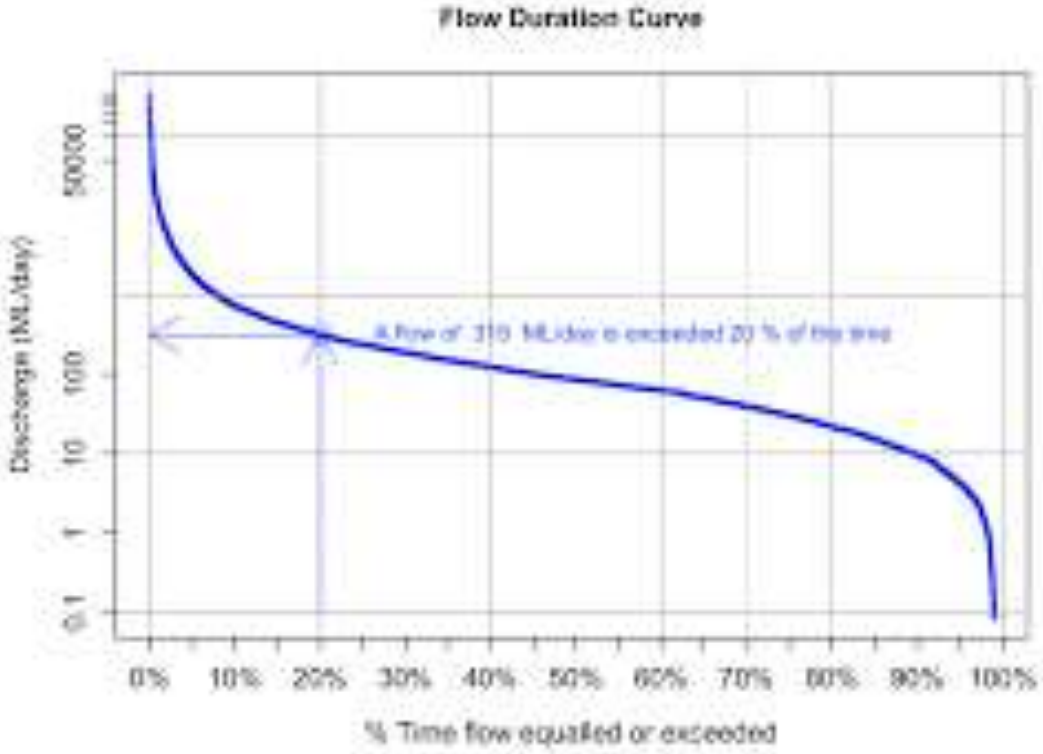


Exceedance Probability

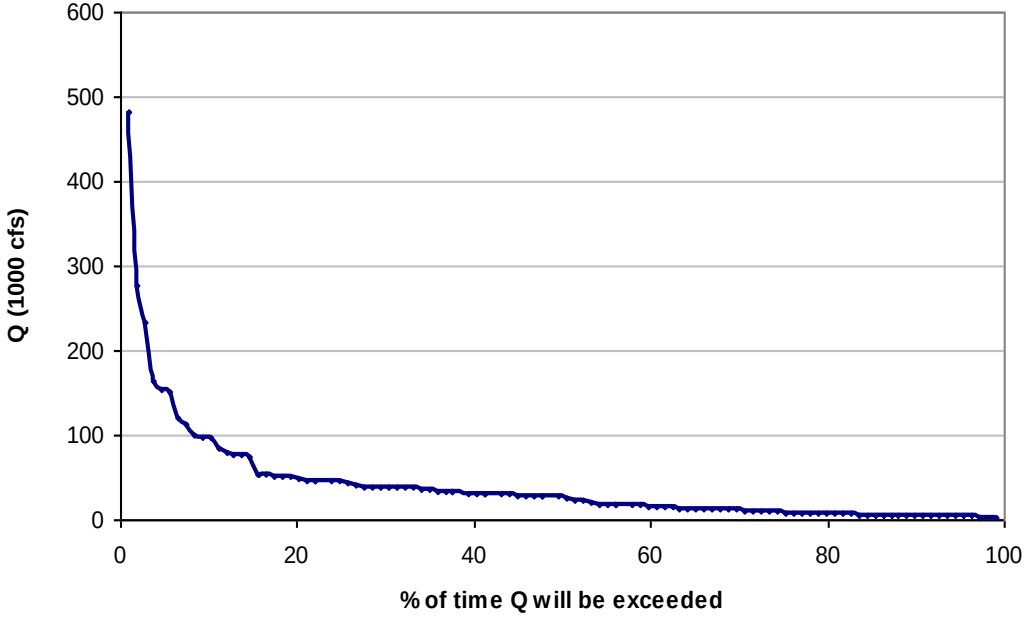
Example: Maximum hourly rainfall per year

year	1hr depth (in)
1972	2.32
1971	1.16
1970	1.75
1969	1.55
1968	1.61
1967	1.55
1966	1.78
1965	0.86
1964	0.65
1963	0.91
1962	1.53
1961	1.38
1960	1.82
1959	2.06
1958	1.04
1957	2.08
1956	1.07
1955	1.13
1954	1.41
1953	1.02

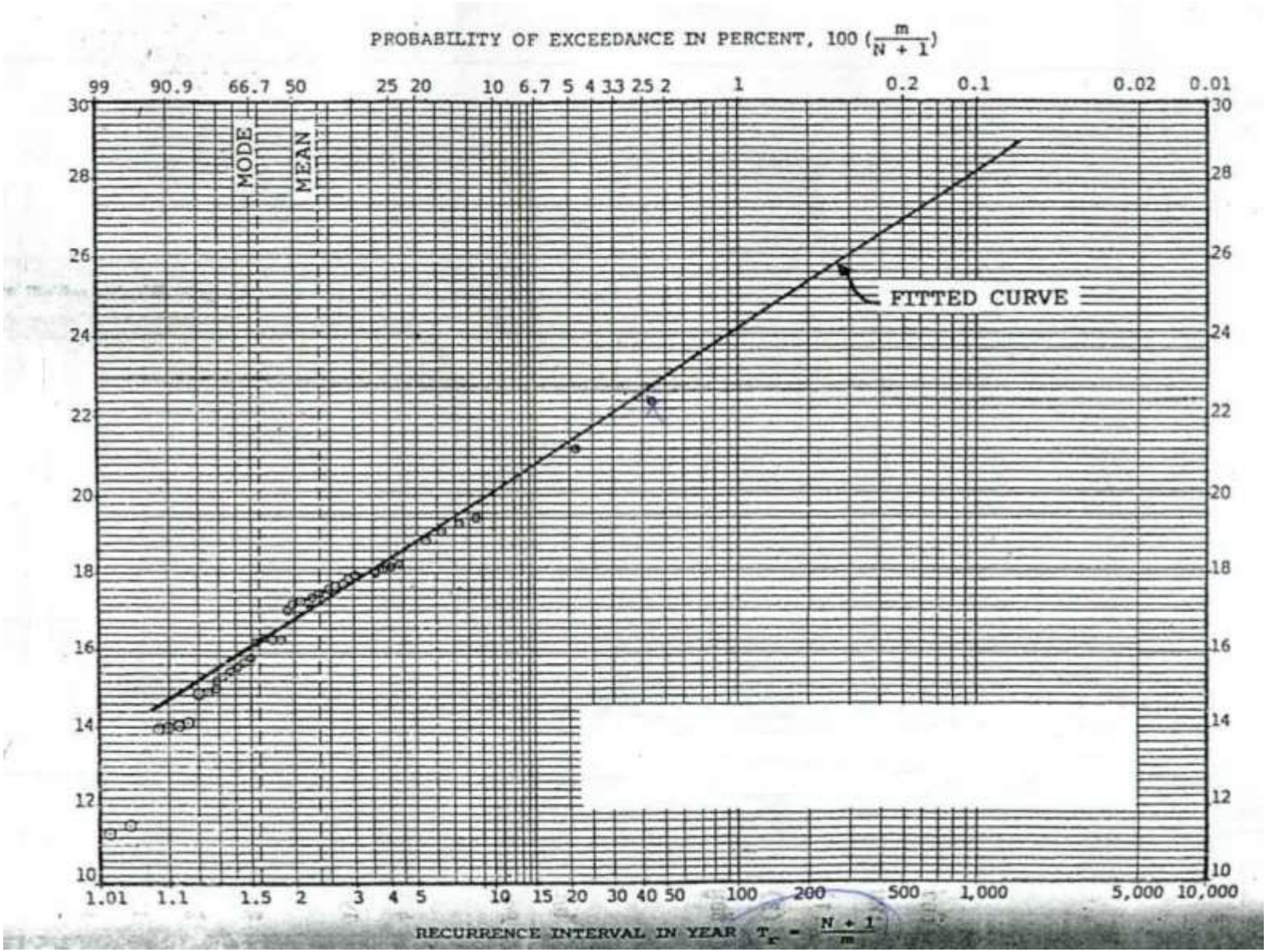
year	1hr depth (in)	Rank	Return Interval (yrs)	probability
1972	2.32	1	21.0	0.05
1957	2.08	2	10.5	0.10
1959	2.06	3	7.0	0.14
1960	1.82	4	5.3	0.19
1966	1.78	5	4.2	0.24
1970	1.75	6	3.5	0.29
1968	1.61	7	3.0	0.33
1969	1.55	8	2.6	0.38
1967	1.55	9	2.3	0.43
1962	1.53	10	2.1	0.48
1954	1.41	11	1.9	0.52
1961	1.38	12	1.8	0.57
1971	1.16	13	1.6	0.62
1955	1.13	14	1.5	0.67
1956	1.07	15	1.4	0.71
1958	1.04	16	1.3	0.76
1953	1.02	17	1.2	0.81
1963	0.91	18	1.2	0.86
1965	0.86	19	1.1	0.90
1964	0.65	20	1.1	0.95



Probability of Exceedanace



Note: Probability from Weibull Method is Exceedance probility (probability of the event being equelled or exceeded.


PROBABILITY OF EXCEEDANCE IN PERCENT, 100 (m/N + 1)
99 90.9 66.7 50 25 20 10 6.7 5 4 3.3 2.5 2 1 0.2 0.1 0.02 0.01
MODE
MEAN
FITTED CURVE
30 28 26 24 22 20 18 16 14 12 10
1.01 1.1 1.5 2 3 4 5 10 15 20 30 40 50 100 200 500 1,000 5,000 10,000
RECURRENCE INTERVAL IN YEAR $T_r = \frac{N+1}{m}$

Probability in Hydrology

Frequency Distribution

The magnitude X of a hydrologic event can be represented as the mean plus the departure of the event from the mean.

$$X = \overline{X} + K_T \sigma_x$$

Where, X is the event of specified probability, $\overline{X}$ is the mean of the event series, σ_x is the standard deviation of the series and K is the frequency factor defined by the specific distribution and T is the return period.

What the above equation means is that an extreme flood with a probability p (or return period of T) will be K standard deviations above the average maximum annual flood.

Frequency distributions can take on many forms according to the equations used to carry out the statistical analyses. Four of the common forms are:

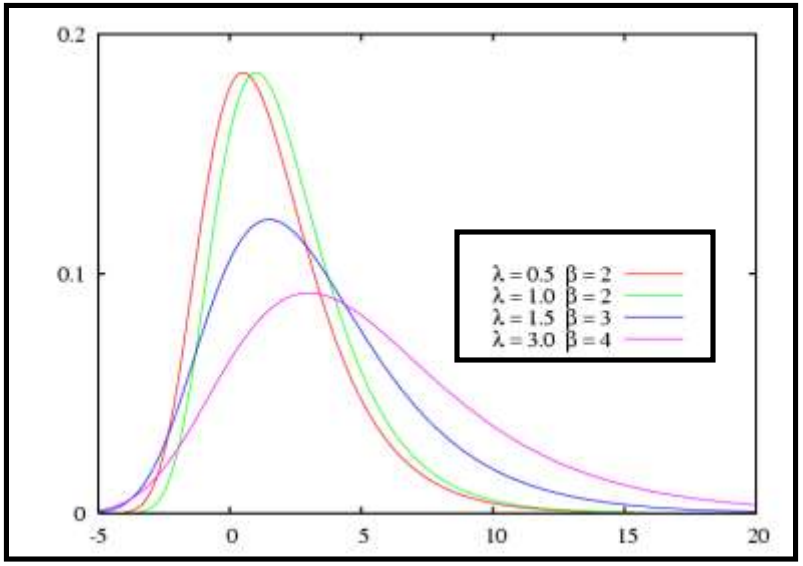
- Normal Distribution
- Log-Normal Distribution
- Gumbel Distribution or Extreme Value Type I (EV 1) Distribution
- Log-Pearson Type III Distribution

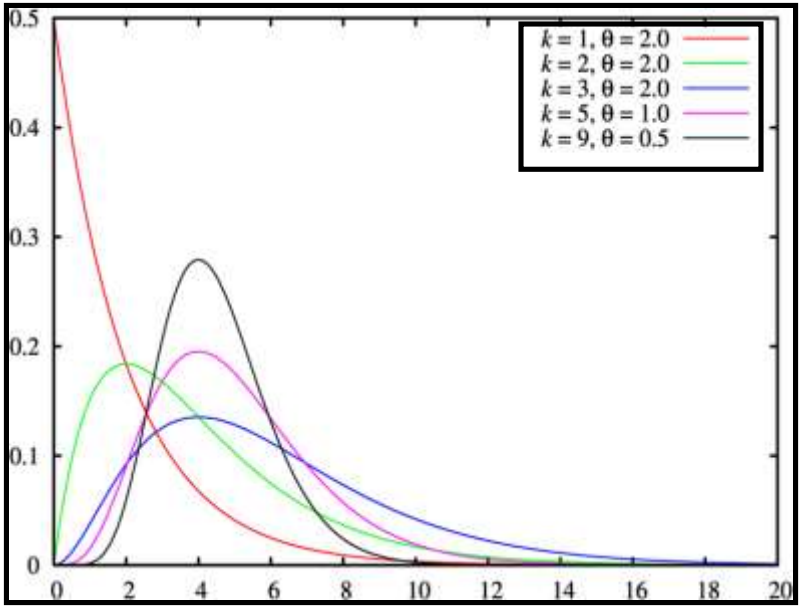
Out of the four distributions, Gumbel and Log-Pearson distributions are widely used for flood frequency analysis (both stage and discharge).

In Bangladesh, Gumbel method is generally used (although there is not much difference in different distributions).

Major Distributions

- Exponential - decays rapidly to low probability - event arrival times
- Normal - Symmetric based on mean and standard deviation
- Lognormal - Log data are normally distributed
- Gumbel - skewed distribution - water data
- Log Pearson III -skewed logs - water data





Frequency Histogram

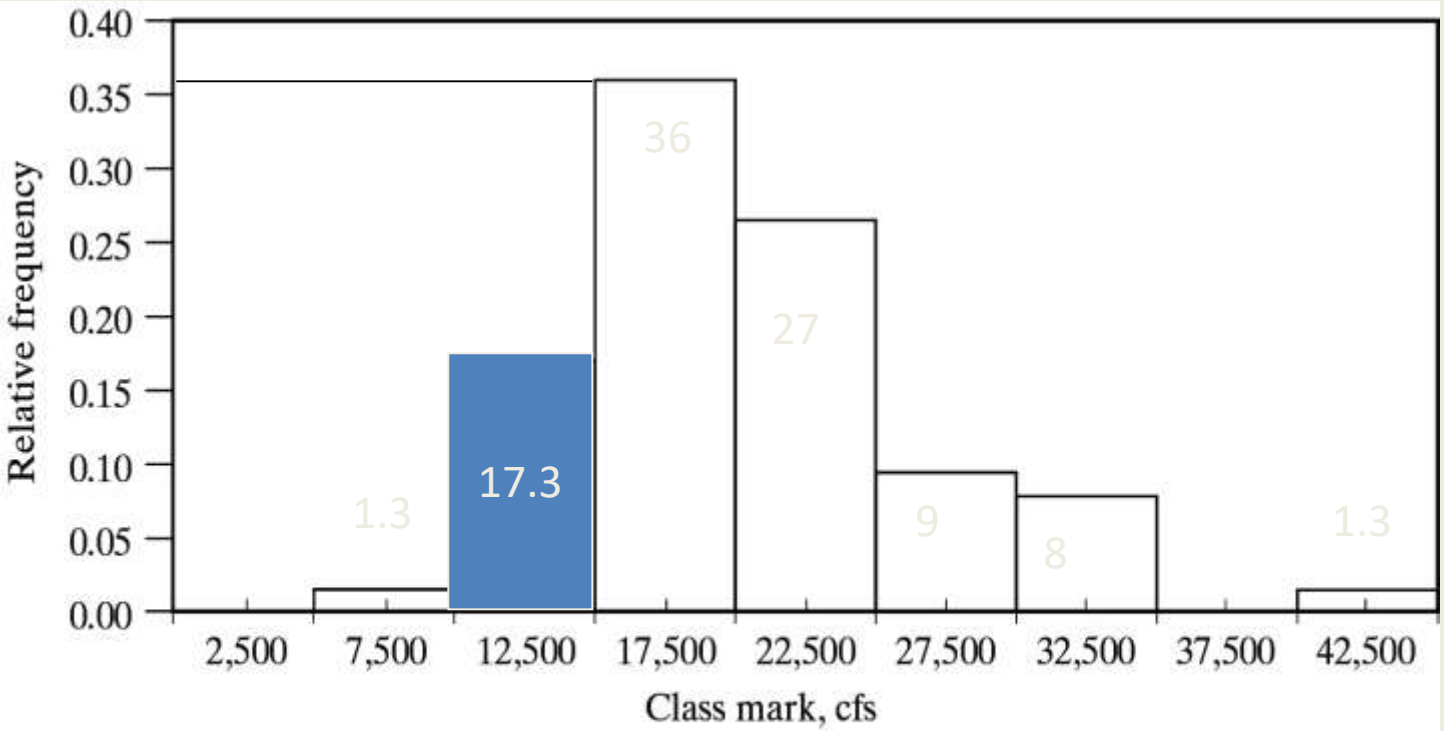
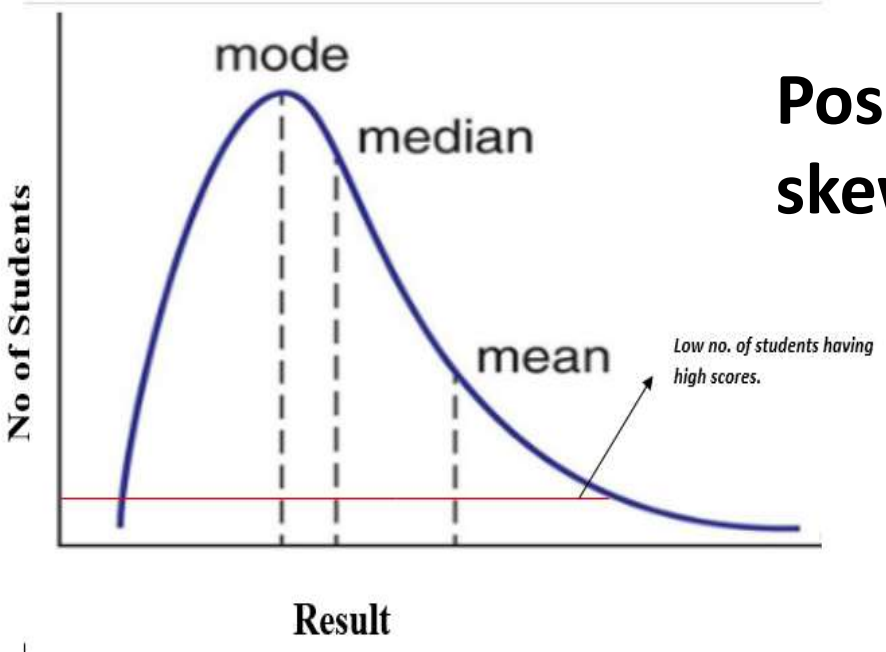


Figure 3.4

Relative frequencies (probabilities) for the Siletz River, plotted vs. their class mark.

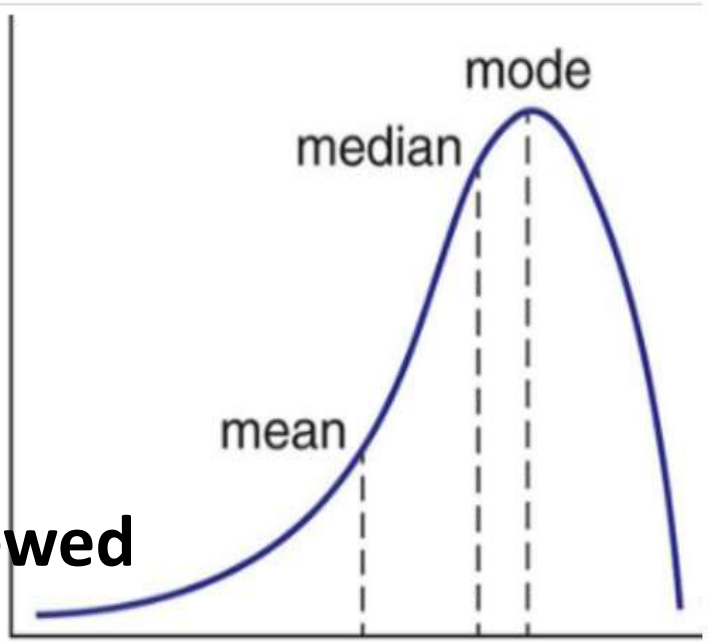
Probability that Q is 10,000 to 15, 000 = 17.3%

Prob that Q < 20,000 = 1.3 + 17.3 + 36 = 54.6%



Positive skewed or right-skewed

Mean > Median > Mode



Negative skewed or left-skewed

Mode > Median > mean

Table No-1 Annual maximum discharges (cumec) with year. Period of record used from 1961 to 2000, Number of years 29, Station Bhairab Bazar (273). River: Meghna. Designation of data set is D1.

Year	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
Maxi Discharge	13141	7590	9487	12300	12100	14400	12700	13300	11500	16400
Year	1972	1973	1974	1975	1976	1981	1982	1983	1984	1985
Maxi Discharge	11500	12400	19500	12700	16700	11200	13500	16000	13600	14300
Year	1986	1987	1988	1989	1990	1991	1992	1993	2000	
Maxi Discharge	11100	15200	19800	15500	11700	14500	12800	19900	12394	

Table-1 Comparison of flood frequency results using data D1 obtained by different methods in various return periods.

Return periods (Years Distribution)	2	5	10	20	50	100
EV1 (cumec)	13584	16101	17767	19366	21435	22985
LN2 (cumec)	13742	15878	17125	18227	19553	20490
LN3 (cumec)	13584	15741	17120	18411	20053	21271
LP3 (cumec)	13599	16598	17039	18300	19914	21121

Probability in Hydrology

Gumbel (EV 1) Distribution

Gumbel's distribution is designed to use the existing data record to tell how large an event (flood) with a return period of T_r will be.

There are three steps in computation:

1. Use the data record to find the mean X, and standard deviation, σ_x of the maximum annual floods.

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\sigma_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \overline{X}\right)^2$$

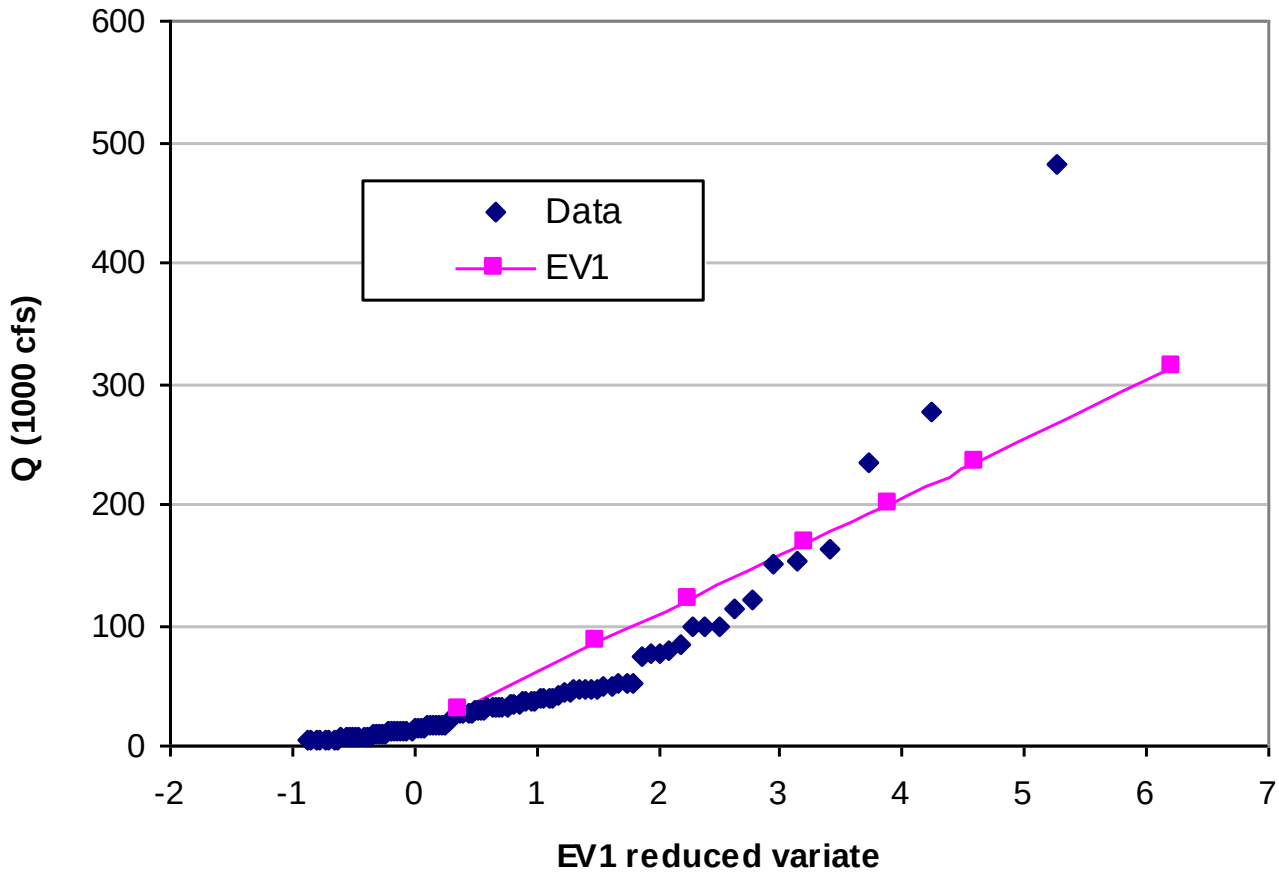
2. Select the return period, T_r you wish and then solve the Gumbel expression for *K*.

$$K = -0.78\left[\,0.577 + \ln\left(\ln\frac{T_r}{T_r - 1}\right)\right]$$

3. To find the discharge of the extreme flood, solve the equation for X.

$$X = \overline{X} + K_T\,\sigma_x$$

Using Plotting position technique, plot the data in probability paper. If the data falls on a straight line derived from EV1 distribution, the data comes from the EV1 distribution. Otherwise, a different distribution has to be tried.



The pink line has been drawn with values obtained from EV1 distribution.

Probability in Hydrology

Example

Find the discharge for the 100-year flood if the mean annual peak flood is 5000 cumec and the standard deviation is σ_q = 1000 cumec.

$X = \bar{X} + K_T \sigma_x$ Here $\bar{X} = 5000$ and $\sigma_x = 1000$

Find K for the 100-year event. (This means T_r = 100).

$$K = -0.78\left[.577 + \ln\left(\ln\frac{100}{99} \right) \right] = -0.78[.577 - 4.60] = 3.14$$

With K = 3.14, the 100 year flood is:

$$q_{100} = \overline{X} + K\,\sigma_q = 5000 + 3.14(1000) = 8140$$

Thus, the 100-year flood is 8140 cumec.

Similarly, it is possible to find out the return period of any flood (for example: the return period of a 6000 cumec flood is 7 years)

Rainfall Analysis

Intensity-Duration-Frequency (IDF) Analysis

In many hydrologic design projects the first step is the determination of the rainfall event to be used which is usually termed the *design storm event*.

The most common approach of determining the design storm event involves a relationship between rainfall intensity (or depth), duration, and the frequency (or return period) appropriate for the facility and site location.

When local rainfall data are available, IDF curves can be developed using frequency analysis.

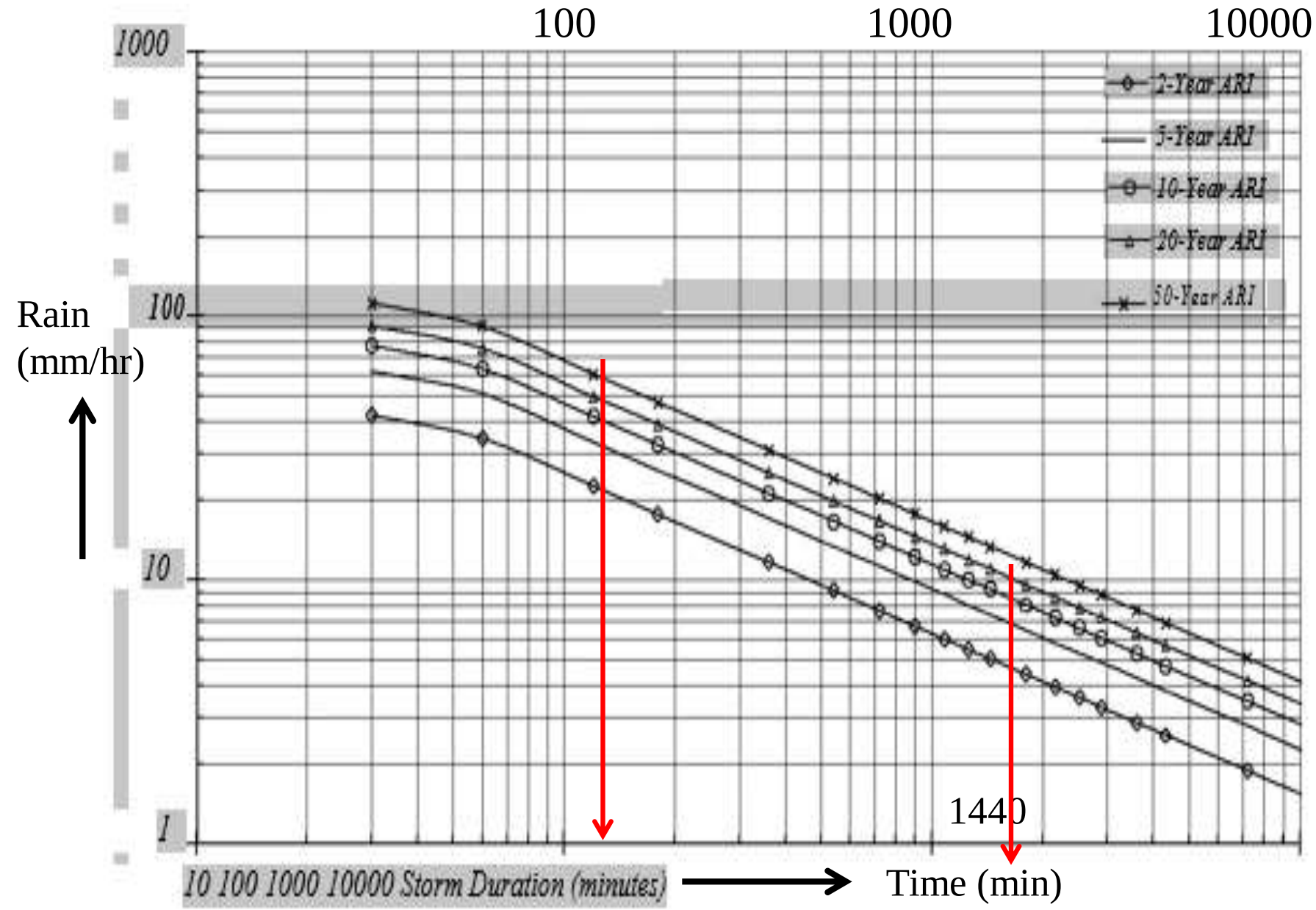
Steps for IDF analysis are:

- Select a design storm duration *D*, say *D*=24 hours.
- Collect the annual maximum rainfall depths of the selected duration from n years of historic data.
- From the data series, determine the mean and the standard deviation of the D-hr annual maximum rainfall.
- Calculate the *D*-hr *T*-yr design storm depth X_T by using the following frequency factor equation:

$$X_T = \mu + K_T \sigma$$

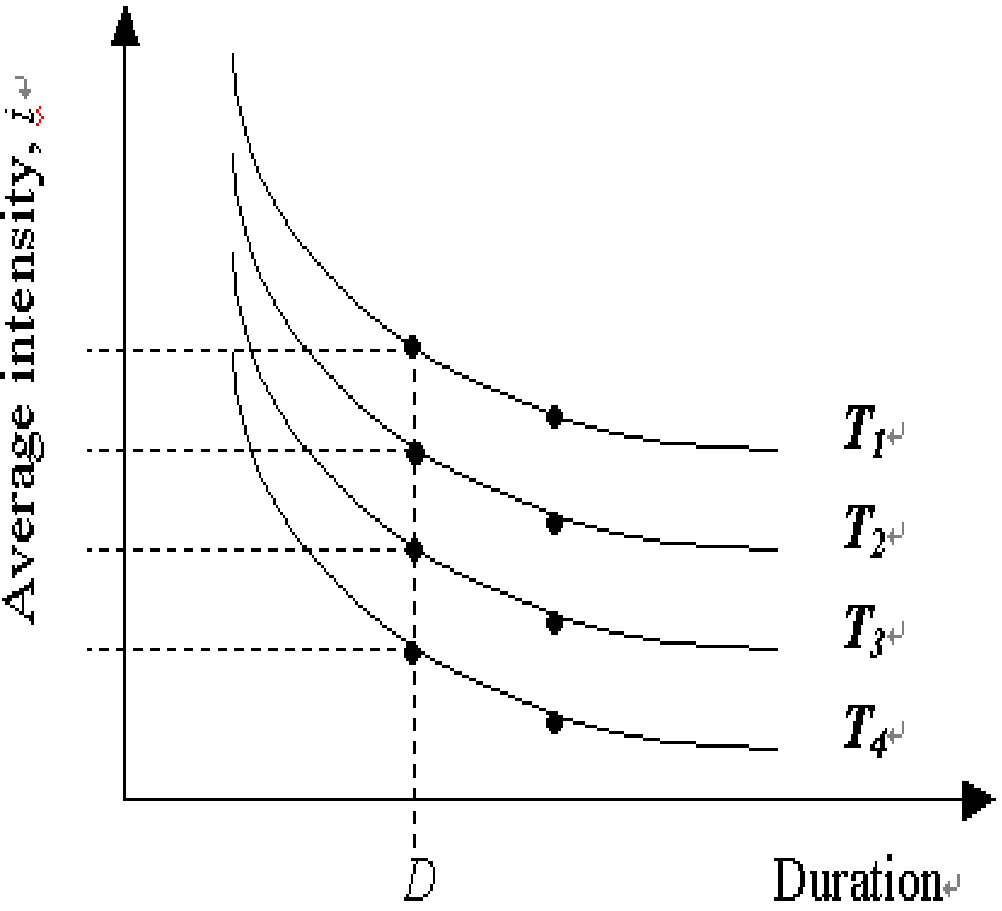
where μ, σ and K_T are mean, standard deviation and frequency factor, respectively. Note that the frequency factor, K is distribution-specific.

IDF Curves for Dhaka City

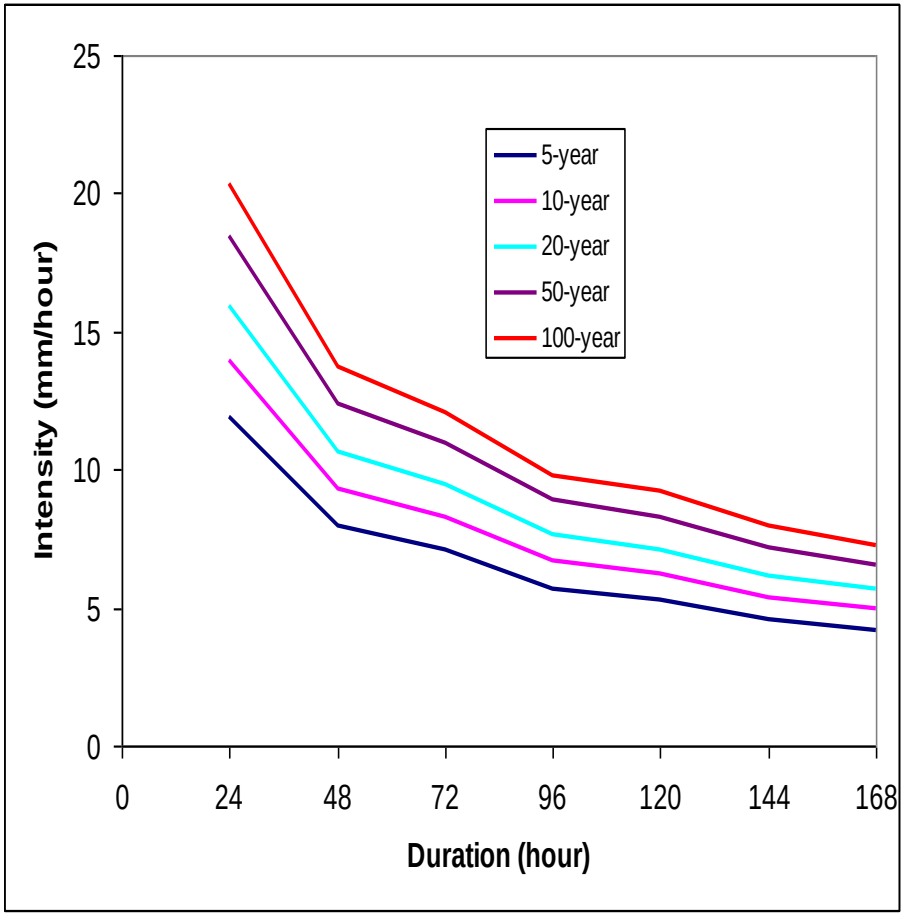


Rainfall Analysis

- Calculate the average intensity i = X_T /D and repeat Steps 1 through 4 for various design storm durations.
- Construct the IDF curves.



Long-duration IDF curves of rainfall at Chittagong



Rainfall Analysis

Example

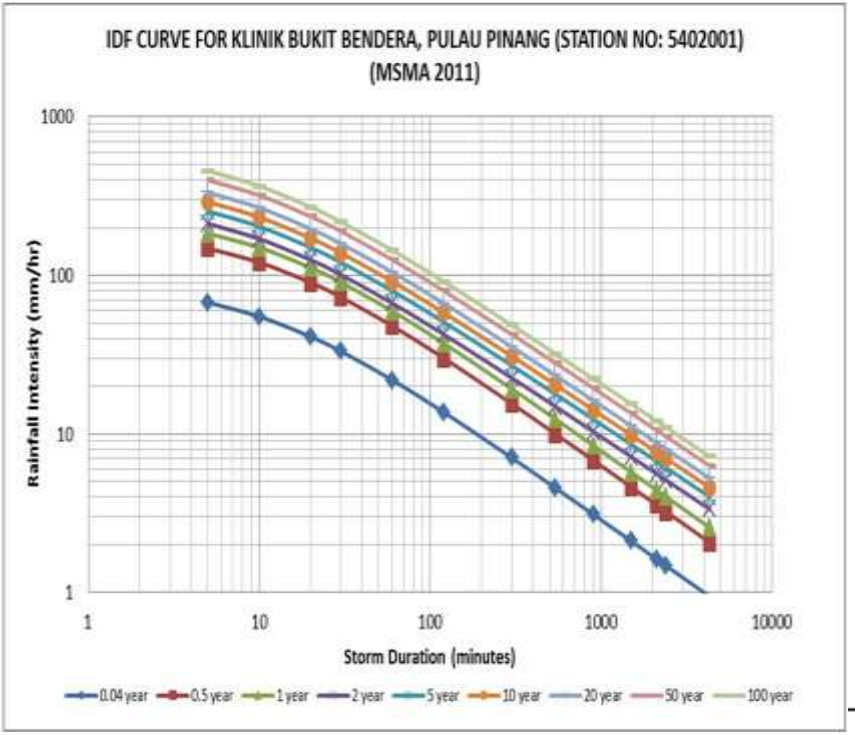
Hourly rainfall intensity data of period 1977-2000 for different durations are shown in table (note that the intensity decreases with the increase of the duration). Next, each duration series is subjected to frequency analysis using the Gumbel or EV1 distribution, as shown for single series in the figure (for 1.2.3….48 hrs).

Then calculate for each duration the intensities for different return periods by using the frequency factor equation:

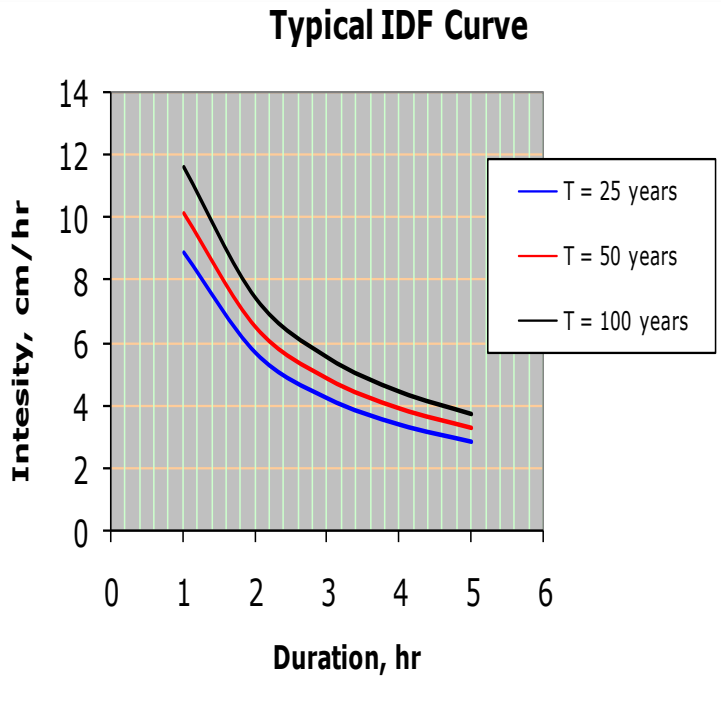
$$X_T = \mu + K_T \sigma$$

Note that the intensity increases with the increase of the return period.

	Durations (hour)									
Year	1	2	3	4	6	9	12	18	24	48
1977	14.40	9.70	7.60	6.55	4.37	2.91	2.18	1.68	1.56	1.03
1978	19.00	14.45	10.03	7.65	5.12	3.41	2.56	1.71	1.66	1.27
1979	23.30	19.40	14.50	10.95	7.30	5.00	3.78	2.81	2.37	1.86
.										
.										
1997	37.00	29.25	27.50	25.50	23.08	18.06	15.71	11.97	9.25	5.41
1998	22.40	15.00	10.00	8.88	6.17	4.46	3.50	2.56	2.38	1.47
1999	30.00	22.60	16.17	12.38	8.30	5.53	4.15	2.77	2.07	1.08
2000	24.00	12.30	8.53	6.70	4.62	4.38	3.28	2.19	2.29	1.48



Duration	Return Periods						
	1	2	4	10	25	50	100
1	11.666	24.020	32.774	42.773	52.212	59.214	66.164
2	7.282	16.280	22.656	29.938	36.813	41.912	46.975
3	5.297	12.428	17.480	23.251	28.698	32.740	36.751
4	4.203	10.040	14.175	18.899	23.358	26.666	29.949
6	3.188	7.615	10.752	14.334	17.716	20.225	22.716
9	2.296	5.614	7.964	10.648	13.183	15.063	16.929
12	1.711	4.399	6.303	8.479	10.532	12.056	13.568
18	1.212	3.098	4.435	5.961	7.403	8.472	9.533
24	1.133	2.636	3.700	4.916	6.064	6.916	7.761
48	0.716	1.586	2.202	2.906	3.571	4.064	4.553



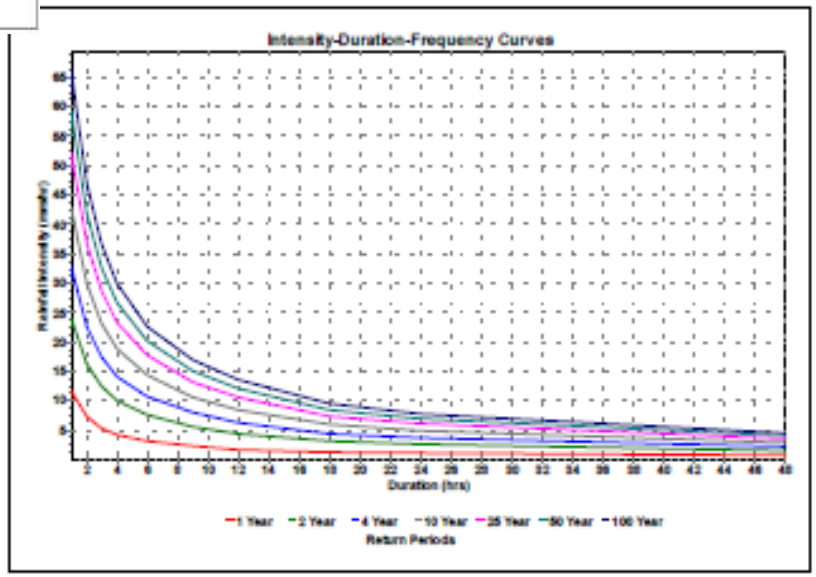


Figure 7.8: Intensity-Density-Frequency curves for Chaskman on linear scale (Annual maximum data)

Example

The 2-hr rainfall data for Mayanagar is shown in the following table. Develop the 2-hr IDF curve.

$$X_T = \mu + K_T \sigma \qquad K = -0.78\left[0.577 + \ln\left(\ln\frac{T_r}{T_r - 1}\right)\right]$$

Year	Depth (mm)
1964	14.6
1965	21.7
1966	12.1
1967	22.4
1968	23.4
1969	13.1
1970	19.2
1971	32.8
1972	11.2
1973	18.2
1974	19.2
1975	11.6
1976	11.6
1977	12.7
1978	7.2
1979	8
1980	10.6
1991	8.2
1982	26.2
1983	9.5

Mean: 15.675

St. dev: 6.92

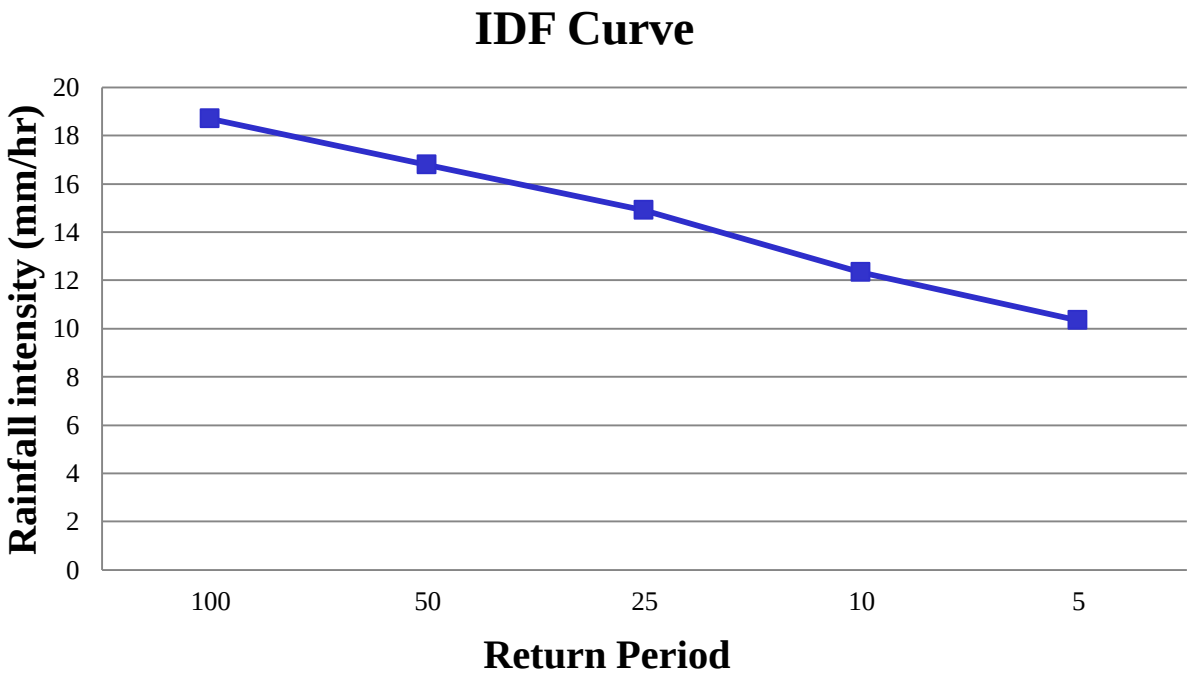
5-year return period rainfall: 15.675 + 0.72 x 6.92 = 20.66 mm or 10.33 mm/hr

10-year return period rainfall: 15.675 + 1.30 x 6.92 = 24.67 = 12.33 mm/hr

25-year return period rainfall: 15.675 + 2.04 x 6.92 = 29.79 = 14.89 mm/hr

50-year return period rainfall: 15.675 + 2.59 x 6.92 = 33.60 = 16.80 mm/hr

100-year return period rainfall: 15.675 + 3.14 x 6.92 = 37.40 = 18.70 mm/hr



Runoff

Rational Method

The rational method is very simple in concept but relies on considerable judgment and experience to evaluate all factors properly. It is used primarily for small drainage areas (less than 250 hectares) and computes the peak discharge as:

$$Q_p = 0.278\ C\ i\ A$$

where Q_p = peak discharge (m^3/s);

C= runoff coefficient

i = average intensity of rainfall of selected frequency (return period) corresponding to the duration of time of concentration (mm/hr)

A = drainage area (km^2)

For Q_p to occur, the rainfall duration must be >= the time of concentartion. If the duration is less than the time of concentration, then the discharge is not the peak for that particular intensity.

The time of concentration can be obtained from Kirpich (1940) equation as:

$$t_c = 0.0195\ L^{0.77}\ S^{-0.385}$$

Where: t_c - time of concentration (minutes)

L - maximum length of travel (meters)

S - slope = (H/L) where H is the difference in elevation between the furthest point in the catchment and the outlet.

Runoff

Example

A 20 ha (suburban area: 12 ha and woodland: 8 ha) catchment has an average slope of 3% and a maximum flow length of 400 m. What would be the peak runoff rate ~~for a 10 mm/hr rainfall of 0.5 hr duration~~? (for 2 –hr 20-year rainfall?

Solution:

For the catchment area of 20 ha, the calculated weighted average C is:

Suburban area C : 12 x 0.30 = 3.6

Woodland area C : 8 x 0.15 = 1.2

The weighted average C for the catchment is = 4.8/20 = 0.24

The time of concentration: $t_c = 0.0195\ L^{0.77}\ S^{-0.385}$

$= 0.0195\ (400)^{0.77}\ (0.03)^{-0.385} = 0.0195 \times (100.83) \times (3.857)$

= 7.58 minutes

Since the rainfall duration (0.5 hr) is greater than t_c, the peak runoff rate is: $Q_p = 0.278\ C\ i\ A = 0.278 \times 0.24 \times 10 \times 20/100 = 0.133$ cumec

Year	Rainfall (mm)
1976	49.5
1977	33.0
1978	36.3
1979	68.9
1980	46.1
1981	13.1
1982	180.3
1983	42.7
1984	86.4
1985	27.9
1986	182.0
1987	21.5
1988	50.3
1989	78.2
1990	21.6
1991	53.3
1992	40.7
1993	38.1
1994	12.8
1995	17.0
1996	34.3
1997	47.8
1998	91.4

Maximum 2-hr Rainfall at Joydebpur

$$x_T = \bar{x} + K_T s$$

K_T values are calculated for different return periods using

$$K_T = -\frac{\sqrt{6}}{\pi}\left\{0.5772 + \ln\left[\ln\left(\frac{T}{T-1}\right)\right]\right\}$$

T (years)	2	5	10	50	100
K_T	-0.164	0.719	1.305	2.592	3.137

Find the 10-yr peak flow for a 2-hr rainfall at Joydebpur.