# Flood Routing

The flood hydrograph is in fact a wave. The stage and discharge hydrographs represent the passage of waves of stream depth and discharge respectively. As this wave moves down, the shape of the wave gets modified due to channel storage, resistance, lateral addition or withdrawal of flows etc. When a flood wave passes through a reservoir its peak is attenuated and the time base is enlarged due to effect of storage

# Flood Routing

Routing is a mathematical technique used to predict the temporal and spatial variations of a flood hydrograph as it moves through a river reach or reservoir. Routing takes into account the effects of storage and flow resistance.

Routing methods combine the continuity equation with some relationship between storage, outflow, and inflow.

These relationships are based on continuity equation and are usually empirical, or analytical in nature.

Continuity equation

The change in storage (dS) equals the difference between inflow (I) and outflow (O):

$$\frac{\Delta S}{\Delta t} = I - O \qquad \text{..... (1)}$$

Routing Applications

Purposes: Flood forecasting, reservoir and channel design, floodplain studies, etc.

Routing Techniques

Hydraulic routing techniques

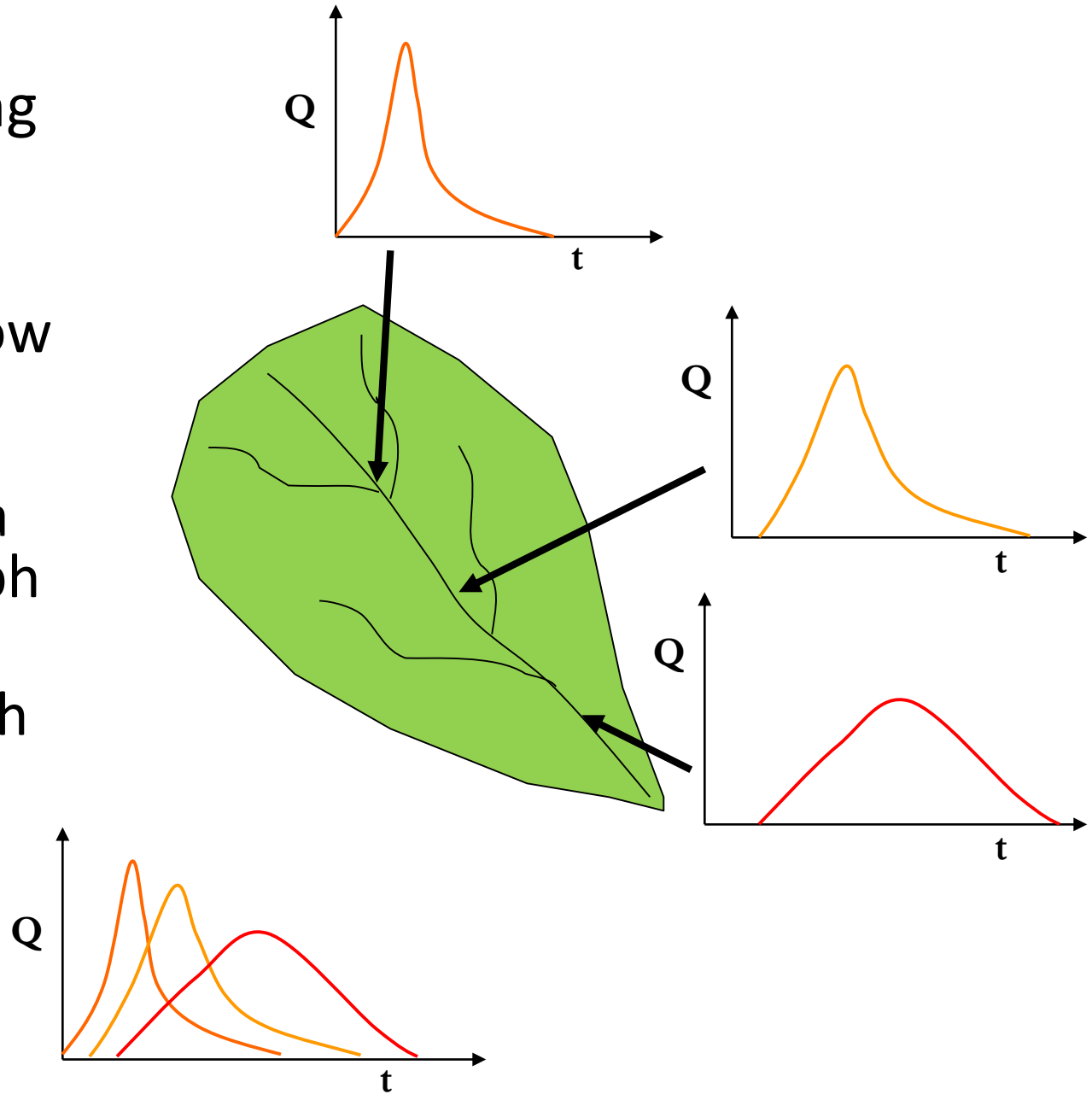
- Involve conservation of mass and momentum
- Range from the full dynamic wave equations to the kinematic wave approximation

Hydrologic routing techniques

- Involve conservation of mass and relationship between storage and outflow

# Flood Routing

- Procedure to determine the flow hydrograph at a point on a watershed from a known hydrograph upstream
- As the hydrograph travels, it
  - attenuates
  - gets delayed



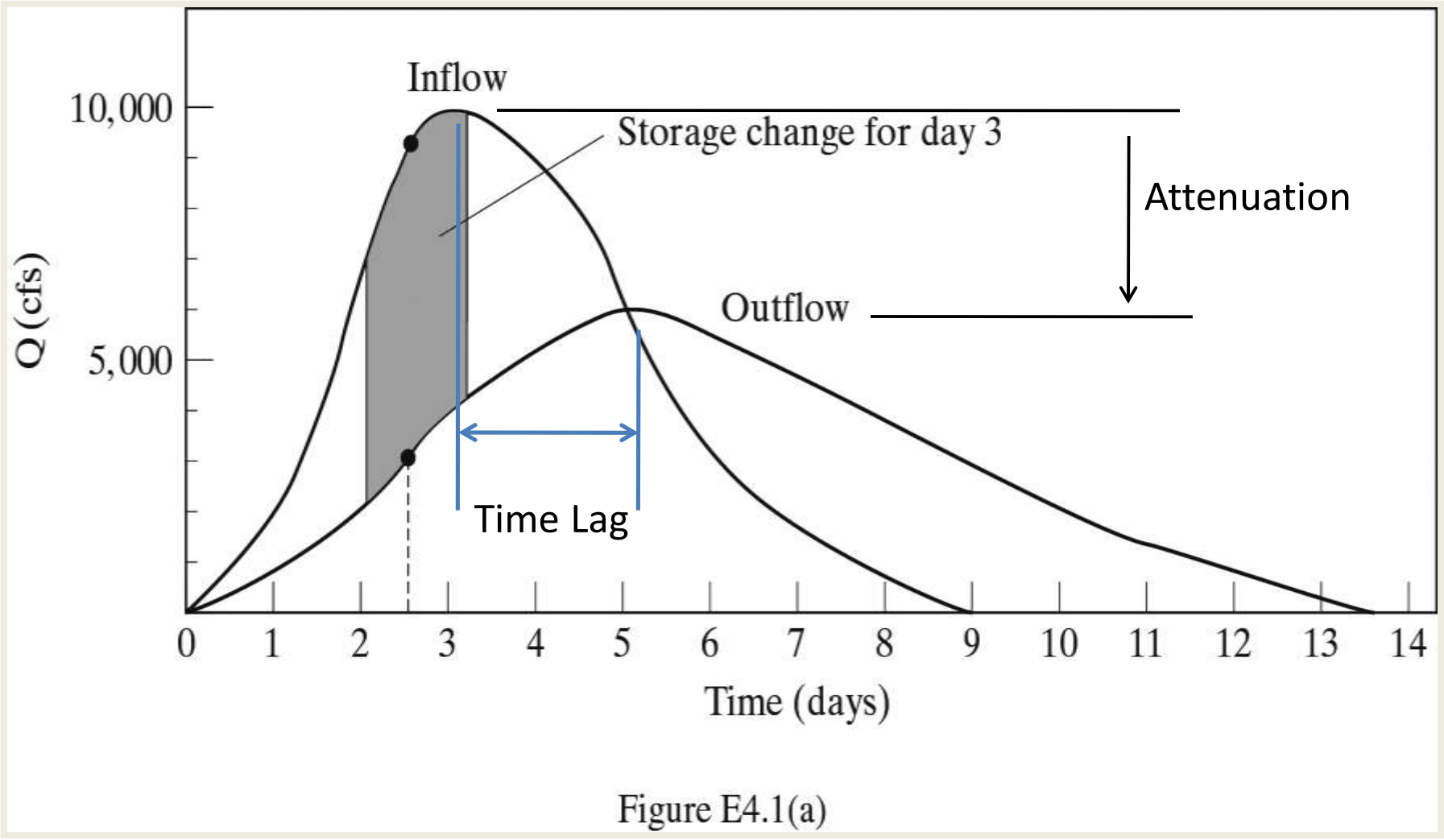



Figure E4.1(a)

The reduction in the peak of the outflow hydrograph due to storage effects is called attenuation.

The time difference between the peaks of inflow and outflow hydrographs is known as the time lag.

# Flood Routing

Hydrologic routing techniques are applied for:

- Reservoir routing
- Channel routing

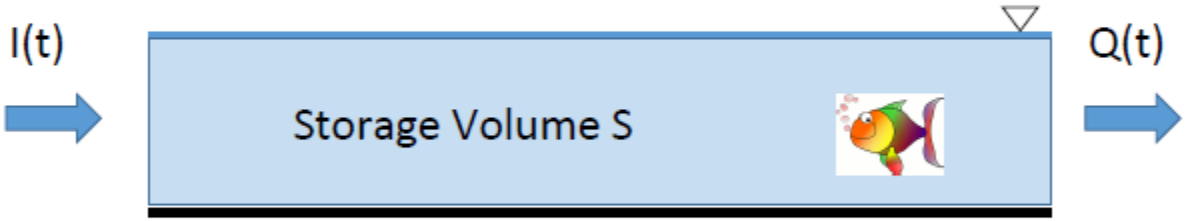
Reservoir routing

In reservoir routing the effect of a flood entering a reservoir is studied. Knowing the storage and outflow characteristics of the reservoir, the effect of inflow on the outflow and storage of the reservoir can be predicted.

Channel routing

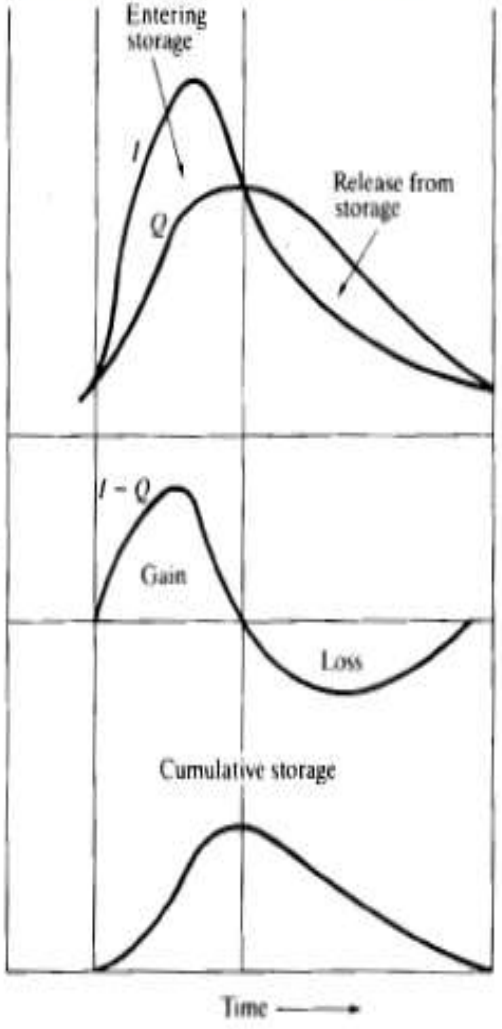
In channel routing the change in the hydrograph as a flood travels along the channel is studied. Information on flood peak attenuation and duration of the high flood level is essential for flood forecasting and for flood protection works.

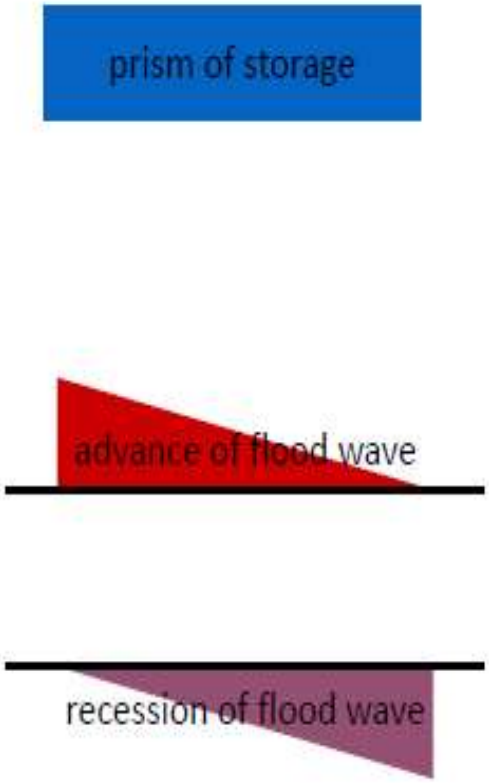
In a steady state condition, there would be a simple ‘prism’ of storage in a river that is never changing.




- If the system is in steady state, I = Q for all time
- Introduce a parameter K so that S(t) = K*Q(t)  (units?)

# Due to the advancing and retreating flood wave, we add or subtract a wedge to the steady-state prism.



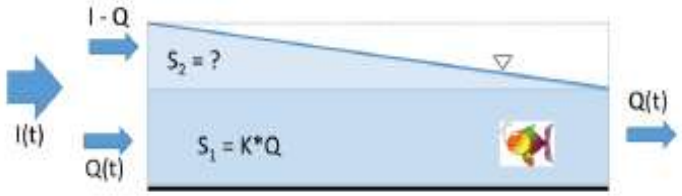



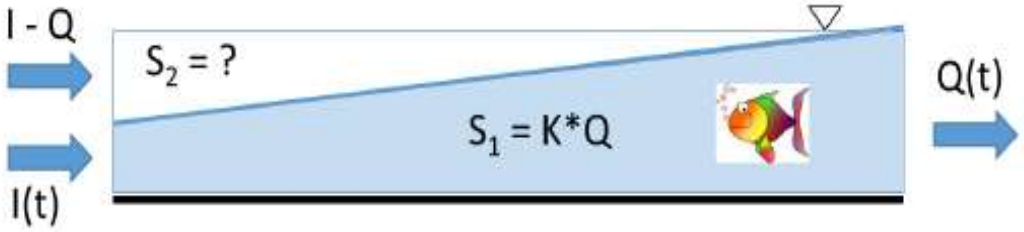

We start with a prism of storage that is formed by a volume of constant cross-section along the length of prismatic channel.

During the advance of a flood wave, inflow exceeds outflow, producing a wedge of storage.

During the recession, outflow exceeds inflow, resulting in a negative wedge.

# Add the consideration of the wedge…

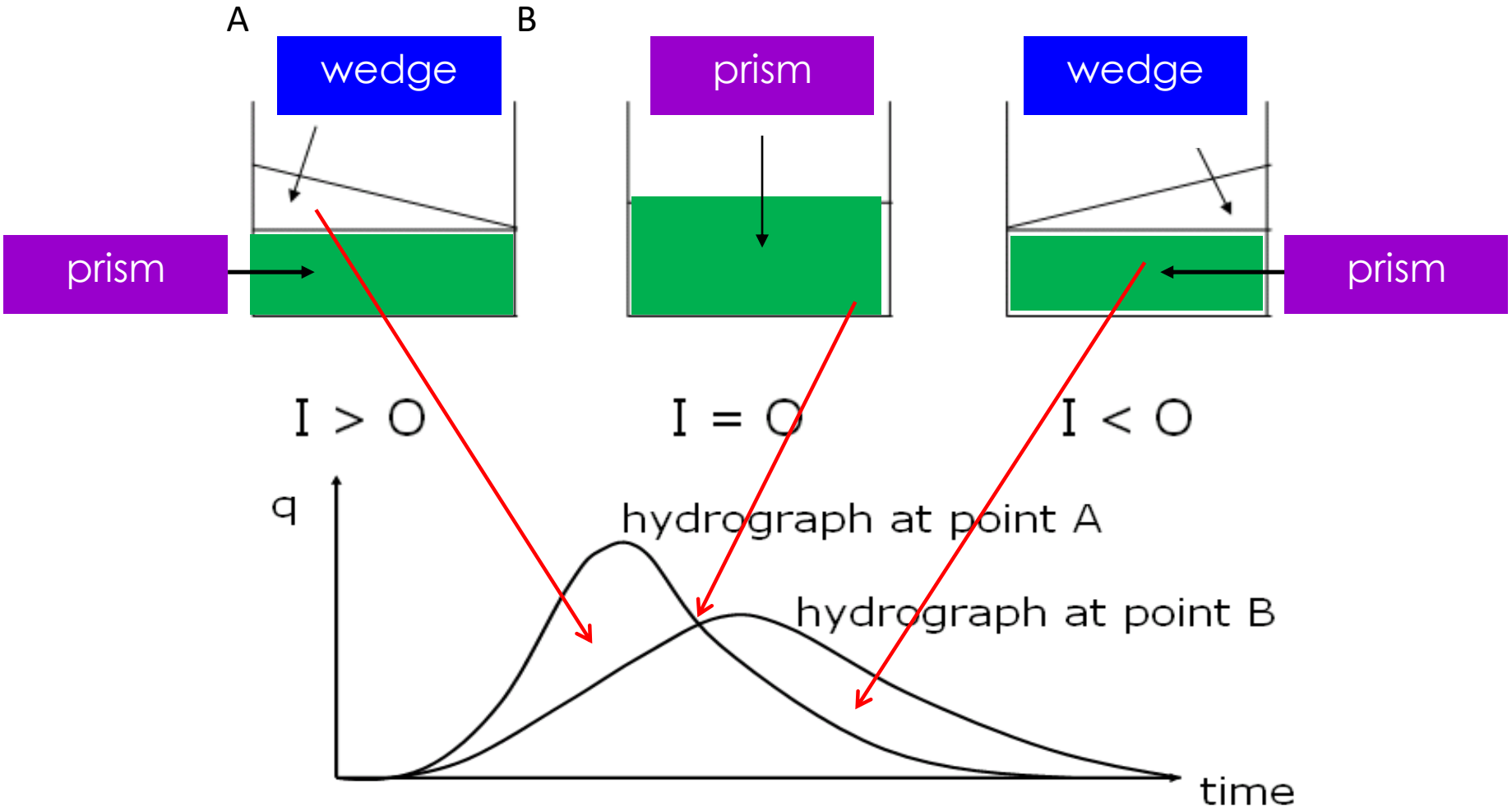



- Muskingum method assumes wedge-shaped storage
- Flow in wedge = I(t) – Q(t). Either positive or negative!
- $S_2 = KX(I - Q)$ in either case (X is a weighting factor, $0 \leq X \leq 0.5$)
- Total storage $S(t) = S_1 + S_2$ for both cases

# Flood Flow in a Channel

As a flood hydrograph moves down a channel, its shape is modified as water is stored in the channel. The channel storage is composed of two parts: the prismatic storage which is the water in the channel when inflow and outflow are equal, and the wedge storage which is proportional to the difference between inflow and outflow.



# River Routing
# (Muskingum Method)

## Prism and wedge storages in reach

$$S_{\text{Prism}} = KQ$$

$$S_{\text{Wedge}} = KX(I - Q)$$

**Advancing Flood Wave** $I > Q$

**K = travel time of peak through the reach**

**X = weight on inflow versus outflow**

**$(0 \le X \le 0.5)$**

**X = 0 ➔ Reservoir, storage depends on outflow, no wedge**

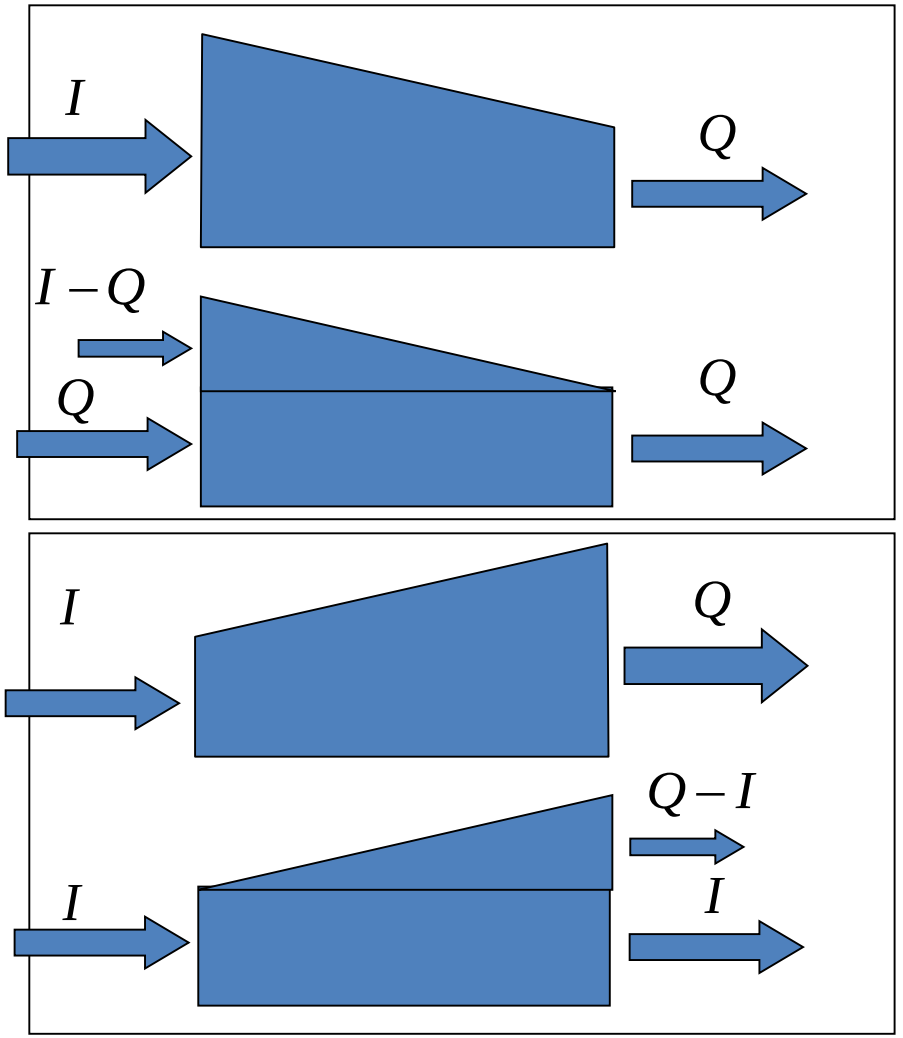
**X = 0.0 - 0.3 ➔ Natural stream**

Storage = $S_{\text{Prism}} + S_{\text{Wedge}}$

$$S = KQ + KX(I - Q)$$

**Receding Flood Wave** $Q > I$

$$S = K[XI + (1 - X)Q]$$



# Muskingum Method

$$S = K[XI + (1-X)Q]$$ ..... Eq. 1

$$S_{j+1} - S_j = K\{[XI_{j+1} + (1-X)Q_{j+1}] - [XI_j + (1-X)Q_j]\}$$

**Recall:**

$$S_{j+1} - S_j = \frac{I_{j+1} + I_j}{2}\Delta t - \frac{Q_{j+1} + Q_j}{2}\Delta t$$ ..... Eq. 2

**Combine:**

$$Q_{j+1} = C_0 I_{j+1} + C_1 I_j + C_2 Q_j$$

$$C_0 = \frac{\Delta t - 2KX}{2K(1-X) + \Delta t}$$

$$C_1 = \frac{\Delta t + 2KX}{2K(1-X) + \Delta t}$$

$$C_2 = \frac{2K(1-X) - \Delta t}{2K(1-X) + \Delta t}$$

If *I(j)*, *K* and *X* are known, *Q(j)* can be calculated using above equations

# Muskingum Method

$$C_0 = \frac{-KX + 0.5\Delta t}{K - KX + 0.5\Delta t}$$

$$C_1 = \frac{KX + 0.5\Delta t}{K - KX + 0.5\Delta t}$$

$$C_2 = \frac{K - KX - 0.5\Delta t}{K - KX + 0.5\Delta t}$$

$$C_0 + C_1 + C_2 = 1$$

$$Q_{j+1} = C_0 I_{j+1} + C_1 I_j + C_2 Q_j$$

**Note:** The denominators of all the equations are same.

**Need K and Δt in the same units**

# Muskingum Method

## Example

- Given:
  - Inflow hydrograph
  - K = 2.3 hr, X = 0.15, $\Delta$t = 1 hour, Initial Q = 85 cfs
- Find:
  - Outflow hydrograph using Muskingum routing method

$$C_0 = \frac{\Delta t - 2KX}{2K(1-X)+\Delta t} = \frac{1-2*2.3*0.15}{2*2.3(1-0.15)+1} = 0.0631$$

$$C_1 = \frac{\Delta t + 2KX}{2K(1-X)+\Delta t} = \frac{1+2*2.3*0.15}{2*2.3(1-0.15)+1} = 0.3442$$

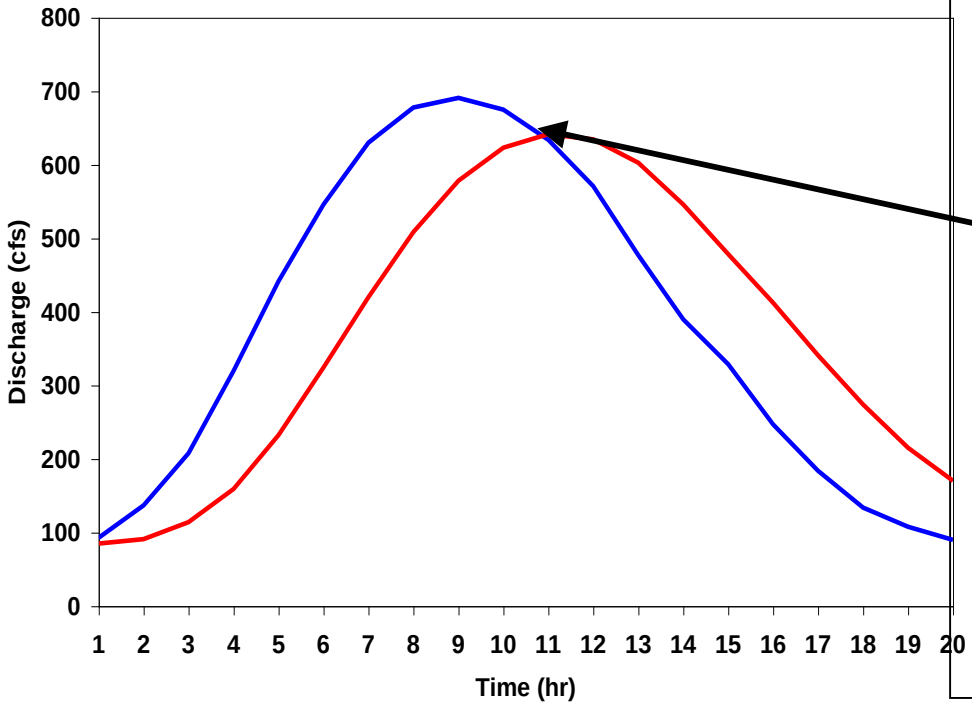
$$C_2 = \frac{2K(1-X)-\Delta t}{2K(1-X)+\Delta t} = \frac{2*2.3*(1-0.15)-1}{2*2.3(1-0.15)+1} = 0.5927$$

| Period (hr) | Inflow (cfs) |
|---|---|
| 1 | 93 |
| 2 | 137 |
| 3 | 208 |
| 4 | 320 |
| 5 | 442 |
| 6 | 546 |
| 7 | 630 |
| 8 | 678 |
| 9 | 691 |
| 10 | 675 |
| 11 | 634 |
| 12 | 571 |
| 13 | 477 |
| 14 | 390 |
| 15 | 329 |
| 16 | 247 |
| 17 | 184 |
| 18 | 134 |
| 19 | 108 |
| 20 | 90 |

# Muskingum Method

$$Q_{j+1} = C_0 I_{j+1} + C_1 I_j + C_2 Q_j$$

$C_0 = 0.0631$, $C_1 = 0.3442$, $C_2 = 0.5927$




| Period (hr) | Inflow (cfs) | $C_0 I_{j+1}$ | $C_1 I_j$ | $C_2 Q_j$ | Outflow (cfs) |
|---|---|---|---|---|---|
| 1 | 93 | 0 | 0 | 0 | 85 |
| 2 | 137 | 9 | 32 | 50 | 91 |
| 3 | 208 | 13 | 47 | 54 | 114 |
| 4 | 320 | 20 | 72 | 68 | 159 |
| 5 | 442 | 28 | 110 | 95 | 233 |
| 6 | 546 | 34 | 152 | 138 | 324 |
| 7 | 630 | 40 | 188 | 192 | 420 |
| 8 | 678 | 43 | 217 | 249 | 509 |
| 9 | 691 | 44 | 233 | 301 | 578 |
| 10 | 675 | 43 | 238 | 343 | 623 |
| 11 | 634 | 40 | 232 | 369 | 642 |
| 12 | 571 | 36 | 218 | 380 | 635 |
| 13 | 477 | 30 | 197 | 376 | 603 |
| 14 | 390 | 25 | 164 | 357 | 546 |
| 15 | 329 | 21 | 134 | 324 | 479 |
| 16 | 247 | 16 | 113 | 284 | 413 |
| 17 | 184 | 12 | 85 | 245 | 341 |
| 18 | 134 | 8 | 63 | 202 | 274 |
| 19 | 108 | 7 | 46 | 162 | 215 |
| 20 | 90 | 6 | 37 | 128 | 170 |

Attenuation: 691 – 642 = 49 cfs  lag time = 11 -9 = 2 hr

# Muskingum Method

$$S = K[XI + (1 - X)Q]$$

**Estimating K and X**

The constant K is called the storage constant. It is the ratio of the storage to the discharge and has the unit of time.

K is estimated to be the travel time through the reach.

The parameter X may be thought of as a weighting coefficient for inflow and outflow.

The value of X must be between 0.0 and 0.5.

When X = 0, the inflow has no effect on storage. When X = 0.5, both inflow and outflow are equally effective.

Values of X = 0.2 to 0.3 are the most common for natural streams. This means that while routing the storage is dominated by the outflow.

If flow data of previous floods are available, then K and X are determined by plotting S and XI + (1 – X)O for various values of X.

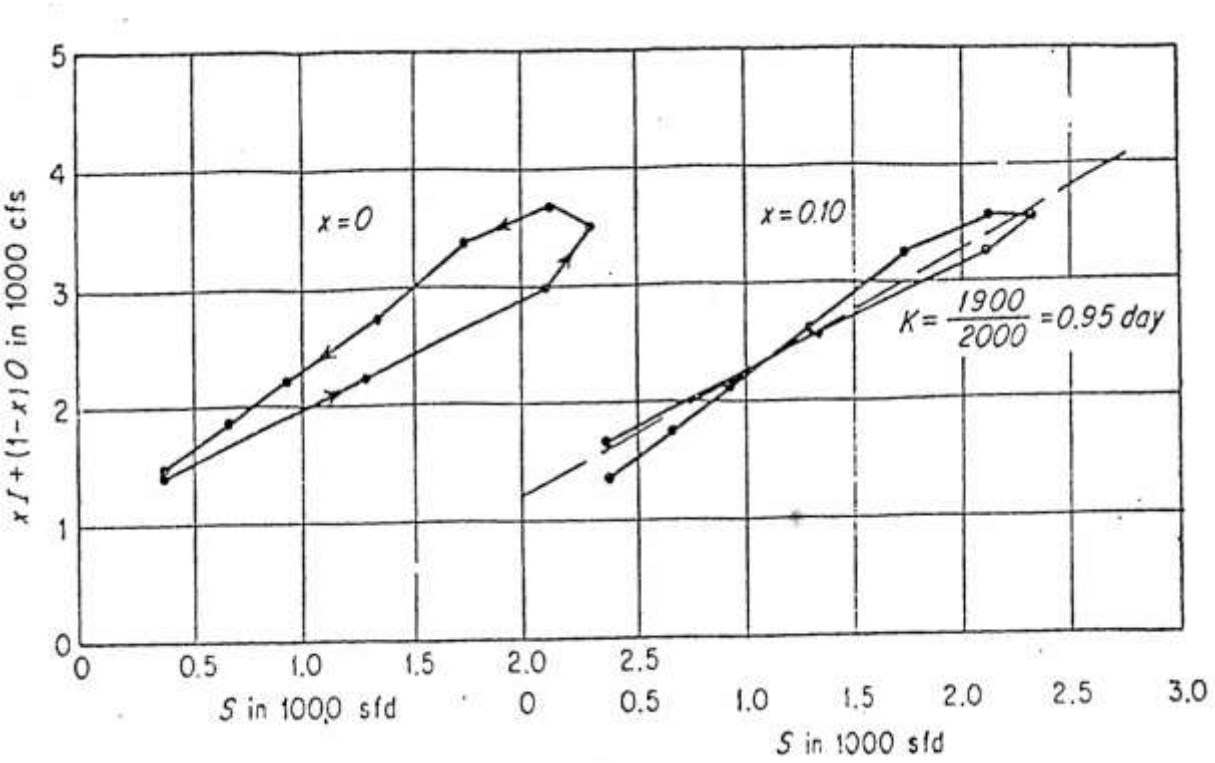
The best value of X is that which causes the data to plot most nearly as a single value curve.

The Muskingum method assumes that this curve is a straight line with slope K.

# Estimating Muskingum Parameters

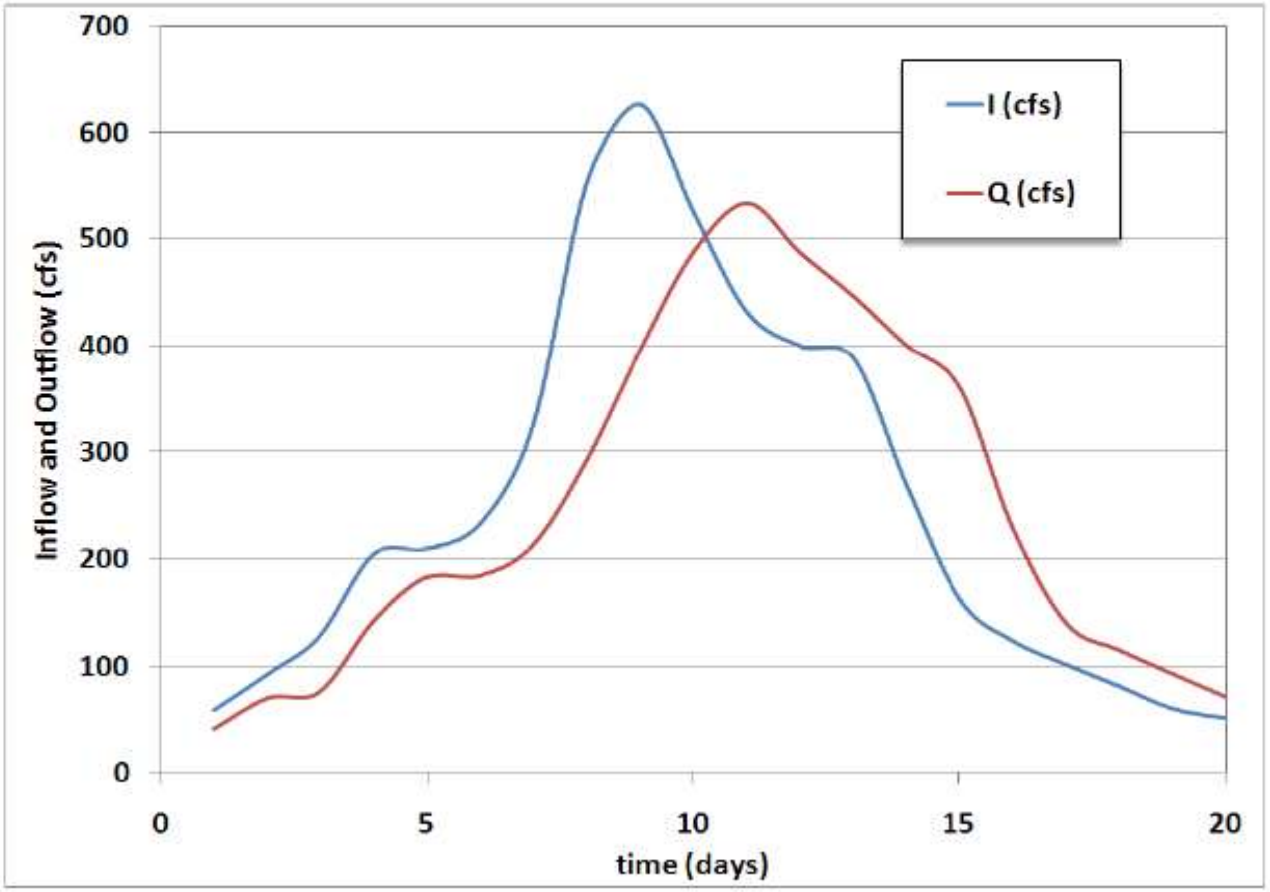
Graphical Method:

Referring to the Muskingum Model, find X such that the plot of $XI_t + (1-X)O_t$ vs $S_t$ behaves almost nearly as a single value curve. The corresponding slope is K.



# Example: Estimating K and X (Muskingum parameters)

- River reach - Know inflow and outflow hydrographs
- Estimate Muskingum Parameters

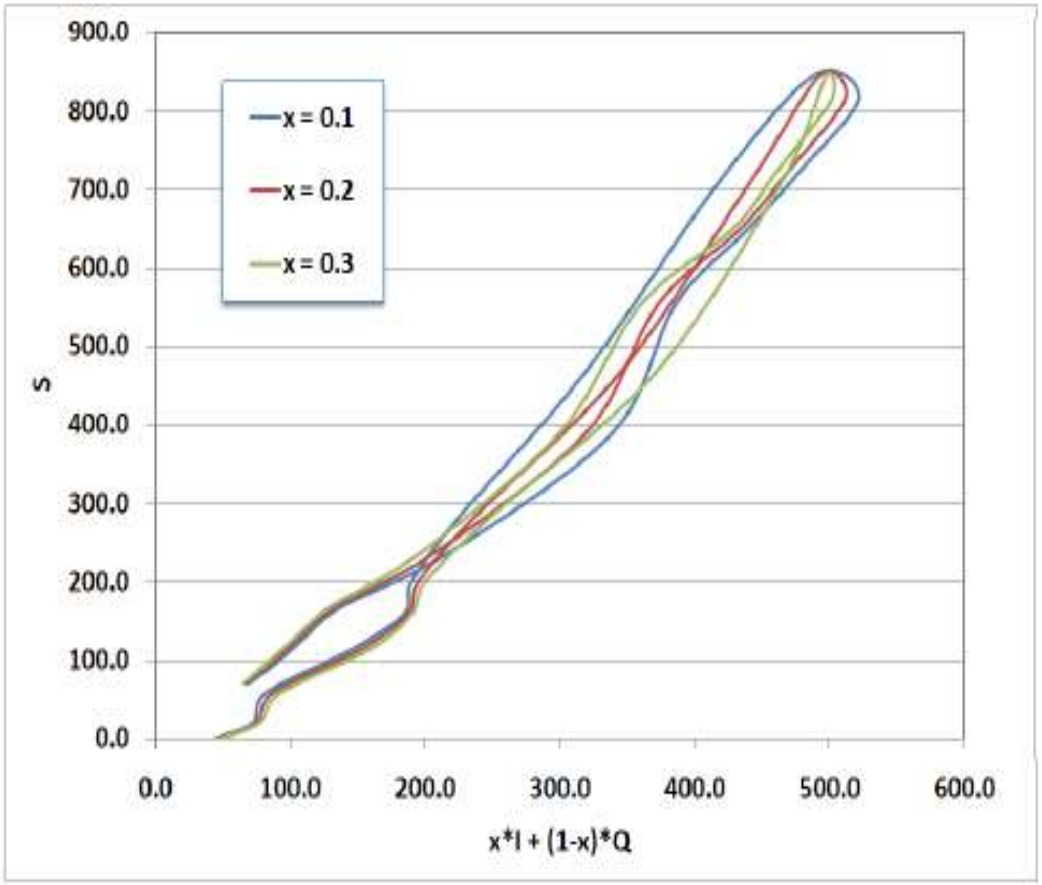


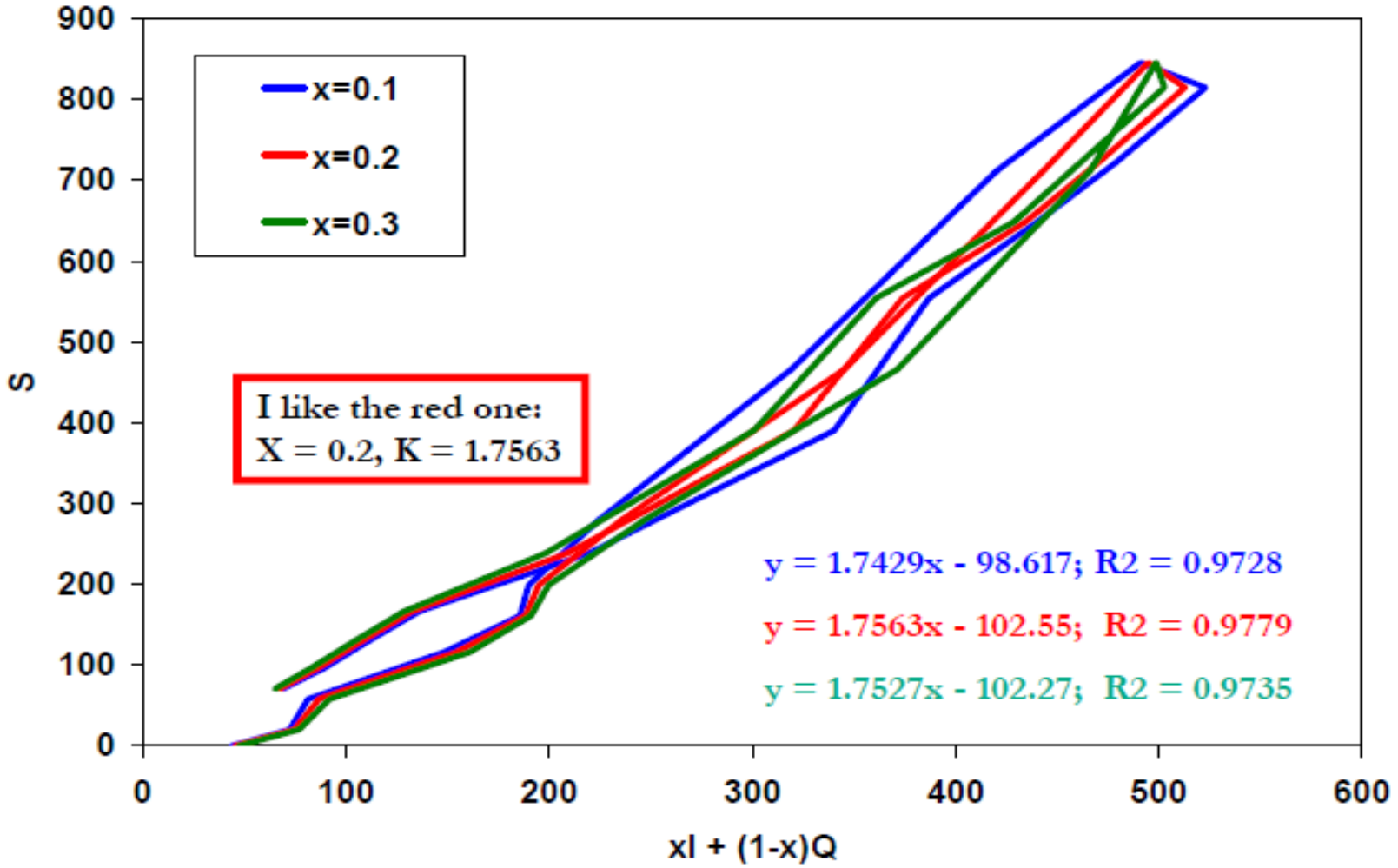

| Time (day) | I (cfs) | Q (cfs) |
|---|---|---|
| 1 | 59 | 42 |
| 2 | 93 | 70 |
| 3 | 129 | 76 |
| 4 | 205 | 142 |
| 5 | 210 | 183 |
| 6 | 234 | 185 |
| 7 | 325 | 213 |
| 8 | 554 | 293 |
| 9 | 627 | 397 |
| 10 | 526 | 487 |
| 11 | 432 | 533 |
| 12 | 400 | 487 |
| 13 | 388 | 446 |
| 14 | 270 | 400 |
| 15 | 162 | 360 |
| 16 | 124 | 230 |
| 17 | 102 | 140 |
| 18 | 81 | 115 |
| 19 | 60 | 93 |
| 20 | 51 | 71 |

- **Know inflow and outflow hydrograph**
- **Compute storage**
- **Compute** $[XI+(1-X)Q]$ **for various values of** $X$

$$S_{j+1} = S_j + \frac{I_{j+1} + I_j}{2}\Delta t - \frac{Q_{j+1} + Q_j}{2}\Delta t$$

| Time t (day) | S S (ft3) | x*I+(1-x)*Q X = 0.1 | x*I+(1-x)*Q X = 0.2 | x*I+(1-x)*Q X = 0.3 |
|---|---|---|---|---|
| 1 | 0 | 44 | 45 | 47 |
| 2 | 20 | 72 | 75 | 77 |
| 3 | 58 | 81 | 87 | 92 |
| 4 | 116 | 148 | 155 | 161 |
| 5 | 161 | 186 | 188 | 191 |
| 6 | 199 | 190 | 195 | 200 |
| 7 | 280 | 224 | 235 | 247 |
| 8 | 466 | 319 | 345 | 371 |
| 9 | 712 | 420 | 443 | 466 |
| 10 | 846 | 491 | 495 | 499 |
| 11 | 815 | 523 | 513 | 503 |
| 12 | 721 | 478 | 470 | 461 |
| 13 | 649 | 440 | 434 | 429 |
| 14 | 555 | 387 | 374 | 361 |
| 15 | 391 | 340 | 320 | 301 |
| 16 | 239 | 219 | 209 | 198 |
| 17 | 167 | 136 | 132 | 129 |
| 18 | 131 | 112 | 108 | 105 |
| 19 | 97 | 90 | 86 | 83 |
| 20 | 71 | 69 | 67 | 65 |






$$S = K[XI + (1 - X)Q]$$

Remember the unit of Δt in the storage equation.