Lecture # 03
Shear and Diagonal Tension in Beams

CE315: DESIGN OF CONCRETE STRUCTURES I

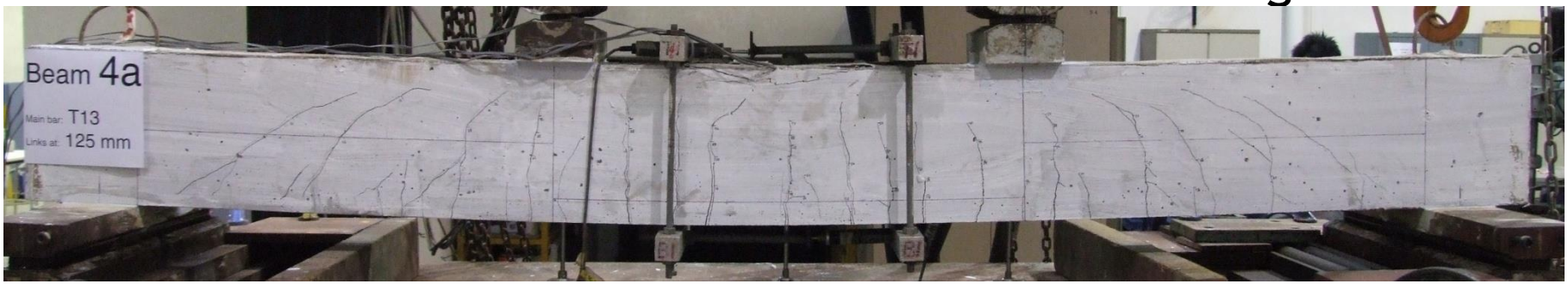
Major Md. Jahidul Islam, PhD, Engrs

e-mail: jahid817@yahoo.com



SHEAR FAILURE

- Besides flexural failure in beam, there are other types of failure that may be more dangerous than flexural failure.
- Shear failure in reinforced concrete beam is one among them. This is because shear is difficult to predict.
- Shear failure is also called diagonal tension failure.
- Reinforced concrete beams are generally provided with shear reinforcement to avoid shear failure altogether.





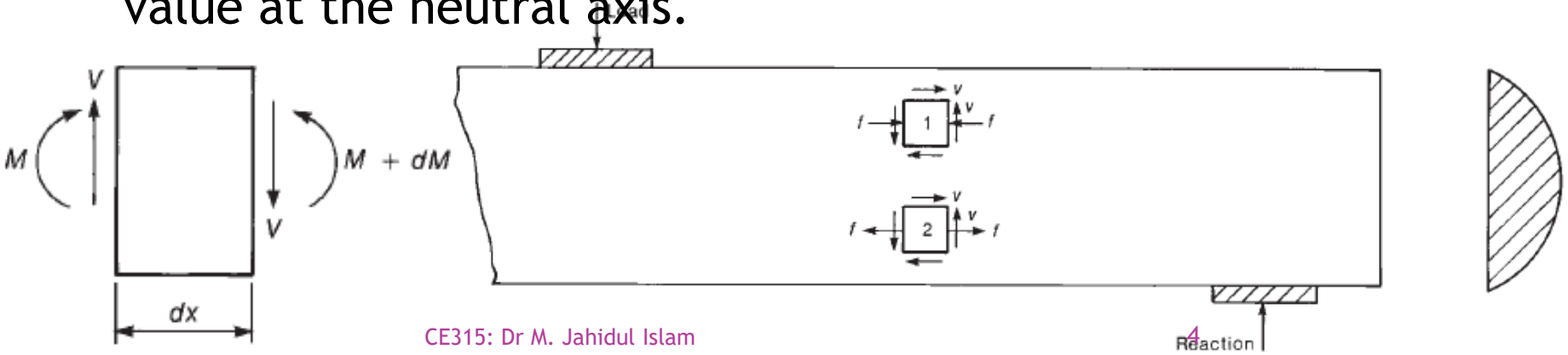
SHEAR ANALYSIS AND DESIGN

- In shear analysis and design shear is not that significant.
- The shear stresses in most beams are far below the direct shear strength of the concrete.
- The real concern is with diagonal tension stress, resulting from the combination of shear stress and longitudinal flexural stress.
- There are several methods for shear design, such as variable angle truss models and diagonal compression field theory.
- However, the modified compression field theory will be discussed in detail.



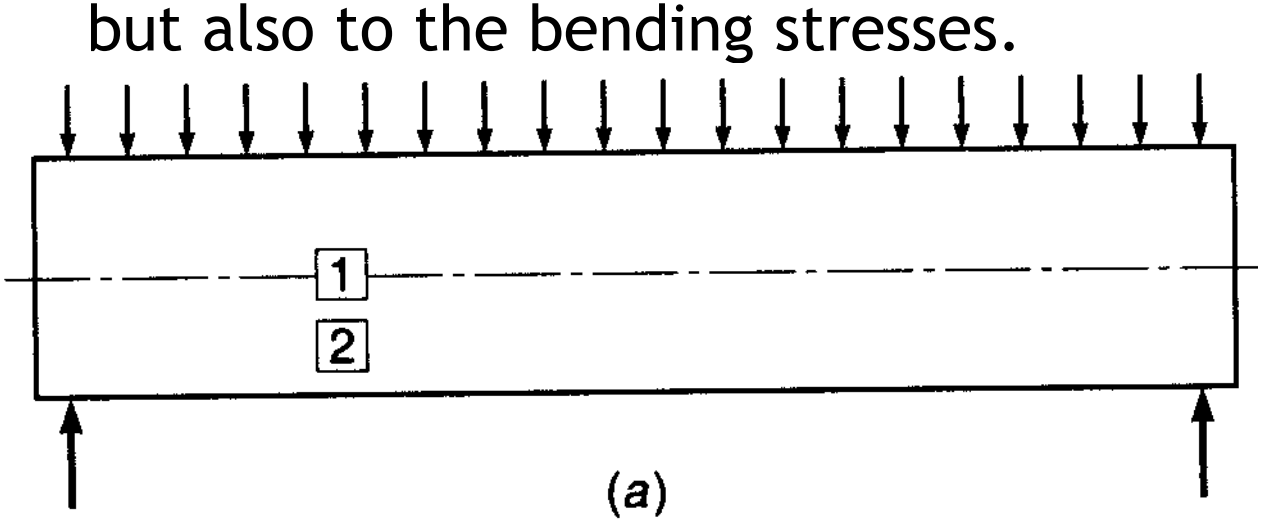
DIAGONAL TENSION IN BEAMS

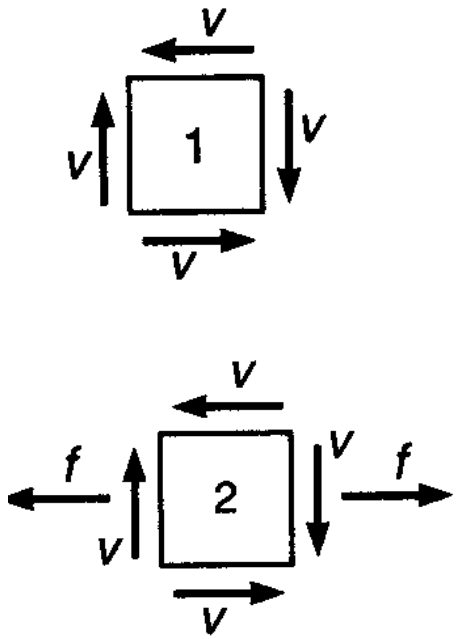
- At any cross section of a homogenous elastic beam there exist internal forces that can be resolved into components normal (bending stress) and tangential (shear stress) to the section.
- For homogenous, elastic and uncracked beams, the shear stress can be defined as, $$v = \frac{VQ}{Ib}$$
- Where, Q = first moment about the centroidal axis of the part of the cross sectional area lying farther from the centroidal axis than the point where the shear stresses are being calculated.
- The shear stress is zero at the outer fibers and has a maximum value at the neutral axis.



DIAGONAL TENSION IN BEAMS

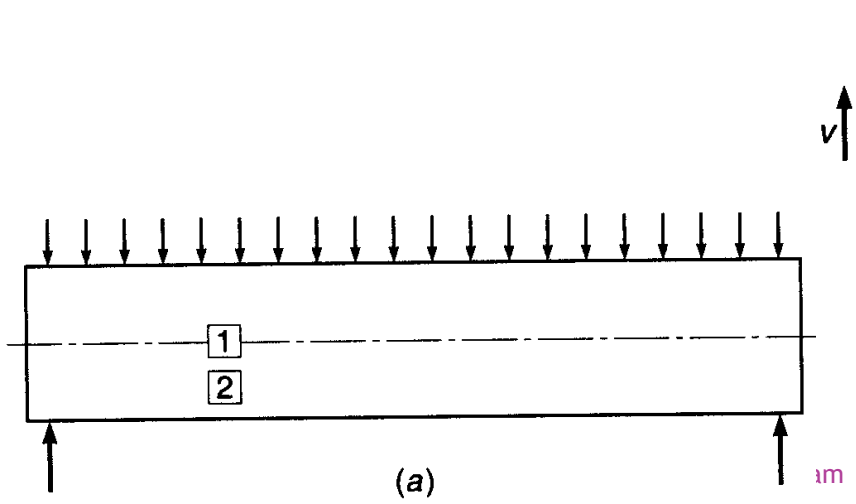
- If a small square element located at the outer fibers and axis of such a beam is isolated, the vertical stresses on it, equal and opposite on the two faces for reasons of equilibrium.
- If an element of the beam is considered that is located neither at the neutral axis nor at the outer edge, its vertical faces are subject not only to the shear stresses but also to the bending stresses.



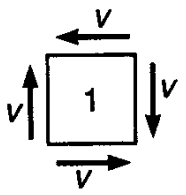


DIAGONAL TENSION IN BEAMS

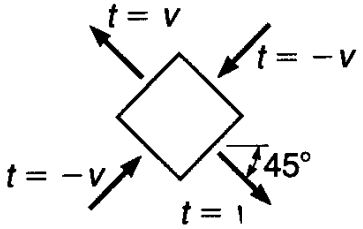
- These stresses can be combined into a pair of inclined compressive stresses and a pair of inclined tensile stresses that can act at right angles to each other. They are known as *principal stresses*.
- Since the value of the shear stresses (v) and the bending stresses (f) changes both along the beam and vertically with distance from the neutral axis, the inclinations as well as the magnitudes of the resulting principal stresses (t) also vary from point to point.



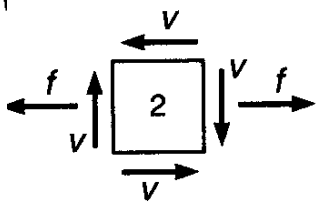
(a)



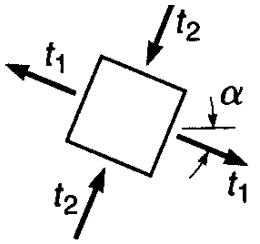
(b)



(c)



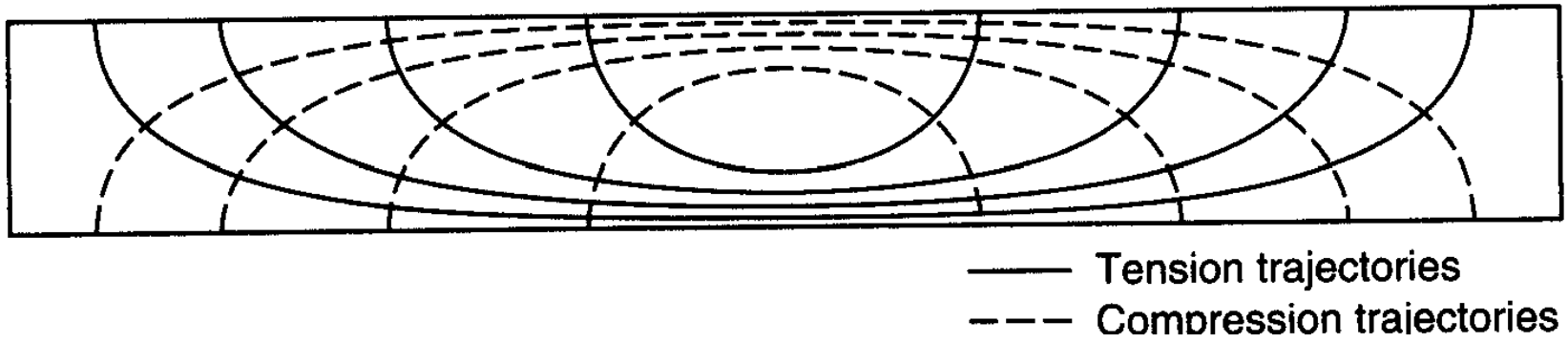
(d)



(e)

DIAGONAL TENSION IN BEAMS

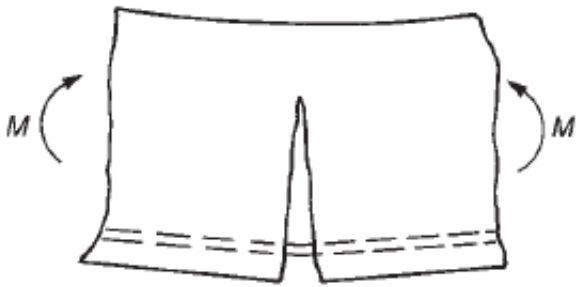
- Stress trajectories for a rectangular beam subjected to uniform load are shown in the figure below.



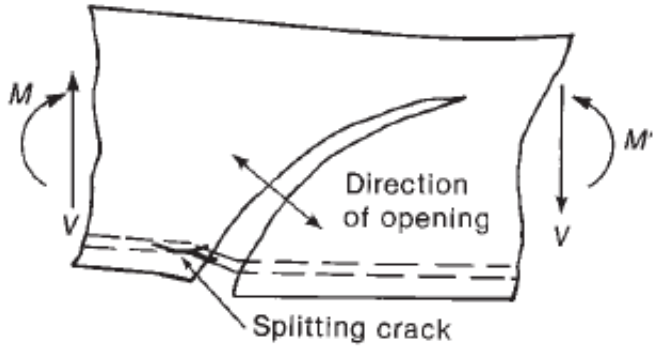
- Tensile stresses of various inclinations and magnitudes, resulting from shear alone or from the combined action of shear and bending, exist in all parts of a beam and can impair its integrity if not adequately provided for.
- Because of that diagonal tension should be carefully considered in reinforced concrete design.

REINFORCED CONCRETE BEAM WITHOUT SHEAR REINFORCEMENT

- The longitudinal tension reinforcement does not reinforce the concrete against the diagonal tension stresses, caused by shear alone or by the combined effect of flexure and shear.
- With increasing load, these stresses increases to open additional tension cracks in a direction perpendicular to the local tension stress.
- These diagonal cracks are most common in regions in which the shear forces are high.

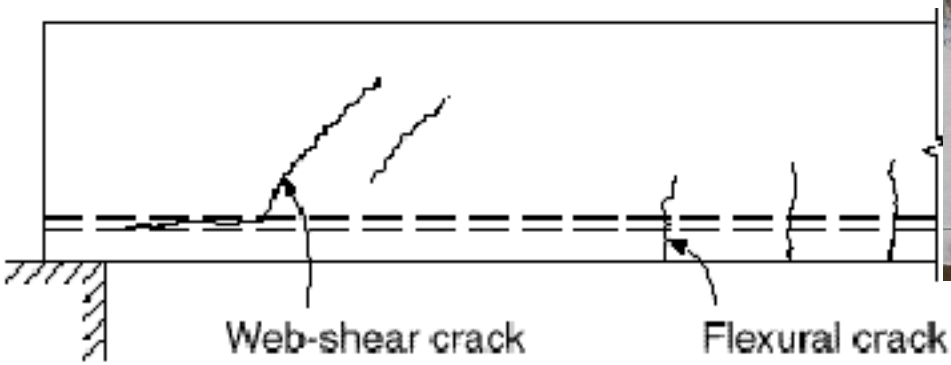


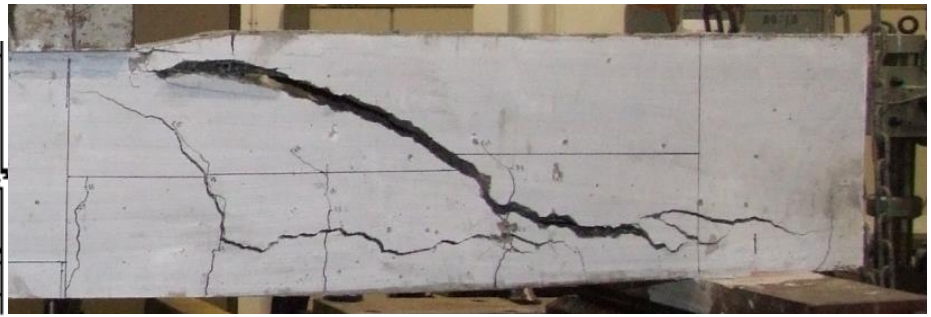
(a) Flexural crack.



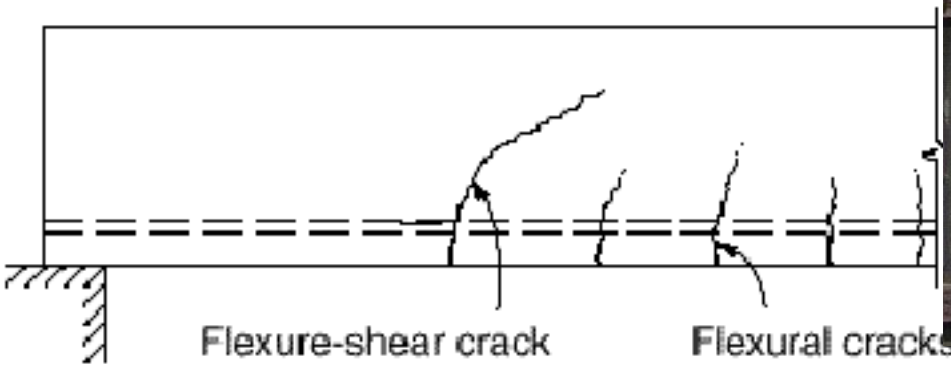
(b) Inclined crack.

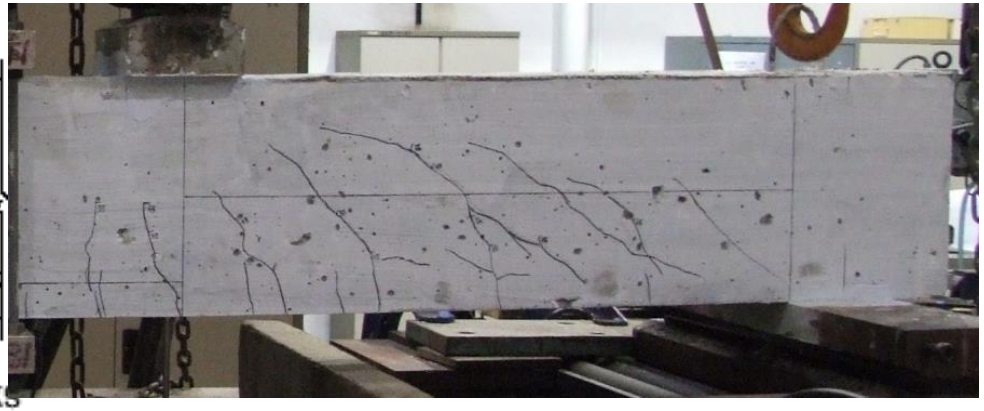
REINFORCED CONCRETE BEAM WITHOUT SHEAR REINFORCEMENT





(*a*) Web-shear cracking

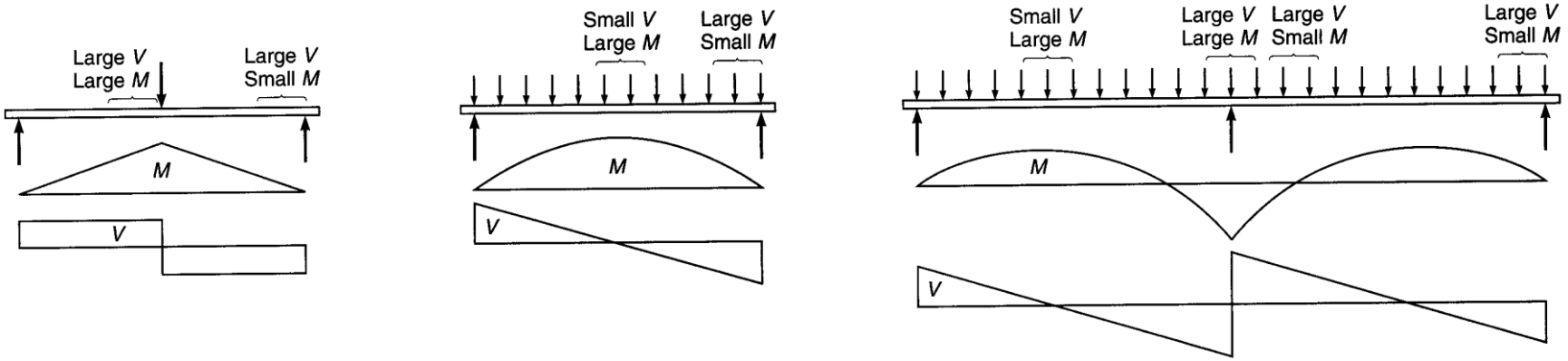




(*b*) Flexure-shear cracking

DIAGONAL TENSION CRACKING

- A given location in a beam may have a large moment combining with a small shear force, or the reverse, or large or small values for both shear and moment.



- Evidently, the relative values of M and V will affect the magnitude as well as the diagonal tension stresses.
- Web-shear cracks can formed when the diagonal tension stress in the vicinity of the neutral axis becomes equal to the tensile strength of the concrete.
- It was found that in regions with large shear and small moment, diagonal tension cracks form at, $v_{cr} = \frac{V_{cr}}{bd} = 3.5\sqrt{f_c'}$

DIAGONAL TENSION CRACKING

- In the presence of large moments (for which adequate longitudinal reinforcement has been provided) the nominal shear stress at which diagonal tension cracks form (also known as flexure-shear cracks) and propagate is,

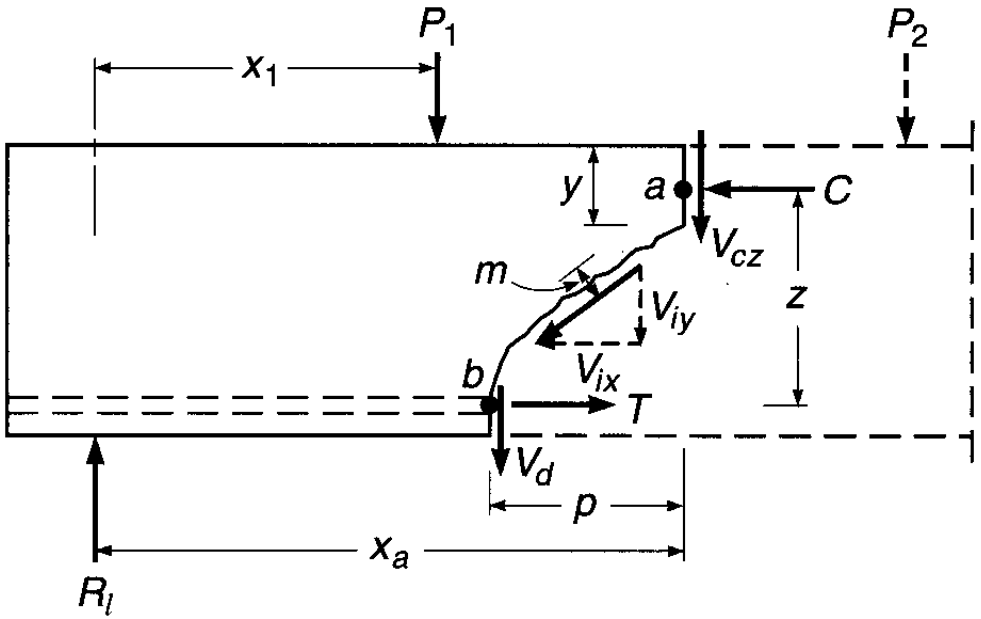
$$v_{cr} = \frac{V_{cr}}{bd} = 1.9\sqrt{f_c'}$$

- Flexure-shear cracks are more common than web-shear cracks.
- From evaluations of large numbers of tests, it has been found that the nominal shear stress at which diagonal flexure-shear cracking develops can be predicted from,

$$v_{cr} = \frac{V_{cr}}{bd} = 1.9\sqrt{f_c'} + 2500\frac{\rho V d}{M} \le 3.5\sqrt{f_c'}$$

BEHAVIOR OF DIAGONALLY CRACKED BEAMS

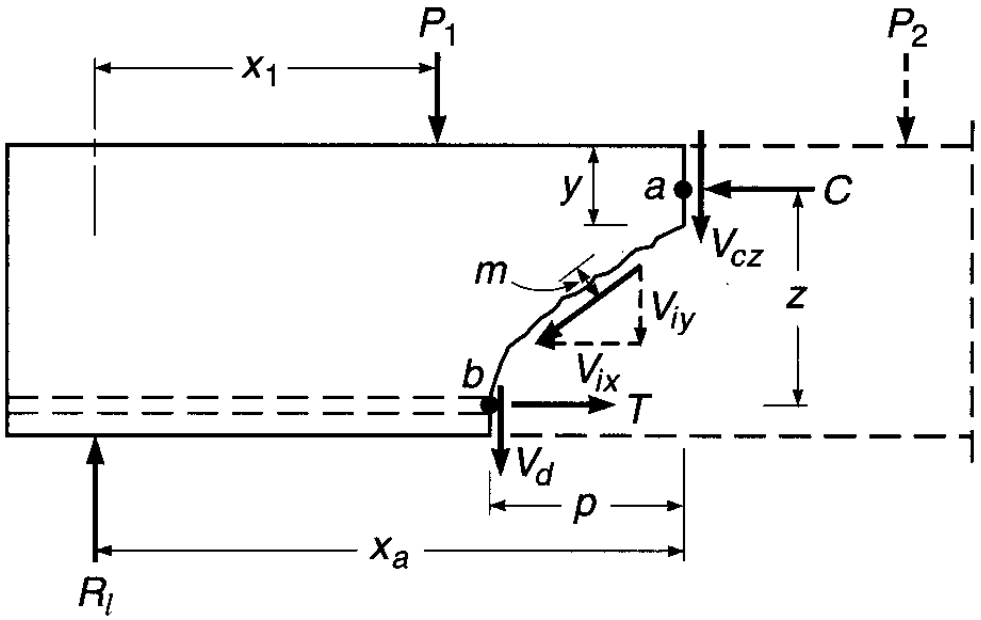
- Consider a part of a beam, arbitrarily loaded, in which a diagonal tension crack has formed.
- There is an external upward shear force, $V_{ext} = R_l - P_1$ acting on this portion.
- Once the crack is formed, no tension force is transmitted perpendicular to the crack. However, as long as the crack is narrow, it can transfer forces through interlocking of the surface roughness. Component of the interlocking force V_i are V_{ix} and V_{iy}.



BEHAVIOR OF DIAGONALLY CRACKED BEAMS

- Other internal forces are those in the uncracked concrete, V_{cz}, and across the longitudinal steel, acting as a dowel, V_d.
- Hence, the internal shear force is,

$$V_{\text{int}} = V_{cz} + V_d + V_{iy}$$

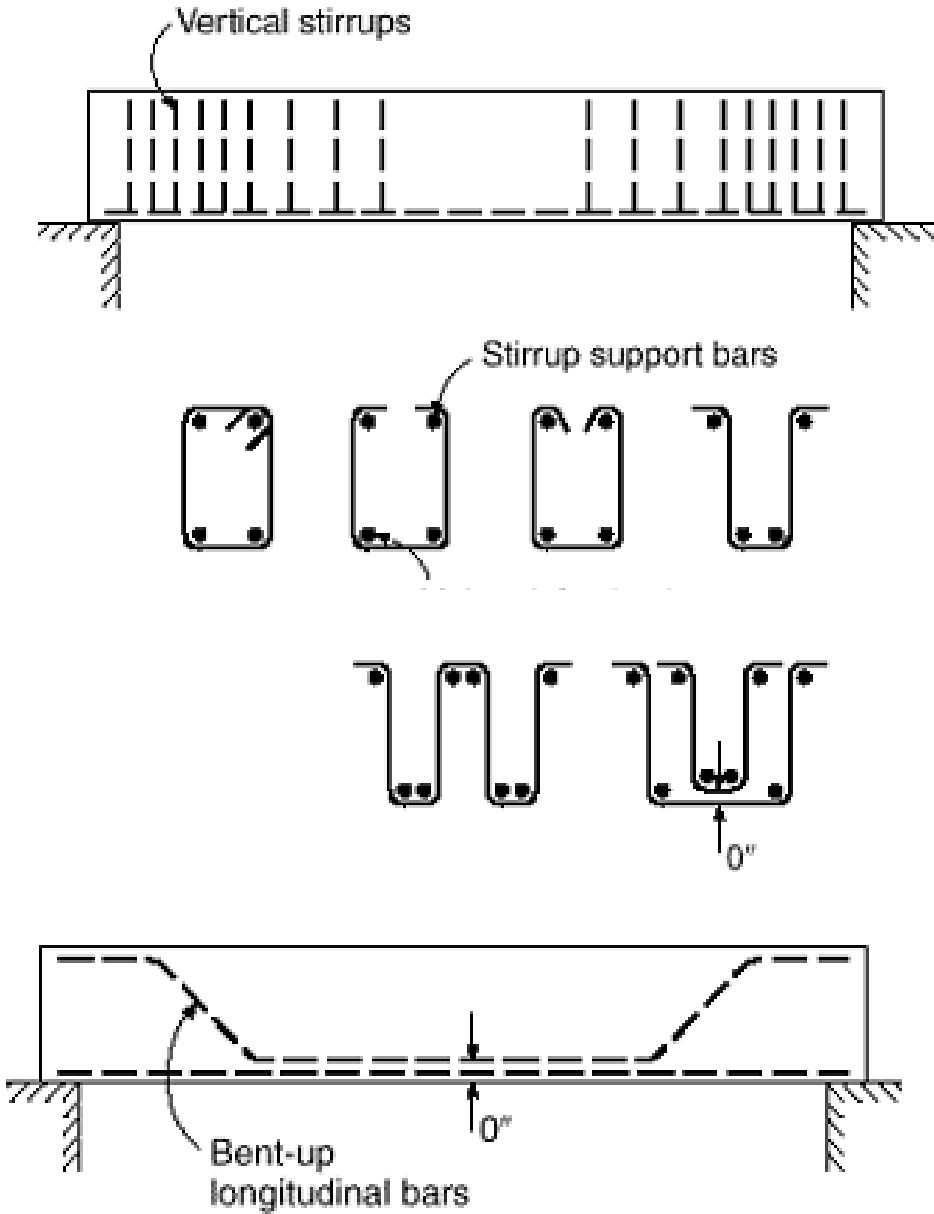


REINFORCED CONCRETE BEAM WITH WEB REINFORCEMENT

- For economical design, it is essential to use full moment capacity of flexural member rather than having its strength limited by premature shear failure.
- It is also desirable to avoid sudden and explosive failure associated with most shear failures.
- Therefore, if concrete does not have adequate safety margin against shear failure, special shear reinforcement, known as web reinforcement, is used to increase its strength.

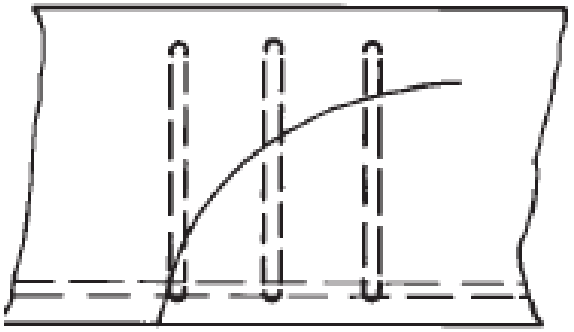
TYPES OF WEB REINFORCEMENT

- Shear reinforcement is provided in the form of vertical stirrups.
- Relatively small sized bars are used, generally No. 3 & No. 4.
- U-shaped bars are most common, although multiple-leg stirrups are sometimes necessary.
- Stirrups are formed to fit around the main longitudinal bars.
- Alternately, longitudinal bars can be bent up to use as shear reinforcement.



BEHAVIOR OF WEB REINFORCEMENT CONCRETE BEAMS

- After diagonal cracks have developed, web reinforcement enhances the shear resistance of a beam in four separate ways:



Vertical shear reinforcement.

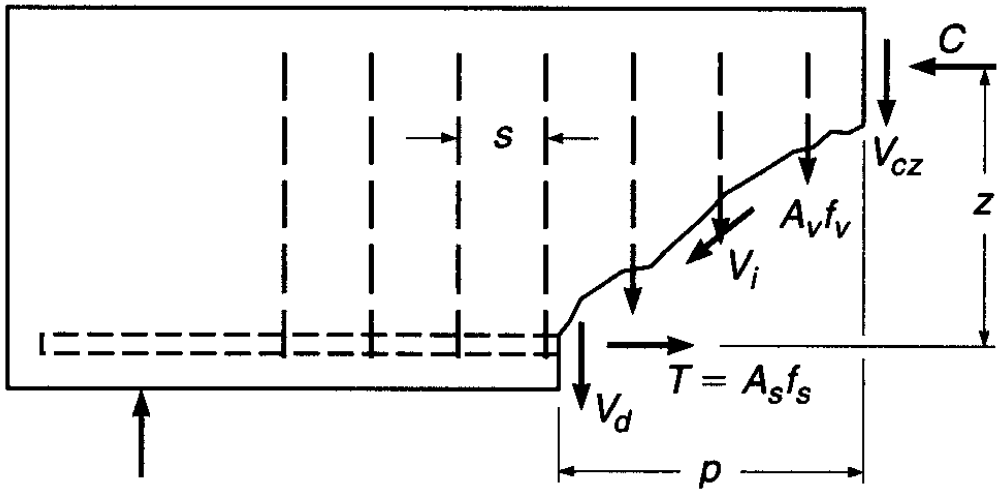
1. Part of the shear force is resisted by the bars that traverse a particular crack.
2. Because of reduced penetration of diagonals cracks more uncracked concrete available at the head of the crack for resisting the combined action of shear and compression.
3. The stirrups also counteract the widening of the cracks, so that the two crack faces stay in close contact.
4. The stirrups along with longitudinal reinforcement tie the main bulk of the concrete, and thus, provides better restraint against the splitting of concrete along the longitudinal reinforcement.

BEAMS WITH VERTICAL STIRRUPS

- Most common type of web reinforcement is vertical stirrups.
- After crack formation, the shear force is resisted by a combination of dowel shear (V_d), interface shear (V_i), shear force by of the remaining uncracked concrete (V_{cz}), and a force $A_v f_v$ by each stirrup traversing the crack.
- Here A_v is the cross-sectional area of the stirrup and f_v is the tensile stress in the stirrup.
- From equilibrium,

$$V_{ext} = V_{cz} + V_d + V_{iy} + V_s = V_c + V_s$$

- where, $V_s = nA_v f_v$ and is the vertical force in the stirrups, n being the number of stirrups traversing the crack and V_c is the contribution of the concrete.



BEAMS WITH VERTICAL STIRRUPS

- The length p, is the horizontal projection of the diagonal crack, is conservatively assumed to be equal to the effective depth of the beam.
- If s is the stirrup spacing, then $n = p/s \approx d/s$, implying a crack somewhat flatter than 45°. Then at failure, when $V_{ext} = V_n$, the nominal shear strength is expressed as,

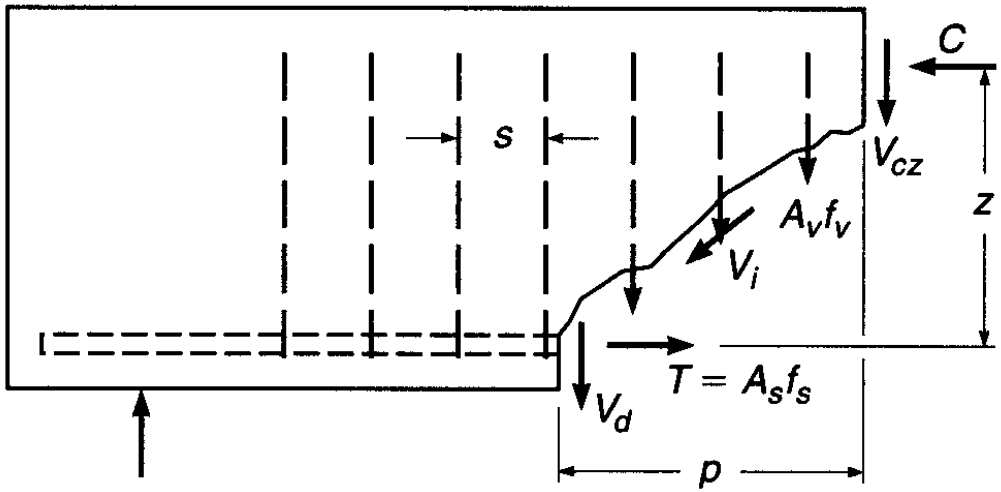
$$V_n = V_c + \frac{A_v f_{yt} d}{s}$$

- where, V_c is taken equal to cracking shear V_{cr} given by,

$$V_c = \left(1.9\sqrt{f_c'} + 2500\frac{\rho V d}{M}\right) bd \le 3.5\sqrt{f_c'} bd$$

- The nominal shear stress (by dividing both side with bd),

$$v_n = \frac{V_n}{bd} = v_c + \frac{A_v f_{yt}}{bs}$$

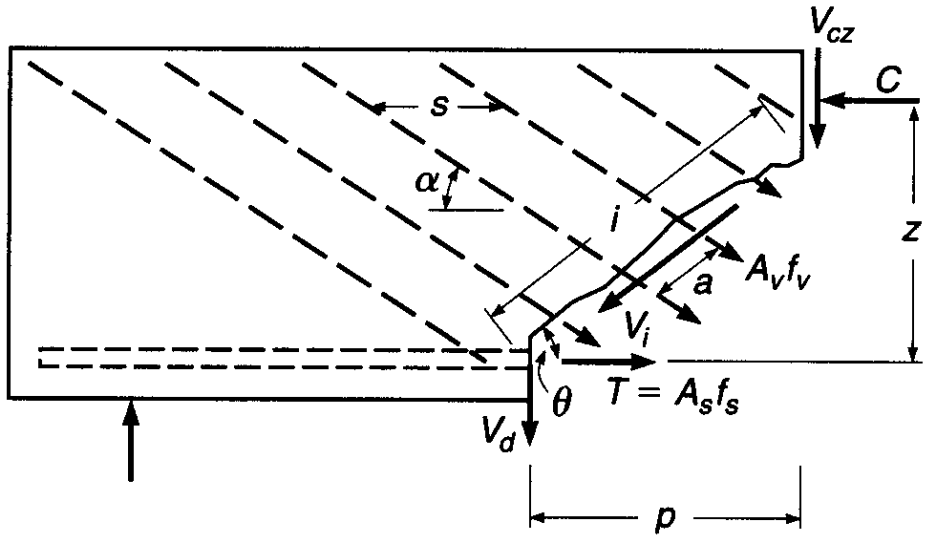


BEAMS WITH INCLINED BARS

- The inclination of bars is α and that of the crack θ.
- The nominal shear strength when failure is caused by shear is,

$$V_n = V_c + \frac{A_v f_{yt} d \left(\sin\alpha + \cos\alpha\right)}{s}$$

- It should be noted that equations for nominal shear strength will only apply if web reinforcement is so spaced that any conceivable diagonal crack is traversed by at least one stirrup or inclined bar.
- Otherwise web reinforcement would be ineffective, because diagonal cracks may form between widely spaced web reinforcement and fail the beam.
- This imposes upper limits on the permissible spacing s to ensure that the web reinforcement is actually effective as calculated.



ACI CODE PROVISION FOR SHEAR DESIGN

- According to the ACI Code, the design of beams for shear is based on the following relation:

$$V_u \le \phi V_n$$

  - where, V_u = total shear force applied in a given section
  - $V_n = V_c + V_s$ = nominal shear strength.
- For vertical stirrups:

$$V_u \le \phi V_c + \frac{\phi A_v f_{yt} d}{s}$$

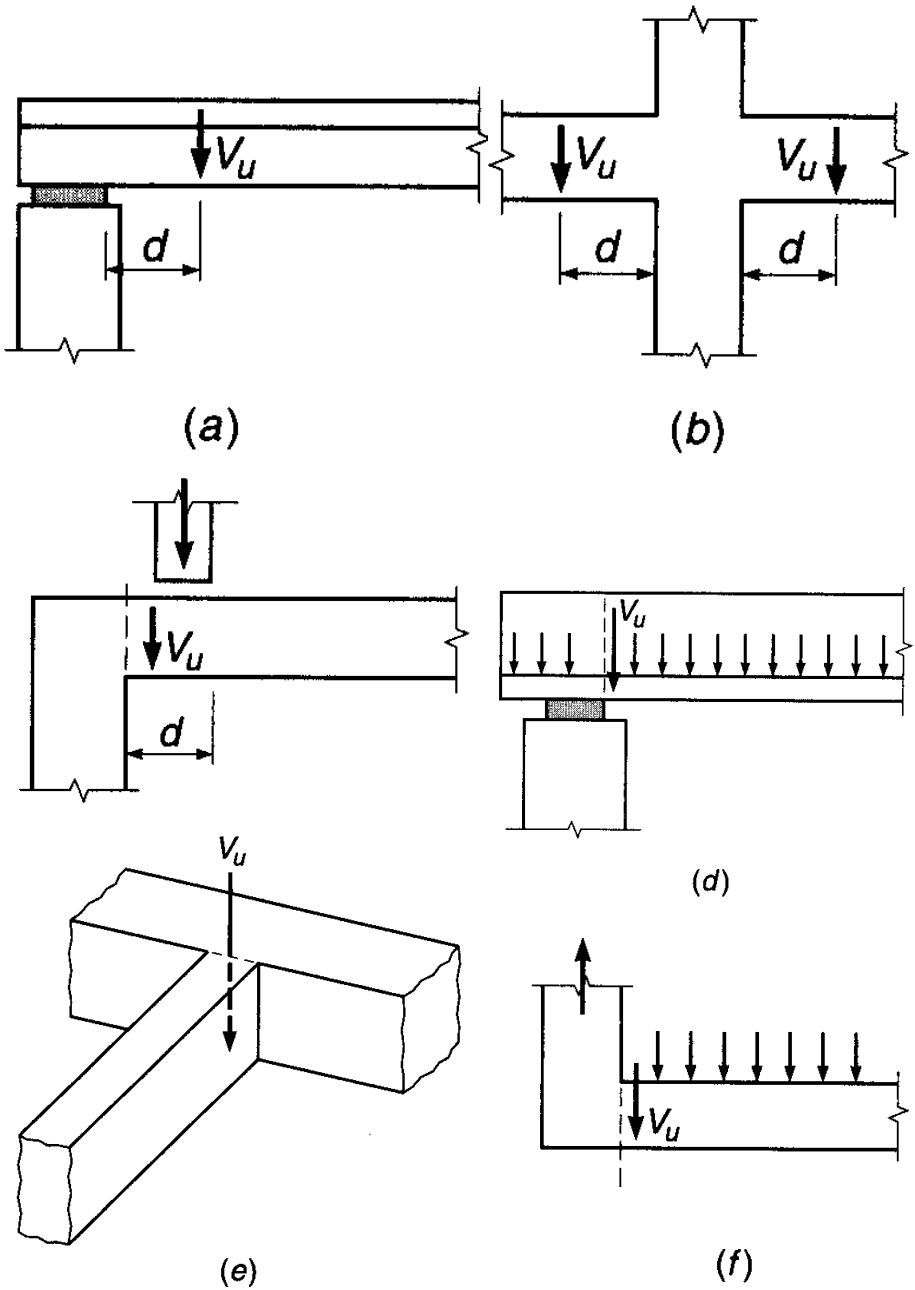
- For inclined bars:

$$V_u \le \phi V_c + \frac{\phi A_v f_{yt} d \left(\sin\alpha + \cos\alpha\right)}{s}$$

- The strength reduction factor, ϕ, is taken as 0.75.

LOCATIONS OF CRITICAL SECTION FOR SHEAR DESIGN

- For typical support conditions, sections located less than a distance d from the face of the support may be designed for the same shear V_u as that computed at a distance d.
- However, the critical design section should be taken at the face of the support
  - If concentrated loads act within that distance,
  - If the beam is loaded near its bottom edge (as may occur for an inverted T beam),
  - If the reaction causes vertical tension rather than compression (e.g., if the beam supported by a girder of similar depth) or at the end of a monolithic vertical element.



SHEAR STRENGTH PROVIDED BY CONCRETE

- For members subjected to shear and flexure, according to ACI Code 11.3.2, the concrete contribution to shear strength is,

$$V_c = \left(1.9\lambda\sqrt{f_c'} + 2500\frac{\rho_w V_u d}{M_u} \right) b_w d \le 3.5\lambda\sqrt{f_c'} b_w d$$

- To account for T beams, the rectangular beam width b is replaced by b_w. For normal weight concrete, λ = 1.0. For light weight concrete, λ = 0.75 to 1.0 for various types.
- However, the above mention equation is bit tedious for manual calculations, and thus, an alternative equation is proposed by ACI Code 11.3.1:

$$V_c = 2\lambda\sqrt{f_c'} b_w d$$

- Because of its simplicity, it is often used in practice.

ACI CODE PROVISION FOR SHEAR DESIGN

- If the factored shear force (V_u) is less than ϕV_c, then theoretically no web reinforcement is required.
- However, ACI Code 11.5.5 requires provision of at least a minimum area of web reinforcement equal to,

$$A_v = 0.75\sqrt{f_c'}\frac{b_w s}{f_y} \geq 50\frac{b_w s}{f_y}$$

- Where, s = longitudinal spacing of web reinforcement, in.
- A_v = total cross-sectional area of web steel within distance s, in^2.
- This provision necessary unless V_u is one-half or less of the design shear strength ϕV_c, provided by the concrete.
- Specific exceptions to this are made for beam with depth not greater than 10 in., 2.5 times the thickness of the flange, or one half the web width (whichever is greatest).

REGION IN WHICH SHEAR REINFORCEMENT IS REQUIRED

- If $V_u > \phi V_c$ in any portion of a beam, it is essential to provide shear reinforcement for the excess.
- A minimum shear reinforcement must be provided for factored shear force greater than $½\phi V_c$.
- The requirement of shear reinforcement can be found by superimposing the plot of the shear strength of the concrete over the shear diagram for the beam span.

DESIGN OF WEB REINFORCEMENT

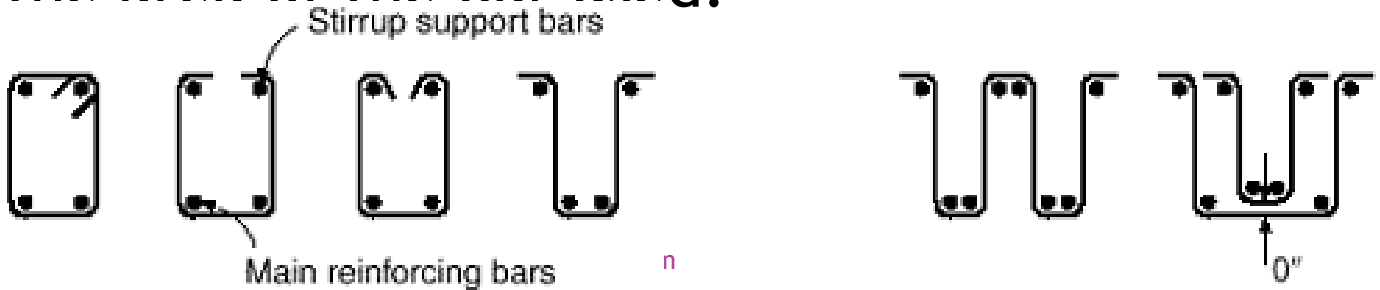
- In design, first select a trail web-steel area A_v based on stirrup sizes (No. 3 or 4) for which the required spacing s can be calculated.
- The required spacing of shear reinforcement for vertical stirrups:

$$s = \frac{\phi A_v f_y d}{V_u - \phi V_c}$$

- For bent bars:

$$s = \frac{\phi A_v f_y d\left(\sin\alpha + \cos\alpha\right)}{V_u - \phi V_c}$$

- For U stirrups, the web area A_v provided by each stirrup is *twice* the cross-sectional area of the bar. For stirrups with multiple legs, like shown in the figure below, A_v is 4 times the area of the bar used.



DESIGN OF WEB REINFORCEMENT

- It is undesirable to space vertical stirrups closer than about 4 in.; the size of the stirrups should be chosen to avoid a closer spacing.
- When vertical stirrups are required over a comparatively short distance, it is good practice to space them uniformly over the entire distance, the spacing being calculated for the point of greatest shear.
- If the web reinforcement is required over a long distance, it is more economical to compute the spacings required at several sections and to place the stirrups accordingly, in groups of varying spacing.
- For the usual case of vertical stirrups, with $V_s \le 4\sqrt{f_c'}b_w d$, the maximum spacing of stirrups (smaller of the three):

$$s_{\max} = \frac{A_v f_{yt}}{0.75\sqrt{f_c'}b_w} \le \frac{A_v f_{yt}}{50 b_w}$$

$$s_{\max} = \frac{d}{2}$$

$$s_{\max} = 24 \text{ in.}$$

EXAMPLE 3.1

- A simply supported rectangular beam of 16 in. wide with an effective depth of 22 in.
- Carries a total factored load of 9.4 kips/ft on a 20 ft. clear span.
- It is reinforced with 7.62 in^2 of tensile steel, which continues uninterrupted into the supports.
- If f'_c = 4,000 psi, throughout what part of the beam is web reinforcement required?

EXAMPLE 3.1 (SOLUTION)

- Maximum external shear force occurs at the ends of the span, where,

$$V_u = 9.4 \times (20/2) = 94 \text{ kips}$$

- At the critical section for shear, a distance d (=22 in. = 1.83 ft.) from the support,

$$V_u = 9.4 \times (20/2 - 1.83) = 76.8 \text{ kips}$$

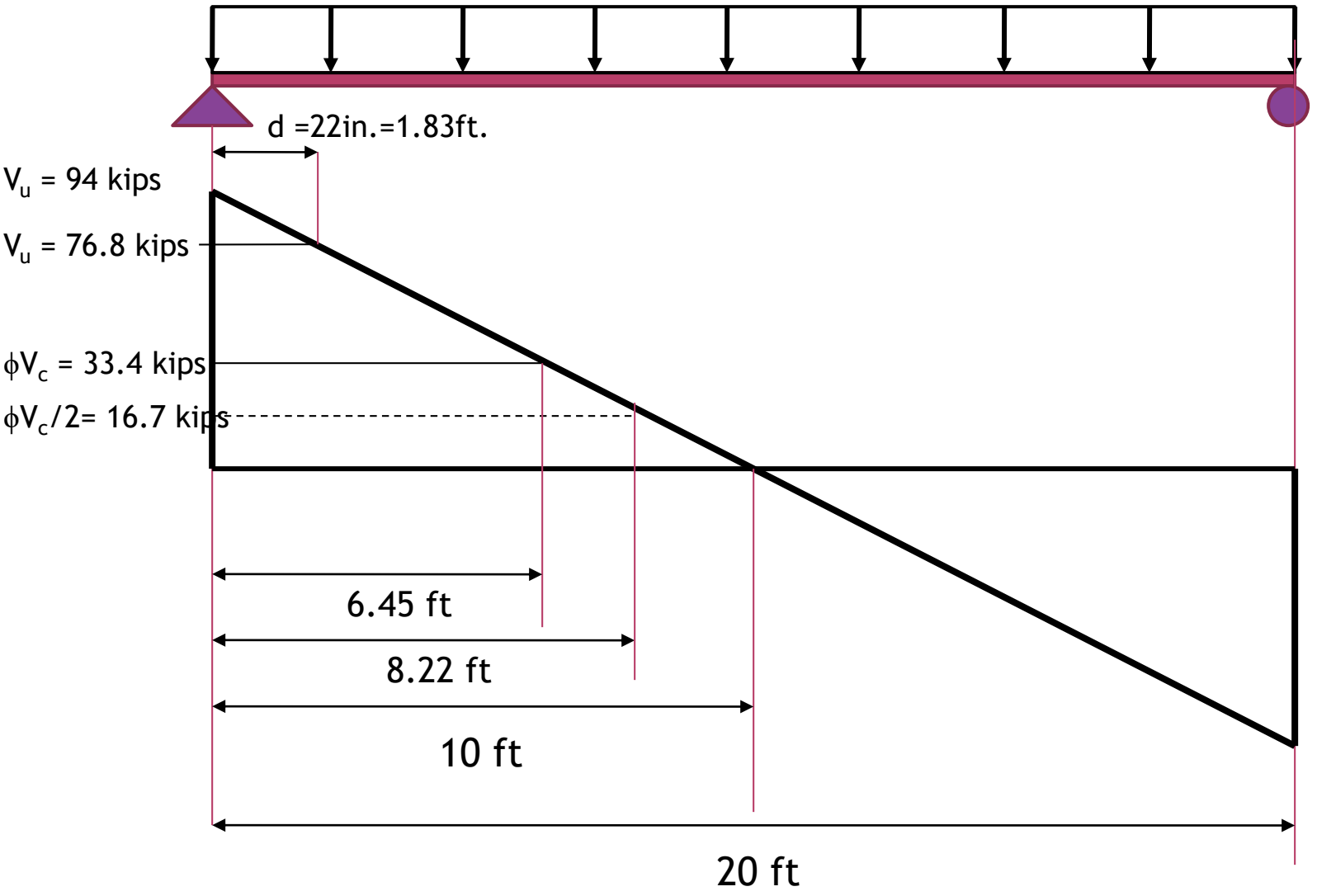
- The shear force varies linearly from maximum at the support to zero at midspan.
- According the ACI Code shear strength provided by concrete:

$$V_c = 2\lambda\sqrt{f_c'}b_w d = 2\sqrt{4,000} \times 16 \times 22 = 44,500 \text{ lb}$$

- Hence, $\phi V_c = 0.75 \text{ x } 44.5 = 33.4 \text{ kips}$
- These values are superimposed on the shear diagram, and, from geometry, the point at which web reinforcements is required is found.

EXAMPLE 3.1

w_u = 9.4 kips/ft



EXAMPLE 3.1 (SOLUTION)

- The point at which shear reinforcement (theoretically) is no longer required is,

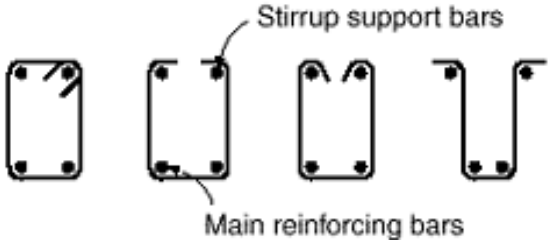
$$10 \times \frac{94 - 33.4}{94} = 6.45 \text{ ft.}$$

- However, according to the ACI Code, at least a minimum amount of shear reinforcement is required wherever shear force exceeds $\phi V_c/2$ or 16.7 kips in this case.
- This applies to a distance (from the support face),

$$10 \times \frac{94 - 16.7}{94} = 8.22 \text{ ft.}$$

- To summarize,
  - At least the minimum shear steel must be provided within a distance of 8.22 ft from the supports.
  - And, within 6.45 ft the shear steel must be provided for the shear force.

EXAMPLE 3.2



- Using vertical **U** stirrups with f_y = 60,000 psi, design the shear reinforcement for the beam in Example 3.1.

- Solution:
- Use No. 3 bars for stirrups.
- So, A_v = 2x0.11 = 0.22 in2.

$$\phi V_s = V_u - \phi V_c \le 4\phi\sqrt{f_c'}\,b_w d$$

$$76,800 - 33,400 \le 4 \times 0.75\sqrt{4000} \times 16 \times 22$$

$$43,400 \text{ lb} \le 66,790 \text{ lb}$$

- Maximum spacing must be smallest of these,

$$s_{\max} = \frac{A_v f_y}{0.75\sqrt{f_c'}\,b_w} \le \frac{A_v f_y}{50 b_w} = \frac{0.22 \times 60,000}{0.75\sqrt{4,000} \times 16} \le \frac{0.22 \times 60,000}{50 \times 16} = 17.4 \le 16.5 = 16.5 \text{ in.}$$

$$s_{\max} = \frac{d}{2} = \frac{22}{2} = 11 \text{ in.}$$

$$s_{\max} = 24 \text{ in.}$$

- So the maximum spacing is 11 in.

EXAMPLE 3.2

- From the supports to a distance d from the support, the required spacing is,

$$s = \frac{\phi A_v f_y d}{V_u - \phi V_c} = \frac{0.75 \times 0.22 \times 60,000 \times 22}{76.8 - 33.4} = 5 \text{ in.}$$

- Note: Vertical stirrups spacing should not be less than 4 in. If it is less than 4 in., higher number of bars should be used.

