Lecture-3

Continuous Random Variables and Probability Distributions

Learning Objectives

After careful study of this lecture, you should be able to do the following:

1. Determine probabilities from probability density functions
2. Determine probabilities from cumulative distribution functions and cumulative distribution functions from probability density functions, and the reverse
3. Calculate means and variances for continuous random variables
4. Understand the assumptions for some common continuous probability distributions
5. Select an appropriate continuous probability distribution to calculate probabilities in specific applications
6. Calculate probabilities, determine means and variances for some common continuous probability distributions
7. Standardize normal random variables
8. Use the table for the cumulative distribution function of a standard normal distribution to calculate probabilities
9. Approximate probabilities for some binomial and Poisson distributions

Continuous Random Variables:

Suppose that a dimensional length is measured on a manufactured part selected from a day's production. In practice, there can be small variations in the measurements due to many causes, such as vibrations, temperature luctuations, operator differences, calibrations, cutting tool wear, bearing wear, and raw material changes. In an experiment such as this, the measurement is naturally represented as a random variable X, and it is reasonable to model the range of possible values of X with an interval of real numbers. A continuous random variable is a random variable with an interval (either finite or infinite) of real numbers for its range. The model provides for any precision in length measurements.

Because the number of possible values of X is uncountably infinite, X has a distinctly different distribution from the discrete random variables studied previously. But as in the discrete case, many physical systems can be modeled by the same or similar continuous random variables. These random variables are described, and example computations of probabilities, means, and variances are provided in this lecture.

Probability Distributions and Probability Density Functions:

Density functions are commonly used in engineering to describe physical systems. For example, consider the density of a loading on a long, thin beam as shown in Fig. 3-1. For any point x along the beam, the density can be described by a function (in grams/cm). Intervals with large loadings correspond to large values for the function. The total loading between points a and b is determined as the integral of the density function from a to b. This integral is the area under the density function over this interval, and it can be loosely interpreted as the sum of all the loadings over this interval.

Similarly, a probability density function f(x) can be used to describe the probability distribution of a continuous random variable X. If an interval is likely to contain a value for X, its probability is large and it corresponds to large values for f (x). The probability that X is between a and b is determined as the integral of f (x) from a to b. See Fig. 3-2.

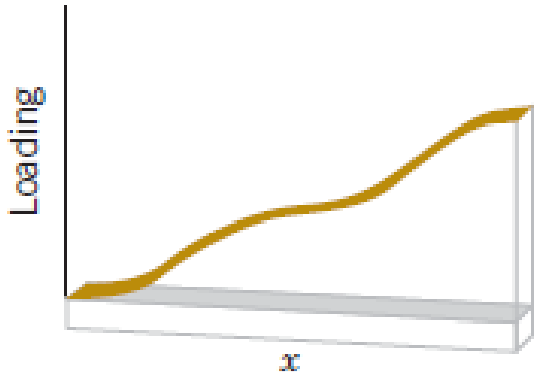


FIGURE 3-1 Density function of a loading on a long, thin beam.

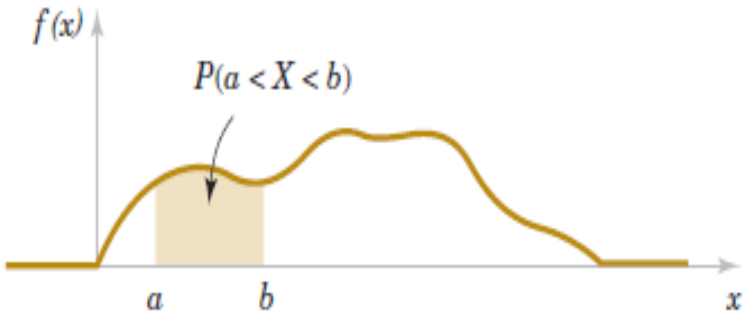


FIGURE 3-2 Probability determined from the area under f(x).

Probability Density Function:

For a continuous random variable X, a probability density function is a function such that

(1) $f(x) \geq 0$

(2) $\int_{-\infty}^{\infty} f(x)dx = 1$

(3) $P(a \leq X \leq b) = \int_a^b f(x)\,dx$ = area under f(x) from a to b for any a and b.

Example 3-1:

Let the continuous random variable X denote the current measured in a thin copper wire in milliamperes. Assume that the range of X is [4.9, 5.1] mA, and assume that the probability density function of X is f (x) = 5 for 4.9 ≤ x ≤ 5.1. What is the probability that a current measurement is less than 5 milliamperes?

The probability density function is shown in Fig. 3-3. It is assumed that f(x) = 0 wherever it is not specifically defined. The shaded area in Fig. 3-3 indicates the probability.

$P(X < 5) = \int_{4.9}^{5} f(x)\,dx = \int_{4.9}^{5} 5\,dx = 0.5$

As another example, $P(4.95 < X < 5.1) = \int_{4.95}^{5.1} f(x)\,dx = \int_{4.95}^{5.1} 5\,dx = 0.75$

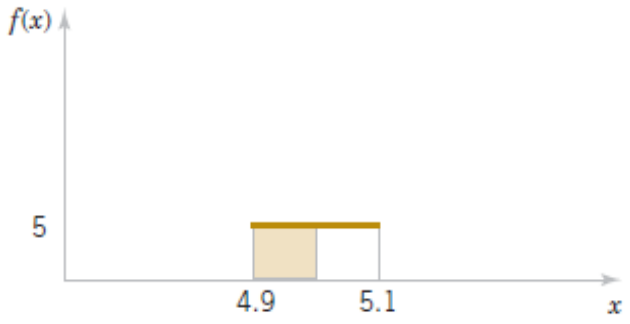


FIGURE 3-3 Probability density function

Example 3-2:

Let the continuous random variable X denote the diameter of a hole drilled in a sheet metal component. The target diameter is 12.5 millimeters. Most random disturbances to the process result in larger diameters. Historical data show that the distribution of X can be modeled by a probability density function $f(x) = 20e^{-20(x-12.5)}\ for\ x \geq 12.5$

If a part with a diameter greater than 12.60 mm is scrapped, what proportion of parts is scrapped? The density function and the requested probability are shown in Fig. 3-

4. A part is scrapped if X > 12.60. Now,

$$P(X > 12.60) = \int_{12.60}^{\infty} f(x)\,dx = \int_{12.6}^{\infty} 20e^{-20(x-12.5)}dx = -e^{-20(x-12.5)}\Big|_{12.6}^{\infty} = 0.135$$

What proportion of parts is between 12.5 and 12.6 millimeters? Now

$$P(12.5 < X > 12.6) = \int_{12.5}^{12.6} f(x)\,dx = \int_{12.5}^{12.6} 20e^{-20(x-12.5)}dx = -e^{-20(x-12.5)}\Big|_{12.5}^{12.6}$$

$$= 0.865$$

Because the total area under f(x) equals 1, we can also calculate$P(12.5 < X < 12.6) = 1 - P(X > 12.6) = 1 - 0.135 = 0.865.$

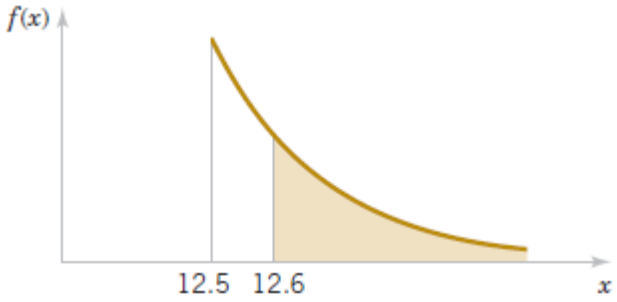


FIGURE 3-4 Probability density function

Practical Interpretation: Because 0.135 is the proportion of parts with diameters greater than 12.60 mm, a large proportion of parts is scrapped. Process improvements are needed to increase the proportion of parts with dimensions near 12.50 mm.

Cumulative Distribution Functions:

The cumulative distribution function of a continuous random variable X is

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)du \quad for -\infty < x < \infty$$

The cumulative distribution function is defined for all real numbers. The following example illustrates the definition.

Example 3-3:

For the copper current measurement in Example 3-1, find the cumulative distribution function of the random variable X.

Solution:

The cumulative distribution function of the random variable X consists of three expressions.

If $x < 4.9, f(x) = 0$. Therefore $F(x) = 0, for\ x < 4.9$ and $F(x) = \int_{4.9}^{x} f(u)du = 5x - 24.5,$ for $4.9 \le x \le 5.1$.

Finally, $F(x) = \int_{5.1}^{\infty} f(u)du = 1, for\ x \ge 5.1$

Therefore, $F(x) = \begin{cases} 0; & x < 4.9 \\ 5x - 24.5; & 4.9 \le x \le 5.1 \\ 1; & x \ge 5.1 \end{cases}$

Example 3-4:

For the drilling operation in Example 3-2, find the cumulative distribution function of the random variable X.

Solution:

The cumulative distribution function of the random variable X consists of two expressions.

$$F(x) = \begin{cases} 0; & x < 12.5 \\ 1 - e^{-20(x-12.5);} & x \ge 12.5 \end{cases}$$

Practical Interpretation: The cumulative distribution function enables one to easily calculate the probability a diameter in less than a value (such as 12.60 mm). Therefore, the probability of a scrapped part can be easily determined.

Probability Density Function from the Cumulative Distribution Function:

The probability density function of a continuous random variable can be determined from the cumulative distribution function by differentiating. The fundamental theorem of calculus states that

$$\frac{d}{dx}\int_{-\infty}^{x} f(u)\,du = \frac{dF(x)}{dx} = f(x)$$

Example 3-5:

The time until a chemical reaction is complete (in milliseconds) is approximated by the cumulative distribution function

$$F(x) = \begin{cases} 0; & x < 0 \\ 1 - e^{-0.01x}; & x \ge 0 \end{cases}$$

Determine the probability density function of X. What proportion of reactions is complete within 200 milliseconds?

Solution:

Using the result that the probability density function is the derivative of F(x), we obtain

$$f(0) = \begin{cases} 0; & x < 0 \\ 0.01e^{-0.01x}; & x \ge 0 \end{cases}$$

The probability that a reaction completes within 200 milliseconds is

$P(X < 200) = F(200) = 1 - e^{-2} = 0.8647$

Mean and Variance of a Continuous Random Variable:

The mean and variance can also be defined for a continuous random variable.

Suppose that X is a continuous random variable with probability density function f(x).

The mean or expected value of X, denoted as μ or E(X), is

$$\mu = E(X) = \int_{-\infty}^{\infty} xf(x)\,dx$$

The variance of X, denoted as V (X) or σ^2 is

$\sigma^2 = V(X) = \int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$

The standard deviation of X is σ

Example 3-6:

For the copper current measurement in Example 3-1, the mean of X is

$E(X) = \int_{4.9}^{5.1} xf(x)\,dx = 5\frac{x^2}{2}\Big|_{4.9}^{5.1} = 5$

The variance of X is $V(X) = \int_{4.9}^{5.1}(x-10)^2 f(x)\,dx = 5\frac{(x-10)^3}{3}\Big|_{4.9}^{5.1} = 0.0033$

Example 3-7:

For the drilling operation in Example 3-2, the mean of x is

$E(X) = \int_{12.5}^{\infty} xf(x)dx = \int_{12.5}^{\infty} x20e^{-20(x-12.5)}\,dx = -xe^{-20(x-12.5)} - \frac{e^{-20(x-12.5)}}{20}\Big|_{12.5}^{\infty} =$

$12.5 + 0.05 = 12.55.$

The variance of X is

$V(X) = \int_{12.5}^{\infty}(x-12.55)^2 f(x)dx = 0.0025$ and $\sigma = 0.05$

Practical Interpretation: The scrap limit at 12.60 mm is only 1 standard deviation greater than the mean. This is generally a warning that the scrap may be unacceptably high.

Normal Distribution:

The most important continuous probability distribution in the entire field of statistics is the normal distribution. Its graph, called the normal curve, is the bell-shaped curve of Figure 6.5, which approximately describes many phenomena that occur in nature, industry, and research. For example, physical measurements in areas such as meteorological experiments, rainfall studies, and measurements of manufactured parts are often more than adequately explained with a normal distribution. In addition, errors in scientific measurements are extremely well approximated by a normal

distribution. In 1733, Abraham De-Moivre developed the mathematical equation of the normal curve. It provided a basis from which much of the theory of inductive statistics is founded. The normal distribution is often referred to as the Gaussian distribution, in honor of Karl Friedrich Gauss (1777–1855), who also derived its equation from a study of errors in repeated measurements of the same quantity.

A continuous random variable X having the bell-shaped distribution of Figure 3.2 is called a normal random variable. The mathematical equation for the probability distribution of the normal variable depends on the two parameters μ and σ, its mean and standard deviation, respectively. Hence, we denote the values of the density of X by n(x; μ, σ).

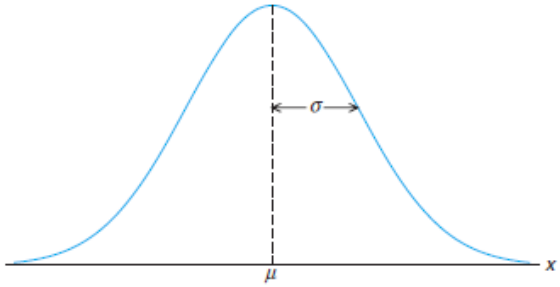


Figure 3.5: The normal curve.

The density of the normal random variable X, with mean μ and variance σ^2, is

$n(x;\ \mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$, $-\infty < x < \infty$ where π = 3.1416 and e = 2.71828 are two constants.

Once μ and σ are specified, the normal curve is completely determined. For example, if μ = 50 and σ = 5, then the ordinates n(x; 50, 5) can be computed for various values of x and the curve drawn. In Figure 3.6, we have sketched two normal curves having the same standard deviation but different means. The two curves are identical in form but are centered at different positions along the horizontal axis.

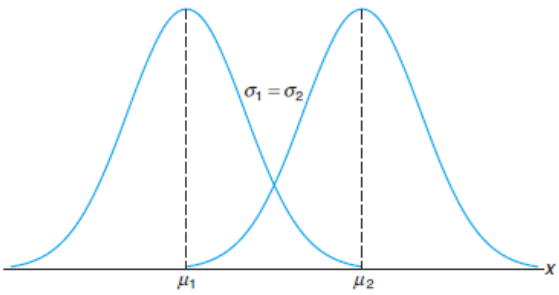


Figure 3.6: Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 = \sigma_2$

In Figure 3.7, we have sketched two normal curves with the same mean but different standard deviations. This time we see that the two curves are centered at exactly the same position on the horizontal axis, but the curve with the larger standard deviation is lower and spreads out farther. Remember that the area under a probability curve must be equal to 1, and therefore the more variable the set of observations, the lower and wider the corresponding curve will be.

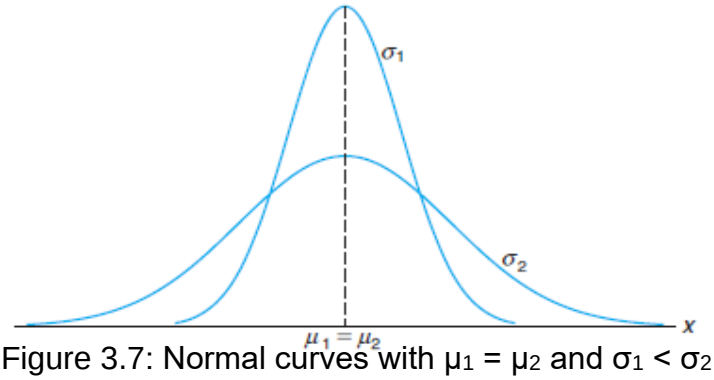


Figure 3.7: Normal curves with $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$.

Figure 3.8 shows two normal curves having different means and different standard deviations. Clearly, they are centered at different positions on the horizontal axis and their shapes reflect the two different values of σ.

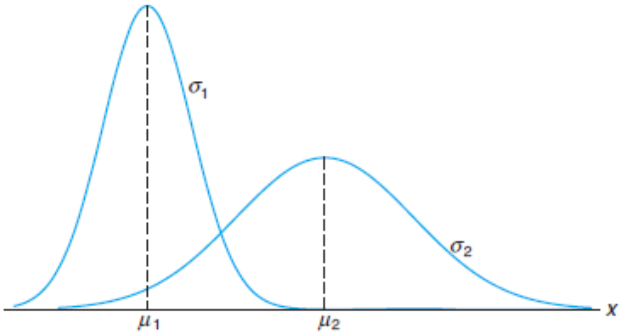


Figure 3.8: Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 < \sigma_2$.

Based on inspection of Figures 3.6 through 3.8 and examination of the first and second derivatives of n(x; μ, σ), we list the following properties of the normal curve:

1. The mode, which is the point on the horizontal axis where the curve is a maximum, occurs at x = μ.
2. The curve is symmetric about a vertical axis through the mean μ.
3. The curve has its points of inflection at x = μ ± σ; it is concave downward if $\mu - \sigma < X < \mu + \sigma$ and is concave upward otherwise.

4. The normal curve approaches the horizontal axis asymptotically as we proceed in either direction away from the mean.

5. The total area under the curve and above the horizontal axis is equal to 1.

The mean and variance of n(x; μ, σ) are μ and σ^2, respectively. Hence, the standard deviation is σ.

The curve of any continuous probability distribution or density function is constructed so that the area under the curve bounded by the two ordinates $x = x_1$ and $x = x_2$ equals the probability that the random variable X assumes a value between $x = x_1$ and $x = x_2$. Thus, for the normal curve in Figure 3.9

$$P(x_1 < X < x_2) = \int_{x_1}^{x_2} n(x;\ \mu, \sigma)dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{x_1}^{x_2} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx$$

is represented by the area of the shaded region.

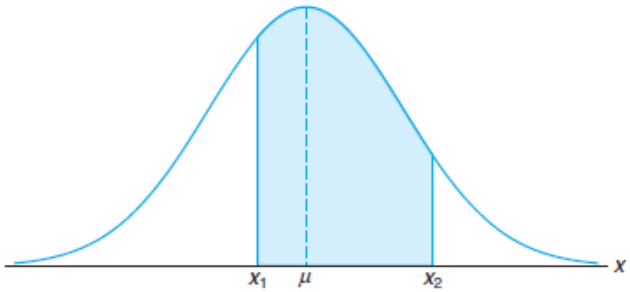


Figure 3.9: P (x_1 < X < x_2) = area of the shaded region

There are many types of statistical software that can be used in calculating areas under the normal curve. The difficulty encountered in solving integrals of normal density functions necessitates the tabulation of normal curve areas for quick reference. However, it would be a hopeless task to attempt to set up separate tables for every conceivable value of μ and σ. Fortunately, we are able to transform all the observations of any normal random variable X into a new set of observations of a normal random variable Z with mean 0 and variance 1. This can be done by means of the transformation $Z = \frac{X-\mu}{\sigma}$

Whenever X assumes a value x, the corresponding value of Z is given by $z = (x - \mu)/\sigma$. Therefore, if X falls between the values $x = x_1$ and $x = x_2$, the random variable Z will fall between the corresponding values $z_1 = (x_1 - \mu)/\sigma$ and $z_2 = (x_2 - \mu)/\sigma$. Consequently, we may write

$$P(x_1 < X < x_2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{x_1}^{x_2} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-\frac{z^2}{2}} dz$$

$$= \int_{z_1}^{z_2} f(z; 0, 1)\, dz$$

$$= P(z_1 < Z < z_2)$$

where Z is seen to be a normal random variable with mean 0 and variance 1.

The distribution of a normal random variable with mean 0 and variance 1 is called a standard normal distribution.

The original and transformed distributions are illustrated in Figure 3.10. Since all the values of X falling between x_1 and x_2 have corresponding z values between z_1 and z_2, the area under the X-curve between the ordinates $x = x_1$ and $x = x_2$ in Figure 3.10 equals the area under the Z-curve between the transformed ordinates $z = z_1$ and $z = z_2$.

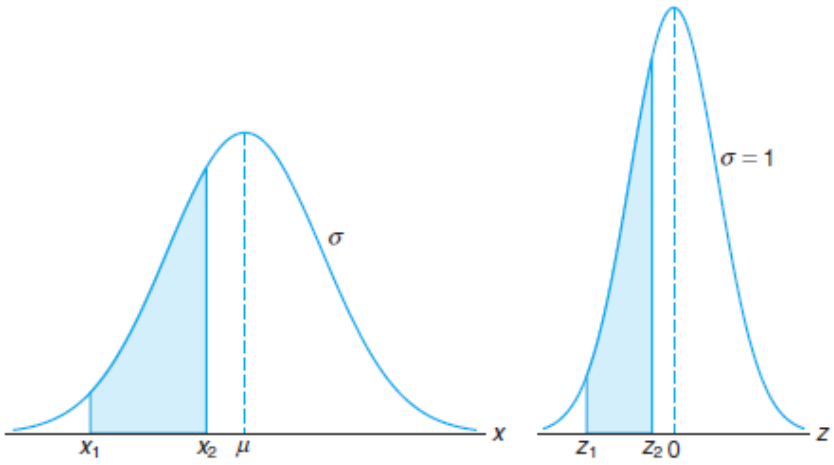


Figure 3.10: The original and transformed normal distributions

Example 3-8:

A certain type of storage battery continues, on average, 3.0 years with a standard deviation of 0.5 year. Assuming that battery life is normally distributed, find the probability that a given battery will continues less than 2.3 years.

Solution: First construct a diagram such as Figure 3.11, showing the given distribution of battery lives and the desired area. To find P(X < 2.3), we need to evaluate the area under the normal curve to the left of 2.3. This is accomplished by finding the area to the left of the corresponding z value. Hence, we find that

$z = \frac{2.3-3}{0.5} = -1.4$ and then, using Table A.3, we have

$P(X < 2.3) = P(Z < -1.4) = 0.0808$

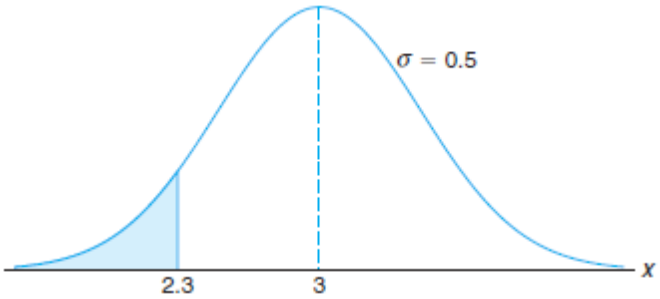


Figure 3.11: Area for Example 3.8

Example 3-9:

An electrical firm manufactures light bulbs that have a life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.

Solution:

The distribution of light bulb life is illustrated in Figure 3.12. The z values corresponding to x_1 = 778 and x_2 = 834 are

$z_1 = \frac{778-800}{40} = -0.55$ and $z_2 = \frac{834-800}{40} = 0.85$

Hence, $P(778 < X < 834) = P(-0.55 < Z < 0.85) = P(Z < 0.85) - P(Z < -0.55) = 0.8023 - 0.2912 = 0.5111$

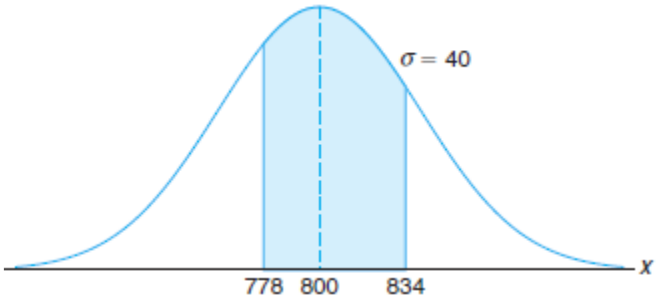


Figure 3.12: Area for Example 3.9

Example 3-10:

In an industrial process, the diameter of a ball bearing is an important measurement. The buyer sets specifications for the diameter to be 3.0 ± 0.01 cm. The implication is that no part falling outside these specifications will be accepted. It is known that in the process the diameter of a ball bearing has a normal distribution with mean μ = 3.0 and standard deviation σ = 0.005. On average, how many manufactured ball bearings will

be scrapped?

Solution:

The distribution of diameters is illustrated by Figure 3.13. The values corresponding to the specification limits are $x_1 = 2.99$ and $x_2 = 3.01$. The corresponding z values are

$z_1 = \frac{2.99-3.0}{0.005} = -2.0$ and $z_2 = \frac{3.01-3.0}{0.005} = +2.0$

Hence, $P(2.99 < X < 3.01) = P(-2.0 < Z < 2.0)$

From Table A.3, P(Z < −2.0) = 0.0228. Due to symmetry of the normal distribution, we find that $P(Z < -2.0) + P(Z > 2.0) = 2(0.0228) = 0.0456$

As a result, it is projected that, on average, 4.56% of manufactured ball bearings will be scrapped.

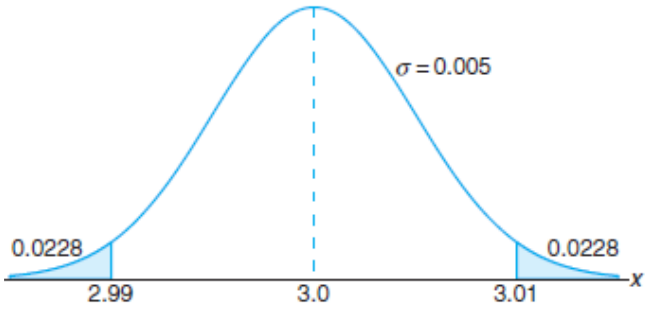


Figure 3.13: Area for Example 3.10

Example 3-11:

A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assuming that the resistance follows a normal distribution and can be measured to any degree of accuracy, what percentage of resistors will have a resistance exceeding 43 ohms?

Solution:

A percentage is found by multiplying the relative frequency by 100%. Since the relative frequency for an interval is equal to the probability of a value falling in the interval, we must find the area to the right of x = 43 in Figure 3.18. This can be done by transforming x = 43 to the corresponding z value, obtaining the area to the left of z from Table A.3, and then subtracting this area from 1. We find

$$z = \frac{43 - 40}{2} = 1.5$$

Therefore, P(X > 43) = P(Z > 1.5) = 1 − P(Z < 1.5) = 1 − 0.9332 = 0.0668.

Hence, 6.68% of the resistors will have a resistance exceeding 43 ohms.

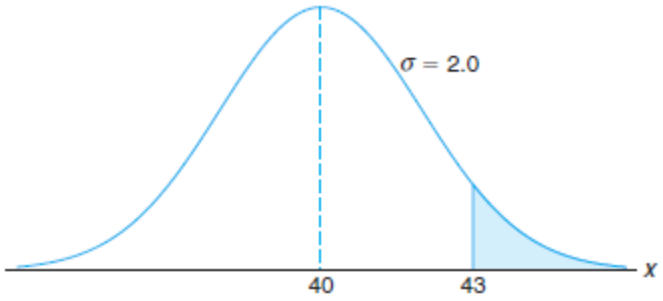


Figure 3.14: Area for Example 3.11

Example 3-12: The grade-point average score of 80 students of the Department of Computer Science of Dhaka University in their term-final examination was found to follow approximately a normal distribution with a mean of 2.1 and a standard deviation 0.6. How many of these students are expected to have a score between 2.5 and 3.5?

Solution: The z values corresponding to the scores 2.5 and 3.5 are respectively

$$z_1 = \frac{2.5 - 2.1}{0.6} = 0.67$$

$$z_2 = \frac{3.5 - 2.1}{0.6} = 2.33$$

Therefore

$$\begin{aligned} P(2.5 < X < 3.5) &= P(0.67 < Z < 2.33) \\ &= P(Z < 2.33) - P(Z < 0.67) \\ &= 0.9901 - 0.7480 = 0.2421 \end{aligned}$$

Hence 24.21% or approximately 0.2421 × 80 = 20 students of the 80 are expected to make a score between 2.5 and 3.5.

Example 3-13: A company pays its employees an average wage of $5.25 per hour with a standard deviation of 60 cents. If the wage are approximately normally distributed (i) what percentage of the employees receive wages between $4.75 and $4.75 and $5.69 per hour?

Solution: The *z* values corresponding to x_1 and x_2 are

$$z_1 = \frac{4.75 - 5.25}{0.6} = -0.83$$

$$z_z = \frac{5.69 - 5.25}{0.6} = 0.73$$

Hence

$$\begin{aligned} P(4.75 < X < 5.69) &= P(-0.83 < Z < 0.73) \\ &= P(Z < 0.73) - P(Z < -0.83) \end{aligned}$$

$= P(Z < 0.73) - [1 - P(Z < 0.83)] = 0.7673 - [1 - 0.7967]$ (From Appendix Table]

= 0.564 = \$56.4%

Hence 56.4 percent of the employees are expected to receive wage between \$4.75 and \$5.69.

Example 3-14: The Bridgestone Tire Company from an actual road test with tires has estimated the mean tire mileage at 36,000 miles and the standard deviation at 5,000 miles. In addition, the data collected indicate that a normal distribution is a reasonable assumption for the duration of the life of tires. What percentage of the tire can be expected to last more than 40,000 miles?

Solution: At $x = 40,000$, we have

$$z = \frac{x - \mu}{\sigma} = \frac{40,000 - 36,000}{5,000} = 0.70$$

Thus $P(X > 50,000) = P(Z > 0.70)$

$= 1 - P(Z < 0.70) = 1 - 0.7580$ = 0.2420 = 25.2%

Hence a little over 24 percent of the tires are expected to last more than 40,000 miles.

Exponential Distribution

The random variable X that equals the distance between successive events from a Poisson process with mean number of events λ > 0 per unit interval is an exponential random variable with parameter λ. The probability density function of X is

$f(x) = \lambda e^{-\lambda x}$ for $0 \leq x < \infty$

The exponential distribution obtains its name from the exponential function in the probability density function. See plots of the exponential distribution for selected values of λ in Fig. 3-15. For any value of λ, the exponential distribution is quite skewed.

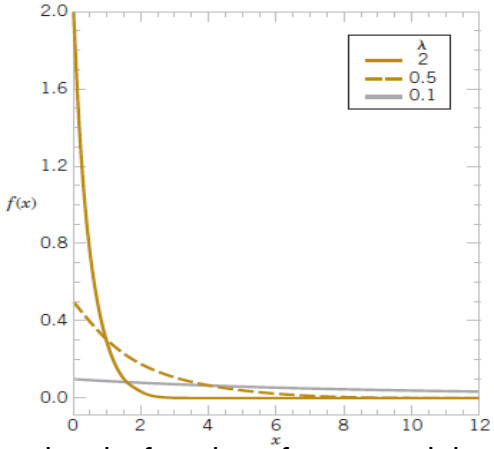


FIGURE 3-15 Probability density function of exponential random variables for selected values of λ.

Mean and Variance

If the random variable X has an exponential distribution with parameter λ,

$$\mu = E(X) = \frac{1}{\lambda} \; and \; \sigma^2 = V(X) = \frac{1}{\lambda^2}$$

Example 3-15:

In a large corporate computer network, user log-ons to the system can be modeled as a Poisson process with a mean of 25 log-ons per hour. What is the probability that there are no log-ons in an interval of six minutes?

Solution:

Let X denote the time in hours from the start of the interval until the first log-on. Then X has an exponential distribution with λ = 25 log-ons per hour. We are interested in the probability that X exceeds 6 minutes. Because λ is given in log-ons per hour, we express all time units in hours. That is, 6 minutes = 0.1 hour. The probability requested is shown as the shaded area under the probability density function in Fig. 3-16. Therefore,

$$P(X > 0.1) = \int_{0.1}^{\infty} 25e^{-25x}dx = e^{-25(0.1)} = 0.082$$

The cumulative distribution function also can be used to obtain the same result as follows:

$$P(X > 0.1) = 1 - F(0.1) = e^{-25(0.1)} = 0.082$$

An identical answer is obtained by expressing the mean number of log-ons as 0.417 log-ons per minute and computing the probability that the time until the next log-on exceeds six minutes. Try it.

What is the probability that the time until the next log-on is between two and three minutes? Upon converting all units to hours,

$$P(0.033 < X < 0.05) = \int_{0.033}^{0.05} 25e^{-25x}dx = -e^{-25x}|_{0.033}^{0.05} = 0.152$$

An alternative solution is $P(0.033 < X < 0.05) = F(0.05) - F(0.033) = 0.152$

Determine the interval of time such that the probability that no log-on occurs in the interval is 0.90. The question asks for the length of time x such that P(X>x) = 0 90. Now,

$$P(X > x) = e^{-25x} = 0.90$$

Take the (natural) log of both sides to obtain$-25x = ln(0.90) = -0.1054$. Therefore,

x = 0.00421 hour = 0.25 minute

Furthermore, the mean time until the next log-on is

$\mu = 1/25 = 0.04 \text{ hour} = 2.4 \text{ minutes}$

The standard deviation of the time until the next log-on is σ = 1 / 25 hours = 2.4 minutes

Practical Interpretation: Organizations make wide use of probabilities for exponential random variables to evaluate resources and staffing levels to meet customer service needs.

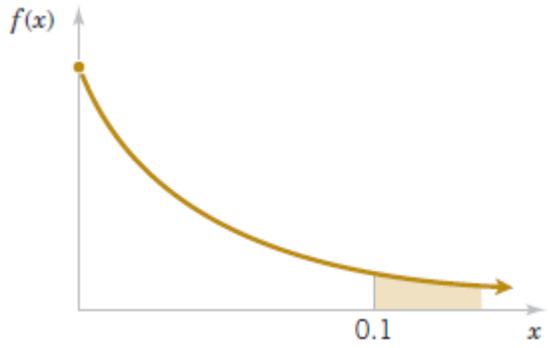


FIGURE 3-16 Probability for the exponential distribution in Example 3-15.

For Practice

1. A city installs 2000 electric lamps for street lighting. These lamps have a mean burning life of 1000 hours with a standard deviation of 200 hours. The normal distribution is a close approximation to this case.

a) What is the probability that a lamp will fail in the first 700 burning hours?

b) What is the probability that a lamp will fail between 900 and 1300 burning hours?

c) How many lamps are expected to fail between 900 and 1300 burning hours?

d) What is the probability that a lamp will burn for exactly 900 hours?

e) What is the probability that a lamp will burn between 899 hours and 901 hours before it fails?

f) After how many burning hours would we expect 10% of the lamps to be left?

g) After how many burning hours would we expect 90% of the lamps to be left?

2. Diameters of bolts produced by a particular machine are normally distributed with mean 0.760 cm and standard deviation 0.012 cm. Specifications call for diameters from 0.720 cm to 0.780 cm.

a) What percentage of bolts will meet these specifications?

b) What percentage of bolts will be smaller than 0.730 cm?

3. An engineer working for a manufacturer of electronic components takes a large number of measurements of a particular dimension of components from the production line. She finds that the distribution of dimensions is normal, with a mean of 2.340 cm and a coefficient of variation of 2.4%.

a) What percentage of measurements will be less than 2.45 cm?

b) What percentage of dimensions will be between 2.25 cm and 2.45 cm?

c) What value of the dimension will be exceeded by 98% of the components?

4. 30% of the soil samples obtained from a proposed construction site gave test results for compressive strength of more than 3.5 tons per square foot. The coefficient of variation of the strengths is known to be 20%. Calculate:

a) The mean soil strength,

b) The standard deviation of soil strengths,

c) The probability of soil strengths falling between 2.7 and 4.0 tons per square foot. State any assumptions made.

5. Steel used for water pipelines is sometimes coated on the inside with cement mortar to prevent corrosion. In a study of the mortar coatings of a pipeline used in a water transmission project, the mortar thicknesses were measured for a very large number of specimens. The mean and the standard deviation were found to be 0.62 inch and 0.13 inch, respectively, and the thickness was found to be normally distributed.

a) In what percentage of the pipelines is the thickness of mortar less than 0.5 inch?

b) If four pipes are selected at random, what is the probability that two or more have mortar thickness less than 0.5 inch?

c) 100 pipes are taken and their mortar thicknesses are measured individually. If the mortar thickness of a pipe is found to be less than 0.5 inch, 10% less is paid to the manufacturer for that pipe. If the normal price of a pipe is S125.00, what is the expected cost of 100 pipes?

6. For a certain type of fluorescent light in a large building, the cost per bulb of replacing bulbs all at once is much less than if they are replaced individually as they burn out. It is known that the lifetime of these bulbs is normally distributed, and that 60% last longer than 2500 hours, while 30% last longer than 3000 hours.

a) What are the approximate mean and standard deviation of the lifetimes of the bulbs?

b) If the light bulbs are completely replaced when more than 20% have burned out, what is the time between complete replacements?