Lecture # 02

Flexural Analysis and Design of Reinforced Concrete Beam

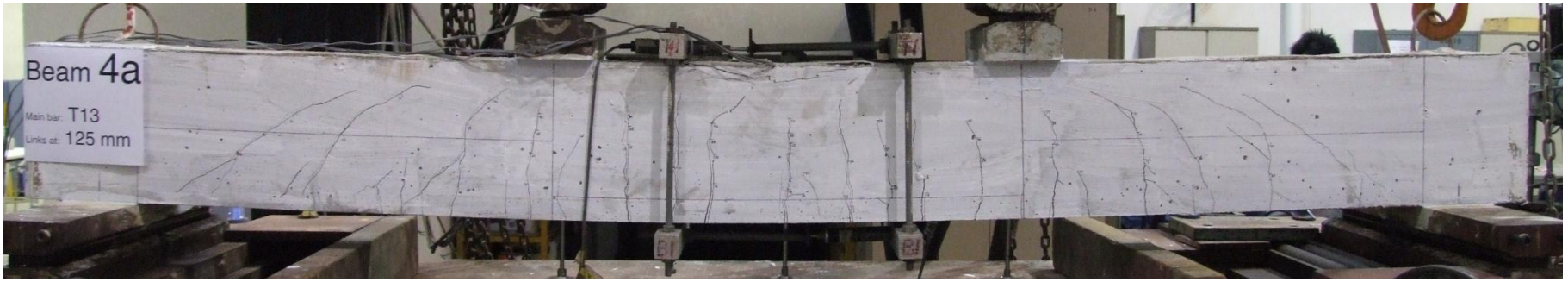
CE315: DESIGN OF CONCRETE STRUCTURES I

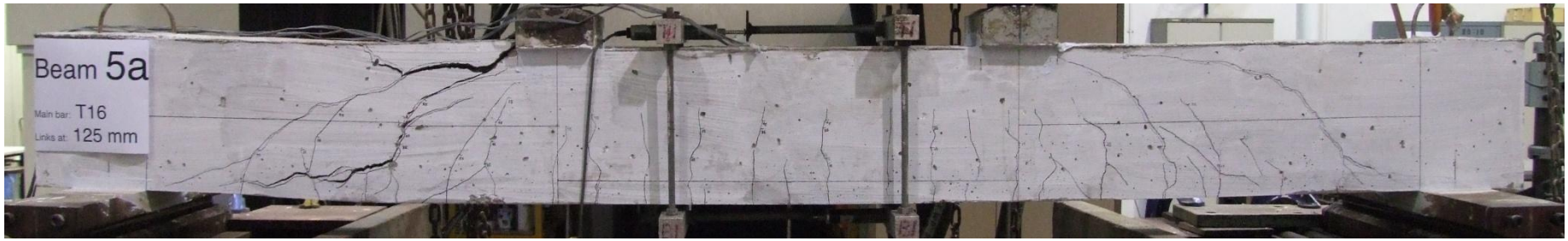
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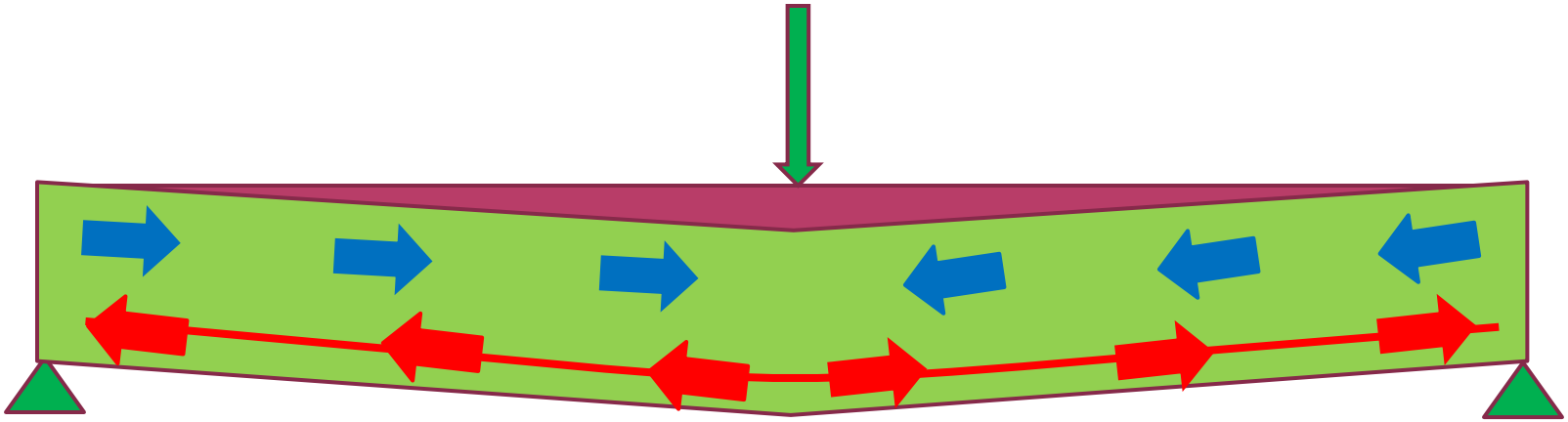
RCC BEAM BEHAVIOR





REINFORCED CONCRETE (RCC) BEAM BEHAVIOR

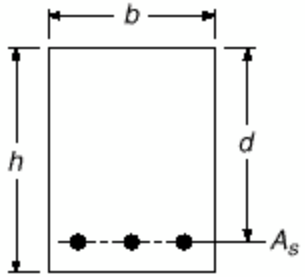
- When a simply supported plain concrete beam subjected a vertically downward load it deforms downward.
- In such a beam, tension is developed at the bottom and compression is developed at the top.



REINFORCED CONCRETE (RCC) BEAM BEHAVIOR

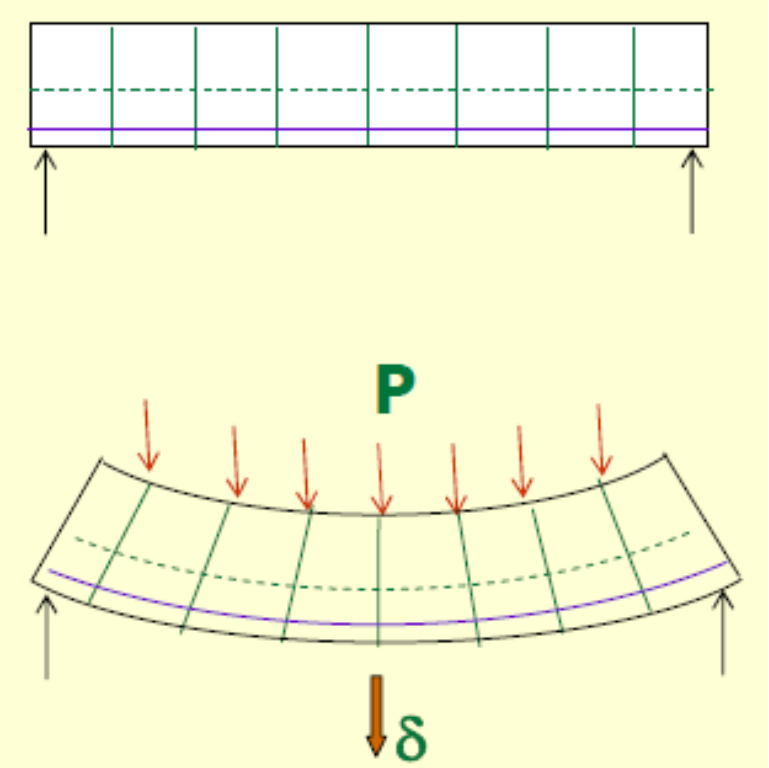
- Plain concrete beams are inefficient as flexural member because they are weak in tension during bending.
- For this reason, steel reinforcing bars are placed on the tension side as close to the extreme tension fiber.
- In such a RCC beam, the tension caused by the bending moments is chiefly resisted by the steel reinforcement, while the concrete alone is capable of resisting the corresponding compression.

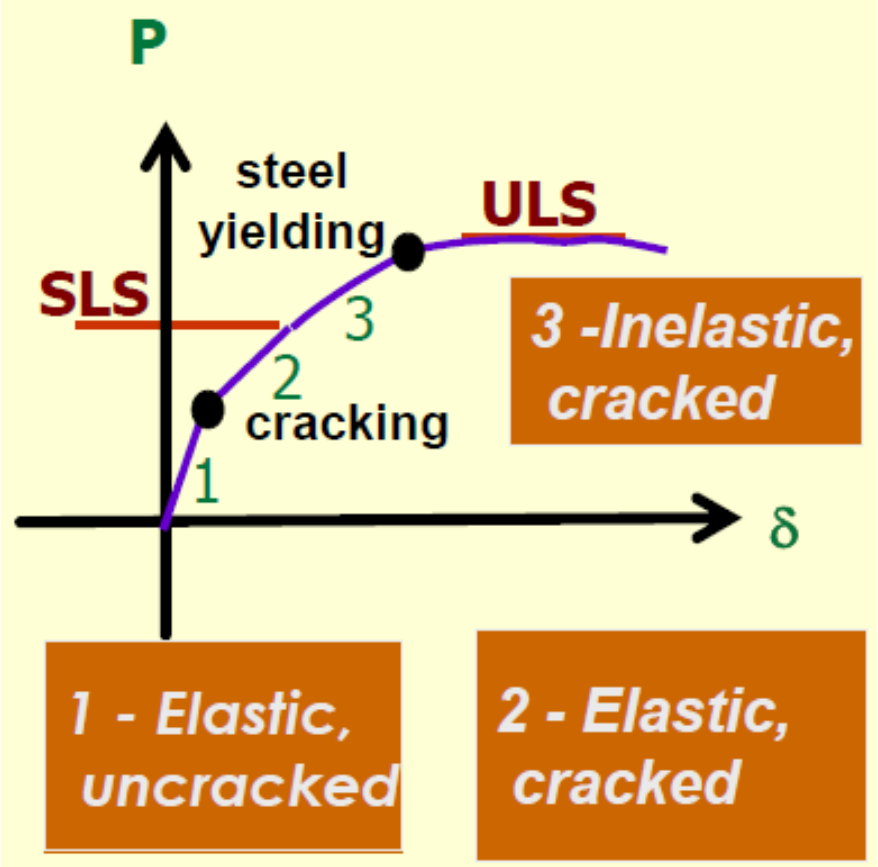




RCC BEAM BEHAVIOR

- When the load in a beam is gradually increased from zero to the magnitude that will cause the beam to fail, several stages of behavior can be clearly distinguished.

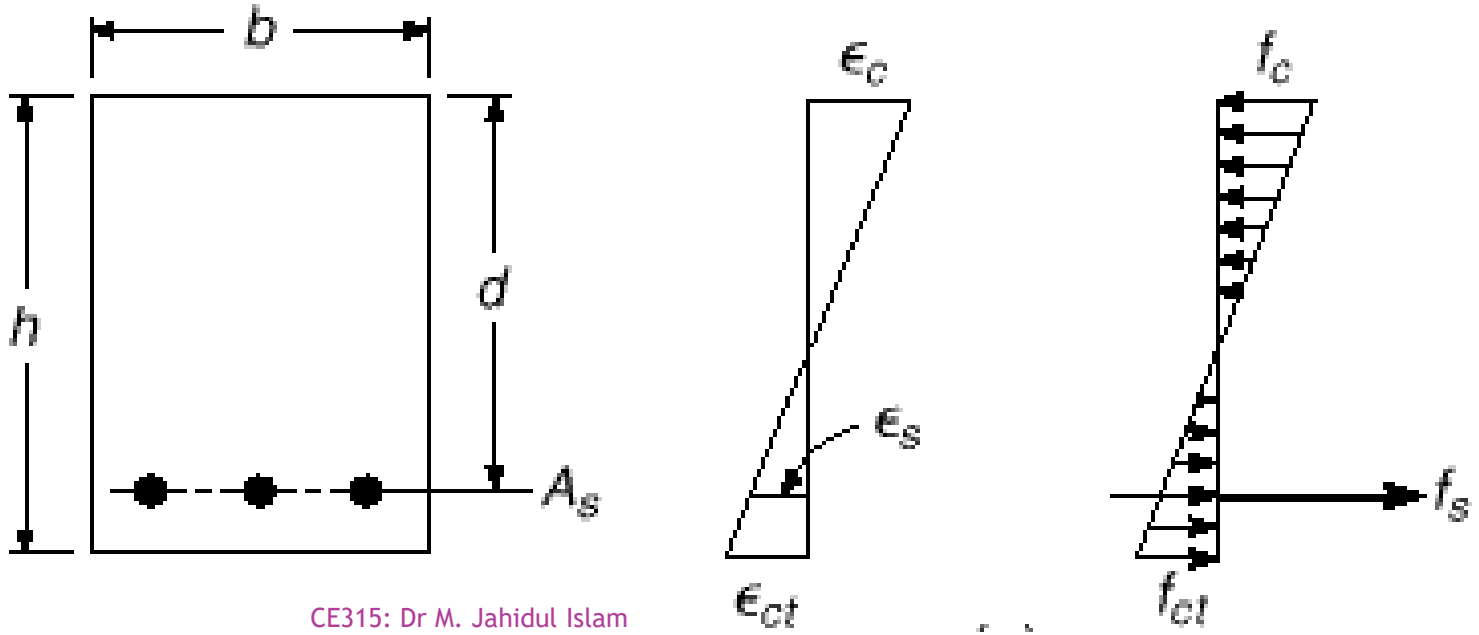




RCC BEAM BEHAVIOR



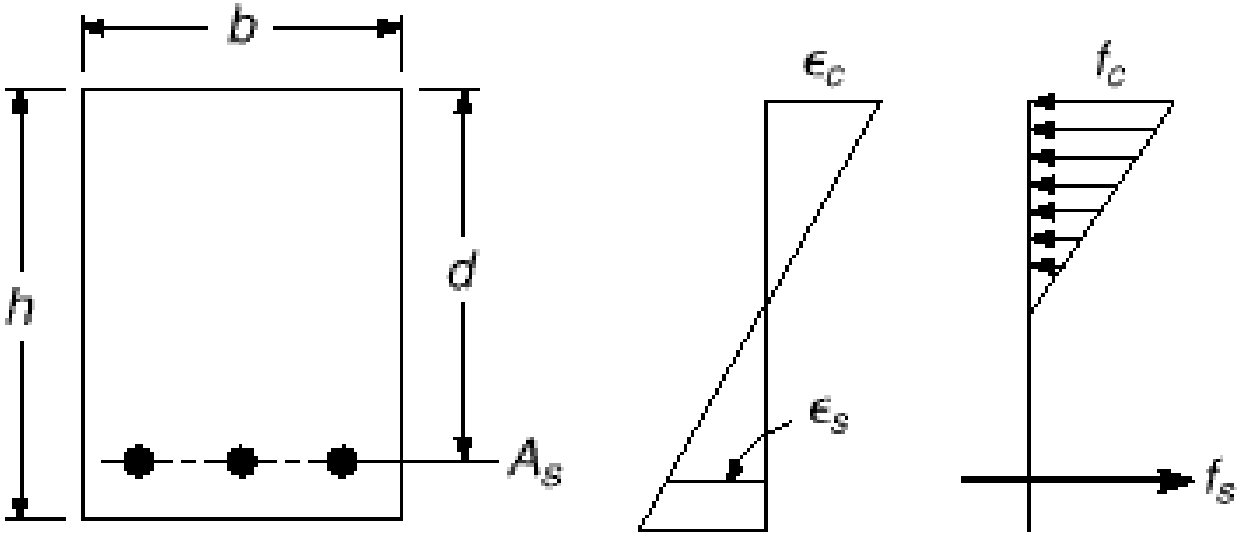
- **Stage1 (Stresses Elastic & Section Uncracked):**
  - At low loads, as long as the tensile stress in the concrete is smaller than the modulus of rupture, the entire concrete is effective in resisting stress.
  - At this stage, all stresses in the concrete are of small magnitude and are proportional to the strains.



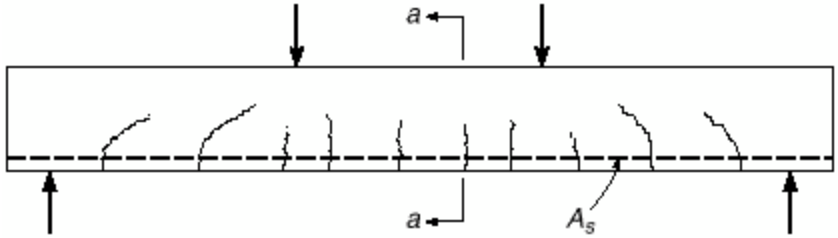
RCC BEAM BEHAVIOR



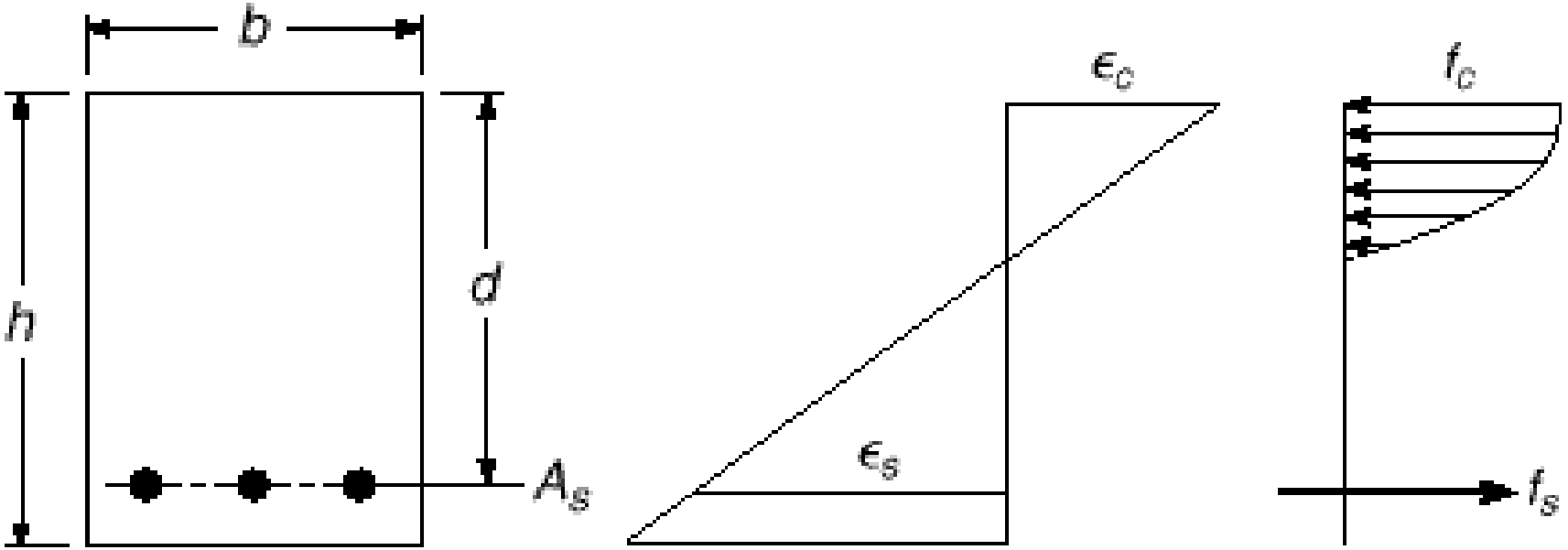
- **Stage2 (Elastic & Cracked):**
  - When the load is further increased the tensile strength of concrete is reached and tension crack is developed.
  - In a cracked section, concrete does not transmit any tensile stresses and steel resists the entire tension.
  - At moderate load, if the concrete stresses do not exceed approximately $f_c'/2$, stresses and strains continue to be proportional.



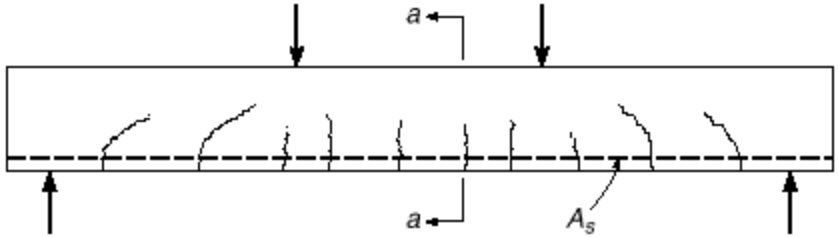
RCC BEAM BEHAVIOR



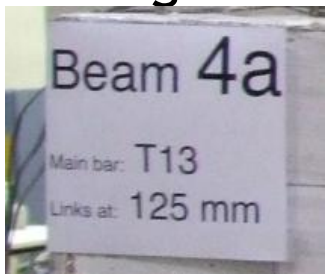
- **Stage3 (Inelastic & Cracked):**
  - When the load is further increased, stress and strain rise correspondingly and no longer proportional.
  - The resulting nonlinear relation between stresses and strains is that given by the concrete stress-strain curve.

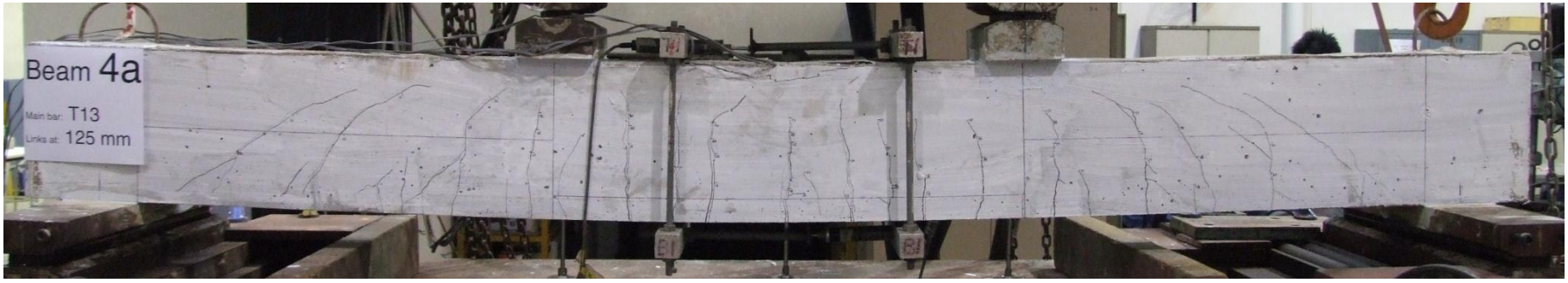


RCC BEAM BEHAVIOR

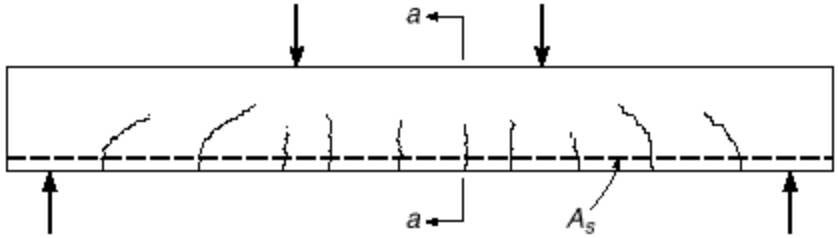


- **Stage4 (Failure):**
  - Failure can be caused in one of two ways.
  - With moderate amount of reinforcement, when steel reach its yield point, reinforcement stretches a large amount, and the tension cracks widen visibly and propagate upward, with simultaneous significant deflection of the beam. This is a gradual failure.



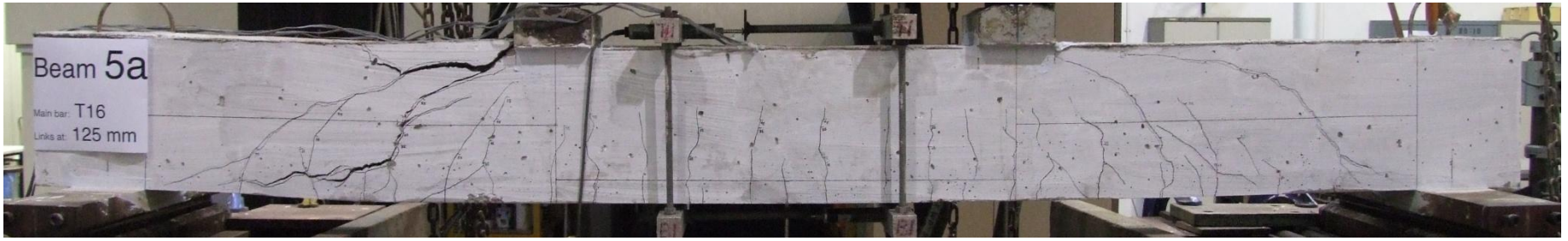


RCC BEAM BEHAVIOR



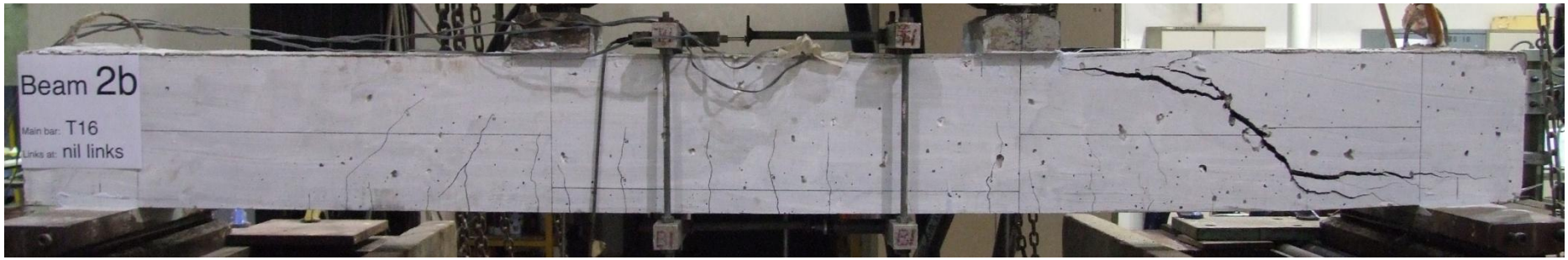
- **Stage4 (Failure):**
  - Failure can be caused in one of two ways.
  - If large amount of reinforcement are used, the compressive strength of concrete may be failed before the steel starts yielding. Compressive failure of concrete is sudden and explosive in nature.





RCC BEAM BEHAVIOR

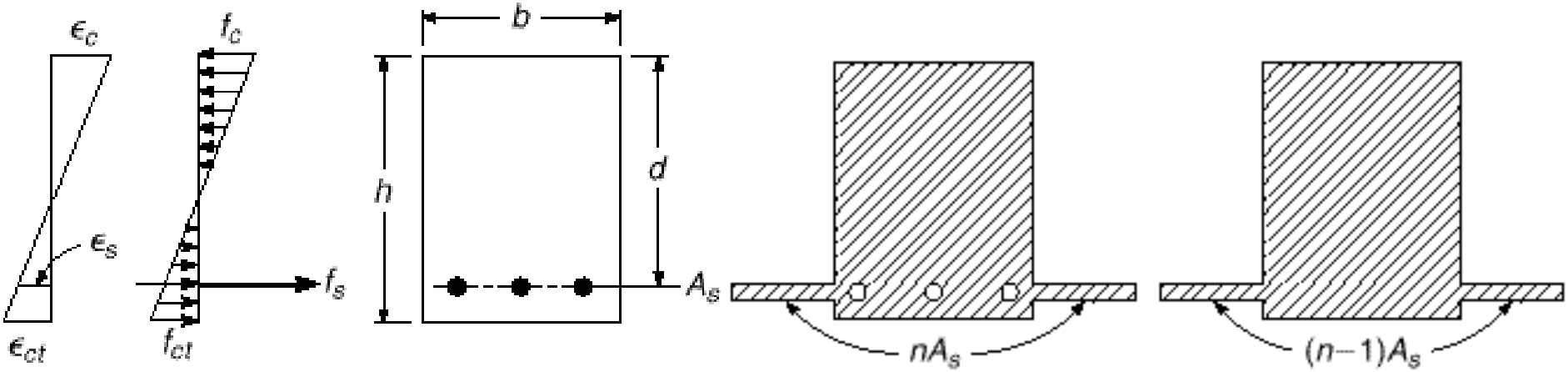




STRESSES ELASTIC & SECTION UNCRACKED

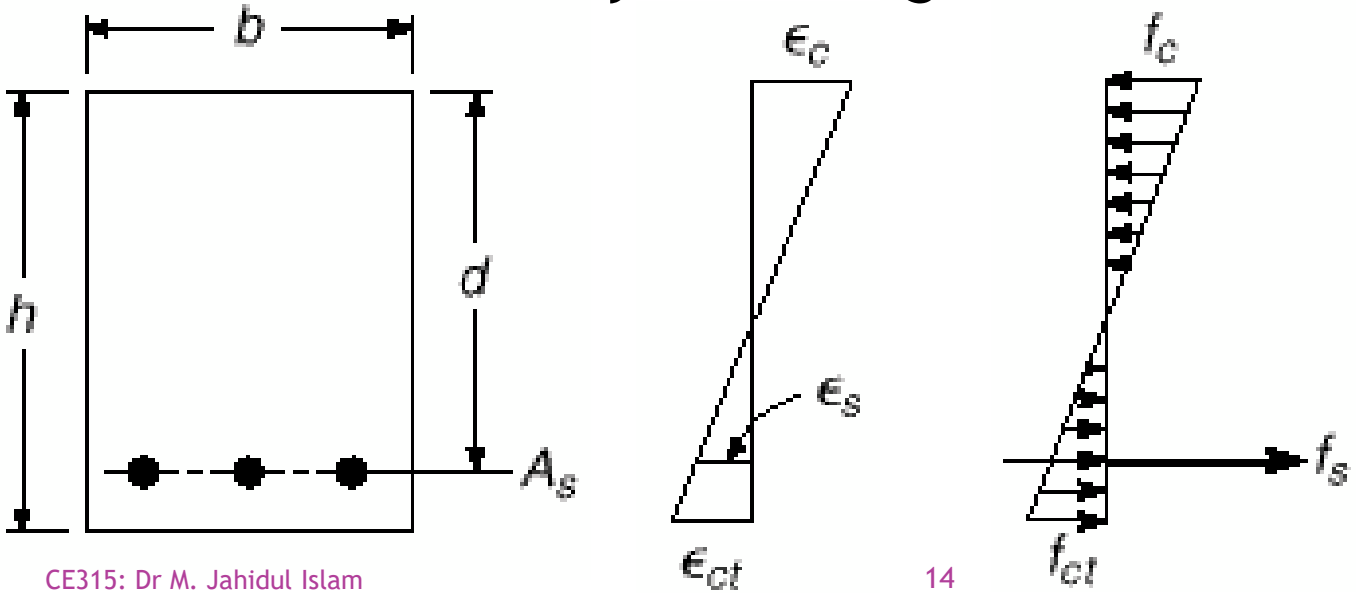
STRESSES ELASTIC & SECTION UNCRACKED

- As long as the tensile stress in the concrete is smaller than the modulus of rupture, so that no tension crack developed, the stress and strain distribution remain linear as shown in the figure.
- This beam can be solved by adopting "transformed section" method, where the actual area of the reinforcement is replaced with an equivalent concrete area to nA_s located at the level of the steel.

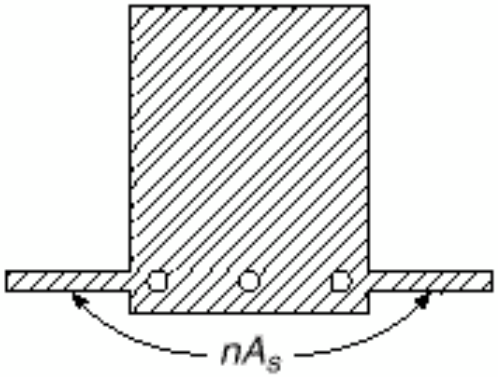


EXAMPLE 2.1

- A rectangular beam has the dimensions b = 10 in., h = 25 in., and d = 23 in. and is reinforced with three No. 8 (No. 25) bars so that A_s = 2.37 in^2.
- The concrete cylinder strength (f_c') is 4,000 psi, and tensile strength in bending (modulus of rupture) is 475 psi. The yield point of the steel (f_y) is 60,000 psi. And n = 8.
- Determine the stresses caused by a bending moment M = 45 ft-kips.

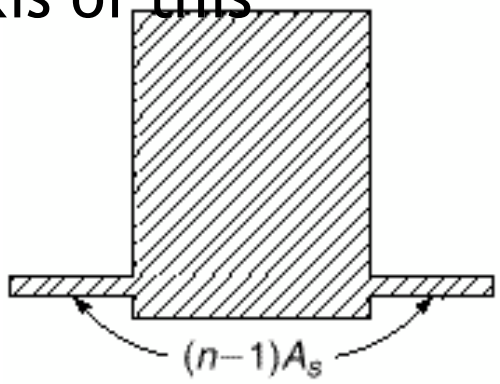


EXAMPLE 2.1 (SOLUTION)



- $n = E_s/E_c$ = 29,000,000 / 3,600,000 = 8
- $(n\text{-}1)A_s$ = (8-1) x 2.37 = 16.59 in^2
- Calculation of the location of the neutral axis of this section:
- From the top,

$$\bar{y} = \frac{10 \times 25 \times 12.5 + 16.59 \times 23}{10 \times 25 + 16.59} = 13.2 \text{ in}$$

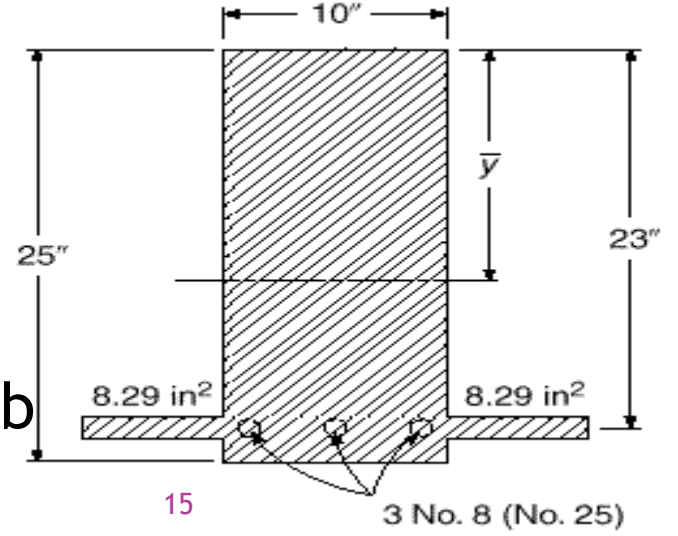


- Moment of inertia:

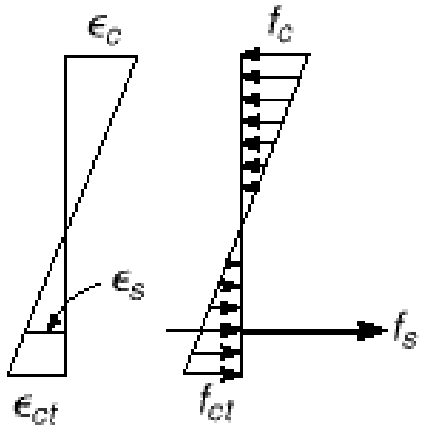
$$I = \frac{10 \times 25^3}{12} + 10 \times 25 \times (13.2 - 12.5)^2$$

$$+16.59 \times (23 - 13.2)^2 = 14,737 \text{ in}^4$$

- Moment, M = 45 ft-kips = 540,000 in-lb



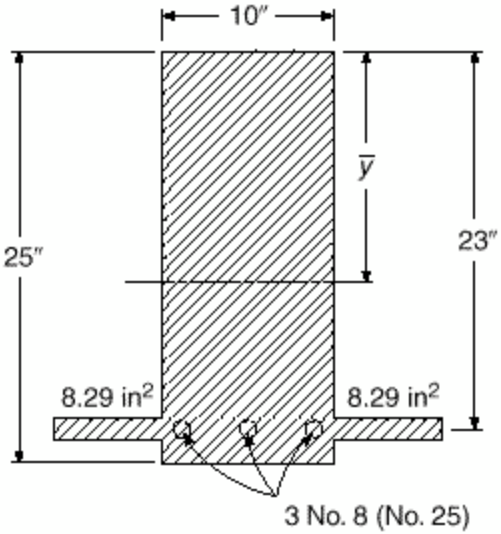
EXAMPLE 2.1 (SOLUTION)



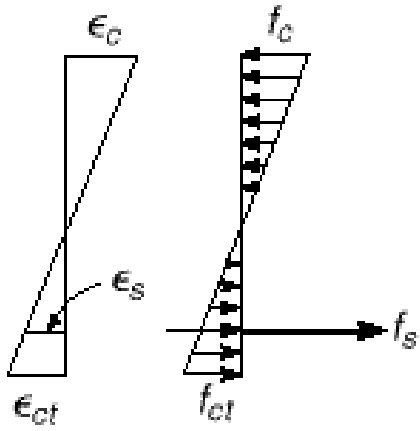
- According to the flexural beam theory, the maximum bending stress occurs at the outer fibers and is equal to $f_{\max} = \dfrac{Mc}{I}$
- The concrete compression stress at top fiber is, $f_c = \dfrac{540,000 \times 13.2}{14737} = 484 \text{ psi}$
- The concrete tension stress at bottom fiber is,

$$f_c = \frac{540,000 \times (25 - 13.2)}{14737} = 432 \text{ psi}$$

- Which is less than the tensile strength of the concrete, 475 psi, so no tensile cracks will form.



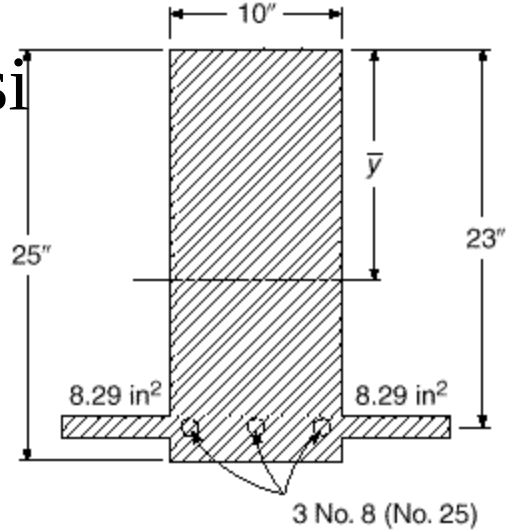
EXAMPLE 2.1 (SOLUTION)



- The stress in the steel reinforcement is,

$$f_s = n\frac{My}{I}$$

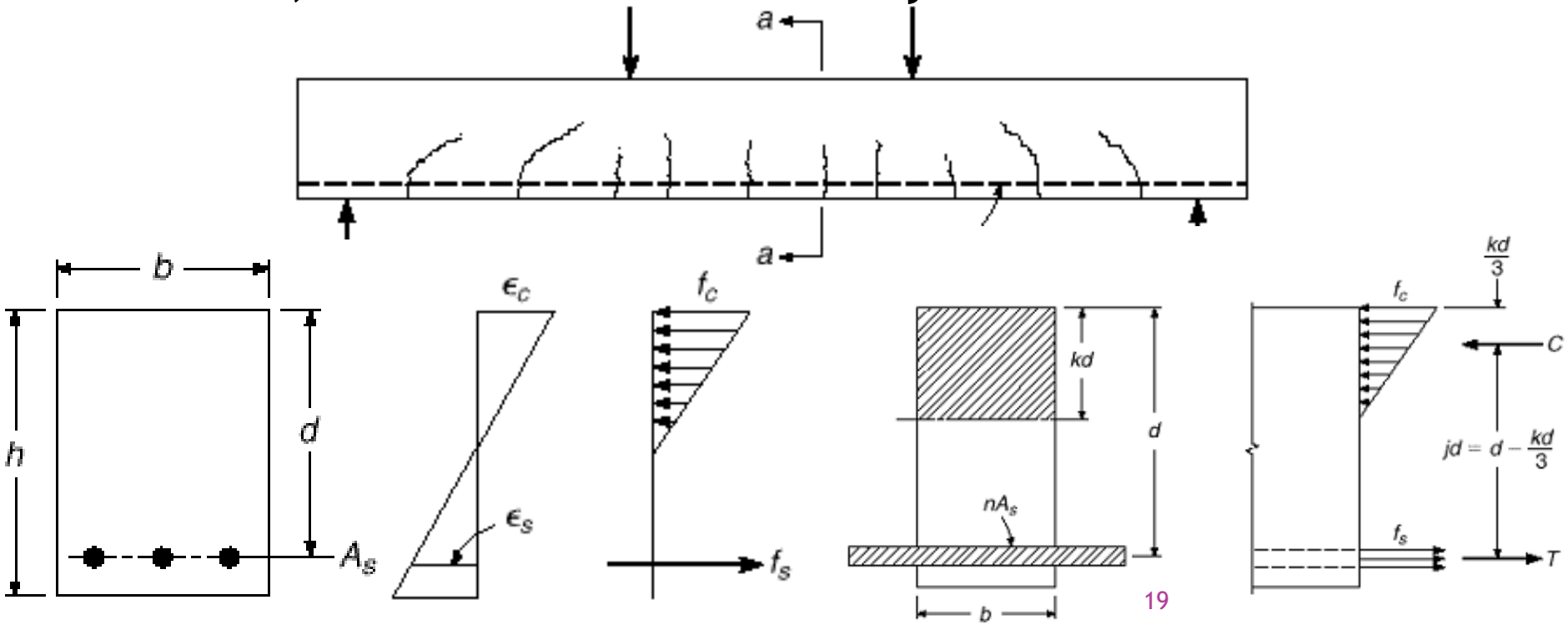
$$f_s = 8 \times \frac{540,000 \times (23 - 13.2)}{14737} = 2873 \text{ psi}$$



STRESSES ELASTIC AND SECTION CRACKED

STRESSES ELASTIC AND SECTION CRACKED

- When the tensile stresses exceeds the modulus of rupture, cracks form. If the concrete compressive stress is less than approximately $f_c'/2$ and the steel has not yet reached the yield point, both material behave elastically.
- All the concrete that is stressed in tension is assumed cracked, and therefore effectively absent.



STRESSES ELASTIC & SECTION CRACKED

- To compute stresses, the transformed section will be used.

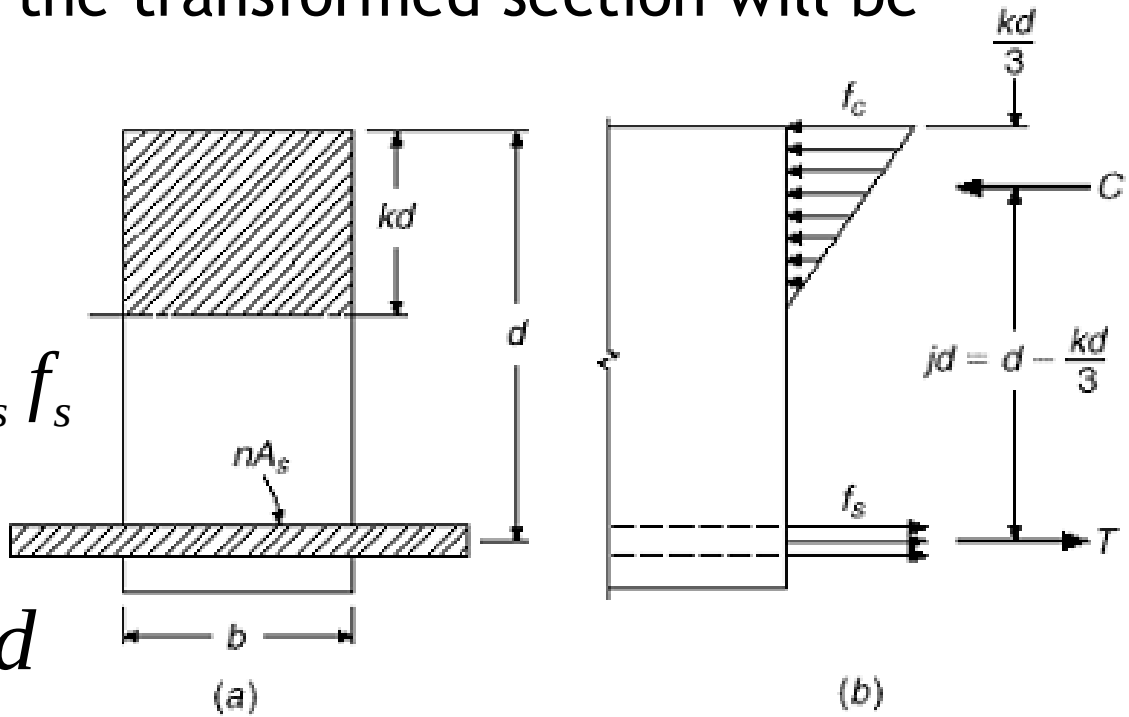
Compression force:

$$C = \frac{f_c}{2} \cdot bkd$$

Tensile force: $T = A_s f_s$

Taking moment about C gives:

$$M = Tjd = A_s f_s jd$$



Therefore, the steel stress is:

$$f_s = \frac{M}{A_s jd}$$

Taking moment about T gives:

$$M = Cjd = \frac{f_c}{2} \cdot bkdjd = \frac{f_c}{2} kjbd^2$$

The concrete stress is

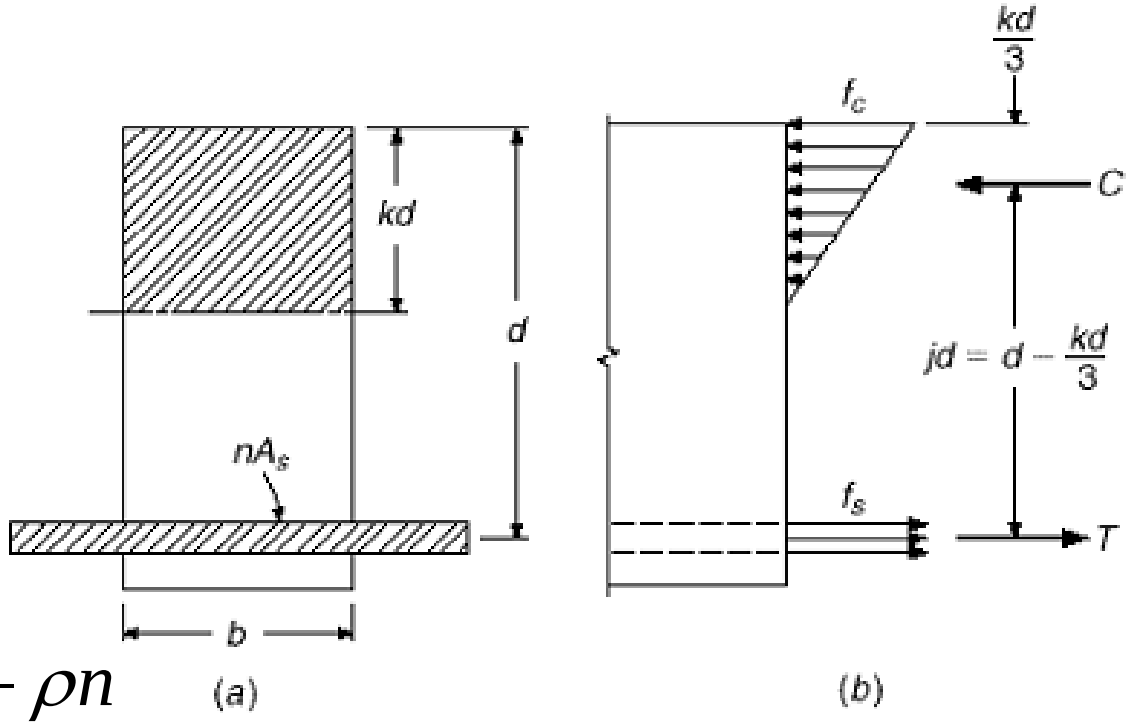
$$f_c = \frac{2M}{kjbd^2}$$

STRESSES ELASTIC & SECTION CRACKED

Reinforcement ratio:

$$\rho = \frac{A_s}{bd}$$

Expression for ***k*** and ***j***



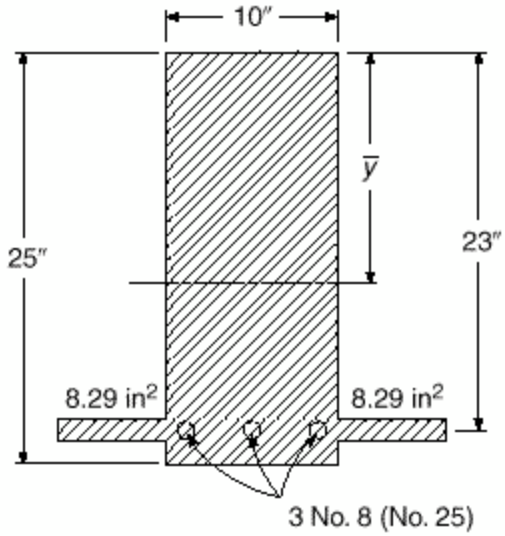
$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

$$j = 1 - \frac{k}{3}$$

EXAMPLE 2.2

- A rectangular beam has the dimensions b = 10 in., h = 25 in., and d = 23 in. and is reinforced with three No. 8 (No. 25) bars so that A_s = 2.37 in^2.
- The concrete cylinder strength (f_c') is 4000 psi, and tensile strength in bending (modulus of rupture) is 475 psi. The yield point of the steel (f_y) is 60,000 psi. Given, E_s = 29,000 ksi and E_c = 3,600 ksi.
- Determine the stresses caused by a bending moment M = 90 ft-kips.

EXAMPLE 2.2 (SOLUTION)



- $n = E_s/E_c$ = 29,000,000 / 3,600,000 = 8
- A_s = 3 x 0.79 = 2.37 in²
- Moment, M = 90 ft-kips = 1,080,000 in-lb
- The concrete compression stress at top fiber is,

$$f_c = \frac{1,080,000 \times 13.2}{14737} = 968 \text{ psi}$$

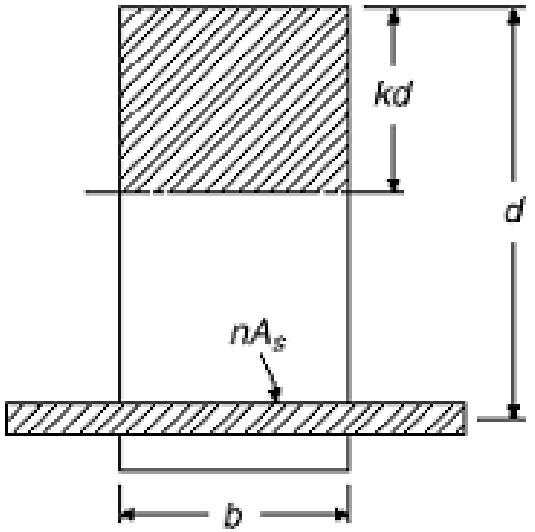
$$f_{\max} = \frac{Mc}{I}$$

- The concrete tension stress at bottom fiber is,

$$f_c = \frac{1,080,000 \times (25 - 13.2)}{14737} = 865 \text{ psi}$$

- Which is greater than the tensile strength of the concrete, 475 psi, so tensile cracks will form and analysis should be adapted considering cracked section.

EXAMPLE 2.2 (SOLUTION)



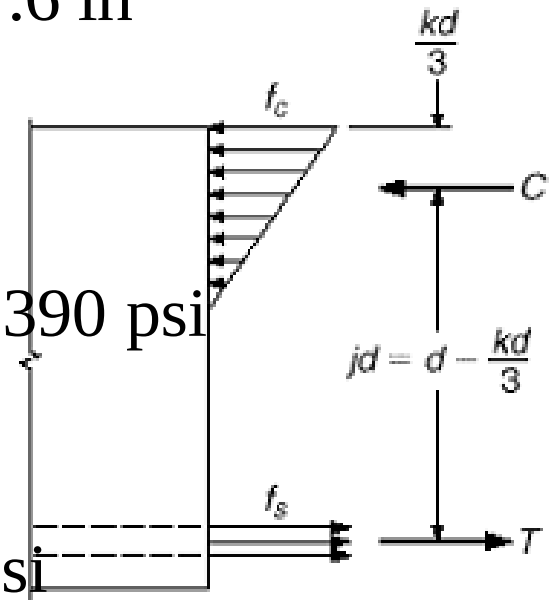
- $$\rho = \frac{A_s}{bd} = \frac{2.37}{10 \times 23} = 0.0103$$

- $$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

$$= \sqrt{(0.0103 \times 8)^2 + 2 \times 0.0103 \times 8} - 0.0103 \times 8 = 0.33$$

$$kd = 0.33 \times 23 = 7.6 \text{ in}$$

- $$j = 1 - \frac{k}{3} = 1 - \frac{0.33}{3} = 0.89$$

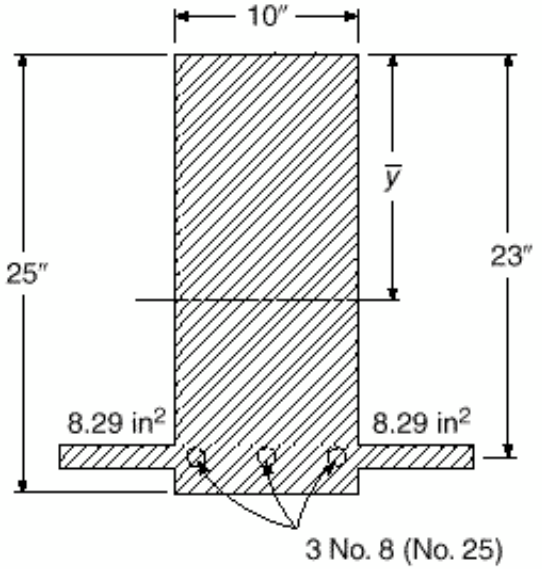


$$f_c = \frac{2M}{kjbd^2} \Rightarrow f_c = \frac{2 \times 1,080,000}{0.33 \times 0.89 \times 10 \times 23^2} = 1,390 \text{ psi}$$

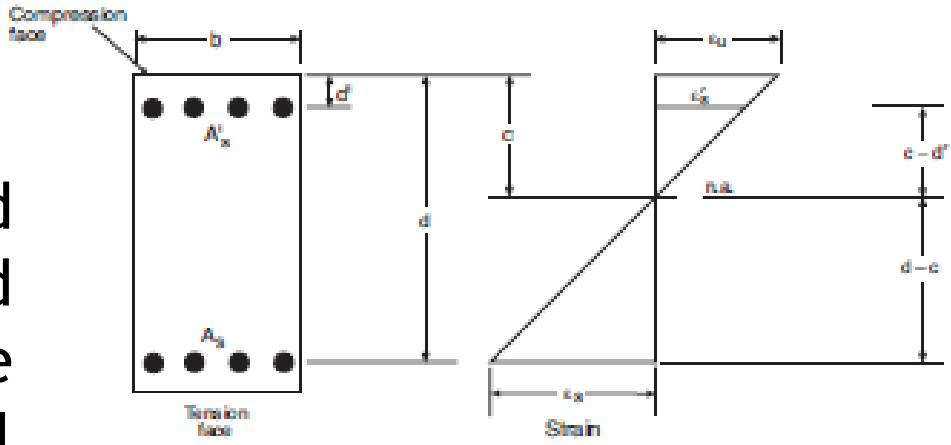
$$f_s = \frac{M}{A_s jd} \Rightarrow f_s = \frac{1,080,000}{2.37 \times 0.89 \times 23} = 22,262 \text{ psi}$$

COMPARISON OF UNCRACKED & CRACKED SECTIONS

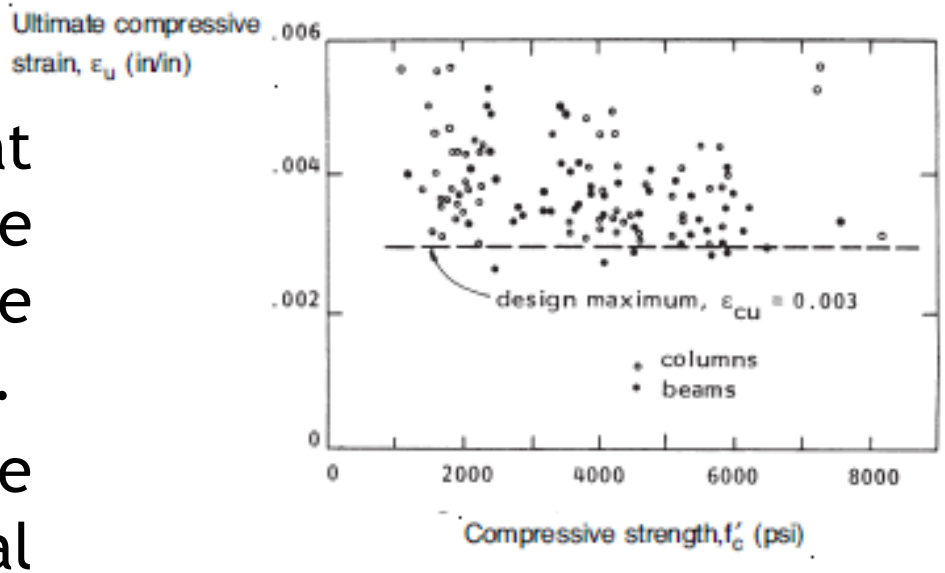
Description	Uncracked Section	Cracked Section
Neutral axis	13.2 in	7.6 in
Moment	45 ft-kips	90 ft-kips
Concrete compression stress	484 psi	1390 psi
Steel stress	2,873 psi	22,262 psi



DESIGN ASSUMPTIONS



1. Strain in reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis.
2. Maximum usable strain at extreme concrete compression fiber shall be assumed equal to ε_u = 0.003.
3. Tensile strength of concrete shall be neglected in axial and flexural calculations of reinforced concrete.

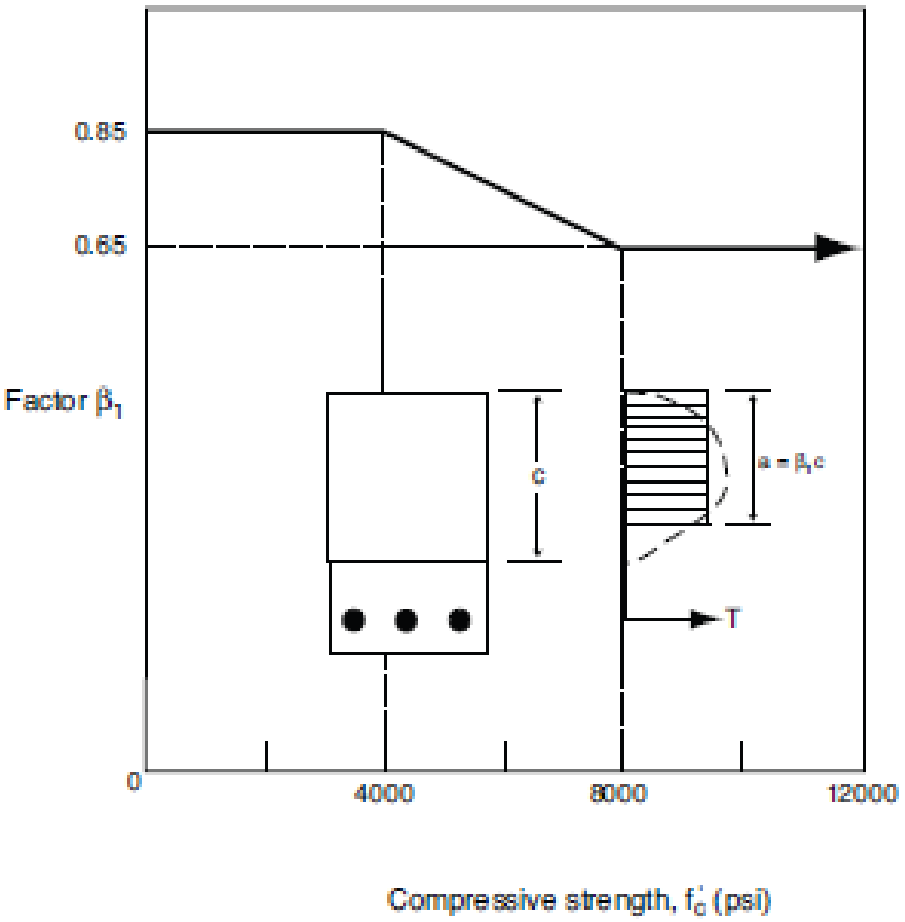


DESIGN ASSUMPTIONS

4. Stress in reinforcement f_s below the yield strength f_y shall be taken as E_s times the steel strain ε_s. For strains greater than f_y/E_s, stress in reinforcement shall be considered independent of strain and equal to f_y.
5. Relationship between concrete compressive stress distribution and concrete strain shall be assumed to be rectangular, trapezoidal, parabolic, or any other shape that results in prediction of strength in substantial agreement with results of compressive tests.

DESIGN ASSUMPTIONS

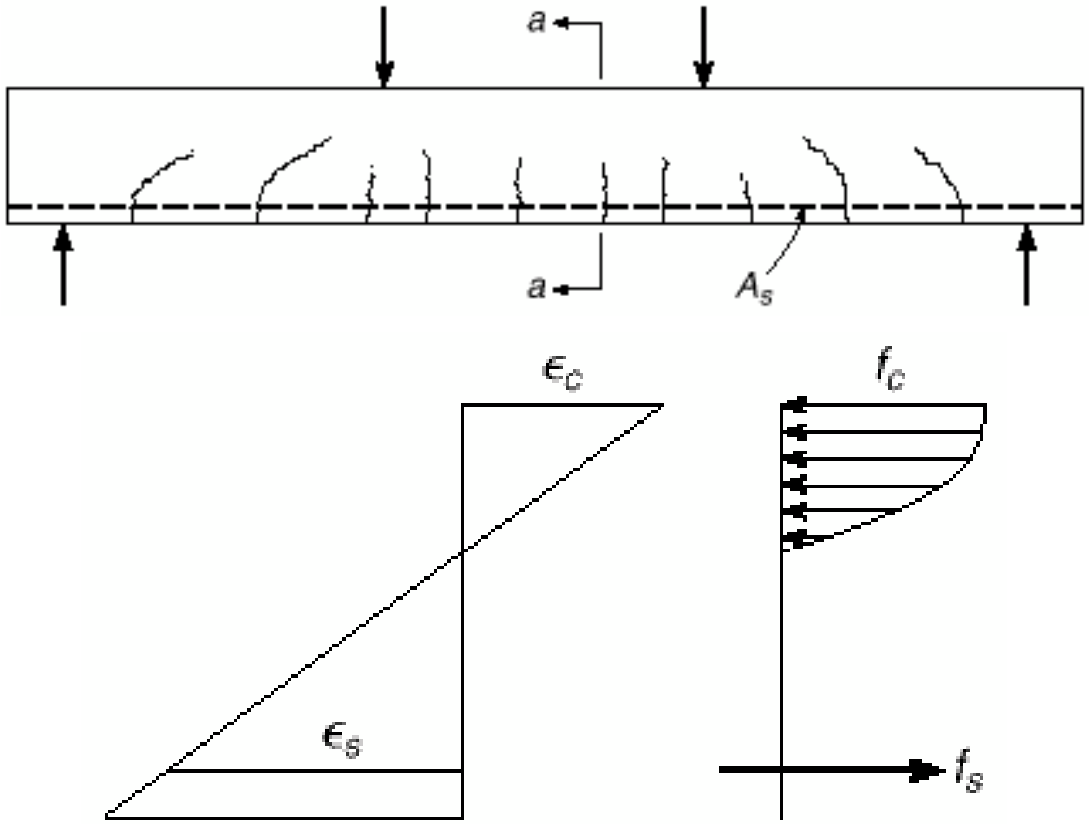
4. Requirements of the above assumption may be considered satisfied by an equivalent rectangular concrete stress distribution defined as follows: A concrete stress of $0.85f_c'$ shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross-section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain.



STRESSES INELASTIC & SECTION CRACKED

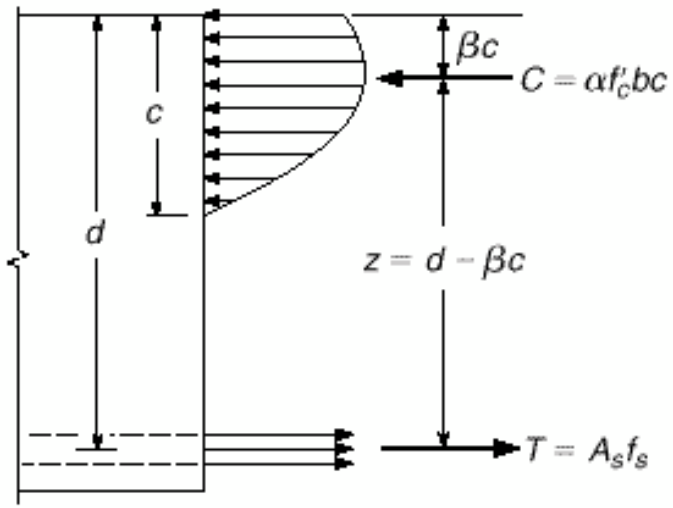
STRESSES INELASTIC & SECTION CRACKED

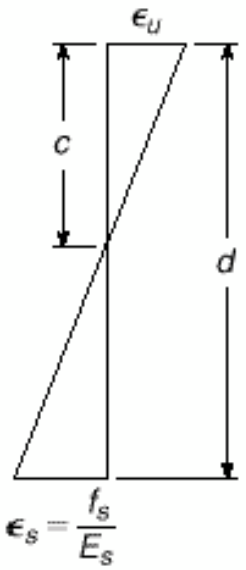
- At high loads, close to failure, the distribution of stresses and strains are no longer elastic distribution.

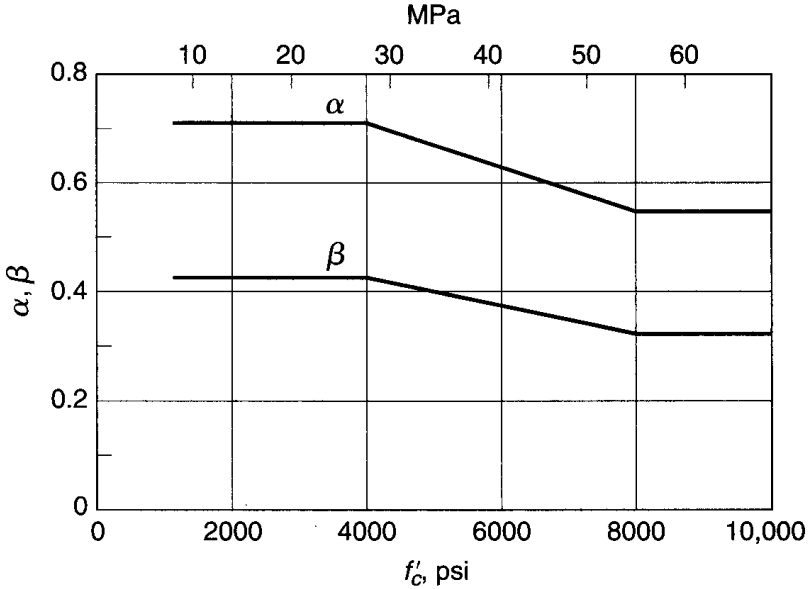


FLEXURAL STRENGTH

- The following figure represents the distribution of internal stresses and strains when the beam is about to fail.



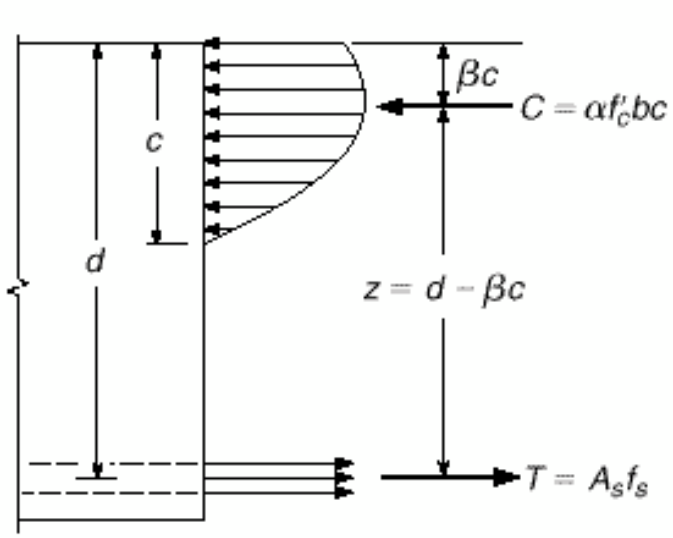


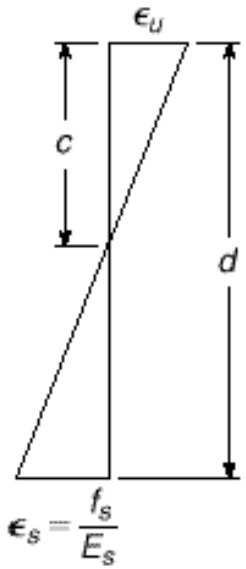


- $\alpha = 0.72$ for $f_c' \leq 4000$ psi and decreases by 0.04 for every 1,000 psi above 4,000 up to 8,000 psi. For $f_c' > 8{,}000$ psi, $\alpha = 0.56$.
- $\beta = 0.425$ for $f_c' \leq 4000$ psi and decreases by 0.025 for every 1000 psi above 4000 up to 8000 psi. For $f_c' > 8000$ psi, $\beta = 0.325$.

FLEXURAL STRENGTH

- For a nominal moment (M_n), the beam will fail either by
  - Tension yielding of the steel
  - Crushing of the concrete in the outer compression fiber.
- For tension yielding of the steel failure, $f_s = f_y$.
- For the second mode of failure, it is assumed that concrete fail when maximum strain ε_u reaches 0.003.





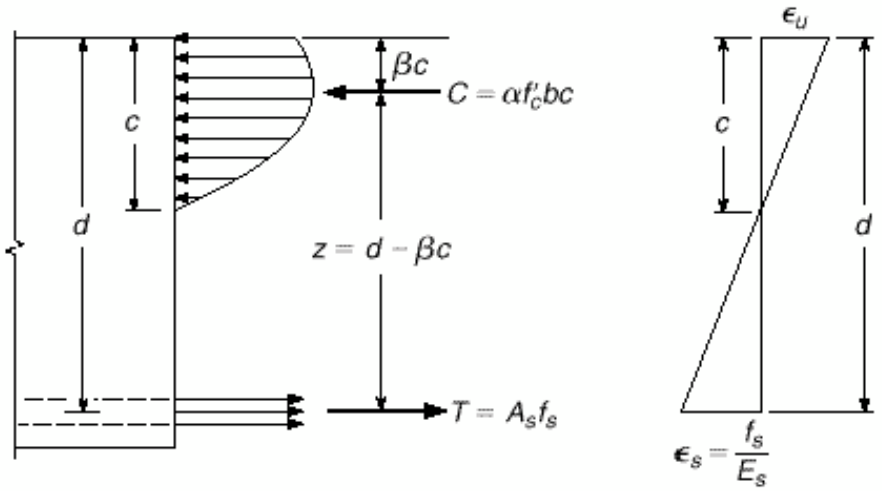
$$C = \alpha f_c' bc$$

$$T = A_s f_s$$

FLEXURAL STRENGTH

- **For failure initiated by yielding of the tension steel:**
- Distance to the neutral axis:

$$C = T \Rightarrow \alpha c f_c' b = A_s f_y \therefore c = \frac{A_s f_y}{\alpha f_c' b}$$



- Total compression force on concrete:

$$C = \alpha f_c' b c$$

$$c = \frac{A_s f_y}{\alpha f_c' b} = \frac{\rho b d f_y}{\alpha f_c' b} = \frac{\rho d f_y}{\alpha f_c'}$$

- The nominal moment:

$$M_n = Cz = \alpha f_c' b c (d - \beta c) = \alpha f_c' b \frac{\rho f_y d}{\alpha f_c'} \left(d - \beta \frac{\rho f_y d}{\alpha f_c'} \right) = \rho f_y b d^2 \left(1 - \frac{\beta \rho f_y}{\alpha f_c'} \right)$$

$$M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'} \right)$$

FLEXURAL STRENGTH

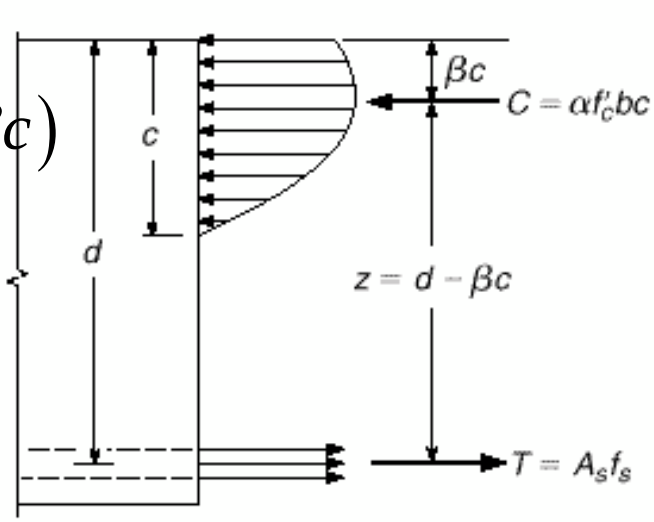
- **For failure by compression of concrete:**
- Distance to the neutral axis:

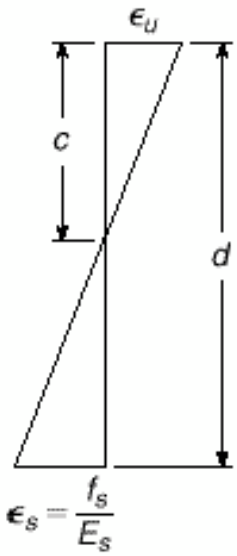
$$C = T \Rightarrow \alpha c f_c' b = A_s f_s \therefore c = \frac{A_s f_s}{\alpha f_c' b}$$

- The nominal moment:

$$M_n = Cz = \alpha f_c' bcz = \alpha f_c' bc(d - \beta c)$$

$$M_n = Tz = A_s f_s z = A_s f_s (d - \beta c)$$



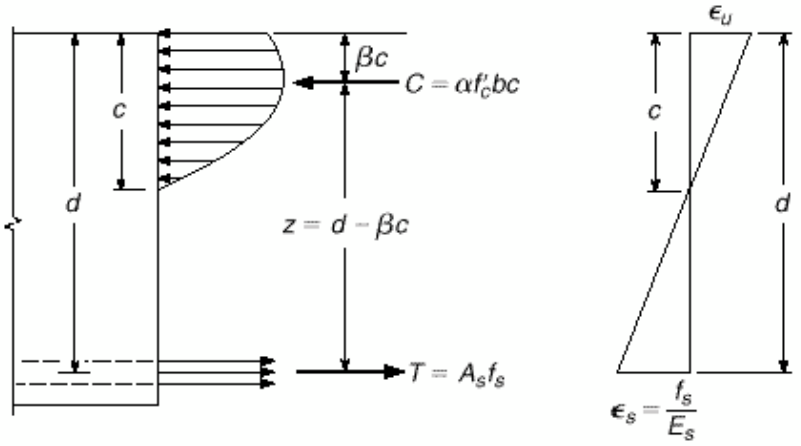


- Steel stress:

$$f_s = \varepsilon_s E_s = \varepsilon_u \frac{d - c}{c} E_s$$

- Hence, $$\alpha c f_c' b = A_s \varepsilon_u \frac{d - c}{c} E_s$$
- Solve this quadratic equation for c.

FLEXURAL STRENGTH



Balanced Steel Ratio, ρ_b

- The balanced steel ratio , representing that amount of reinforcement necessary for beam to fail by crashing of the concrete at the same load that causes the steel to yield.
- This means concrete reaches its compressive strain, ε_u and for steel, $f_s = f_y$.
- For this case, $c = \dfrac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} d$

Whether or not the steel has yielded can be determined by comparing the actual reinforcement ratio with the *balanced reinforcement ratio*, ρ_b.

$$\rho_b = \frac{\alpha f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$$

EXAMPLE 2.3

- A rectangular beam has the dimensions b = 10 in., h = 25 in., and d = 23 in. and is reinforced with three No. 8 (No. 25) bars so that A_s = 2.37 in^2.
- The concrete cylinder strength is f_c' is 4,000 psi. The yield point of the steel f_y is 60,000 psi. Given, E_s = 29,000 ksi and E_c = 3,600 ksi. Also ε_u = 0.003 and ε_y = 0.002
- Determine the nominal moment M_n at which the beam will fail.

EXAMPLE 2.3 (SOLUTION)

- Given: b = 10 in., h = 25 in., and d = 23 in. and A_s = 2.37 in^2.
- f_c' = 4,000 psi, f_y = 60,000 psi.
- Actual reinforcement ratio:

$$\rho = \frac{A_s}{bd} = \frac{2.37}{10 \times 23} = 0.0103$$

- Balanced reinforcement ratio:

$$\rho_b = \frac{\alpha f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \Rightarrow \rho_b = \frac{0.72 \times 4,000}{60,000} \frac{0.003}{0.003 + 0.002} \therefore \rho_b = 0.0284$$

- Since, $\rho < \rho_b$, the beam will fail in tension by yielding of the steel.
- Nominal moment: $M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$

$$= 0.103 \times 60,000 \times 10 \times 23^2 \left(1 - 0.59 \frac{0.0103 \times 60,000}{4,000}\right)$$

$$= 2,971,214 \text{ in-lb} = 248 \text{ ft-kips}$$

DESIGN OF TENSION-REINFORCED BEAMS

- The ***nominal strength*** of a proposed member is calculated, based on the best current knowledge of member and material behavior.
- This nominal strength is modified by a *strength reduction factor* ϕ, less than unity, to obtain the ***design strength***.
- Thus reinforced concrete members are proportioned so that,

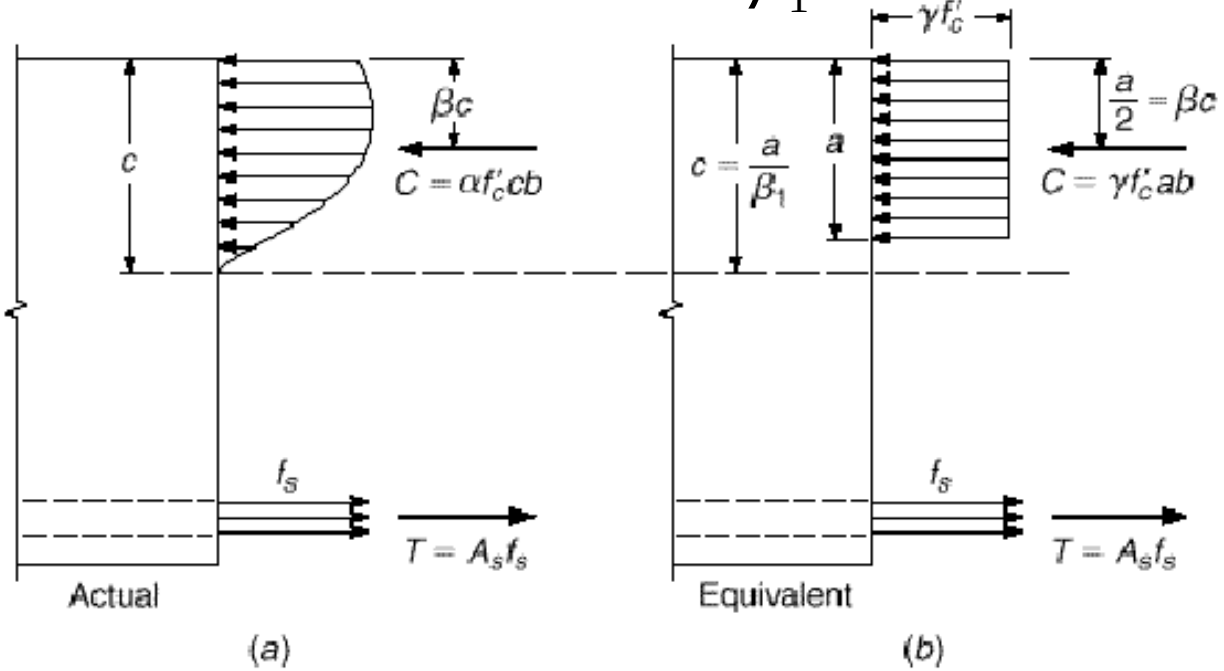
$$M_u \leq \phi M_n$$

$$V_u \leq \phi V_n$$

- Where the subscripts n denote the nominal strengths in flexural and shear respectively, and the subscript u denote the factored load moment and shear.

EQUIVALENT RECTANGULAR STRESS DISTRIBUTION

- In this case, the actual stress distribution is replaced by an equivalent simple rectangular outline.
- The intensity of stress is $\gamma f_c'$ of this equivalent constant stress and its depth, $a = \beta_1 c$.
- Note that, $\gamma = \alpha \dfrac{c}{a} = \dfrac{\alpha}{\beta_1}$ $\qquad \beta_1 = 2\beta$



$$C = \gamma f_c' ab$$

$$T = A_s f_s$$

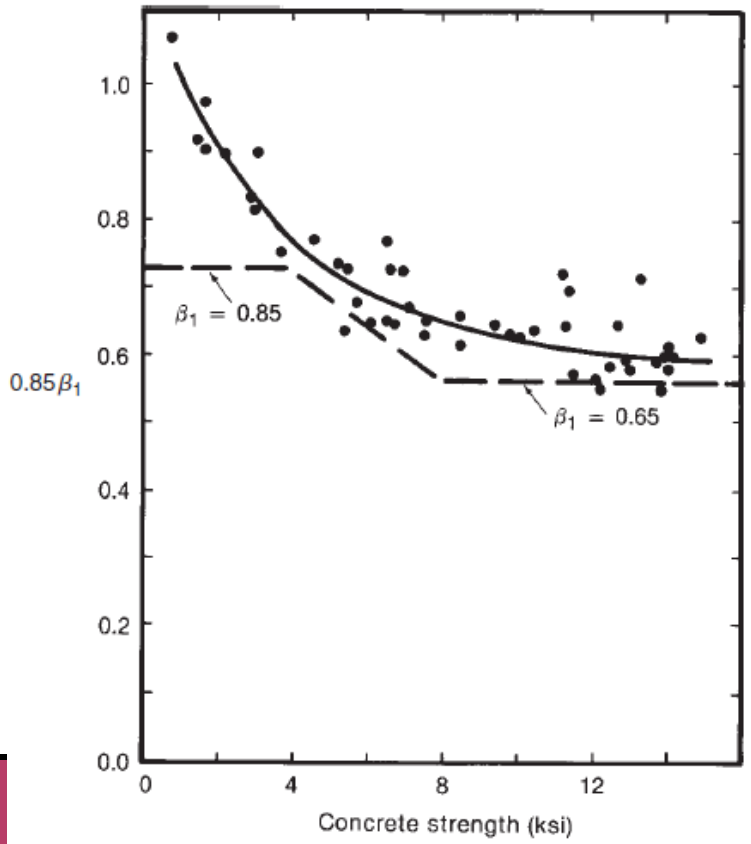
$$M_n = T\left(d - \frac{a}{2}\right)$$

$$M_n = A_s f_s\left(d - \frac{a}{2}\right)$$

EQUIVALENT RECTANGULAR STRESS DISTRIBUTION

$$\beta_1 = 0.85 - 0.05\frac{f_c' - 4000}{1000}$$

$$\text{and } 0.65 \le \beta_1 \le 0.85$$



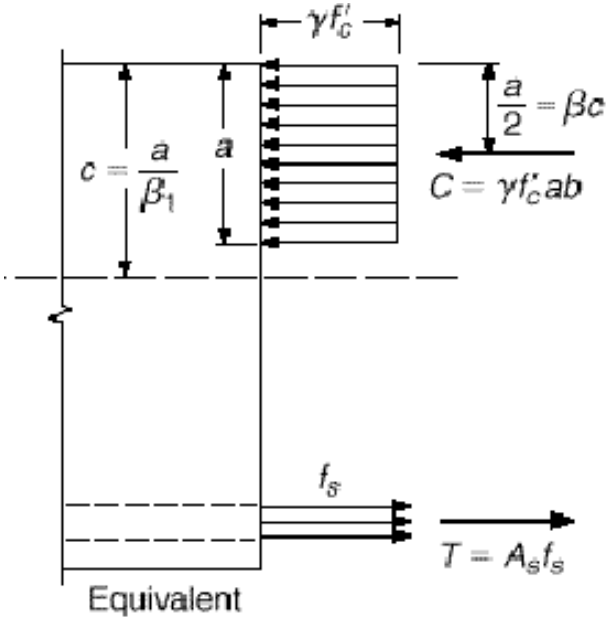
Concrete stress block parameters

	f_c', psi				
	≤ 4000	5000	6000	7000	≥ 8000
α	0.72	0.68	0.64	0.60	0.56
β	0.425	0.400	0.375	0.350	0.325
$\beta_1=2\beta$	0.85	0.80	0.75	0.70	0.65
$\gamma=\alpha/\beta_1$	0.85	0.85	0.85	0.86	0.86

BALANCED STRAIN CONDITION

- At balance failure, the steel strain = ε_y and the strain in the concrete, ε_u = 0.003.
- Balanced reinforcement ratio can be calculated from the equilibrium requirement, $C = T$.

$$\rho_b f_y b d = 0.85 f_c' a b = 0.85 \beta_1 f_c' b c$$



$$\rho_b = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \qquad c = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} d$$

- For,

$$\varepsilon_u = 0.003$$

$$\& \quad \varepsilon_y = \frac{f_y}{29 \times 10^6} \qquad \rho_b = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{87{,}000}{87{,}000 + f_y} \quad (b)$$

UNDER-REINFORCED BEAMS

- A *compression failure* gives little if any warning of distress, while a *tension failure* typically is gradual.
- With gradual failure observing large deflections and widening of concrete cracks associated with yielding of the steel reinforcement it is possible to take measures to avoid total collapse.
- Because of that, it is prudent to design beams in such a way that failure, if it occurs, will be by yielding of the steel, not by crushing of the concrete.
- This can be achieved by requiring that the reinforcement ratio, ρ be less than the balanced ratio, ρ_b.

$$\rho_b = 0.85\beta_1 \frac{f_c'}{f_y} \frac{87,000}{87,000 + f_y}$$

ACI CODE PROVISIONS

- To ensure underreinforced behavior, ACI Code provides the maximum reinforcement ratio for beams with net tensile strain $\varepsilon_t = 0.004$:

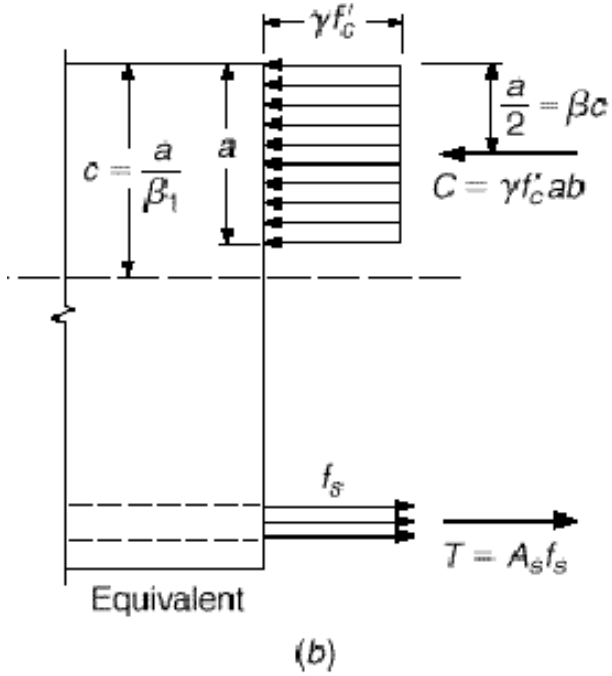
$$\rho_{\max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.004}$$

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t}$$

- For underreinforced concrete, $f_s = f_y$,

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) \qquad A_s = \rho b d$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$



ACI CODE PROVISIONS

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

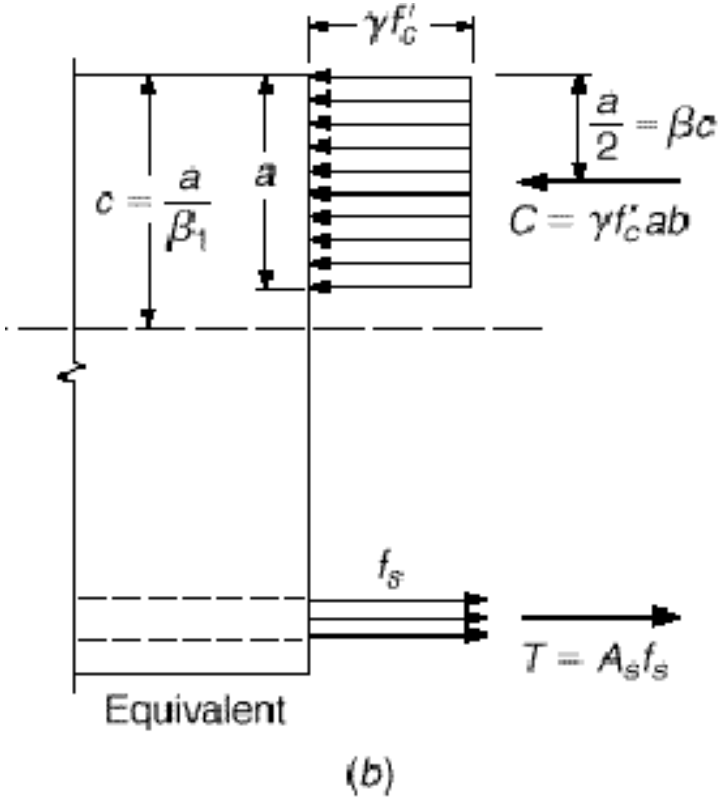
$$a = \frac{A_s f_y}{0.85 f_c' b}$$

- Substituting a into M_n we have,

$$M_n = A_s f_y \left(d - \frac{1}{2} \frac{A_s f_y}{0.85 f_c' b} \right) = \rho b d f_y \left(d - \frac{1}{2} \frac{\rho b d f_y}{0.85 f_c' b} \right) \qquad A_s = \rho b d$$

$$M_n = \rho f_y \left(1 - 0.59 \frac{\rho f_y}{f_c'} \right) b d^2 = R b d^2$$

$$R = \rho f_y \left(1 - 0.59 \frac{\rho f_y}{f_c'} \right)$$



ACI CODE PROVISIONS

$$R = \rho f_y \left(1 - 0.59 \frac{\rho f_y}{f'_c}\right)$$

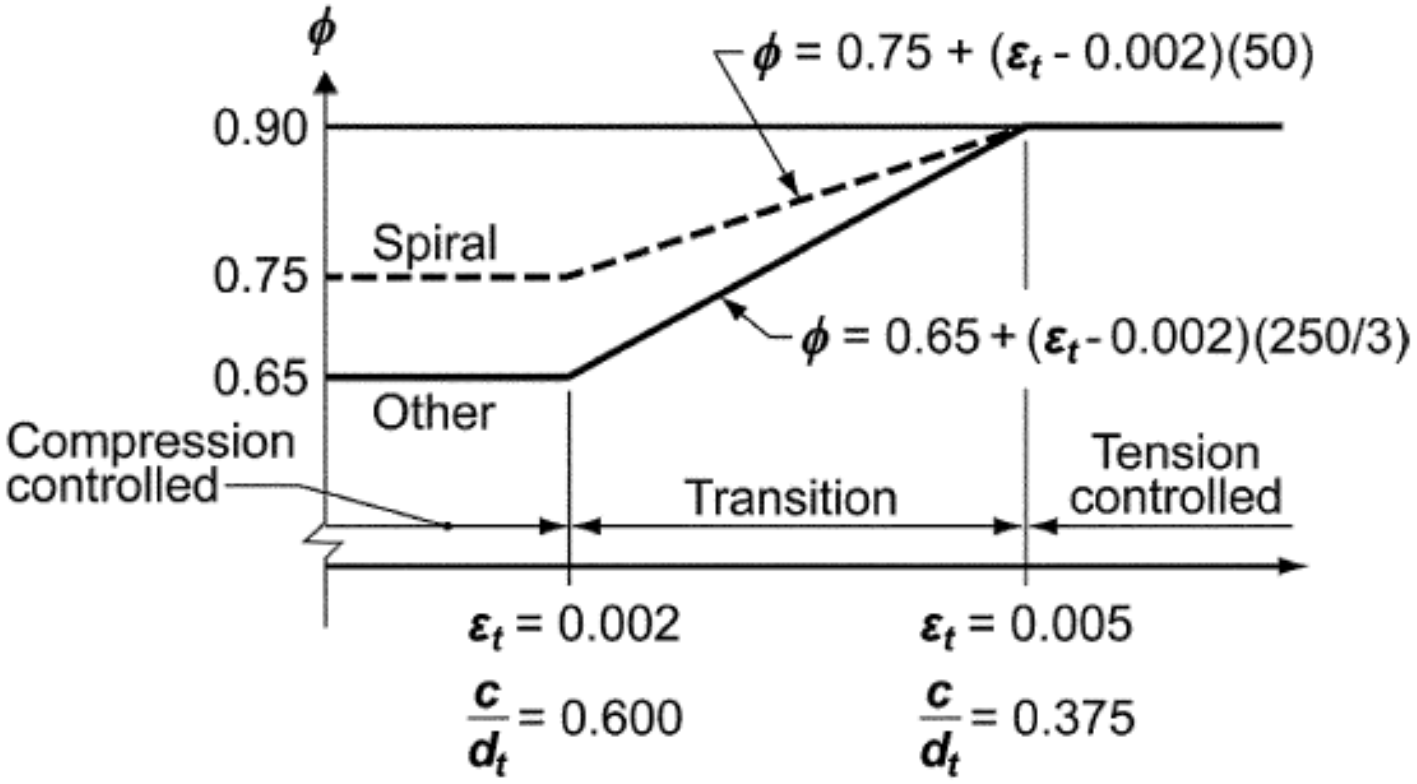
- The flexural resistance factor R depends only on the reinforcement ratio and the strength of the materials and is easily tabulated (Table A.5a and A.5b of Appendix A).

	= 40,000 psi				= 60,000 psi			
	, psi				, psi			
	3000	4000	5000	6000	3000	4000	5000	6000
0.0005	20	20	20	20	30	30	30	30
0.0010	40	40	40	40	59	59	60	60
0.0015	59	59	60	60	88	89	89	89
0.0020	79	79	79	79	117	118	118	119
0.0025	98	99	99	99	146	147	147	148
0.0030	117	118	118	119	174	175	176	177
0.0035	136	137	138	138	201	204	205	206
0.0040	155	156	157	157	229	232	233	234
0.0045	174	175	176	177	256	259	261	263
0.0050	192	194	195	196	282	287	289	291
0.0055	211	213	214	215	309	314	317	319
0.0060	229	232	233	234	335	341	345	347
0.0065	247	250	252	253	360	368	372	375
0.0070	265	268	271	272	385	394	399	403
0.0075	282	287	289	291	410	420	426	430

ACI CODE PROVISIONS

- The ACI Code encourages the use of lower reinforcement ratios.
- A tension-controlled member as one with a net tensile strain greater than or equal to 0.005.
- The corresponding strength reduction factor is $\phi = 0.9$.
- A compression-controlled member as having a net tensile strain of less than 0.002 and the corresponding reduction factor is 0.65 (0.75 for spirally reinforced members).
- The strength reduction factor varies linearly.

ACI CODE PROVISIONS



Interpolation on c/d_t: Spiral $\phi = 0.75 + 0.15[(1/c/d_t) - (5/3)]$
Other $\phi = 0.65 + 0.25[(1/c/d_t) - (5/3)]$

Fig. R9.3.2—Variation of ϕ with net tensile strain in extreme tension steel, ε_t, and c/d_t for Grade 60 reinforcement and for prestressing steel.

ACI CODE PROVISIONS

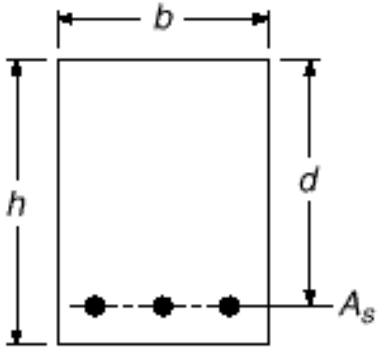
- The design strength:

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right)$$

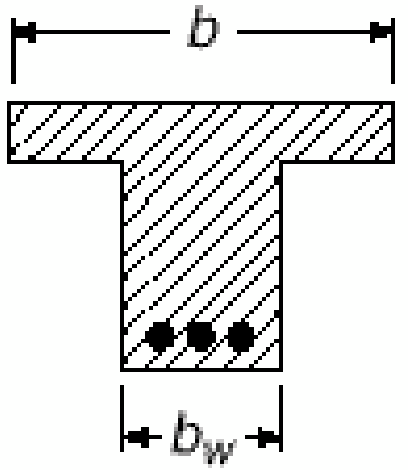
$$\phi M_n = \phi \rho f_y \left(1 - 0.59 \frac{\rho f_y}{f_c'} \right) bd^2$$

$$\phi M_n = \phi R b d^2$$

MINIMUM REINFORCEMENT RATIO



- For a rectangular section: $A_{s,\min} = \dfrac{1.6\sqrt{f'_c}}{f_y} bd$
- For T-beam (compression in flange): $A_{s,\min} = \dfrac{2.7\sqrt{f'_c}}{f_y} b_w d$
- For T-beam (tension in flange): $A_{s,\min} = \dfrac{6.2\sqrt{f'_c}}{f_y} b_w d$



- According to ACI Code (for any section):

$$A_{s,\min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \geq \frac{200 b_w d}{f_y}$$

- According to ACI Code 10.5, the requirement of above equation need not be imposed if, at every section, the area of tensile reinforcement provided is at least one-third greater than the required by analysis. Specially applicable for grade beams, where the usual equations may require excessive amounts of steel.

RECTANGULAR BEAM ANALYSIS AND DESIGN

Flexural beam problem can be broadly classified as:

- Analysis problem
  - In this problem, the section dimensions, reinforcement, and material strengths are known.
  - The moment capacity is required.
- Design problem
  - In this case, the required moment capacity is given, as are the material strengths.
  - It is required to find the section dimensions and reinforcement.

EXAMPLE 2.4

- A rectangular beam has width 12 in. and effective depth 17.5 in. It is reinforced with four No. 9 (No. 29) bars in one row. If f_y = 60,000 psi and f_c' = 4000 psi, what is the nominal flexural strength, and what is the maximum moment that can be utilized in design, according to the ACI Code?
- **Solution:**
- Area of four No. 9 bars, A_s = 4 x 1.00 = 4.00 in^2.
- Now, the actual reinforcement ratio,

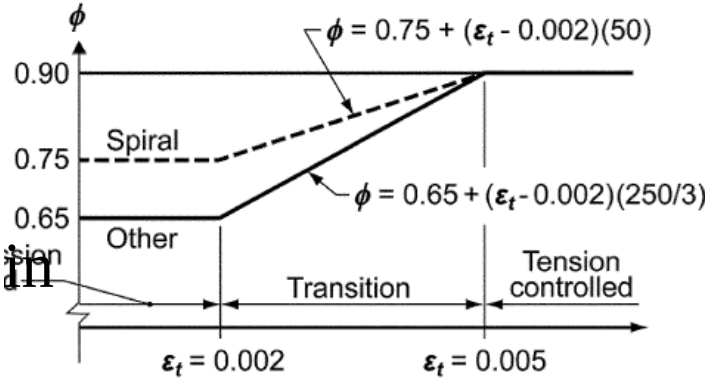
$$\rho = \frac{A_s}{bd} = \frac{4.00}{12 \times 17.5} = 0.019$$

- Maximum reinforcement ratio,

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.004} = 0.85 \times 0.85 \frac{4000}{60000} \frac{0.003}{0.003 + 0.004} = 0.0206$$

- Since, $\rho < \rho_{max}$, this is an under-reinforced beam.

EXAMPLE 2.4 (SOLUTION)



- $a = \dfrac{A_s f_y}{0.85 f_c' b} = \dfrac{4.00 \times 60000}{0.85 \times 4000 \times 12} = 5.88$ in
- Nominal moment,

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) = 4.00 \times 60,000 \left(17.5 - \frac{5.88}{2} \right) = 3490,000 \text{ in-lb} = 3490 \text{ in-kip}$$

- Depth of N.A.: $c = a/\beta_1 = 5.88/0.85 = 6.92$
- Net tensile strain: $\varepsilon_t = \varepsilon_u \dfrac{d-c}{c} = 0.003 \times \dfrac{17.5 - 6.92}{6.92} = 0.00459$
- The strength reduction factor: $\phi = 0.65 + (0.00459 - 0.002)(250/3) = 0.87$
- Now, design moment,

$$\phi M_n = 0.87 \times 3,490 = 3,036 \text{ in-kip}$$

- Check: $\rho_{max} = 0.0206 > \rho$

$$\rho_{\min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y} = \frac{3\sqrt{4,000}}{60,000} \geq \frac{200}{60,000} = 0.0033 \quad < \rho$$

EXAMPLE 2.5 (DESIGN PROBLEM)

- Find the cross section of concrete and area of steel required for a simply supported rectangular beam with a span of 15ft that is to carry a computed dead load of 1.27 kips/ft and service live load of 2.15 kips/ft. Material strengths are f_y = 60,000 psi and f_c' = 4000 psi.
- **Solution:**
- Factored load, $w_u = 1.2 \text{ x } 1.27 + 1.6 \text{ x } 2.15 = 4.96$ kips/ft.
- Factored moment,

$$M_u = \frac{1}{8} w_u l^2 = \frac{1}{8} \times 4.96 \times 15^2 = 139.5 \text{ ft-kips} = 1674 \text{ in-kips}$$

- Maximum reinforcement ratio,

$$\rho_{\max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.004} = 0.85 \times 0.85 \frac{4000}{60000} \frac{0.003}{0.003 + 0.004} = 0.0206$$

$$\rho_{\min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y} = 0.0033$$

EXAMPLE 2.5 (DESIGN PROBLEM)

- Consider reinforcement ratio (for ε_t = 0.005),

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.005} = 0.85 \times 0.85 \frac{4000}{60000} \frac{0.003}{0.003 + 0.005} = 0.0181$$

- Setting the required flexural strength equal to the design strength,

$$M_u = \phi M_n \qquad \phi M_n = \phi \rho f_y \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right) bd^2$$

$$1674 = 0.9 \times 0.0181 \times 60 \left(1 - 0.59 \frac{0.0181 \times 60}{4}\right) bd^2$$

$$bd^2 = 2040 \text{ in}^3$$

- For a beam with, b = 10 in
- d = 14.3 in
- Assuming 2.5 in concrete cover from the centroid of the bars, the required total depth is h = 14.3 + 2.5 = 16.8 in.

EXAMPLE 2.5 (DESIGN PROBLEM)

- The required steel area,

$$A_s = \rho bd = 0.0181 \times 10 \times 14.3 = 2.59 \text{ in}^2$$

- Two No. 10 bars provide A_s = 2.54 in^2 and is very close to the required area.
- However, the concrete dimensions b and h are always rounded upward to the nearest inch, and often to the nearest multiple of 2 inch.
- Hence, for the present problem, b = 10 in and h = 18 in.
- So, $d = h - 2.5 = 18 - 2.5 = 15.5$ in.
- With the increment of the concrete proportion, steel requirement will be less, and calculated based on the actual effective depth (d = 15.5 in).

EXAMPLE 2.5 (DESIGN PROBLEM)

- Setting the required flexural strength equal to the design strength,

$$M_u = \phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right)$$

- Assume, a = 5 in.

$$1674 = 0.9 \times A_s \times 60 \left(15.5 - \frac{5}{2} \right)$$

$$A_s = 2.38 \text{ in}^2$$

- Checking the assumed a:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2.38 \times 60}{0.85 \times 4 \times 10} = 4.2 \text{ in}$$

- Which is close enough to the assumed value that no further trial is need.
- Now, three No. 8 bars with (A_s = 2.37 in^2) can be used.

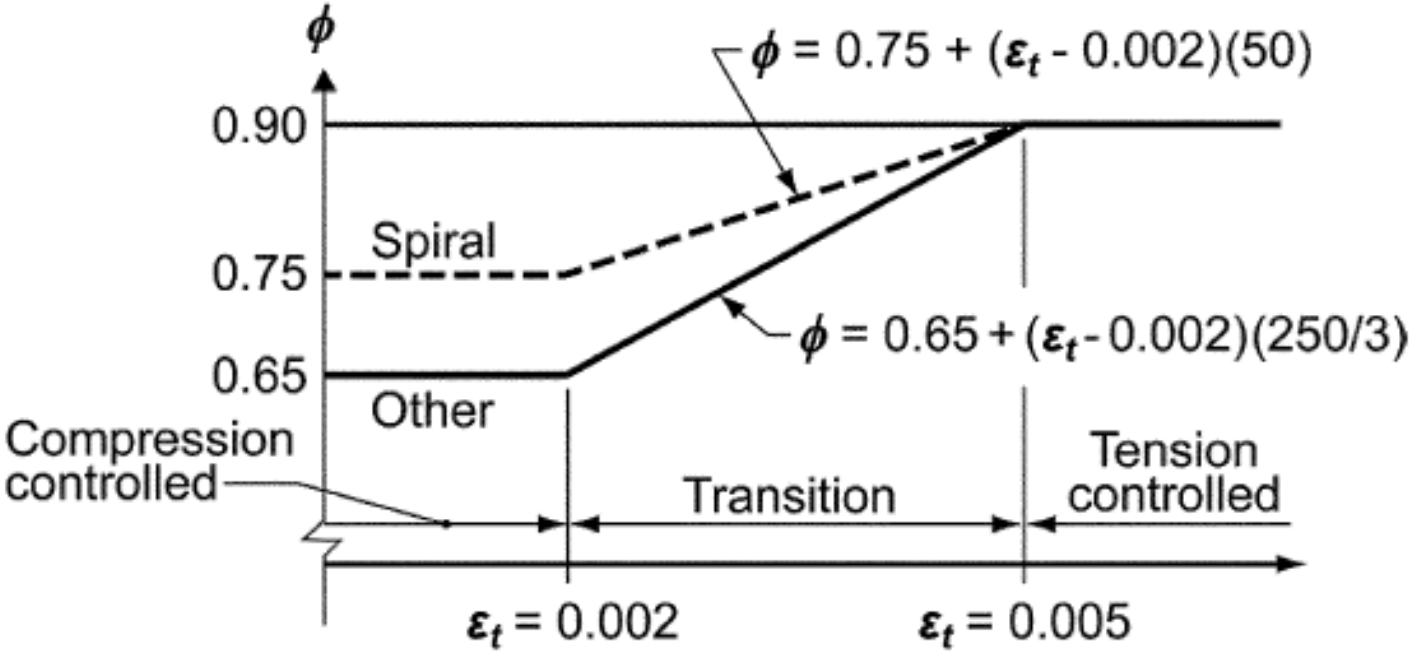
VARIATION OF STRENGTH REDUCTION FACTOR

- The ACI Code encourage the use of lower reinforcement ratios by allowing higher strength reduction factors in beams.
- The Code defines, for $\varepsilon_t = 0.005$, $\Phi = 0.9$.

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t}$$

For spiral: $\phi = 0.75 + (\varepsilon_t - 0.002)(50)$

For other: $\phi = 0.65 + (\varepsilon_t - 0.002)(250/3)$



DESIGN AIDS

- In practice, the design of beams and other reinforced concrete members is greatly facilitated by the use of aids.
- For design purposes, where concrete dimensions and reinforcement are to be found and the factored load moment M_u is to be resisted, there are two possible approaches:
  - One starts with selecting the optimum reinforcement ratio, and then calculating concrete dimensions.
  - The alternative approach starts with selecting concrete dimensions, after which the required reinforcement is found.

DESIGN AIDS (FIRST APPROACH)

1. Set the required strength M_u equal to the design strength ϕM_n.

$$M_u = \phi R b d^2 \qquad R = \rho f_y \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$$

2. With the aid for Table A.4, select an appropriate reinforcement ratio between ρ_{max} and ρ_{min}. For practical purpose use, $\rho = 0.6\rho_{max}$. Selection of $\rho \le \rho$ for $\varepsilon_t = 0.005$ assures that $\phi = 0.90$. For $\rho < \rho_{max}$ and for ρ for $\varepsilon_t = 0.005$, an iterative solution will be necessary.

$$\phi = 0.65 + (\varepsilon_t - 0.002)(250/3)$$

3. From table A.5 for a selected ρ in second step chooses value of R for a given f'_c and f_y. Now calculate, $bd^2 = \dfrac{M_u}{\phi R}$
4. Choose b and d. Unless otherwise restricted, d ~ 2 to 3 times b.
5. Calculate required steel area, $A_s = \rho b d$
6. Then from Table A.1, A.2 and A.7 find size, number of bars and spacing.

DESIGN AIDS (SECOND APPROACH)

1. Select beam width b and effective depth d. Then calculate the required R:

$$R = \frac{M_u}{\phi b d^2}$$

2. From Table A.5 for a given f'_c and f_y, find the reinforcement ratio between $\rho < \rho_{max}$ that will provide the required value of R and verify the selected value of ϕ.

$$\phi = 0.65 + (\varepsilon_t - 0.002)(250/3)$$

$$A_s = \rho b d$$

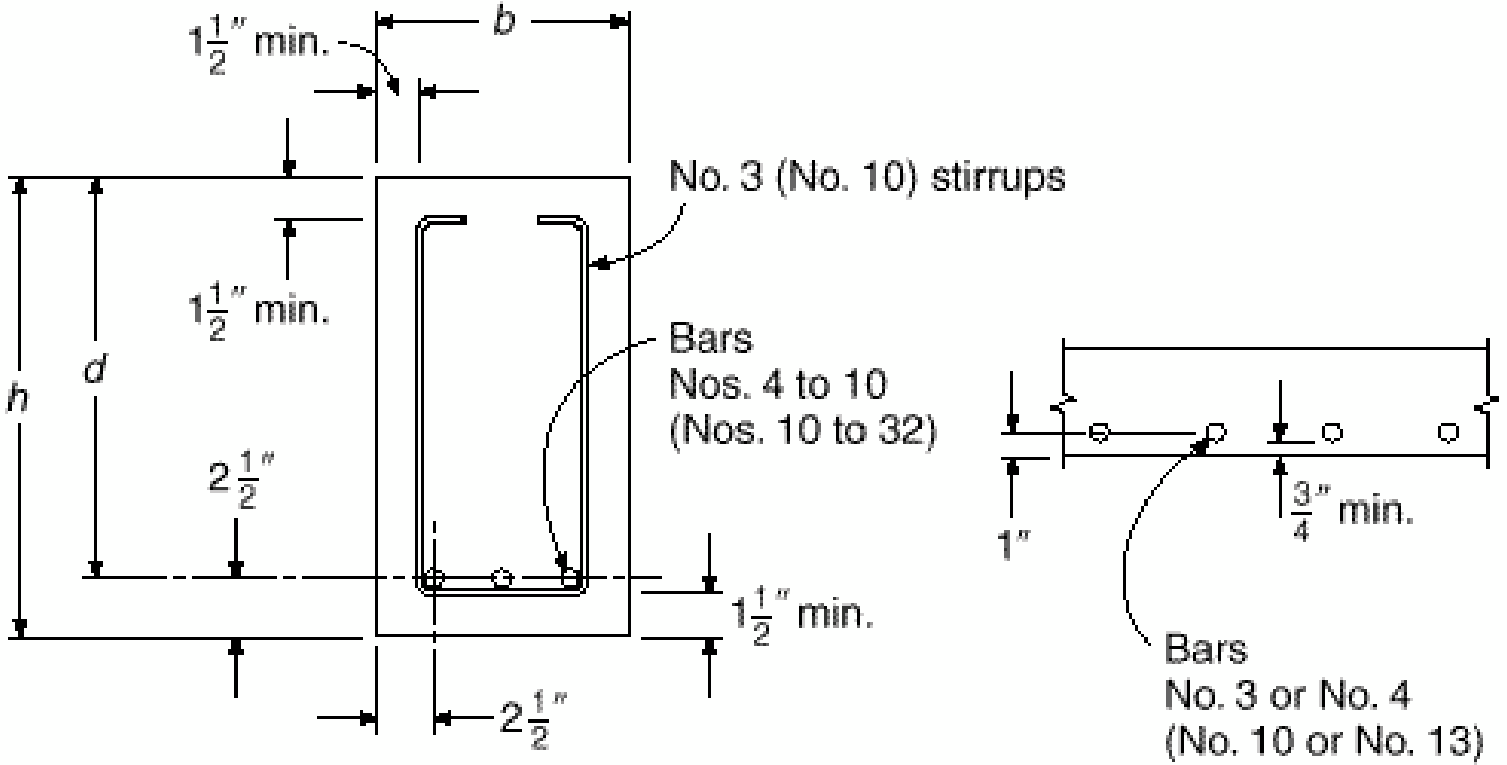
3. Calculate required steel area,
4. Then from Table A.1, A.2 and A.7 find size, number of bars and spacing.

EXAMPLE - 2A

- Using the concrete dimensions, b = 10 in, d = 17.5 in, and h = 20 in, and material strengths are f_y = 60,000 psi and f_c' = 4000 psi, find the steel area required to resist a moment Mu of 1,300 in-kips.

CONCRETE PROTECTION FOR REINFORCEMENT

- To provide the steel with adequate concrete protection against fire and corrosion, the designer must maintain a certain minimum thickness of concrete cover outside the outermost steel.



(*a*) Beam with stirrups

(*b*) Slab

CONCRETE PROTECTION FOR REINFORCEMENT

- To provide the steel with adequate concrete protection against fire and corrosion, the designer must maintain a certain minimum thickness of concrete cover outside the outermost steel.
- Cover:
  - At least ¾ inch for slabs and walls.
  - At least 1½ inch for beams and columns.
  - If the concrete is exposed to weather or in contact with ground, a protective measure of at least 2 inch is required.
  - If the concrete is in contact with ground without the use of forms, a protective measure of at least 3 inch is required.

SELECTION OF BARS AND BAR SPACING

- The common reinforcing bar sizes range from No. 3 to No. 11.
- The two larger sizes, No. 14 & No. 18 are used mainly in columns.
- It is often desirable to mix bar sizes to meet steel area requirements more closely, but they have to of comparable diameter.
- It is necessary to maintain a certain minimum distance between the adjacent bars to ensure proper placement of concrete around them.
  - ACI Code 7.6 specifies that the minimum clear distance between adjacent bars shall not be less than the bars diameter or 1 inch.
  - For two or more layers of beam reinforcement, the clear distance between layers must not be less than 1 inch, and the bars in the upper layer should be placed directly above those in the bottom layer.