requirements for both minimum and maximum spacing of principal reinforcement, however.

The locations of bend and cutoff points shown in Fig. 4.3*a* were obtained using Graph 11, Appendix B, as explained in Art. 3.20. A check of the development length provided against Table 8, Appendix B, makes evident the fact that flexural bond stress need not be considered.

The arrangement of Fig. 4.3*b* uses only straight bars. While apparently satisfactory according to the ACI Code, cutting off the shorter positive and negative bars as shown leads to an undesirable condition at the ends of those bars, where there will be concentrations of stress in the concrete. The design would be improved if the negative bars were cut off at 2 ft 11 in. from the face of supports, rather than 2 ft 3 in. and 2 ft 6 in. as shown, and if the positive steel were cut off at 2 ft 3 in., rather than at 2 ft 11 in. This would result in an overlap of approximately $2d$ of the cut positive and negative bars.

Reinforcement to control temperature and shrinkage cracking will be included in the amount of

$$A_s = 0.0018 \times 12 \times 5.25 = 0.11 \text{ in.}^2$$

per 12-in. strip. This requirement is met by using No. 3 bars, at 12 in. spacing, placed directly on top of the main reinforcement in the positive-moment region and directly underneath the main reinforcement in the negative-moment zone.

PROBLEMS

1. A small bridge consisting of a concrete slab supported by steel stringers is to carry a uniformly distributed load of 300 psf in addition to its own weight. The four stringers of the bridge, spanning in the long direction, are spaced 8 ft on centers. The concrete slab spans in the transverse direction and is continuous over the two interior stringers. Find the required thickness of the slab, and design and detail the bar reinforcement, using $f_s = 20{,}000$ psi and $f_c = 1350$ psi, with $n = 9$. Bent bars will be used in preference to all-straight-bar reinforcement. The ACI moment coefficients do not apply.

2. Redesign the bridge of Prob. 1, using ultimate-strength methods, with overload factors of 1.5 and 1.8 applied to dead and live loads, respectively. For the steel, $f_y = 40{,}000$ psi, and the concrete strength is $f'_c = 3000$ psi. Use a maximum steel ratio of $0.50p_b$.

TWO-WAY SLABS

4.5 Performance of Two-way Slabs Most rectangular reinforced-concrete slabs are supported on all four sides by beams, girders, or walls. Figure 1.1*a* shows a typical example of a beam-and-girder floor in which each individual slab panel is so supported. If the proportions of the panel are not too far from square (side ratio up to about 2:1), the slab bends under load into a dished surface rather than into a cylindrical one, as

would be the case for one-way slabs supported only at two opposite sides. This means that at any point the slab is curved in both principal directions, and since bending moments are proportional to curvatures, moments also exist in both directions. To resist these moments, the slab must be reinforced in both directions, by two mutually perpendicular layers of bars parallel, respectively, to the two pairs of edges, one layer resting directly on the other. The slab, then, transmits loads to all four supporting beams, the relative amounts depending on the proportions of the slab and the conditions of continuity at the four edges.

In order to visualize the flexural performance of such a slab, it is convenient to think of it as consisting of two sets of parallel strips, in each of the two directions, intersecting each other. Evidently, part of the load is carried by one set and transmitted to one pair of edge supports, and the remainder by the other. Figure 4.4 shows the two center strips of a rectangular plate with short span A and long span B. If the uniform load is w per ft^2 of slab, each of the two strips acts approximately like a simple beam uniformly loaded by its share of w. Because these imaginary strips actually are part of the same monolithic slab, their deflections at the intersection point must be the same. Equating the center deflections of the short and long strips,

$$\frac{5w_s A^4}{384EI} = \frac{5w_l B^4}{384EI}$$

where w_s = share of load w carried in short direction
w_l = share of load w carried in long direction

Consequently,

$$\frac{w_s}{w_l} = \frac{B^4}{A^4} \tag{4.1}$$

One sees that the larger share of the load is carried in the short direction, the ratio of the two portions of the total load being proportional to the fourth power of the ratio of the spans. This result is approximate because the actual behavior of a slab is more complex than that of the two intersecting strips, but it is a good approximation when the edge conditions are the same on all four sides (e.g., simply supported or rotationally fixed). The two pairs of edge beams can then be designed for the loads w_s and w_l transmitted to them by the slab.

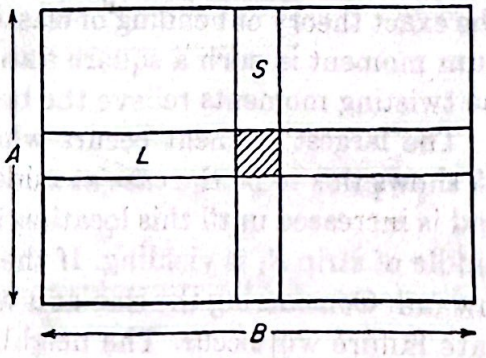


Fig. 4.4

An understanding of the flexural behavior of the slab itself can be

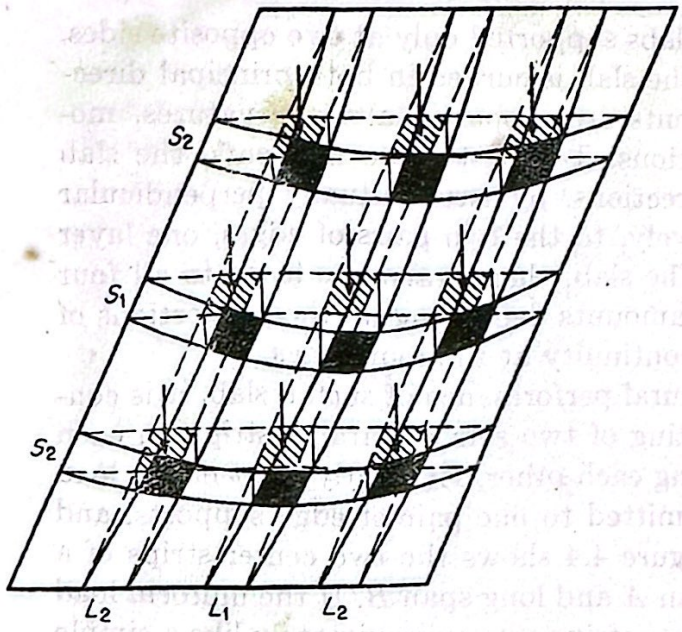


***Fig.* 4.5** Grid model of uniformly loaded, simply supported slab.

gained from Fig. 4.5, which shows a slab model consisting of two sets of three strips each. It is seen that the two central strips S_1 and L_1 bend in a manner similar to that of Fig. 4.4. The outer strips S_2 and L_2, however, are not only bent, but also twisted. Consider, for instance, one of the intersections of S_2 with L_2. It is seen that at the intersection the exterior edge of strip L_2 is at a higher elevation than the interior edge, while at the nearby end of strip L_2 both edges are at the same elevation; the strip is twisted. This twisting results in torsional stresses and torsional moments which are seen to be most pronounced near the corners. Consequently, the total load on the slab is carried not only by the bending moments in the two directions, but also by the twisting moments. For this reason bending moments in elastic slabs are smaller than would be computed for sets of unconnected strips loaded by w_s and w_l. For instance, for a simply supported square slab, $w_s = w_l = w/2$ [Eq. (4.1)]. If only bending were present, the maximum moment in each strip would be

$$(w/2)A^2/8 = 0.0625wA^2$$

The exact theory of bending of elastic plates shows that actually the maximum moment in such a square slab is only $0.048wA^2$, so that in this case the twisting moments relieve the bending moments by about 25 per cent.

The largest moment occurs where the curvature is sharpest. Figure 4.5 shows this to be the case at midspan of the short strip S_1. Suppose the load is increased until this location is overstressed, so that the steel at the middle of strip S_1 is yielding. If the strip were an isolated beam, it would now fail. Considering the slab as a whole, however, one sees that no immediate failure will occur. The neighboring strips (those parallel as well as those perpendicular to S_1), being actually monolithic with it, will take over that share of any additional load which strip S_1 can no longer carry,

until they, in turn, start yielding. This inelastic redistribution will continue until, in a rather large area in the central portion of the slab, all the steel in both directions is yielding. Only then will the entire slab fail. From this reasoning, which is confirmed by tests, it follows that slabs need not be designed for the absolute maximum moment in each of the two directions (such as $0.048wA^2$ in the example of the preceding paragraph), but only for a smaller average moment in each of the two directions in the central portion of the slab. For instance, one of the several analytical methods provided in the ACI Code permits the above square slab to be designed for a moment of $0.036wA^2$. By comparison with the actual elastic maximum moment, $0.048wA^2$, it is seen that, owing to inelastic redistribution, a moment reduction of 25 per cent is provided.

The flexural behavior of a simply supported slab is summarized in the solid lines of Fig. 4.6. The largest moments occur in the center strips, along 1-1 and 2-2. The moment diagram in the short direction along 1-1 is given by Fig. 4.6*a* and shows a maximum $M_{A,\max}$ at midspan. From Fig. 4.5 it is seen that short strips located closer to the edges (such as S_2) deflect less, are less sharply curved, and consequently have smaller bending moments than the central strip. Therefore, the maximum moment

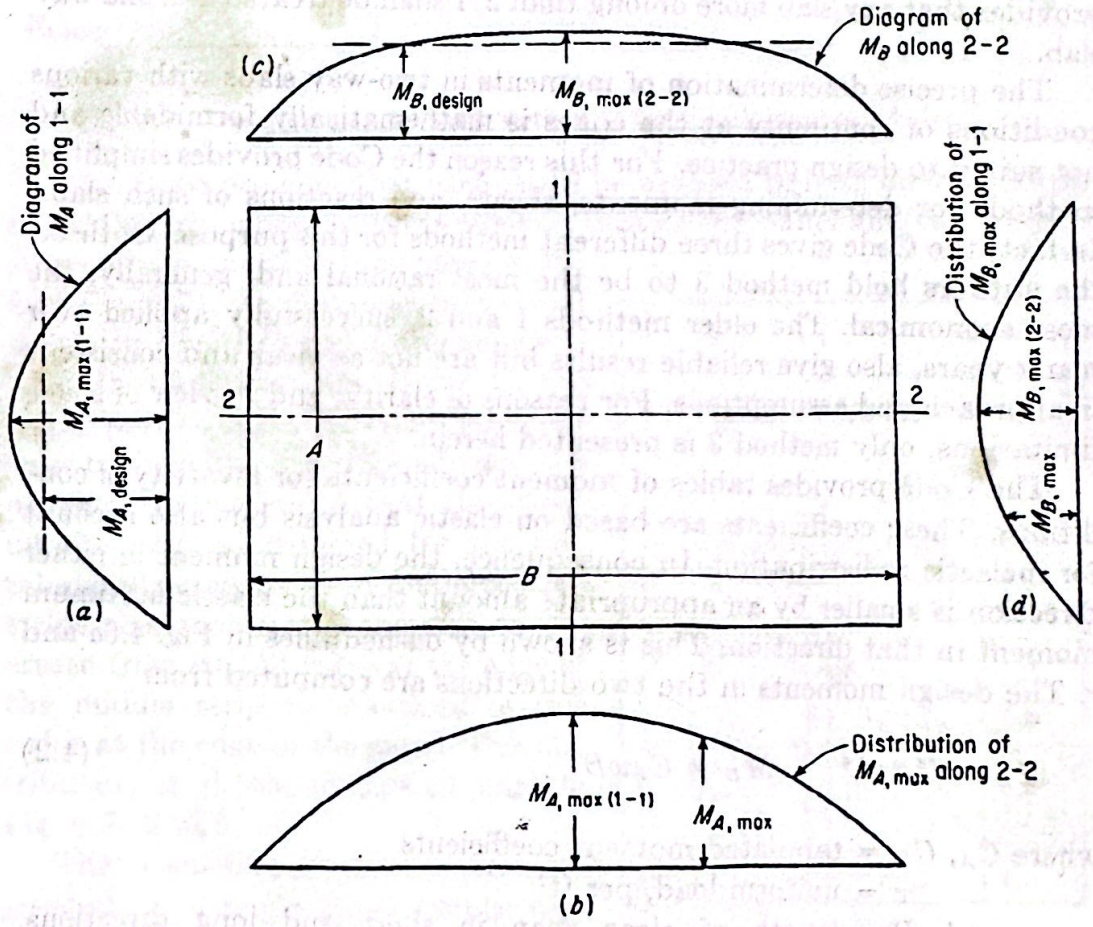


Fig. 4.6 **Moments in uniformly loaded, simply supported slab.**

$M_{A,max}$ at the center of the short strips is distributed longitudinally, as in Fig. 4.6*b*; it is largest at the center and decreases toward zero at the edges. For a slab which departs significantly from square, the moment M_B in the center strip in the long direction is fairly constant in the middle half of the slab. This is seen from Fig. 4.6*c*, which gives the moment diagram along 2-2. Again, inspection of Fig. 4.5 shows that in long strips located closer to the edges (such as L_2) deflection, curvatures, and therefore bending moments are smaller than in the central strip L_1. The distribution of $M_{B,max}$ as it varies in the short direction is shown in Fig. 4.6*d*. As explained before, in view of inelastic redistribution it is not necessary to design for the absolute maximum values of M_A and M_B. The manner in which this and other features are considered in design is discussed in the next article.

4.6 Design of Two-way Slabs by ACI Code The Code provides that slabs with side ratios no larger than 2:1 shall be designed as two-way slabs. From Eq. (4.1) it is seen that for a slab of this proportion the share of the load carried in the long direction is only of the order of one-sixteenth that in the short direction. Hence, such a slab acts almost as if it were spanning in the short direction only. In agreement with this, the Code provides that any slab more oblong than 2:1 shall be treated as a one-way slab.

The precise determination of moments in two-way slabs with various conditions of continuity at the edges is mathematically formidable and not suited to design practice. For this reason the Code provides simplified methods for determining moments, shears, and reactions of such slabs. In fact, the Code gives three different methods for this purpose. Of these, the authors hold method 3 to be the most rational and, generally, the most economical. The older methods 1 and 2, successfully applied over many years, also give reliable results but are not as clear and consistent in approach and assumptions. For reasons of clarity, and in view of space limitations, only method 3 is presented herein.

The Code provides tables of moment coefficients for a variety of conditions. These coefficients are based on elastic analysis but also account for inelastic redistribution. In consequence, the design moment in either direction is smaller by an appropriate amount than the elastic maximum moment in that direction. This is shown by dashed lines in Fig. 4.6*a* and *c*. The design moments in the two directions are computed from

$$M_A = C_A w A^2 \qquad M_B = C_B w B^2 \tag{4.2}$$

where C_A, C_B = tabulated moment coefficients
w = uniform load, per ft^2
A, B = length of clear span in short and long directions, respectively

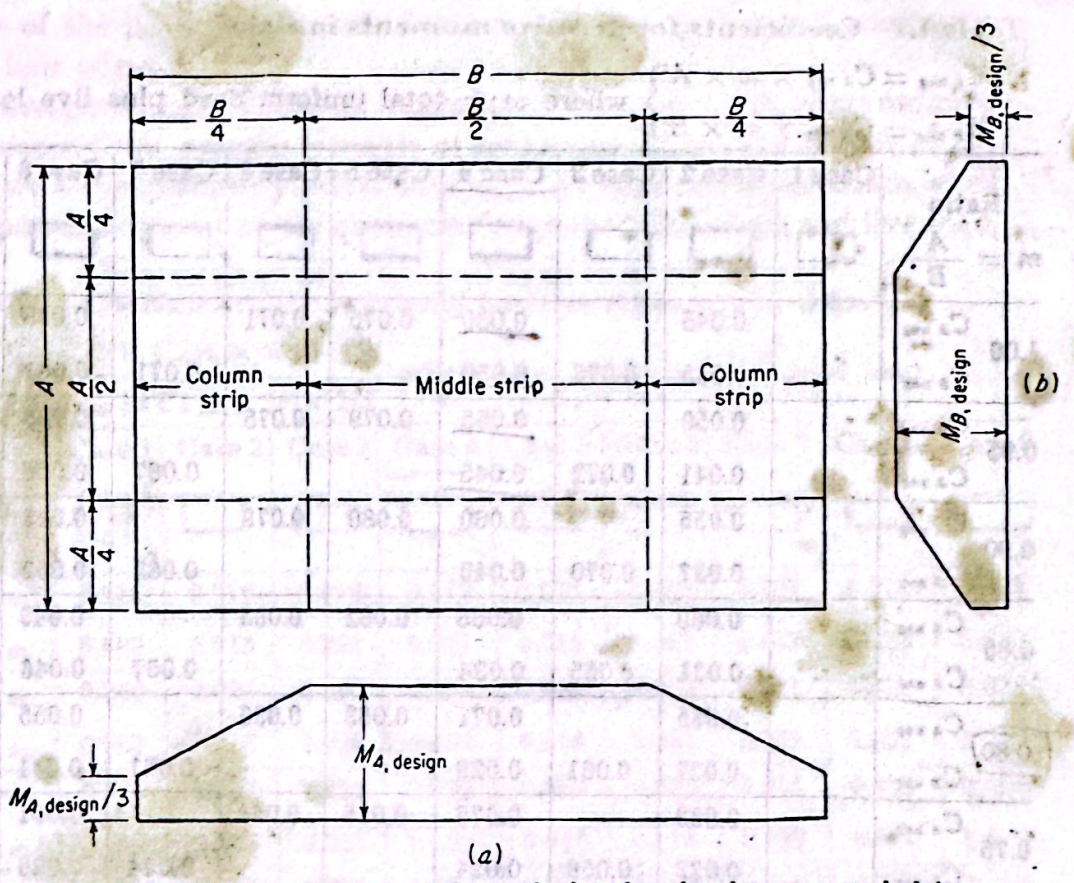


Fig. 4.7 Design moments, ACI method 3, for simply supported slab.

The Code provides that each panel be divided in both directions into a middle strip whose width is one-half that of the panel and two column strips of one-quarter of the panel width (see Fig. 4.7). As discussed before and shown in Fig. 4.6*b* and *d*, the moments in both directions are larger in the center portion of the slab than in regions close to the edges. Correspondingly, it is provided in method 3 that the entire middle strip be designed for the full, tabulated design moment. In the column strips this moment is assumed to decrease from its full value at the edge of the middle strip to one-third of this value at the edge of the panel. This distribution is shown in dashed lines in Fig. 4.7*a* and *b*.

The discussion so far has been restricted to a single panel simply supported at all four edges. An actual situation is shown in Fig. 4.8, in

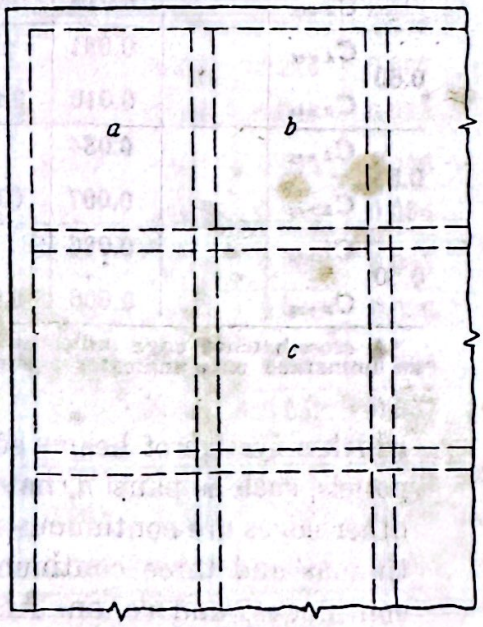


Fig. 4.8 Portion of typical two-way slab floor.

Table 4.2 **Coefficients for negative moments in slabs***

$M_{A\,neg} = C_{A\,neg} \times w \times A^2$
$M_{B\,neg} = C_{B\,neg} \times w \times B^2$
where w = total uniform dead plus live load

Ratio $m = \frac{A}{B}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{A\,neg}$		0.045		0.050	0.075	0.071		0.033	0.061
	$C_{B\,neg}$		0.045	0.076	0.050			0.071	0.061	0.033
0.95	$C_{A\,neg}$		0.050		0.055	0.079	0.075		0.038	0.065
	$C_{B\,neg}$		0.041	0.072	0.045			0.067	0.056	0.029
0.90	$C_{A\,neg}$		0.055		0.060	0.080	0.079		0.043	0.068
	$C_{B\,neg}$		0.037	0.070	0.040			0.062	0.052	0.025
0.85	$C_{A\,neg}$		0.060		0.066	0.082	0.083		0.049	0.072
	$C_{B\,neg}$		0.031	0.065	0.034			0.057	0.046	0.021
0.80	$C_{A\,neg}$		0.065		0.071	0.083	0.086		0.055	0.075
	$C_{B\,neg}$		0.027	0.061	0.029			0.051	0.041	0.017
0.75	$C_{A\,neg}$		0.069		0.076	0.085	0.088		0.061	0.078
	$C_{B\,neg}$		0.022	0.056	0.024			0.044	0.036	0.014
0.70	$C_{A\,neg}$		0.074		0.081	0.086	0.091		0.068	0.081
	$C_{B\,neg}$		0.017	0.050	0.019			0.038	0.029	0.011
0.65	$C_{A\,neg}$		0.077		0.085	0.087	0.093		0.074	0.083
	$C_{B\,neg}$		0.014	0.043	0.015			0.031	0.024	0.008
0.60	$C_{A\,neg}$		0.081		0.089	0.088	0.095		0.080	0.085
	$C_{B\,neg}$		0.010	0.035	0.011			0.024	0.018	0.006
0.55	$C_{A\,neg}$		0.084		0.092	0.089	0.096		0.085	0.086
	$C_{B\,neg}$		0.007	0.028	0.008			0.019	0.014	0.005
0.50	$C_{A\,neg}$		0.086		0.094	0.090	0.097		0.089	0.088
	$C_{B\,neg}$		0.006	0.022	0.006			0.014	0.010	0.003

*A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

which a system of beams supports a two-way slab. It is seen that some panels, such as panel *a*, have two discontinuous exterior edges, while the other edges are continuous with their neighbors. Panel *b* has one discontinuous and three continuous edges, the interior panel *c* has all edges continuous, and so on. At a continuous edge in a slab, moments are negative, just as at interior supports of continuous beams. Also, the mag-

nitude of the positive moments depends on the conditions of continuity at all four edges.

Correspondingly, Table 4.2, reproduced from the Code, gives moment coefficients C for *negative moments at continuous edges.* The details of the table are self-explanatory. Maximum negative edge moments obtain when both panels adjacent to the particular edge carry full dead and live load.

Table* 4.3 Coefficients for dead-load positive moments in slabs

$$M_{A\,pos\,DL} = C_{A\,DL} \times w \times A^2$$
$$M_{B\,pos\,DL} = C_{B\,DL} \times w \times B^2$$
where w = total uniform dead load

Ratio $m = \frac{A}{B}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{A\,DL}$	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023
	$C_{B\,DL}$	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020
0.95	$C_{A\,DL}$	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
	$C_{B\,DL}$	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
0.90	$C_{A\,DL}$	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
	$C_{B\,DL}$	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.85	$C_{A\,DL}$	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
	$C_{B\,DL}$	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.80	$C_{A\,DL}$	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029
	$C_{B\,DL}$	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010
0.75	$C_{A\,DL}$	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
	$C_{B\,DL}$	0.019	0.009	0.018	0.013	0.007	0.012	0.020	0.013	0.007
0.70	$C_{A\,DL}$	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.033
	$C_{B\,DL}$	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.65	$C_{A\,DL}$	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.034
	$C_{B\,DL}$	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.60	$C_{A\,DL}$	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.036
	$C_{B\,DL}$	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.55	$C_{A\,DL}$	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.037
	$C_{B\,DL}$	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.50	$C_{A\,DL}$	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.038
	$C_{B\,DL}$	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002

*A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

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Table 4.4 **Coefficients for live-load positive moments in slabs***

$$M_{A\ pos\ LL} = C_{A\ LL} \times w \times A^2$$
$$M_{B\ pos\ LL} = C_{B\ LL} \times w \times B^2$$
} where w = total uniform live load

Ratio $m = \frac{A}{B}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{A\ LL}$	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030
	$C_{B\ LL}$	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.028
0.95	$C_{A\ LL}$	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
	$C_{B\ LL}$	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
0.90	$C_{A\ LL}$	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
	$C_{B\ LL}$	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.85	$C_{A\ LL}$	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
	$C_{B\ LL}$	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.80	$C_{A\ LL}$	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042
	$C_{B\ LL}$	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017
0.75	$C_{A\ LL}$	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046
	$C_{B\ LL}$	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013
0.70	$C_{A\ LL}$	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050
	$C_{B\ LL}$	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.65	$C_{A\ LL}$	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
	$C_{B\ LL}$	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.60	$C_{A\ LL}$	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
	$C_{B\ LL}$	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.55	$C_{A\ LL}$	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063
	$C_{B\ LL}$	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.50	$C_{A\ LL}$	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
	$C_{B\ LL}$	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.004

*A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

Hence, the moment is computed for this total load. The Code provides that *negative moments at discontinuous edges* shall be assumed equal to one-third of the positive moments for the same direction. One must provide for such moments because some degree of restraint is provided discontinuous edges by the torsional rigidity of the edge beam or by the supporting wall.

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As for *positive moments*, there will be little, if any, rotation at the continuous edges if *dead load* alone is acting, because the loads on both adjacent panels tend to produce opposite rotations which cancel, or nearly so. Hence, for this condition the continuous edges can be regarded as fixed, and the appropriate coefficients for the dead-load moments are given in Table 4.3. On the other hand, the maximum *live-load moments* are obtained when live load is placed only on the particular panel, and

Table 4.5 **Ratio of load w in A and B directions for shear in slab and load on supports***

Ratio $m = \frac{A}{B}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	W_A	0.50	0.50	0.17	0.50	0.83	0.71	0.29	0.33	0.67
	W_B	0.50	0.50	0.83	0.50	0.17	0.29	0.71	0.67	0.33
0.95	W_A	0.55	0.55	0.20	0.55	0.86	0.75	0.33	0.38	0.71
	W_B	0.45	0.45	0.80	0.45	0.14	0.25	0.67	0.62	0.29
0.90	W_A	0.60	0.60	0.23	0.60	0.88	0.79	0.38	0.43	0.75
	W_B	0.40	0.40	0.77	0.40	0.12	0.21	0.62	0.57	0.25
0.85	W_A	0.66	0.66	0.28	0.66	0.90	0.83	0.43	0.49	0.79
	W_B	0.34	0.34	0.72	0.34	0.10	0.17	0.57	0.51	0.21
0.80	W_A	0.71	0.71	0.33	0.71	0.92	0.86	0.49	0.55	0.83
	W_B	0.29	0.29	0.67	0.29	0.08	0.14	0.51	0.45	0.17
0.75	W_A	0.76	0.76	0.39	0.76	0.94	0.88	0.56	0.61	0.86
	W_B	0.24	0.24	0.61	0.24	0.06	0.12	0.44	0.39	0.14
0.70	W_A	0.81	0.81	0.45	0.81	0.95	0.91	0.62	0.68	0.89
	W_B	0.19	0.19	0.55	0.19	0.05	0.09	0.38	0.32	0.11
0.65	W_A	0.85	0.85	0.53	0.85	0.96	0.93	0.69	0.74	0.92
	W_B	0.15	0.15	0.47	0.15	0.04	0.07	0.31	0.26	0.08
0.60	W_A	0.89	0.89	0.61	0.89	0.97	0.95	0.76	0.80	0.94
	W_B	0.11	0.11	0.39	0.11	0.03	0.05	0.24	0.20	0.06
0.55	W_A	0.92	0.92	0.69	0.92	0.98	0.96	0.81	0.85	0.95
	W_B	0.08	0.08	0.31	0.08	0.02	0.04	0.19	0.15	0.05
0.50	W_A	0.94	0.94	0.76	0.94	0.99	0.97	0.86	0.89	0.97
	W_B	0.06	0.06	0.24	0.06	0.01	0.03	0.14	0.11	0.03

*A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

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not on any of the adjacent panels. In this case some rotation will occur at all continuous edges. As an approximation, the Code assumes 50 per cent restraint for calculating these live-load moments. The corresponding coefficients are given in Table 4.4. Finally, for computing shear in the slab and loads on the supporting beams, Table 4.5 gives the fractions of the total load w which are transmitted in the two directions. It is easily checked that for cases 1 and 2 these fractions are in exact accord with Eq. 4.1. In the other cases they are different, depending on the particular edge conditions.

Since positive-moment steel is placed in two layers, the distance d for the upper layer is smaller than that for the lower layer by one bar diameter (when the same bar size is used for both layers.) Because the moments in the long direction are the smaller ones, it is economical to place the steel in that direction on top of the bars in the short direction. The twisting moments discussed in Art. 4.5 are usually of consequence only in exterior corners, where they tend to crack the slab along 45° lines of the corner panels. For this reason the Code provides that, in addition to the calculated reinforcement, special reinforcement shall be provided at exterior corners in both bottom and top of the slab, for a distance in each direction from the corner equal to one-fifth the long span. The reinforcement in the top of the slab shall be parallel to the diagonal from the corner. The reinforcement in the bottom of the slab shall be at right angles to the diagonal, or it may consist of bars in two directions parallel to the sides of the slab. The reinforcement in each band shall be of size and spacing equivalent to that required for the maximum positive moment in the slab.

The precise locations of inflection points in two-way slabs are not easily determined, since they depend on the side ratio, ratio of live to dead load, and continuity conditions at the edges. A reasonable rule is to assume that a line of inflection exists parallel to any continuous edge at a distance from it of one-sixth the span. The rule may be applied to both the long and the short directions. If bent bars are used, they are generally bent up at these inflection locations.

To limit deflections, the code provides that the minimum thickness of a two-way slab shall be 3.5 in. or the panel perimeter divided by 180, whichever is larger.

4.7 Design example Design a slab panel 20 ft 0 in. by 25 ft 0 in. to support a uniform live load of 125 psf. The panel is a corner panel of a group. A 3000-psi concrete and steel with $f_y = 60{,}000$ psi are used, in connection with ultimate-strength design.

The minimum thickness, by Code, of $^{9}\!/_{180} = \frac{1}{2}$ ft = 6 in. will be used. (For heavy loads, such as in warehouses, deeper slabs lead to steel savings which frequently prove more economical than minimum-thickness slabs.) Consequently, the dead load is $\frac{1}{2} \times 150 = 75$ psf, and the design loads for ultimate-strength

design are

Live load:	1.8 × 125	=	225 psf
Dead load:	1.5 × 75	=	112.5 psf
Total			337.5 psf

MOMENT CALCULATIONS FOR MIDDLE STRIPS. $m = A/B = 20/25 = 0.80$. The negative moments at continuous edges, from Table 4.2, are

$$M_{A,neg} = 0.071 \times 337.5 \times 20^2 = 9580 \text{ ft-lb} = 115{,}000 \text{ in.-lb}$$
$$M_{B,neg} = 0.029 \times 337.5 \times 25^2 = 6110 \text{ ft-lb} = 73{,}400 \text{ in.-lb}$$

The positive moments, from Tables 4.3 and 4.4, are

Dead load:	$M_{A,pos} = 0.039 \times 112.5 \times 20^2$	= 1755 ft-lb	= 21,000 in.-lb
Live load:	$M_{A,pos} = 0.048 \times 225 \times 20^2$	= 4320 ft-lb	= 51,800 in.-lb
Total			$M_{A,pos}$ = 72,800 in.-lb
Dead load:	$M_{B,pos} = 0.016 \times 112.5 \times 25^2$	= 1125 ft-lb	= 13,500 in.-lb
Live load:	$M_{B,pos} = 0.020 \times 225 \times 25^2$	= 2820 ft-lb	= 33,800 in.-lb
Total			$M_{B,pos}$ = 47,300 in.-lb

The negative moments at discontinuous edges are

$$M_{A,neg} = \tfrac{1}{3}(72{,}800) = 24{,}300 \text{ in.-lb}$$
$$M_{B,neg} = \tfrac{1}{3}(47{,}300) = 15{,}800 \text{ in.-lb}$$

SELECTION OF REINFORCEMENT. From Graph 1, Appendix B:

1. Short direction

a. Span:

$$\frac{M_u}{\phi bd^2} = \frac{72{,}800}{0.9 \times 12 \times 5^2} = 270 \qquad p = 0.0048$$

A_s is equal to $0.0048 \times 12 \times 5 = 0.288$ in.²/ft. From Table 4, Appendix B, No. 4 bars at 7 in. spacing are selected, giving $A_s = 0.34$ in.²/ft.

b. Continuous edge:

$$\frac{M_u}{\phi bd^2} = \frac{115{,}000}{0.9 \times 12 \times 5^2} = 426; \quad p = 0.0078$$

A_s is equal to $0.0078 \times 12 \times 5 = 0.468$ in.²/ft. If two of every three positive-moment bars are bent up, and likewise for the adjacent panel, the negative-moment steel area furnished at the continuous edge will be four-thirds the positive-moment steel area in the span, or $A_s = \frac{4}{3} \times 0.34 = 0.453$ in.²/ft. It is seen that this is 3.5 per cent less than the required amount of 0.468. On the other hand, the positive-moment steel furnished, 0.34, represents about 15 per cent more than the required amount. As is discussed in Art. 8.15, the Code permits a certain amount of inelastic moment redistribution, within strictly specified limits. In the case at hand the negative steel furnished suffices for only 96.5 per cent of the calculated moment, but the positive steel permits about 115 per cent of the calcu-

lated moment to be resisted. This more than satisfies the conditions for inelastic moment redistribution set by the Code. This situation illustrates how such moment redistribution can be utilized to obtain a simpler and more economical distribution of steel.

c. *Discontinuous edge:* The negative moment at the discontinuous edge is one-third the positive moment in the span; it would be adequate to bend up every third bar from the bottom to provide negative-moment steel at the discontinuous edge. However, this would make for a 21-in. spacing, which is larger than the maximum spacing of $3t = 18$ in. permitted by the Code. Hence, for the discontinuous edge, two of every three bars will be bent up from the bottom steel.

2. Long direction

a. *Span:*

$$\frac{M_u}{\phi bd^2} = \frac{47{,}300}{0.90 \times 12 \times 4.5^2} = 216 \qquad p = 0.0038$$

(The positive-moment steel in the long direction is placed on top of that for the short direction. This makes the depth d for the former one bar diameter smaller than for the latter. This is the reason for using $d = 4.5$ in. for the positive-moment steel in the long direction, and $d = 5$ in. in all other locations.) A_s is equal to $0.0038 \times 12 \times 4.5 = 0.205$ in.²/ft. From Table 4, Appendix B, No. 3 bars at 6 in. spacing are selected, giving $A_s = 0.22$ in.²/ft.

b. *Continuous edge:*

$$\frac{M_u}{\phi bd^2} = \frac{73{,}400}{0.90 \times 12 \times 5^2} = 272; \; p = 0.0048$$

A_s is equal to $0.0048 \times 12 \times 5 = 0.288$ in.²/ft. Again, bending up two of every three bottom bars from both panels adjacent to the continuous edge, one has, at that edge, $A_s = \frac{1}{3} \times 0.22 = 0.29$ in.²/ft.

c. *Discontinuous edge:* For the reasons discussed in connection with the short direction, two out of every three bottom bars will, likewise, be bent up at this edge.

The steel selections above refer to the middle strips in both directions. For the *column strips*, as was pointed out, the moments are assumed to decrease linearly from the full calculated value at the inner edge of the column strip to one-third of this value at the edge of the supporting beam or wall. To simplify steel placement, a uniform spacing will be used in the column strips. The average moments in the column strips being two-thirds of the corresponding moments in the middle strips, adequate column strip steel will be furnished if the spacing of this steel is 1½ times that in the middle strip.

According to the recommendation of Art. 4.6, inflection points may be assumed a distance $L/6$ from the continuous edges, that is, 3 ft 4 in. in the short direction and 4 ft 2 in. in the long direction. Two of every three bars are bent up at approximately these locations. The bent-up bars are carried over the support and beyond the inflection point of the adjacent panel a distance not less than

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one-sixteenth of the span, or the depth of the member, or 12 bar diameters, whichever is greater. It is customary and slightly conservative, instead, to extend these bars to the quarter point of the adjacent span. This extension will also satisfy the bond requirements of these negative-moment bars at the continuous edges. At the discontinuous edges twice as much steel has been furnished as is needed for the design moments. Therefore, only one-half of the development length of Table 8, Appendix B, must be provided. For the No. 4 bars this length is $\frac{1}{2} \times 17 = 8.5$ in. Flexural bond need not be considered if anchorage bond is no larger than 80 per cent of the permissible value. This will be so if actual development length is no less than 1.25 times this minimum length, or $1.25 \times 8.5 = 10.6$ in. An 11-in. anchorage length will be used for both No. 4 and No. 3 bars.

The reactions of the slab constitute loads on the edge beams and are calculated from Table 4.5, which indicates that 71 per cent of the load is transmitted in the short direction, and 29 per cent in the long direction. The total load on the panel being $20 \times 25 \times 337.5 = 169{,}000$ lb, the load per foot on the long beam is $(0.71 \times 169{,}000)/(2 \times 25) = 2400$ lb per ft, and on the short beams $(0.29 \times 169{,}000)/(2 \times 20) = 1220$ lb per ft. However, the Code suggests that the short beams be designed for a uniform load not smaller than $wA/3$; in the present case $337.5 \times \frac{2}{3} = 2250$ lb per ft exceeds 1220 lb per ft and, therefore, governs. In addition, the edge beams evidently carry their own weight and, in the case of the beams along the continuous edges, the corresponding reactions from the adjacent panels.

PROBLEMS

1. A concrete slab roof is to be designed to cover a transformer vault. The outside dimensions of the vault are 17×20 ft, and walls are 8-in. brick. A design live load of 80 psf, uniformly distributed over the roof surface, will be assumed. Design the roof as a two-way slab, using U.S.D. methods, with $f'_c = 4000$ psi and $f_y = 50{,}000$ psi.

2. A concrete warehouse floor is framed by beams on column lines, which are 18 ft on centers in one direction and 24 ft on centers in the other. Beam webs may be assumed to be 12 in. wide. A working live load of 225 psf must be carried. Design a typical interior panel, using U.S.D. methods, with $f'_c = 5000$ psi and $f_y = 60{,}000$ psi.

3. Design a typical exterior and first-interior panel for the warehouse floor of the preceding problem. The 24-ft dimension of the exterior panel is parallel to the outside wall.

FLAT SLABS

4.8 Description of General Type A flat-slab floor, as its name implies, is one consisting of a reinforced-concrete floor slab built monolithically with the columns and supported directly by the columns without the aid of beams and girders. The slab may be of uniform thickness