Department of Civil Engineering
Military Institute of Science and Technology

Lecture - 1

Bond, Anchorage and Development Length

Presented by

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FUNDAMENTALS OF FLEXURAL BOND

- If the reinforced concrete beam of **Fig. 6.1a** were made with greased plain round bars, it would be only slightly stronger than plain concrete without reinforcement.
- If a load were applied, as in **Fig. 6.1b**, the bars would try to maintain their original length while the beam deflected, causing them to slip relative to the surrounding concrete, which would undergo tensile strain due to flexure.
- **Fig. 6.1c** shows bond forces on the concrete due to bending, while **Fig. 6.1d** shows equal and opposite forces on the reinforcement. These interface bond forces prevent the slip shown in **Fig. 6.1b.**

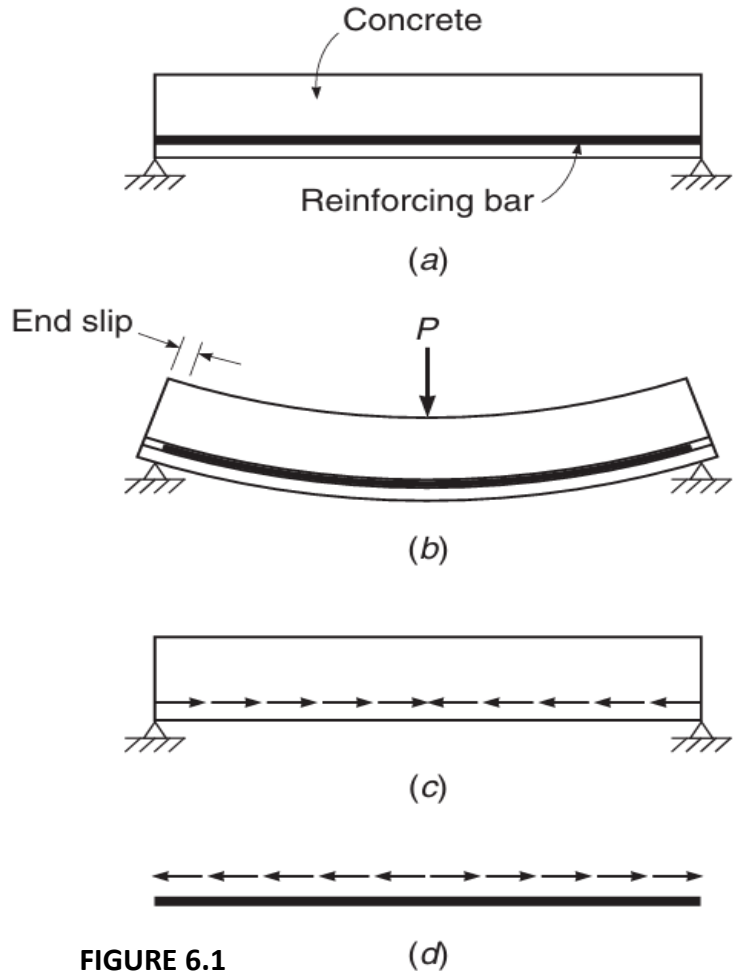


FIGURE 6.1 Bond forces due to flexure: (a) beam before loading; (b) unrestrained slip between concrete and steel; (c) bond forces acting on concrete; and (d) bond forces acting on steel.

FUNDAMENTALS OF FLEXURAL BOND

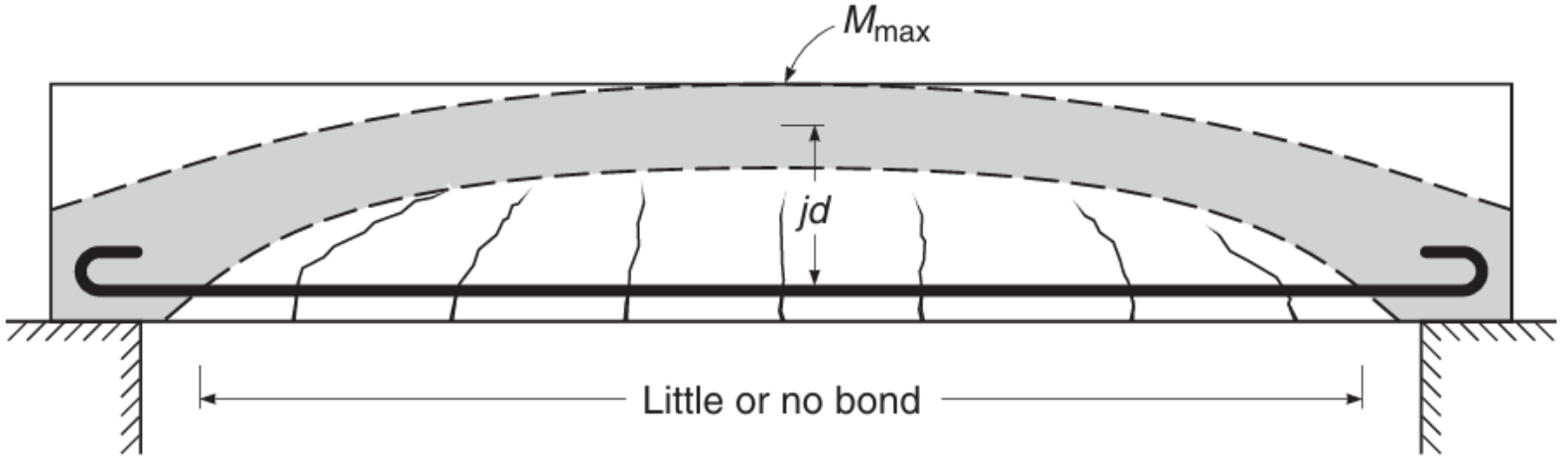


FIGURE 6.2
Tied-arch action in a beam with little or no bond.

In the past, plain bars relied on weak chemical adhesion and friction for bond strength. At higher loads, slip occurred, but the bond was too weak, often leading to failure as bars were pulled through the concrete. To prevent collapse, hooks were used for end anchorage (**Fig. 6.2**). If anchorage is sufficient, the beam acts as a tied arch, with zero bond forces along the unbonded length. The steel force remains constant (**$T = M_{max}/jd$**), resulting in greater elongation, deflection, and crack width compared to beams with maintained bond.

a. Bond Force Based on Simple Cracked Section Analysis

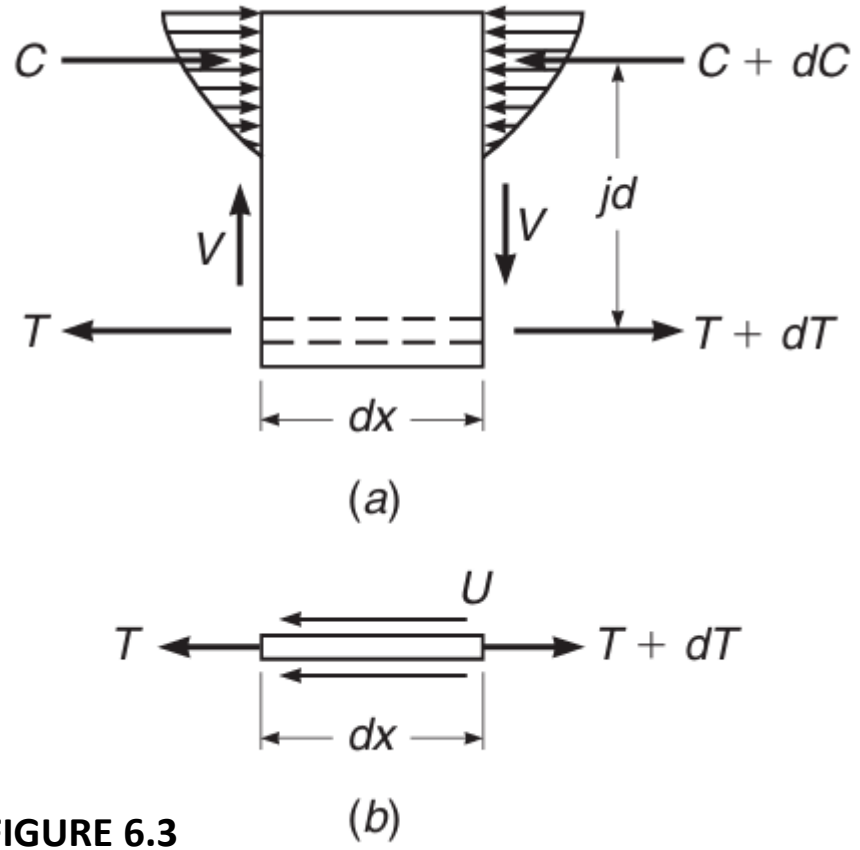


FIGURE 6.3
Forces acting on elemental length of beam: (a) free-body sketch of reinforced concrete element, and (b) free-body sketch of steel element.

- In a short beam segment of length *dx* (**Fig. 6.3a**), the bending moment at one end differs slightly from the other by ***dM***.
- Assuming the concrete no longer resists tension after cracking, internal forces act as shown in **Fig. 6.3a**. The moment change ***dM*** causes a change in bar force, $\mathbf{dT = \frac{dM}{jd}}$ .....(a)

with ***jd*** as the lever arm between tension and compression forces. To maintain equilibrium, this change in bar force is resisted by an equal and opposite bond force at the steel-concrete interface, as shown in **Fig. 6.3b**.

a. Bond Force Based on Simple Cracked Section Analysis

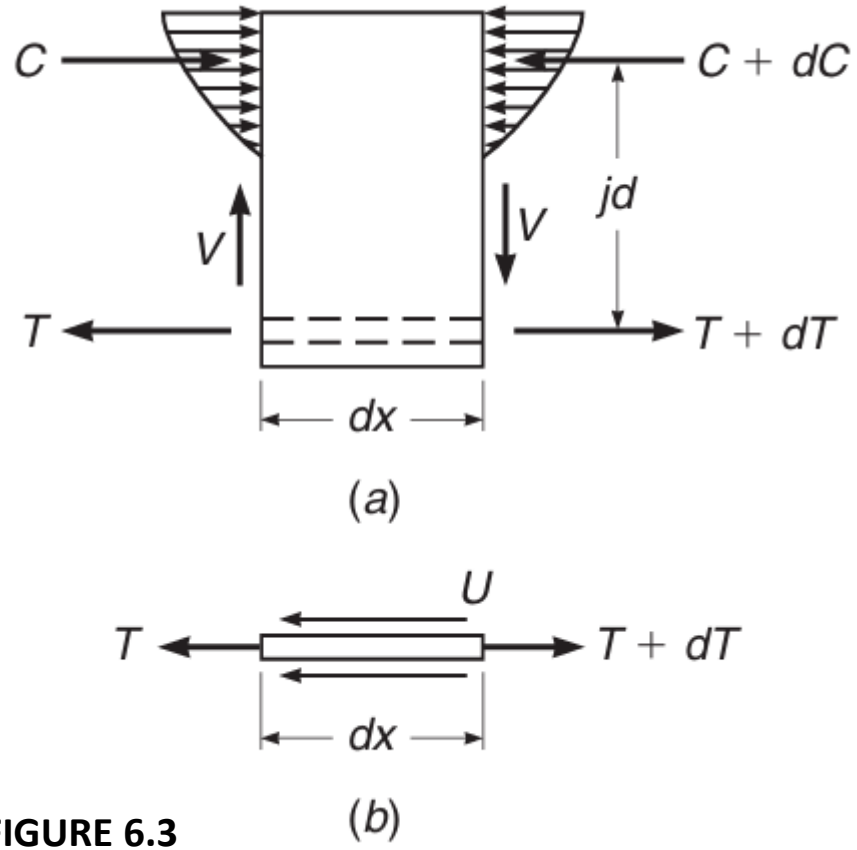


FIGURE 6.3
Forces acting on elemental length of beam: (a) free-body sketch of reinforced concrete element, and (b) free-body sketch of steel element.

- If **U** is the magnitude of the local bond force per unit length of bar, then, by summing horizontal forces

$$\mathbf{U}dx = \mathbf{d}T \ldots\ldots\ldots\ldots (b)$$

- Thus,

$$\mathbf{U} = \frac{\mathbf{d}T}{\mathbf{d}x} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.1)$$

indicating that the local unit bond force is proportional to the rate of change of bar force along the span.

a. Bond Force Based on Simple Cracked Section Analysis

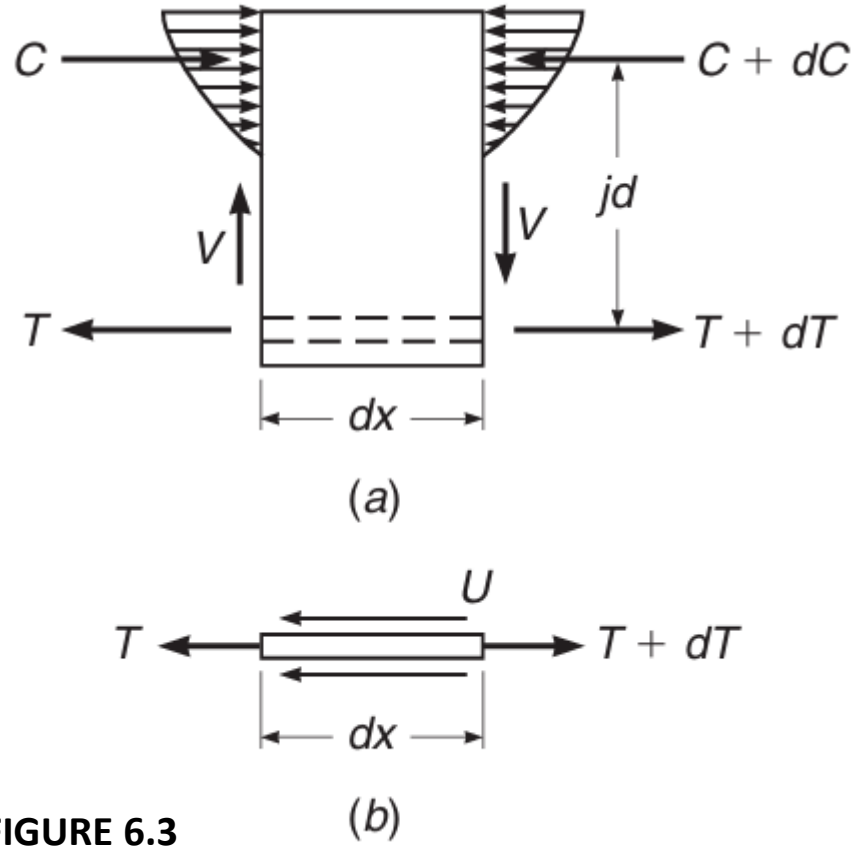


FIGURE 6.3
Forces acting on elemental length of beam: (a) free-body sketch of reinforced concrete element, and (b) free-body sketch of steel element.

- Alternatively, substituting Eq. (a) in Eq. (6.1), the unit bond force can be written as

$$U = \frac{1}{jd}\frac{dM}{dx} \quad \text{.....(c)}$$

from which

$$U = \frac{V}{jd} \quad \text{.....(6.2)}$$

Equation (6.2) is the "elastic cracked section equation" for flexural bond force, and it indicates that the bond force per unit length is proportional to the shear at a particular section, that is, to the rate of change of bending moment.

b. Actual Distribution of Flexural Bond Force

The actual bond force distribution along deformed bars is more complex than suggested by **Eq. (6.2**); **Eq. (6.1)** better reflects real beam behavior.

Eq. 6.1:

$$U = \frac{dT}{dx}$$

Eq. 6.2:

$$U = \frac{V}{jd}$$

b. Actual Distribution of Flexural Bond Force

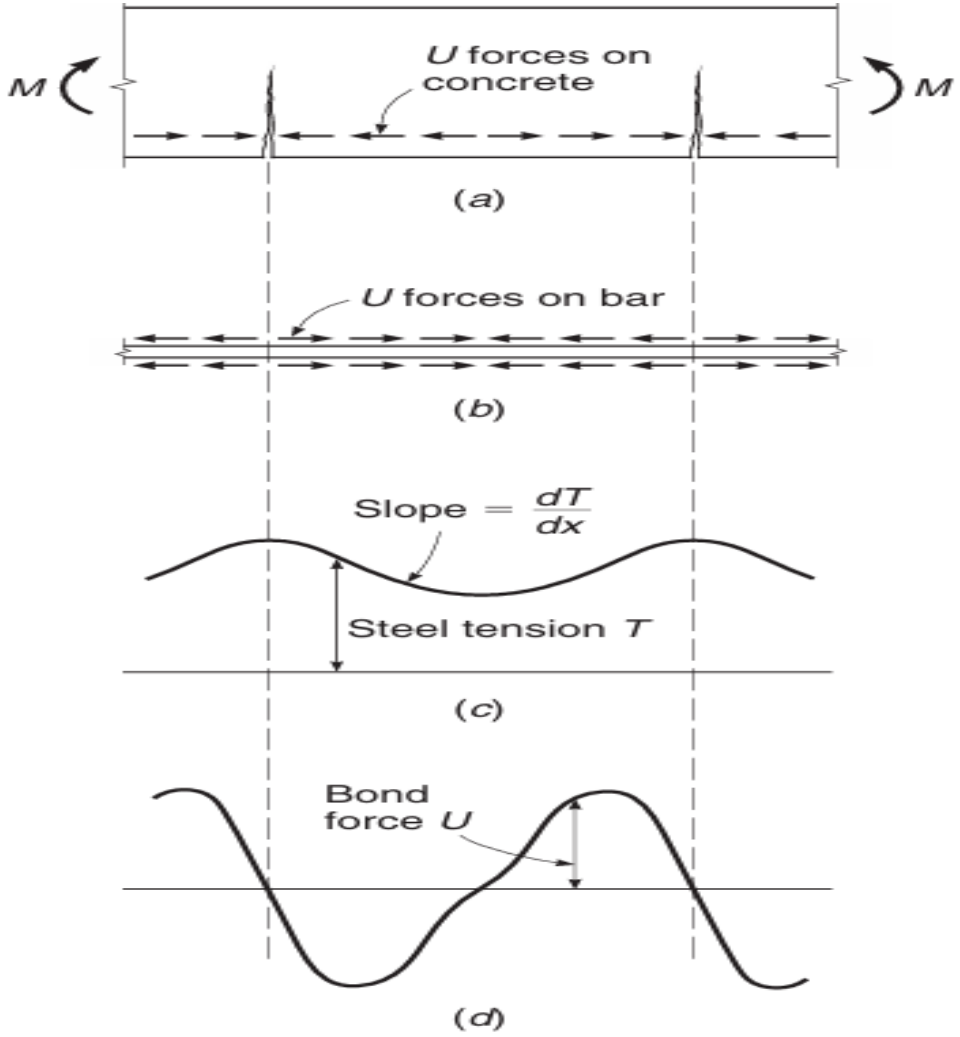
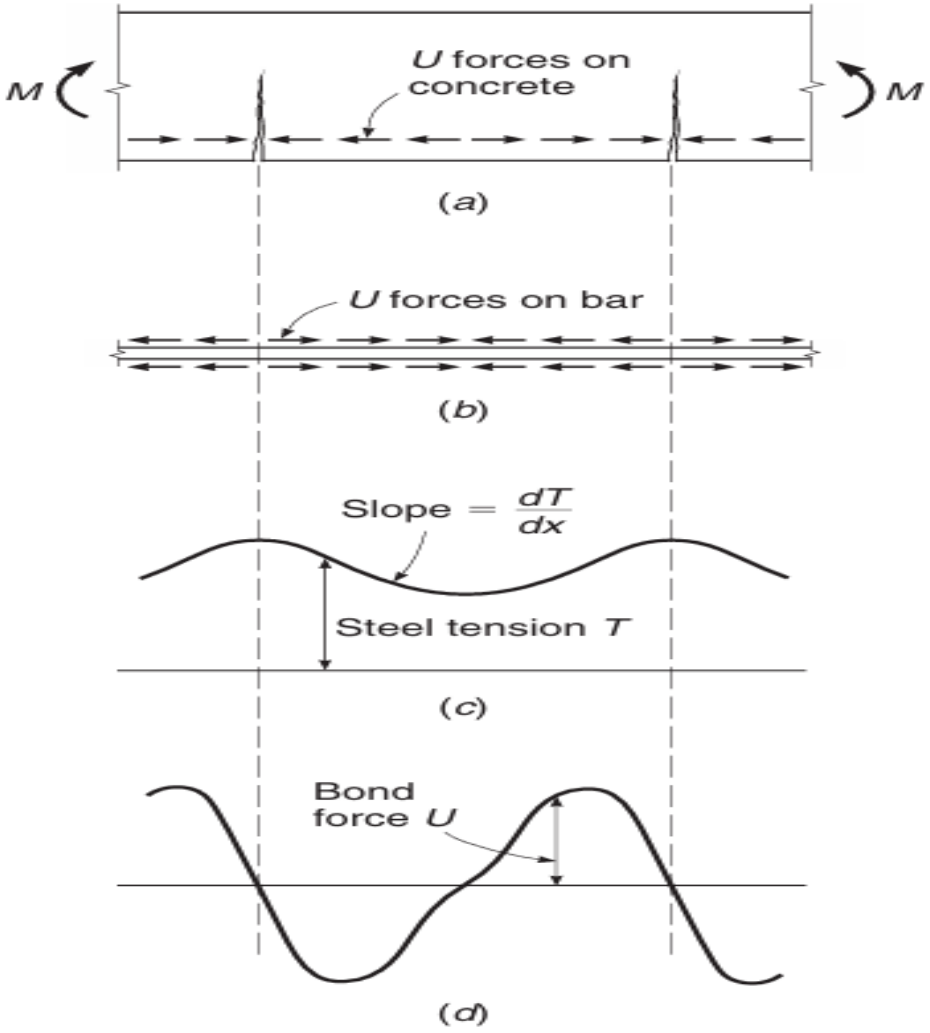


FIGURE 6.4
Variation of steel and bond forces in a reinforced concrete member subject to pure bending: (a) cracked concrete segment; (b) bond forces acting on reinforcing bar; (c) variation of tensile force in steel; and (d) variation of bond force along steel.

b. Actual Distribution of Flexural Bond Force



- **Fig. 6.4** shows a beam under pure bending. Concrete resists *tension* only at uncracked sections, while at cracks, steel tension is *maximum* and equals **T = M/jd**.
- Between cracks, bond forces transfer tension to the concrete (**Fig. 6.4a**), reducing steel tension as shown in **Fig. 6.4c**.
- According to **Eq. (6.1)**, **bond force U** depends on the rate of change of bar force, peaking where the slope is steepest and dropping to zero where it's flat (**Fig. 6.4d**). Tests (**Refs. 6.1 and 6.2**) show very high bond forces near cracks, causing inevitable local slip between steel and concrete.

b. Actual Distribution of Flexural Bond Force

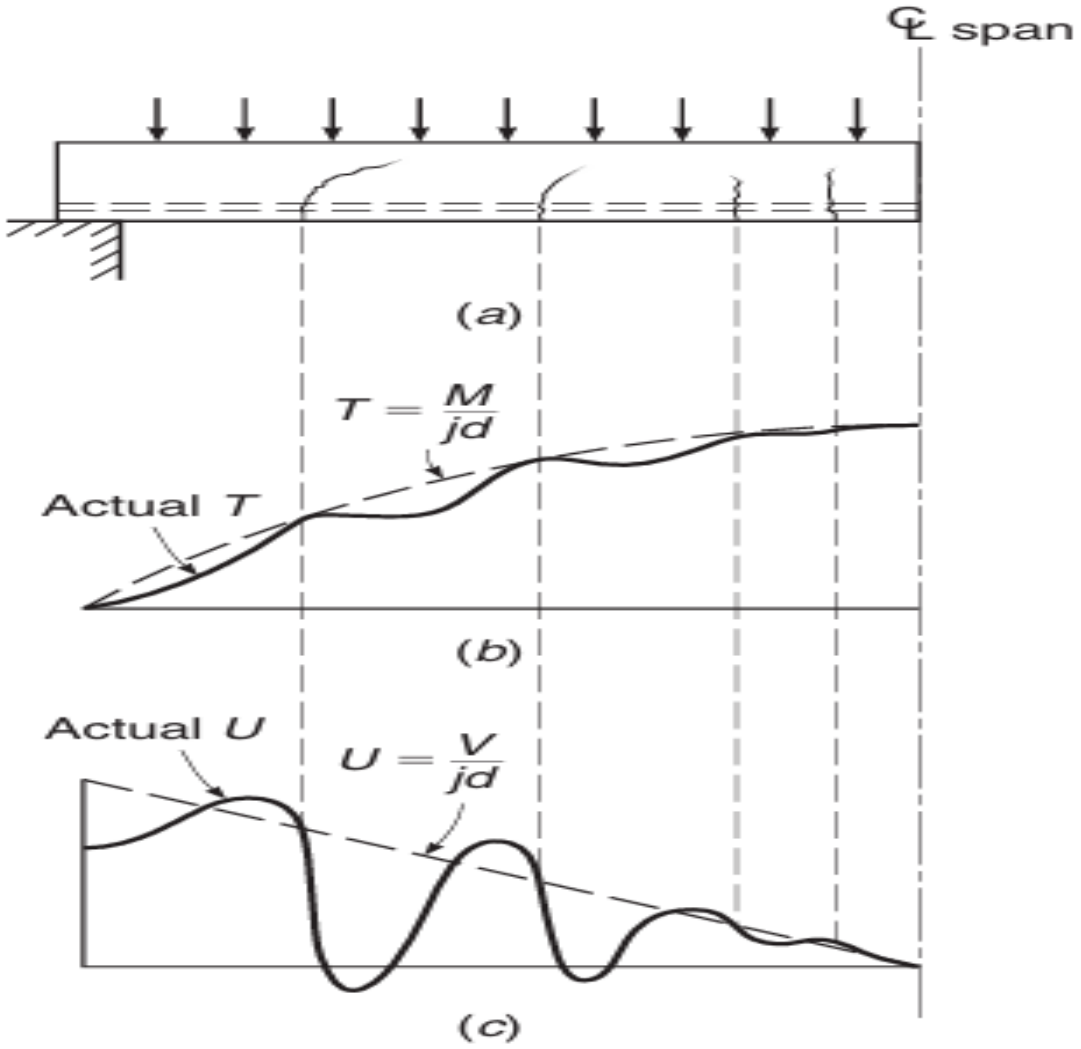
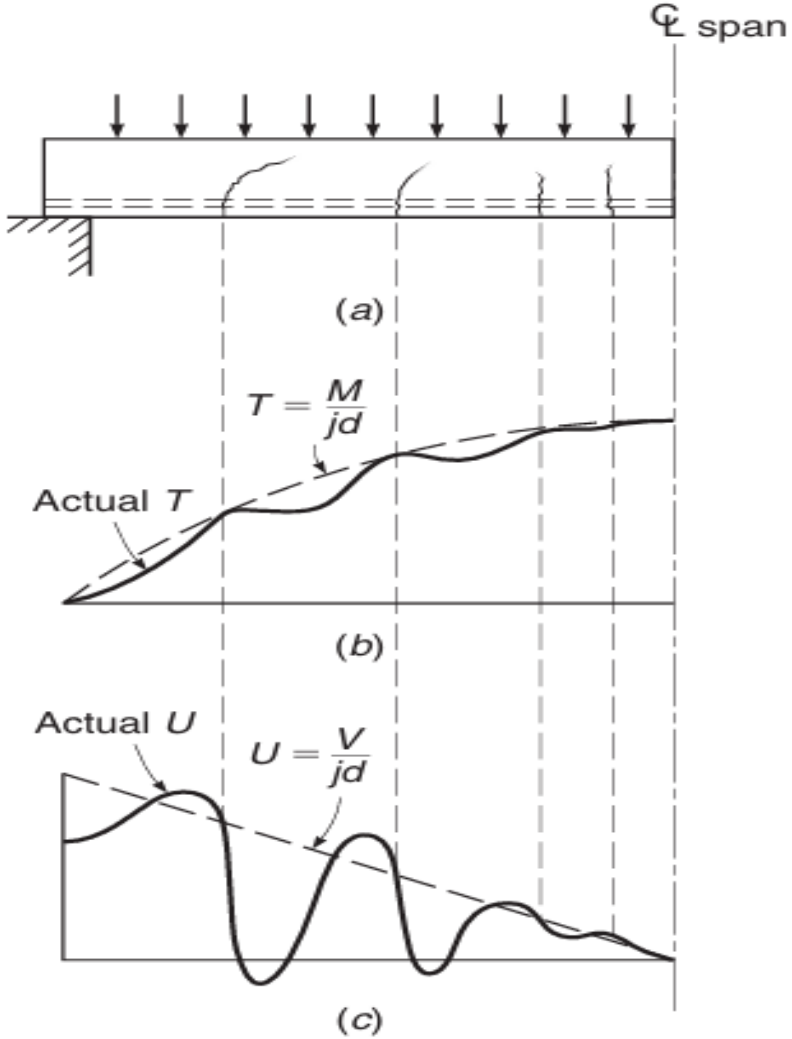


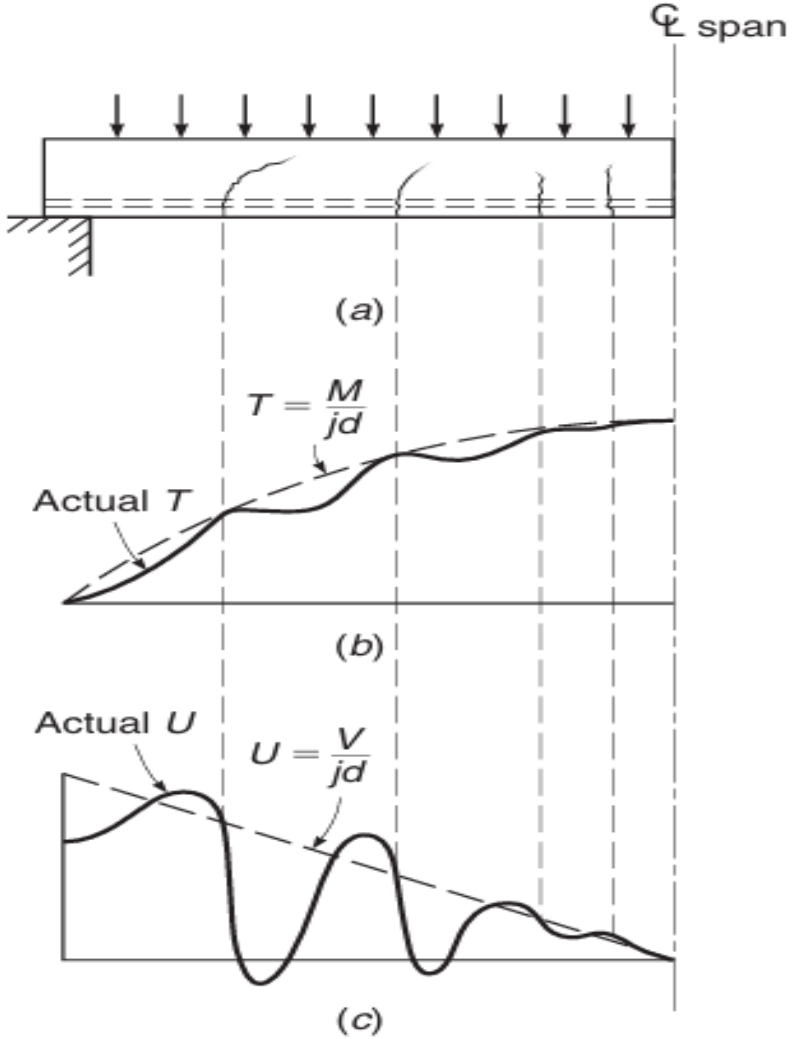
FIGURE 6.5
Effect of flexural cracks on bond forces in beam: (a) beam with flexural cracks; (b) variation of tensile force T in steel along span; and (c) variation of bond force per unit length *U* along span.

b. Actual Distribution of Flexural Bond Force



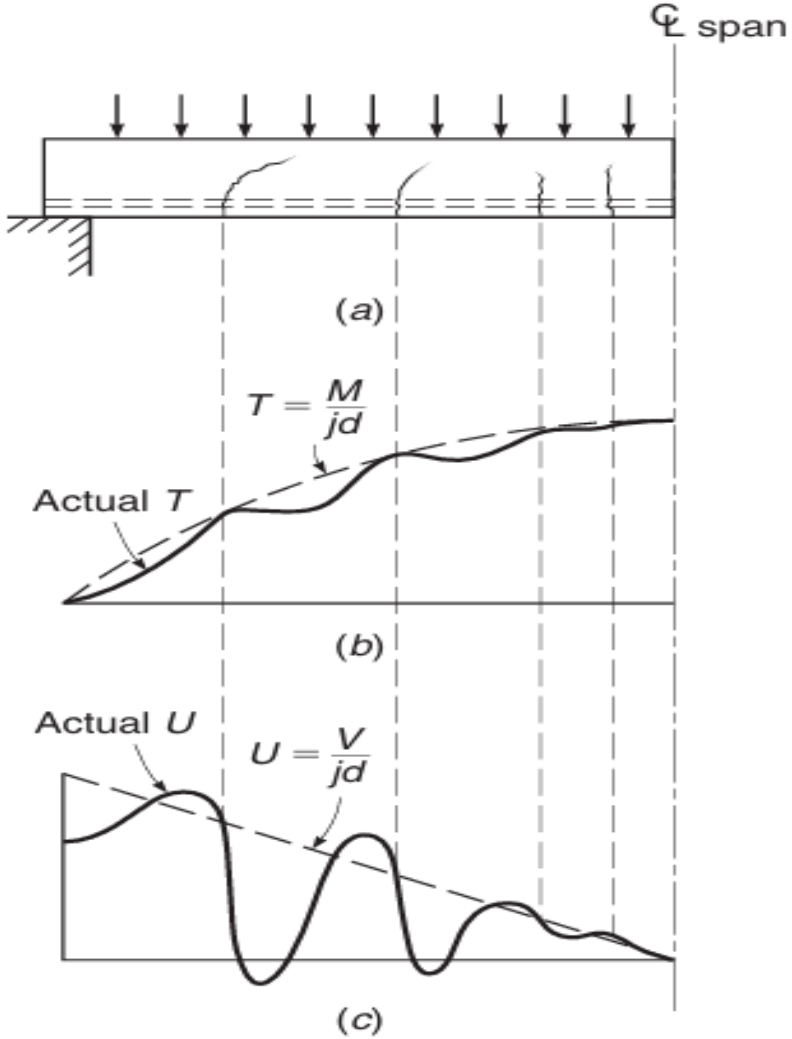
- Beams typically carry **transverse loads**, not pure bending, resulting in **varying shear and moment** along their length.
- **Fig. 6.5a** shows a beam with distributed loading and typical cracking patterns.
- The **theoretical steel force (T)** from simple cracked section analysis follows the **moment diagram** (dashed line in **Fig. 6.5b**).
- The **actual steel force (T)** is ***lower*** than predicted everywhere except at the cracks (solid line in **Fig. 6.5b).**

b. Actual Distribution of Flexural Bond Force



- In **Fig. 6.5c**, the **simplified bond force (U)** is shown by the dashed line, while the **actual bond force** is shown by the solid line.
- The value of **U matches Eq. (6.2)** only where the **slope of the T diagram** equals that from simple theory.
- Elsewhere:
  i. If the slope is **steeper**, actual ***U is greater*** than predicted.
  ii. If the slope is **flatter**, actual ***U is less*** than predicted.

b. Actual Distribution of Flexural Bond Force



- **Left of the cracks**, U is higher than predicted and may cause ***local bond failure***.
- **Right of the cracks**, U is lower and often **negative**, meaning ***bond forces reverse direction***.
- Overall, **actual bond forces differ significantly** from those predicted by **Eq. (6.2)**, except in the general trend of being **highest in high-shear regions**.

For reinforcing bars in tension, **two types** of bond failure have been observed. The first is **direct pullout of the bar**, which occurs when **ample confinement** is provided by the surrounding concrete. This could be expected when relatively **small-diameter bars are used** with sufficiently large concrete cover distances and bar spacing. The second type of failure is **splitting of the concrete** along the bar when **cover, confinement, or bar spacing is insufficient** to resist the lateral concrete tension resulting from the wedging effect of the bar deformations. Present-day design methods require that both possible failure modes be accounted for.

a. Bond Strength

- If the bar is sufficiently confined by a mass of surrounding concrete, then as the tensile force on the bar is **increased**, **adhesive bond** and **friction** are overcome, the concrete eventually crushes locally ahead of the bar deformations, and bar pullout results. The surrounding concrete remains intact, except for the crushing that takes place ahead of the ribs immediately adjacent to the bar interface. For modern deformed bars, *adhesion and friction are much less important* than the mechanical interlock of the deformations with the surrounding concrete.

a. Bond Strength

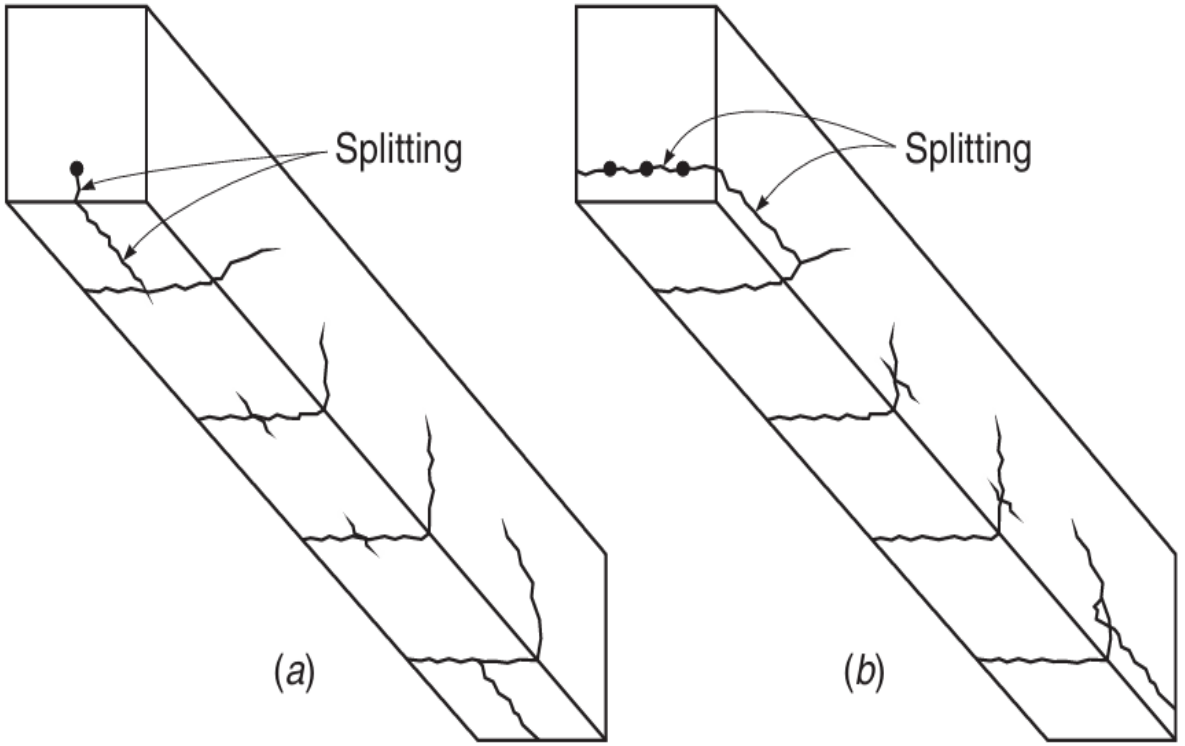


FIGURE 6.6
Splitting of concrete along reinforcement.

- **Splitting bond failure** is more common in beams than direct pullout failures.
- This type of failure is primarily caused by the ***wedging action of deformed bar*** ribs pressing against the surrounding concrete (Refs. 6.3 and 6.4).
- Splitting can occur in **two orientations**:

  i. Vertically, perpendicular to the bars (**Fig. 6.6a**).
  ii. Horizontally, along the plane of the bars (**Fig. 6.6b**), often initiating at a diagonal crack.

a. Bond Strength

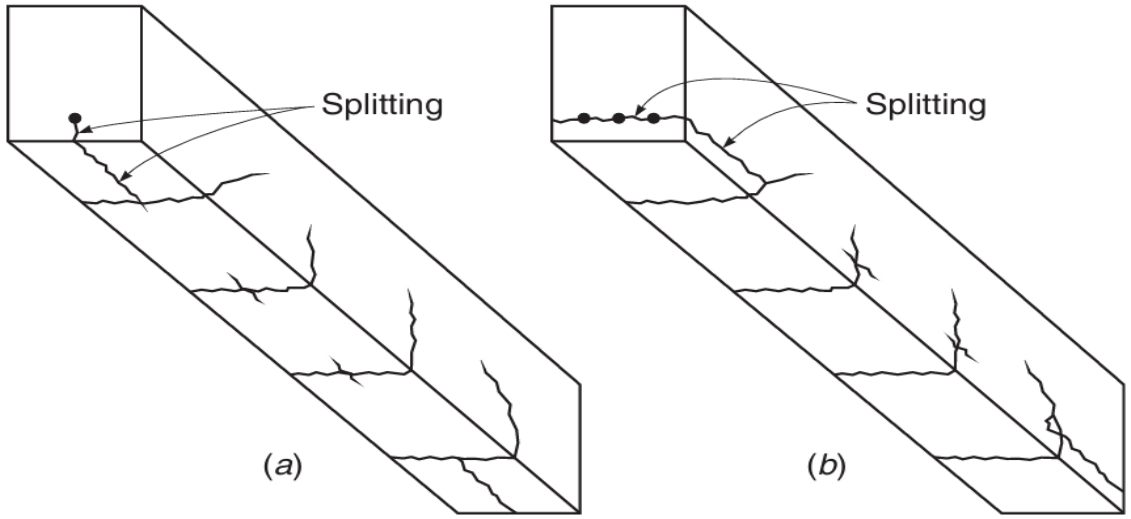


FIGURE 6.6
Splitting of concrete along reinforcement.

- The horizontal type of splitting of **Fig 6.6b** frequently begins at a diagonal crack. In this case, dowel action increases the tendency toward splitting. This indicates that **shear** and **bond failures** are often intricately interrelated.
- When pullout resistance is overcome or when splitting has spread all the way to the end of an unanchored bar, complete bond failure occurs. *Sliding of the steel relative to the concrete leads to immediate collapse of the beam*.

b. Development Length

- The development length is defined as that length of embedment necessary to develop the full tensile strength of the bar, controlled by either pullout or splitting.

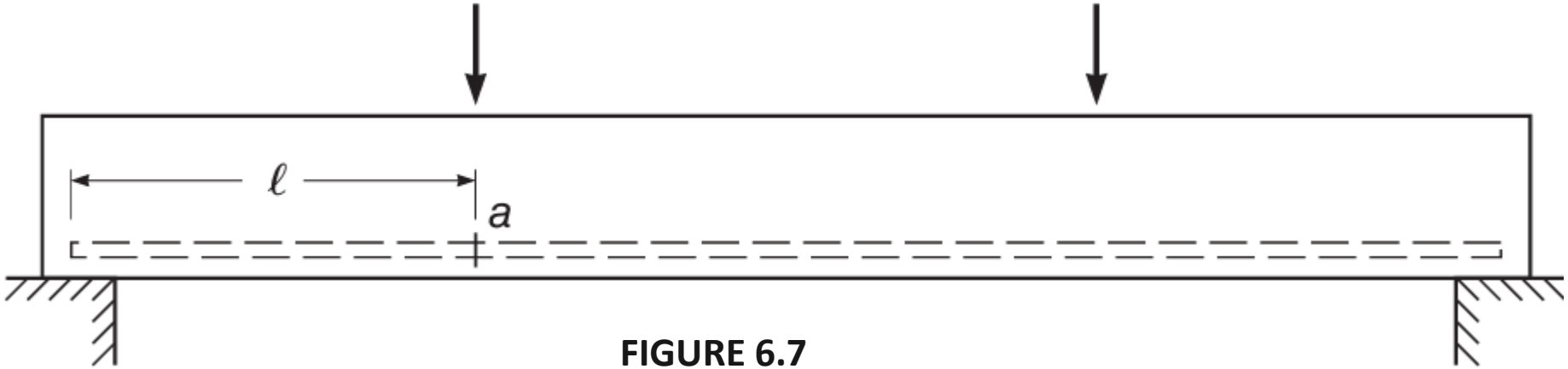


FIGURE 6.7
Development length.

- With reference to **Fig. 6.7**, the moment, and therefore the steel stress, is evidently maximum at point **a** (neglecting the weight of the beam) and **zero at the supports**. If the bar stress is f_s at a, then the total tension force $A_b f_s$ must be transferred from the bar to the concrete in the distance ℓ by bond forces.

b. Development Length

- To fully develop the strength of the bar $A_b f_y$, the distance ℓ must be at least equal to the development length of the bar.
- In the beam of **Fig. 6.7**, if the actual length ℓ is equal to or greater than the development length ℓ_d, **no premature bond failure** will occur. That is, the beam will fail in ***bending*** or ***shear*** rather than by bond failure. This will be so even if in the vicinity of cracks local slip may have occurred over small regions along the beam.
- It is seen that the main requirement for safety against bond failure is this: the length of the bar, from any point of given steel stress (f_s or at most f_y) to its nearby free end, must be at least equal to its development length. If this requirement is satisfied, the magnitude of the nominal flexural bond force along the beam, as given by **Eq. (6.2)**, is of only secondary importance, since the integrity of the member is ensured even in the face of possible minor local bond failures. However, if the actual available length is inadequate for full development, special anchorage, such as by hooks, must be provided.

c. Factors Influencing Development Length

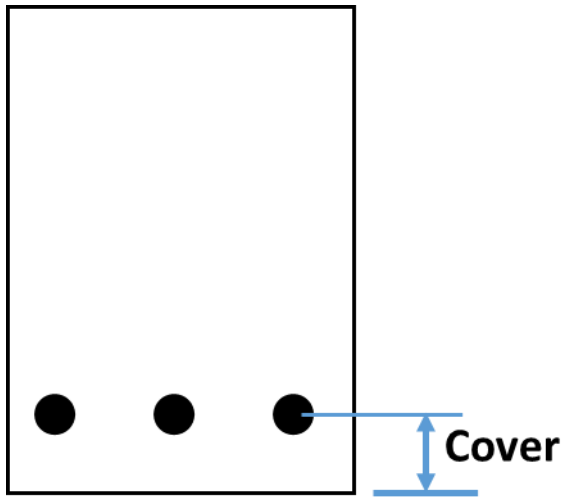
(a) Tensile strength of concrete:

For **lightweight concretes**, the tensile strength is usually less than for **normal density concrete** having the same compressive strength; accordingly, if lightweight concrete is used, development lengths must be increased. Alternatively, if split- cylinder strength is known or specified for lightweight concrete, it can be incorporated in development length equations as follows. For normal concrete, the splitting tensile strength f_{ct} is generally taken as $f_{ct} = 6.7\sqrt{f'c}$. If the splitting tensile strength f_{ct} is known for a particular lightweight concrete, then $\sqrt{f'c}$ in the development length equations is replaced by $f_{ct}/6.7$.

c. Factors Influencing Development Length

(b) Cover Distance:

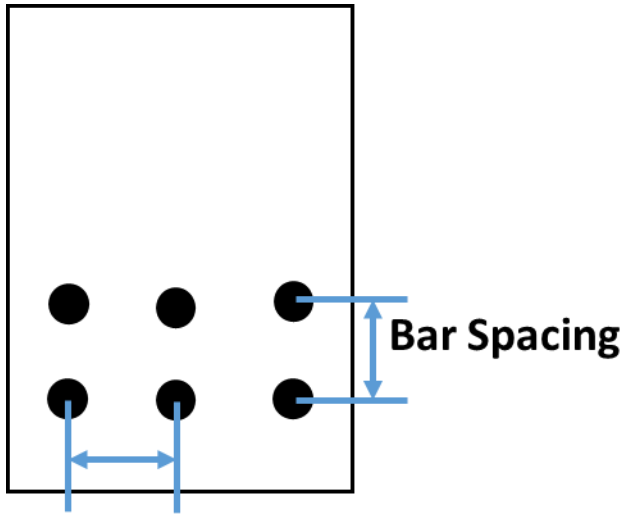
Conventionally measured from the **center of the bar to the nearest concrete face** and measured either in the plane of the bars or perpendicular to that plane—also influences splitting. Clearly, if the vertical or horizontal cover is increased, more concrete is available to resist the *tension* resulting from the wedging effect of the deformed bars, resistance to splitting is improved, and development length is less.



c. Factors Influencing Development Length

(c) Bar Spacing:

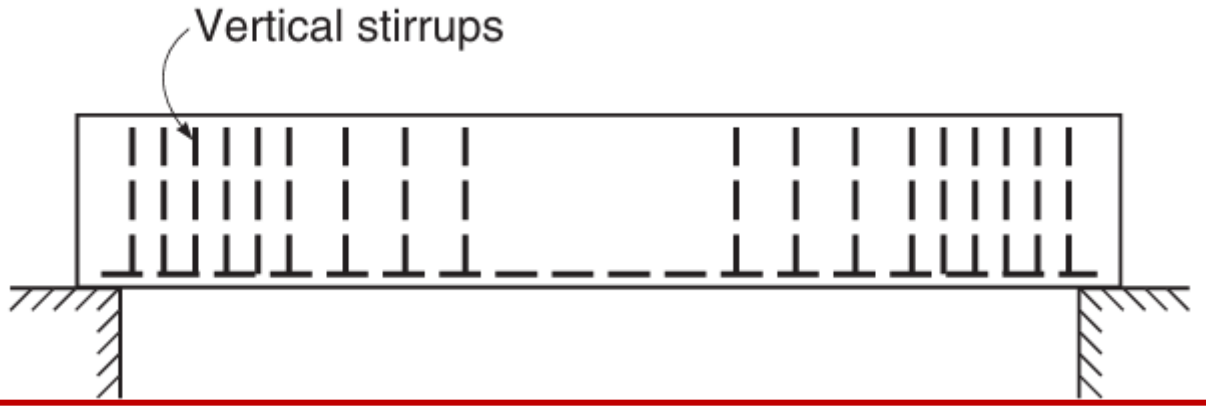
If the bar spacing is increased (such as, if only two instead of three bars are used), more concrete per bar will be available to resist horizontal splitting. In beams, bars are typically spaced about **one or two bar diameters apart**. On the other hand, for slabs, footings, and certain other types of member, bar spacings are typically much greater, and the required development length is reduced, if not limited by cover.



c. Factors Influencing Development Length

(d) Transverse Reinforcement:

Transverse reinforcement, such as that provided by stirrups improves the resistance of tensile bars to both vertical or horizontal splitting failure because the tensile force in the transverse steel tends to prevent opening of the actual or potential crack. The effectiveness of such transverse reinforcement depends on its cross-sectional area and spacing along the development length. Its effectiveness does not depend on its **yield strength** f_{yt}, because transverse reinforcement rarely yields during a bond failure.



c. Factors Influencing Development Length

(e) Other Factors:

- **The vertical location of horizontal bars relative to beam depth** has been found to have an effect. If bars are placed in the forms during construction such that a substantial depth of concrete is placed below those bars, there is a tendency for excess water, often used in the mix for workability, and for entrapped air to rise to the top of the concrete during consolidation. Air and water tend to accumulate on the underside of the bars. Tests have shown a significant loss in bond strength for bars with more than 12 in. of fresh concrete cast beneath them, and accordingly the development length must be increased. This effect increases as the slump of the concrete increases and is greatest for bars cast near the upper surface of a concrete placement.

c. Factors Influencing Development Length

(e) Other Factors:

- **Epoxy-coated reinforcing bars** are used regularly in projects where the structure may be subjected to corrosive environmental conditions or deicing chemicals, such as for highway bridge decks and parking garages. Zinc and epoxy dual-coated bars are also available. Studies have shown that bond strength is reduced because the epoxy coating reduces the friction between the concrete and the bar, and the required development length must be increased substantially. Early research suggested that when concrete cover and bar spacing are large, the negative impact of epoxy coating on bond strength is reduced, justifying a smaller increase in development length. Although later studies disagreed, the ACI Code still permits this smaller increase. However, because epoxy-coated bars have reduced bond strength due to poor adhesion, the ACI Code sets an upper limit on the combined development length factors for certain conditions.

c. Factors Influencing Development Length

(e) Other Factors:

- Finally, based on bars with very short development lengths (most with values of $\ell_d/d_b < 15$), it was observed that smaller-diameter bars required lower development lengths than predicted by **Eq. (6.3)**. As a result, the required development lengths for No. 6 (No. 19) and smaller bars were reduced below the values required by **Eq. (6.3)**.

$$\ell_d = \left(\frac{3}{40} \frac{f_s}{\sqrt{f'_c} \left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \qquad (6.3)$$

where d_b = bar diameter
c_b = smaller of minimum cover or one-half of bar spacing *measured to center of bar*
$K_{tr} = 40A_{tr}/sn$, which represents effect of confining reinforcement
A_{tr} = area of transverse reinforcement normal to plane of splitting through the bars being developed
s = spacing of transverse reinforcement
n = number of bars developed or spliced at same location

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(a) Equation for Development Length for Bars and Wires in Tension:

According to ACI Code 25.4.2.3, for deformed bars or deformed wires,

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \qquad (6.4)$$

in which the term $(c_b + K_{tr})/d_b$ may not be taken greater than 2.5. In Eq. (6.4), the terms are defined and values established as follows.

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(a) Equation for Development Length for Bars and Wires in Tension:

ψ_t = casting position factor	
More than 12 in. of fresh concrete is placed below horizontal reinforcement:	1.3
Other situations:	1.0
ψ_e = epoxy coating factor	
Epoxy-coated or zinc and epoxy dual-coated bars or wires with cover less than $3d_b$ or clear spacing less than $6d_b$:	1.5
All other epoxy-coated or zinc and epoxy dual-coated bars or wires:	1.2
Uncoated and zinc-coated (galvanized) reinforcement:	1.0
However, the product of $\psi_t\psi_e$ need not be taken greater than 1.7.	
ψ_s = reinforcement size factor	
No. 6 (No. 19) and smaller bars and deformed wires:	0.8†
No. 7 (No. 22) and larger bars:	1.0

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(a) Equation for Development Length for Bars and Wires in Tension:

λ = lightweight aggregate concrete factor

When lightweight aggregate concrete is used: 0.75

However, when f_{ct} is specified, $\lambda = f_{ct}/(6.7\sqrt{f_{cm}}) \leq 1.0$, where f_{cm} is the measured compressive strength.

When normalweight concrete is used: 1.0

c_b = spacing or cover dimension, in.

Use the smaller of either the distance from the center of the bar to the nearest concrete surface or one-half the center-to-center spacing of the bars being developed.

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(a) Equation for Development Length for Bars and Wires in Tension:

K_{tr} = transverse reinforcement index: $40A_{tr}/sn$

where A_{tr} = total cross-sectional area of all transverse reinforcement that is within the spacing s and that crosses the potential plane of splitting through the reinforcement being developed, in^2

s = maximum spacing of transverse reinforcement within ℓ_d center to center, in.

n = number of bars or wires being developed along the plane of splitting

As a simplification, the designer is permitted to use $K_{tr} = 0$ even if transverse reinforcement is present.

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(a) Equation for Development Length for Bars and Wires in Tension:

The limit of 2.5 on $(c + K_{tr})/d_b$ is imposed to avoid pullout failure. With that term taken equal to its limit of 2.5, evaluation of Eq. (6.4) results in $\ell_d = 0.03 d_b f_y / \sqrt{f'_c}$, the experimentally derived limit found in earlier ACI Codes when pullout failure controls. Note that in Eq. (6.4) and in all other ACI Code equations relating to the development length and splices of reinforcement, *values of* $\sqrt{f'_c}$ *are not to be taken greater than 100 psi* because, as explained in Section 6.2c and recognized in ACI Commentary 25.4.1.4, bond strength increases more slowly than $\sqrt{f'_c}$. The $\sqrt{f'_c}$, however, is reasonably accurate for values of f'_c up to 10,000 psi.

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(b) Simplified Equations for Development Length:

Calculation of required development length (in terms of bar diameter) by Eq. (6.4) requires that the term $(c_b + K_{tr})/d_b$ be calculated for each particular combination of cover, spacing, and transverse reinforcement. Alternatively, according to the Code, a simplified form of Eq. (6.4) may be used in which $(c_b + K_{tr})/d_b$ is set equal to 1.5, provided that certain restrictions are placed on cover, spacing, and transverse reinforcement. Two cases of practical importance are:

1. Minimum clear cover of $1.0d_b$, minimum clear spacing of $1.0d_b$, and at least the Code required minimum stirrups or ties (see Section 5.5b) throughout ℓ_d
2. Minimum clear cover of $1.0d_b$ and minimum clear spacing of $2d_b$

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(b) Simplified Equations for Development Length:

For either of these common cases, it is easily confirmed from Eq. (6.4) that for No. 7 (No. 22) and larger bars

$$\ell_d = \left(\frac{f_y \psi_t \psi_e}{20\lambda \sqrt{f'_c}} \right) d_b \qquad (6.5a)$$

and for No. 6 (No. 19) bars and smaller (with $\gamma = 0.8$)

$$\ell_d = \left(\frac{f_y \psi_t \psi_e}{25\lambda \sqrt{f'_c}} \right) d_b \qquad (6.5b)$$

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(b) Simplified Equations for Development Length:

If these restrictions on spacing are not met, then, provided that Code-imposed minimum spacing requirements are met (see Section 4.5c), the term $(c_b + K_{tr})/d_b$ will have a value not less than 1.0 (rather than 1.5 as before) whether or not transverse steel is used. The values given by Eqs. (6.5*a*) and (6.5*b*) are then multiplied by the factor 1.5/1.0.

Thus, if the designer accepts certain restrictions on bar cover, spacing, and transverse reinforcement, simplified calculation of development requirements is possible. The simplified equations are summarized in Table 6.1.

Further simplification is possible for the most common condition of normalweight concrete and uncoated reinforcement. Then λ and ψ_e in Table 6.1 take the value 1.0, and the development length, in terms of bar diameters, is simply a function of f_y, f'_c, and the bar location factor ψ_t. Thus, development lengths are easily tabulated for the usual combinations of material strengths and bottom or top bars and for the restrictions on bar spacing, cover, and transverse steel defined.† Results are given in Table A.10 of Appendix A.

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(b) Simplified Equations for Development Length:

TABLE 6.1
Simplified tension development length in bar diameters according to the ACI Code

	No. 6 (No. 19) and Smaller Bars and Deformed Wires†	No. 7 (No. 22) and Larger Bars
Clear spacing of bars or wires being developed or spliced $\geq d_b$, clear cover $\geq d_b$, and stirrups or ties throughout ℓ_d not less than the Code minimum	$\ell_d = \left(\frac{f_y \psi_t \psi_e}{25\lambda\sqrt{f'_c}}\right) d_b$	$\ell_d = \left(\frac{f_y \psi_t \psi_e}{20\lambda\sqrt{f'_c}}\right) d_b$
Clear spacing of bars or wires being developed or spliced $\geq 2d_b$, and clear cover $\geq d_b$	Same as above	Same as above
Other cases	$\ell_d = \left(\frac{3f_y \psi_t \psi_e}{50\lambda\sqrt{f'_c}}\right) d_b$	$\ell_d = \left(\frac{3f_y \psi_t \psi_e}{40\lambda\sqrt{f'_c}}\right) d_b$

ACI CODE PROVISIONS FOR DEVELOPMENT OF TENSION REINFORCEMENT

TABLE A.10
Simplified tension development length in bar diameters ℓ_d/d_b for uncoated bars and normalweight concrete

		No. 6 (No. 19) and Smallera			No. 7 (No. 22) and Larger		
		f'_c, psi			f'_c, psi		
	f_y, ksi	4000	5000	6000	4000	5000	6000
(1) Bottom Bars							
Spacing, cover and ties as per Case *a* or *b*	40	26	23	21	32	29	26
	60	38	34	31	48	43	39
	75	48	43	39	60	54	49
	80	51	46	42	64	57	52
Other cases	40	38	34	31	48	43	39
	60	57	51	47	72	64	59
	75	72	64	59	89	80	73
	80	76	68	62	95	85	78
(2) Top Bars							
Spacing, cover and ties as per Case *a* or *b*	40	33	30	27	42	37	34
	60	50	45	41	62	56	51
	75	62	56	51	78	69	63
	80	66	59	54	83	74	68
Other cases	40	50	45	41	62	56	51
	60	74	67	61	93	83	76
	75	93	83	76	116	104	95
	80	99	89	81	124	111	101

Case *a:* Clear spacing of bars being developed or spliced $\geq d_b$, clear cover $\geq d_b$, and stirrups or ties throughout ℓ_d not less than the Code minimum.
Case *b:* Clear spacing of bars being developed or spliced $\geq 2d_b$, and clear cover not less than d_b.
[a]ACI Committee 408 recommends that the values indicated for bar sizes No. 7 (No. 22) and larger be used for all bar sizes.

EXAMPLE 6.1 **Development length in tension.** Figure 6.8 shows a beam-column joint in a continuous building frame. Based on frame analysis, the negative steel required at the end of the beam is 2.90 in^2 ; two No. 11 (No. 36) bars are used, providing A_s = 3.12 in^2. Beam dimensions are b = 10 in., d = 18 in., and h = 21 in. The design will include No. 3 (No. 10) stirrups spaced four at 3 in., followed by a constant 5 in. spacing in the region of the support, with 1.5 in. clear cover. Normalweight concrete is to be used, with f'_c = 4000 psi, and reinforcing bars have f_y = 60,000 psi. Find the minimum distance ℓ_d at which the negative bars can be cut off, based on development of the required steel area at the face of the column, (a) using the simplified equations of Table 6.1,(b) using Table A.10, of Appendix A, and (c) using the more accurate Eq. (6.4).

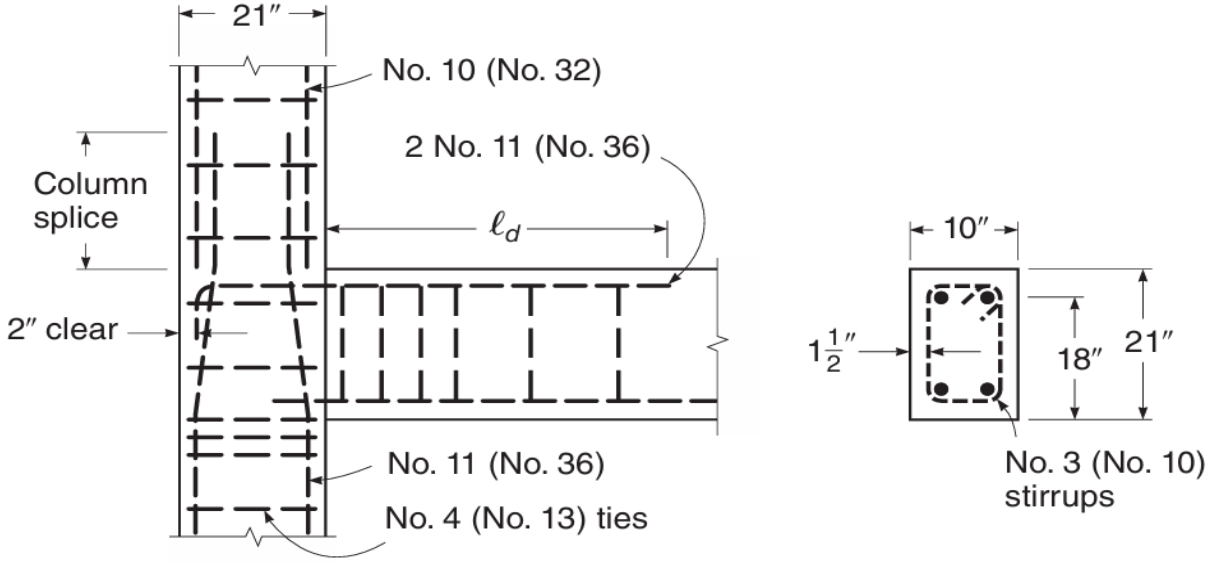


FIGURE 6.8
Bar details at beam-column joint for bar development examples.

Solution:

(a) Using the simplified equations of Table 6.1

Area of No. 3 stirrups

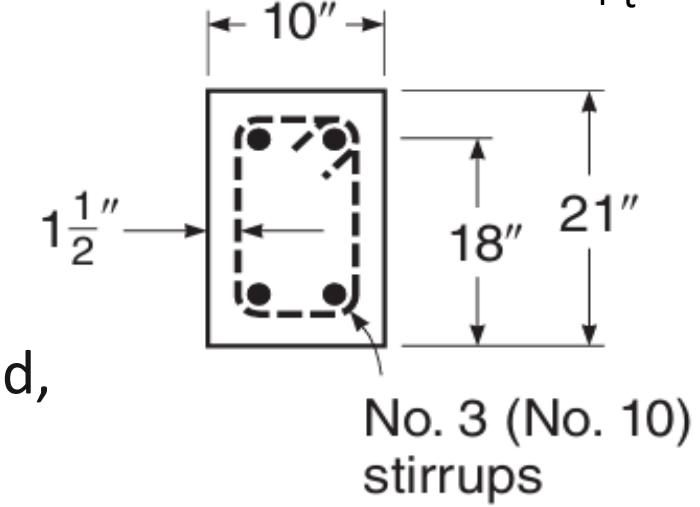
Area of No. 11 bars

The clear distance between the bars is 10 − 2(1.50 + 0.38 + 1.41) = 3.42 in., or 2.43 times the bar diameter d_b .

The clear cover of the No. 11 (No. 36) bars to the side face of the beam is 1.50 + 0.38 = 1.88 in., or 1.33 bar diameters, and that to the top of the beam is 3.00 − 1.41/2 = 2.30 in., or 1.63 bar diameters.

Then for top bars, uncoated, and with normal-density concrete, we have the values of ψ_t = 1.3, ψ_e= 1.0, and λ = 1.0. From **Table 6.1**,

$$\ell_d = \left(\frac{f_y \psi_t \psi_e}{20\lambda\sqrt{f'_c}}\right) d_b = \frac{60{,}000 \times 1.3 \times 1.0}{20 \times 1.0\sqrt{4000}} 1.41 = 62 \times 1.41 = 87 \text{ in.}$$



This can be reduced by the ratio of steel required to that provided, so that the final development length is 87 × 2.90/3.12 = 81 in.

Solution:

(b) using Table A.10, of Appendix A

From Table A.10, for f'_c = 4000 psi, f_y = 60,000 psi and for top bars,

$$\frac{\ell_d}{d_b} = 62$$

The required length to point of cutoff is 62 × 1.41 × 2.90/3.12 = 81 in.

TABLE A.10
Simplified tension development length in bar diameters ℓ_d/d_b for uncoated bars and normalweight concrete

		No. 6 (No. 19) and Smallera			No. 7 (No. 22) and Larger		
		f'_c, psi			f'_c, psi		
	f_y, ksi	4000	5000	6000	4000	5000	6000
(1) Bottom Bars							
Spacing, cover and ties as per Case *a* or *b*	40	26	23	21	32	29	26
	60	38	34	31	48	43	39
	75	48	43	39	60	54	49
	80	51	46	42	64	57	52
Other cases	40	38	34	31	48	43	39
	60	57	51	47	72	64	59
	75	72	64	59	89	80	73
	80	76	68	62	95	85	78
(2) Top Bars							
Spacing, cover and ties as per Case *a* or *b*	40	33	30	27	42	37	34
	60	50	45	41	62	56	51
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	80	66	59	54	83	74	68
Other cases	40	50	45	41	62	56	51
	60	74	67	61	93	83	76
	75	93	83	76	116	104	95
	80	99	89	81	124	111	101

Solution:

(c) using the more accurate Eq. (6.4).

The center-to-center spacing of the No. 11 (No. 36) bars is 10 − 2(1.50 + 0.38 + 1.41/2) = 4.83, one-half of which is 2.42 in.

The side cover to bar centerline is 1.50 + 0.38 + 1.41/2 = 2.59 in., and the top cover is 3.00 in. The smallest of these three distances controls, and c_b = 2.42 in.

Potential splitting would be in the horizontal plane of the bars, and in calculating A_{tr} two times the stirrup bar area is used. Based on the No. 3 (No. 10) stirrups at 5 in. spacing:

$$K_{tr} = \frac{40\,A_{tr}}{sn} = \frac{40 \times 0.11 \times 2}{5 \times 2} = 0.88 \qquad \text{and} \qquad \frac{c_b + K_{tr}}{d_b} = \frac{2.42 + 0.88}{1.41} = 2.34$$

This is less than the limit value of 2.5. Then from Eq. (6.4) with $\psi_s = 1.0$,

$$\ell_d = \left(\frac{3}{40}\frac{f_y}{\lambda\sqrt{f'_c}}\frac{\psi_t\psi_e\psi_s}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right)d_b = \left(\frac{3}{40}\frac{60{,}000}{1.0\sqrt{4000}}\frac{1.3 \times 1.0 \times 1.0}{2.34}\right)1.41 \quad = 40 \times 1.41 = 55.7 \text{ in.}$$

Solution:

(c) using the more accurate Eq. (6.4).

Therefore, the required development length is 55.7 × 2.90/3.12 = 55.7 × 0.93 = 52 in. rather than 81 in. as before.

Clearly, the use of the more accurate Eq. (6.4) permits *a considerable reduction* in development length. Even though its use requires much more time and effort, it is justified if the design is to be repeated many times in a structure.

ANCHORAGE OF TENSION BARS BY HOOKS

(a) Standard Dimensions

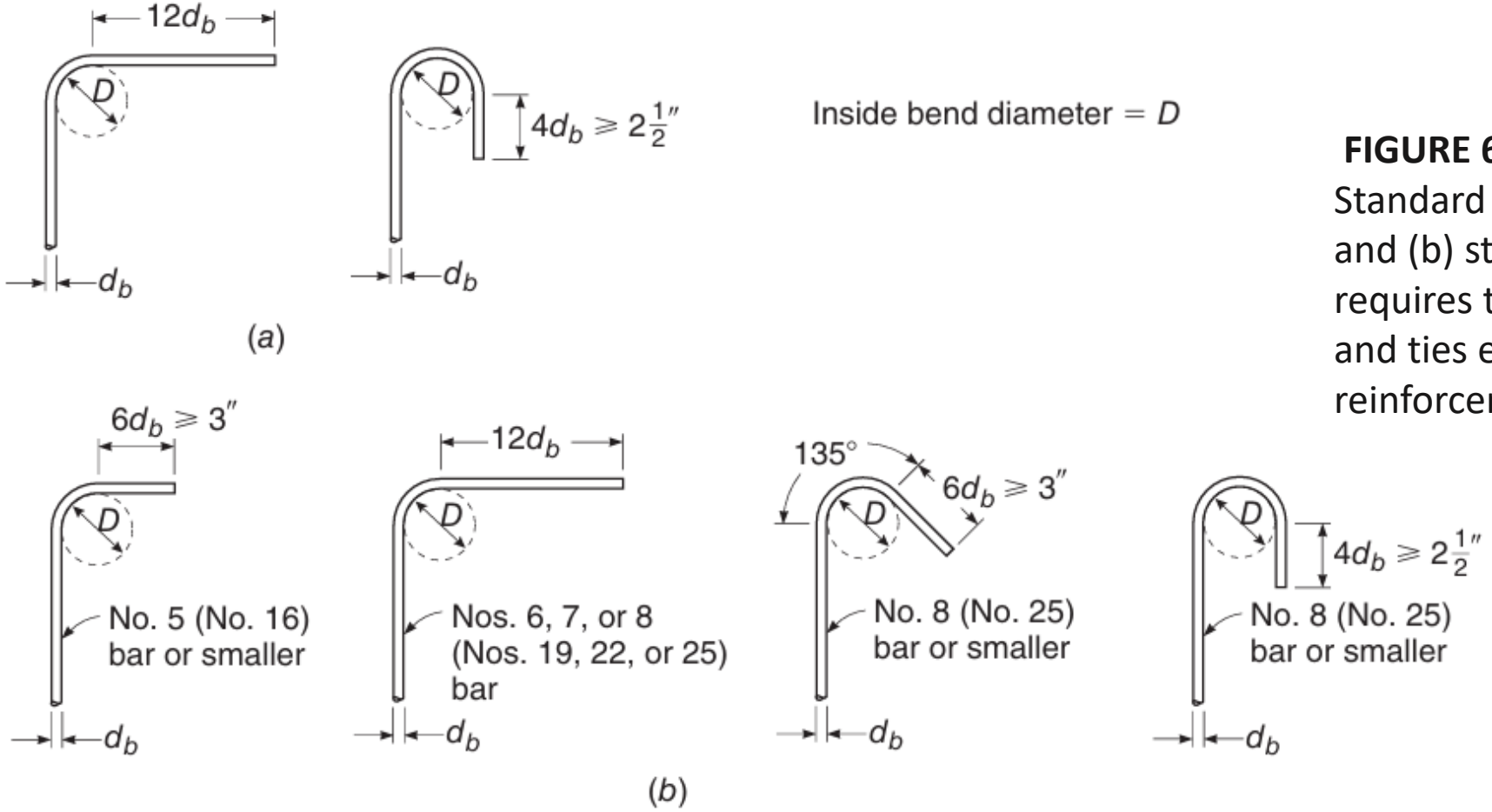


FIGURE 6.9
Standard bar hooks: (a) main reinforcement and (b) stirrups and ties. ACI Code 25.3.2 requires that standard hooks for stirrups and ties enclose longitudinal reinforcement.

ANCHORAGE OF TENSION BARS BY HOOKS

(a) Standard Dimensions

In the event that the ***desired tensile stress in a bar cannot be developed by bond along the length of the bar alone***, it is necessary to provide special anchorage at the ends of the bar, usually by means of a 90° or a 180° hook or a headed bar. The dimensions and bend radii for hooks have been standardized in ACI Code 25.3.1 and 25.3.2 as follows (see **Fig. 6.9**):

1. A 90° bend plus an extension of at least 12 bar diameters at the free end of the bar, or
2. A 180° bend plus an extension of at least 4 bar diameters, but not less than $2\frac{1}{2}$ in. at the free end of the bar, or

ANCHORAGE OF TENSION BARS BY HOOKS

(a) Standard Dimensions

3. For stirrup and tie anchorage only:
   a) For Nos. 3, 4, 5 (Nos. 10, 13, and 16) bars, a 90° bend plus an extension of at least 6 bar diameters, but not less than 3 in., at the free end of the bar, or
   b) For Nos. 6, 7, and 8 (Nos. 19, 22, and 25) bars, a 90° bend plus an extension of at least 12 bar diameters at the free end of the bar, or
   c) For No. 8 (No. 25) bars and smaller, a 135° bend plus an extension of at least 6 bar diameters, but not less than 3 in., at the free end of the bar, or
   d) For No. 8 (No. 25) bars and smaller, a 180° bend plus an extension of at least 4 bar diameters, but not less than 2.5 in., at the free end of the bar.

ANCHORAGE OF TENSION BARS BY HOOKS

(a) Standard Dimensions

The ***minimum diameter of bend***, measured on the inside of the bar, for standard hooks other than for stirrups or ties in sizes Nos. 3 through 5 (Nos. 10 through 16), should be not less than the values shown in **Table 6.2**. For stirrup and tie hooks, for bar sizes No. 5 (No. 16) and smaller, the inside diameter of bend is reduced to 4 bar diameters, according to ACI Code 25.3.2.

TABLE 6.2
Minimum inside bend diameters for standard hooks

Bar Size	Minimum Diameter
Nos. 3 through 8 (Nos. 10 through 25)	6 bar diameters[a]
Nos. 9, 10, and 11 (Nos. 29, 32, and 36)	8 bar diameters
Nos. 14 and 18 (Nos. 43 and 57)	10 bar diameters

[a] Nos. 3, 4, and 5 (Nos. 10, 13, and 16): 4 bar diameters for stirrups and ties.

ANCHORAGE OF TENSION BARS BY HOOKS

(a) Standard Dimensions

When welded wire reinforcement (smooth or deformed wires) is used for stirrups or ties, the inside diameter of bend should not be less than 4 wire diameters for deformed wire larger than D6 and 2 wire diameters for all other wires. *Bends with an inside diameter of less than 8 wire diameters should not be less than 4 wire diameters from the nearest welded intersection*.

ANCHORAGE OF TENSION BARS BY HOOKS

(b) Development Length and Modification Factors for Hooked Bars

Hooked bars resist pullout by the ***combined actions of bond along the straight length of bar leading to the hook and anchorage provided by the hook***. Tests indicate that the failure of hooked bars in tension is accompanied by **splitting of the concrete** both parallel and perpendicular to the plane of the hook. This splitting is due to **slip of the bar** ahead of the hook, transverse tensile stresses in the concrete on the inside of the hook, and tensile stresses in the concrete as portions of the concrete are pulled out in the direction of the tensile force in the hooked bar. Resistance to splitting has been found to depend on the **concrete cover for the hooked bar, measured laterally from the edge of the member to the bar perpendicular to the plane of the hook, and measured to the top (or bottom) of the member from the point where the hook starts, parallel to the plane of the hook**. Hook strength can be increased by providing confinement steel in the form of closed stirrups or ties.

ANCHORAGE OF TENSION BARS BY HOOKS

(b) Development Length and Modification Factors for Hooked Bars

The Code requirements (**ACI Code 25.4.3**) account for the combined contribution of bond along the straight bar leading to the hook, plus the hooked anchorage.For standard hooks, as shown in **Fig. 6.9**, the development length is,

Eq. (6.6)

$$\ell_{dh} = \left(\frac{f_y \psi_e \psi_c \psi_r}{50\lambda\sqrt{f'_c}}\right) d_b$$

In Eq. (6.6), the terms are defined values established as follows.

ψ_e = epoxy coating factor

For epoxy-coated or zinc and epoxy dual-coated reinforcement:	1.2
For uncoated and zinc-coated (galvanized) reinforcement:	1.0

ψ_c = cover factor

For No. 11 bar and smaller hooks with side cover (normal to plane of hook) $\geq 2\frac{1}{2}$ in. and for 90° hooks with cover on bar extension beyond hook $\geq$ 2 in.:	0.7
Other	1.0

ANCHORAGE OF TENSION BARS BY HOOKS

(b) Development Length and Modification Factors for Hooked Bars

Eq. (6.6)

$$\ell_{dh} = \left(\frac{f_y \psi_e \psi_c \psi_r}{50\lambda\sqrt{f'_c}}\right) d_b$$

ψ_r = confining reinforcement factor

For 90° hooks of No. 11 and smaller bars that are either (1) enclosed along ℓ_{dh} within ties or stirrups perpendicular to ℓ_{dh} at $s \le 3d_b$ or (2) enclosed along the bar extension beyond hook including the bend within ties or stirrups perpendicular to the tail extension of the hook at $s \le 3d_b$:	0.8
For 180-degree hooks of No. 11 and smaller bars enclosed along ℓ_{dh} within ties or stirrups perpendicular to ℓ_{dh} at $s \le 3d_b$:	0.8
Other:	1.0

Note: The first tie or stirrup shall enclose the bent portion of the hook within $2d_b$ of the outside of the bend, and d_b is the nominal diameter of the hooked bar.

λ = lightweight concrete factor

For lightweight concrete:	0.75
Other:	1.0

ANCHORAGE OF TENSION BARS BY HOOKS

(b) Development Length and Modification Factors for Hooked Bars

A total development length ℓ_{dh} is defined as shown in **Fig. 6.10** and is measured from the critical section to the farthest point on the bar, parallel to the straight part of the bar.

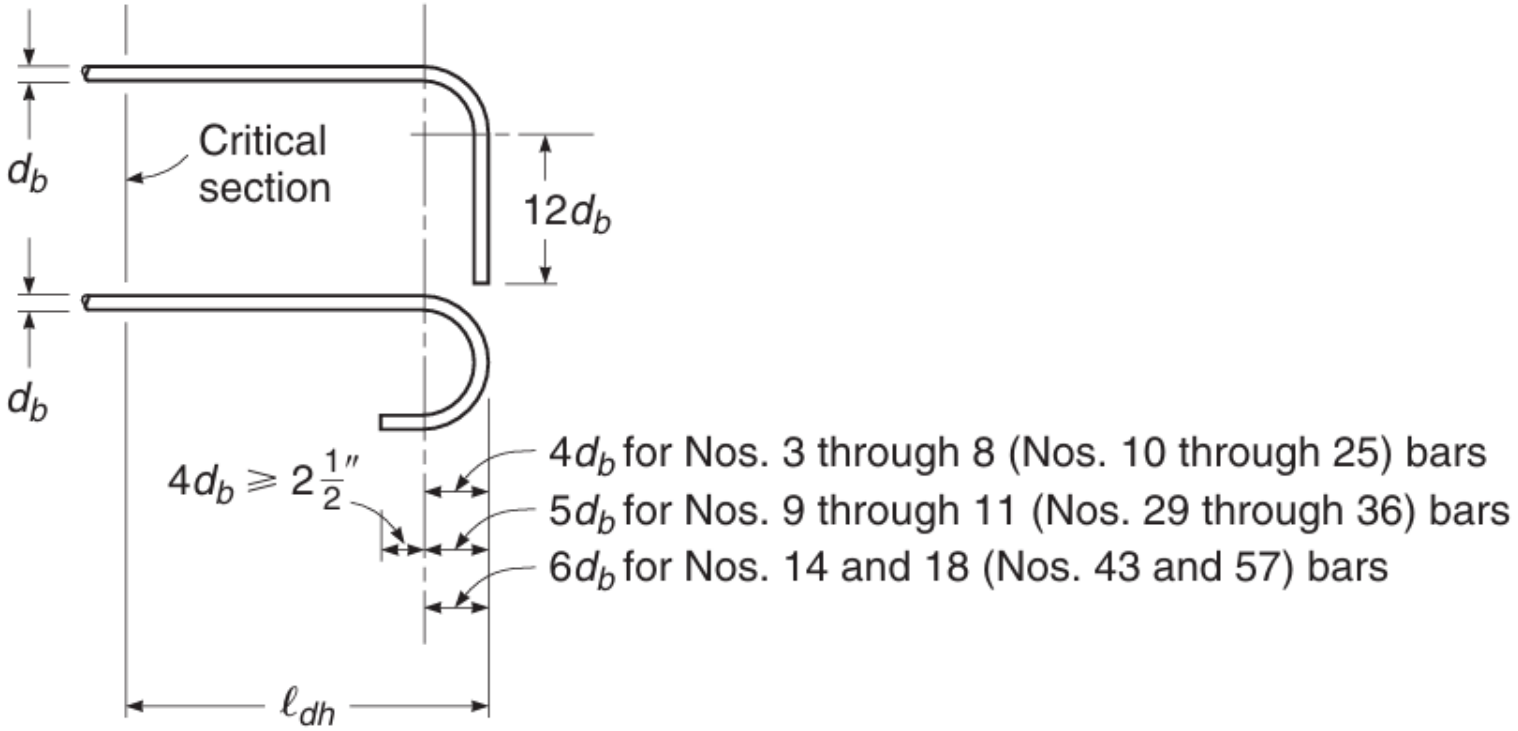


FIGURE 6.10
Bar details for development of standard hooks.

ANCHORAGE OF TENSION BARS BY HOOKS

(b) Development Length and Modification Factors for Hooked Bars

As for straight reinforcement in tension, ACI Code 25.4.10 permits the development length of a hook ℓ_{dh} to be reduced by the ratio (A_s required/A_s provided), except where anchorage or development for f_y is specifically required or in seismic-force-resisting systems in structures assigned to Seismic Design Categories D, E, or F.

Transverse confinement steel is essential if the full bar strength must be developed with *minimum concrete confinement*, such as when hooks may be required at the ends of a simply supported beam or where a beam in a continuous structure frames into an end column and does not extend past the column or when bars must be anchored in a short cantilever, as shown in **Fig. 6.11**.

ANCHORAGE OF TENSION BARS BY HOOKS

(b) Development Length and Modification Factors for Hooked Bars

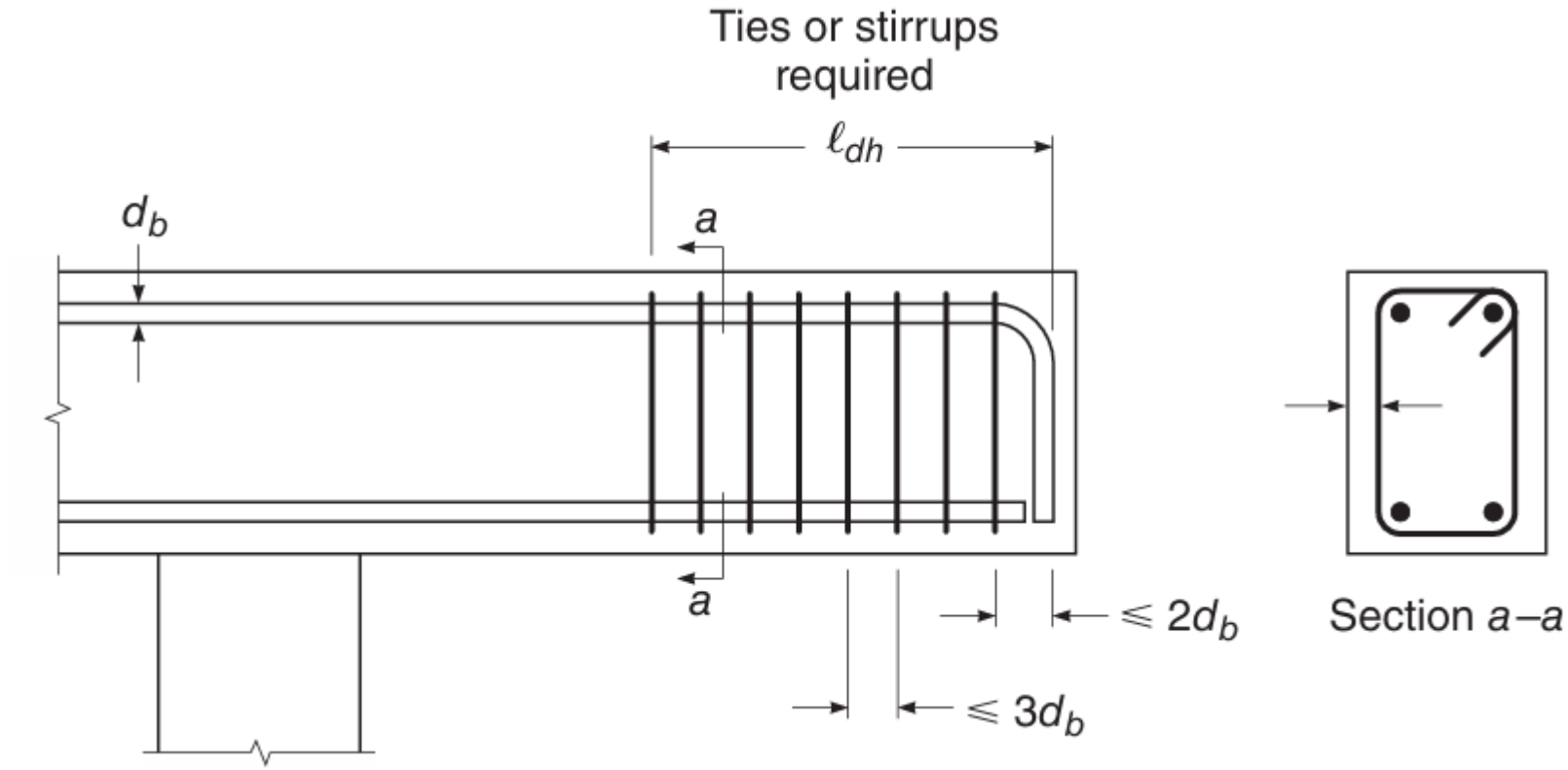


FIGURE 6.11

Transverse reinforcement requirements at discontinuous ends of members with small cover distances.

ANCHORAGE OF TENSION BARS BY HOOKS

(b) Development Length and Modification Factors for Hooked Bars

According to ACI Code 25.4.3.3, for bars hooked at the **discontinuous ends** of members with both side cover and top or bottom cover less than $2\frac{1}{2}$ in., hooks must be enclosed with closed stirrups or ties along the full development length, as shown in **Fig. 6.11**. The spacing of the confinement steel must not exceed **3 times the diameter** of the hooked bar d_b , and the first stirrup or tie must enclose the bent portion of the hook within a distance equal to $2d_b$ of the outside of the bend. In such cases, the confining reinforcement factor ψ_r of 0.8 does not apply.

EXAMPLE 6.2 **Development of hooked bars in tension.** Referring to the beam-column joint shown in Fig. 6.8, the No. 11 (No. 36) negative bars are to be extended into the column and terminated in a standard 90° hook, keeping 2 in. clear to the outside face of the column. The column width in the direction of beam width is 16 in. Find the minimum length of embedment of the hook past the column face, and specify the hook details.

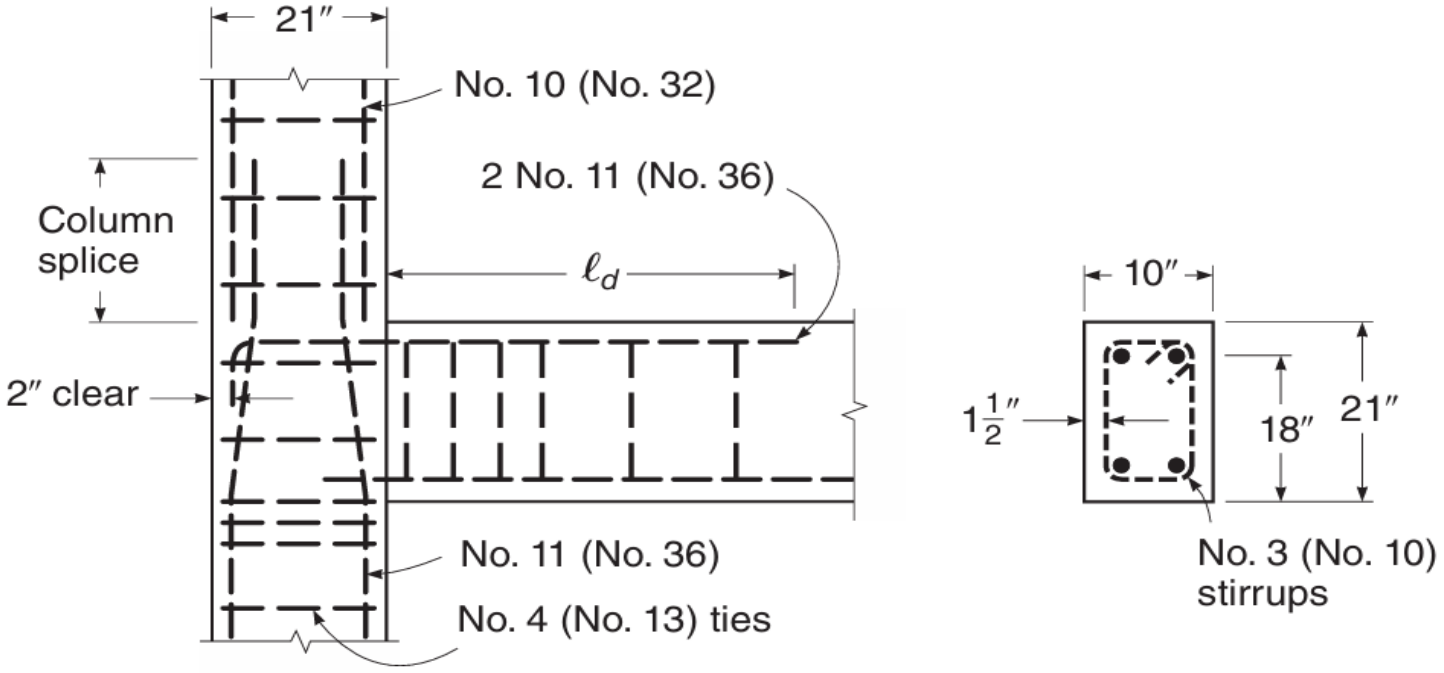


FIGURE 6.8
Bar details at beam-column joint for bar development examples.

Solution:

The development length for hooked bars, measured from the critical section along the bar to the far side of the vertical hook, is given by **Eq. (6.6)**. In this case, the bars are uncoated, giving an epoxy coating factor of $\psi_e = 1.0$, the side cover for the No. 11 (No. 36) bars exceeds 2.5 in. and cover beyond the bent bar is adequate, so the cover factor $\psi_c = 0.7$, and the confining reinforcement is such that $\psi_r = 1.0$. This gives

$$\ell_{dh} = \left(\frac{f_y \psi_e \psi_c \psi_r}{50\lambda\sqrt{f'_c}}\right) d_b = \left(\frac{60{,}000 \times 1.0 \times 0.7 \times 1.0}{50 \times 1.0 \times \sqrt{4000}}\right) 1.41 = 13.28 \times 1.41 = 19 \text{ in.}$$

The only other factor applicable is for excess reinforcement, which is 0.93 as for Example 6.1. Accordingly, the minimum development length for the hooked bars is

$$\ell_{dh} = 19 \times 0.93 = 18 \text{ in.}$$

Solution:

The available length within the column is 21 − 2 = 19 in. and can thus accommodate ℓ_{dh}. [$\because \ell_{dh} = 18$ in.]

Note that although ℓ_{dh} is less that the available length, the hook should be placed to the far side of the column to use the full 19 in. available. The hook will be bent to a **minimum diameter** of 8 × 1.41 = 11.28 in. The bar will continue for 12 bar diameters, or 17 in. past the end of the bend in the vertical direction.

Headed bars to hooked

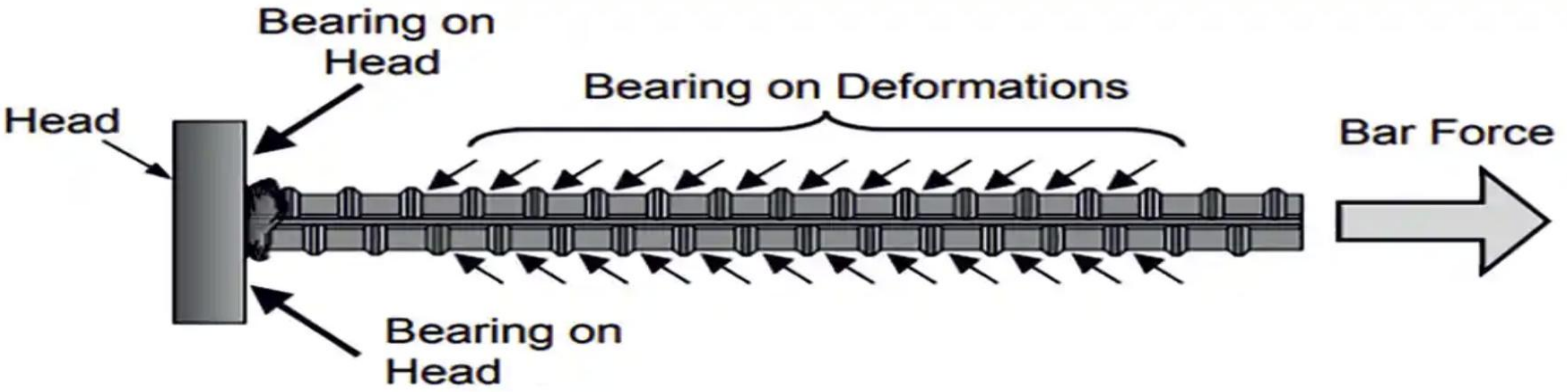


FIGURE 6.14
Development length of headed deformed bars.

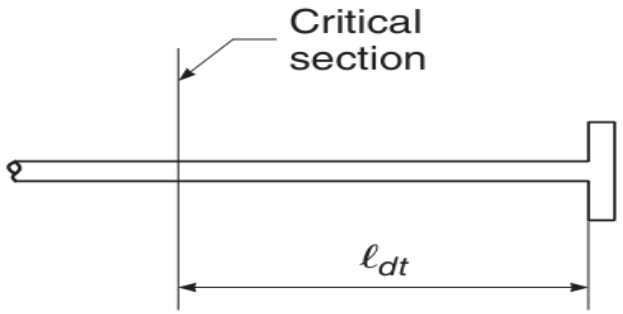


FIGURE 6.15
Headed deformed bar extended to far side of column with anchorage length that exceeds ℓ_{dt}.

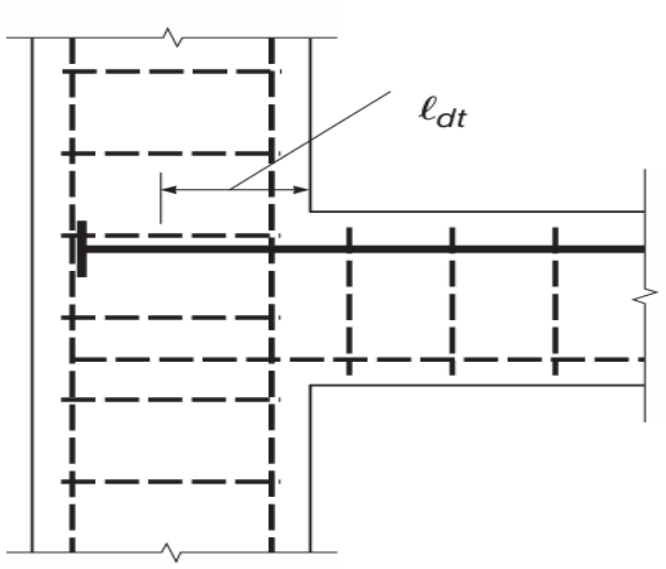


TABLE 6.3
Development lengths for headed deformed bars in tension

A. Development length ℓ_{dt} for headed bars	$\left(\frac{0.016 f_y \psi_e}{\sqrt{f'_c}}\right) d_b$
B. Modification factors applied to ℓ_{dt}	
ψ_e	
For epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
For other bars	1.0

length ℓ_{dt} is limited to 6000 psi. The development length for headed deformed bars in tension is

$$\ell_{dt} = \left(\frac{0.016 f_y \psi_e}{\sqrt{f'_c}} \right) d_b \qquad (6.7)$$

where $\psi_e = 1.2$ for epoxy-coated reinforcement and 1.0 for other cases.

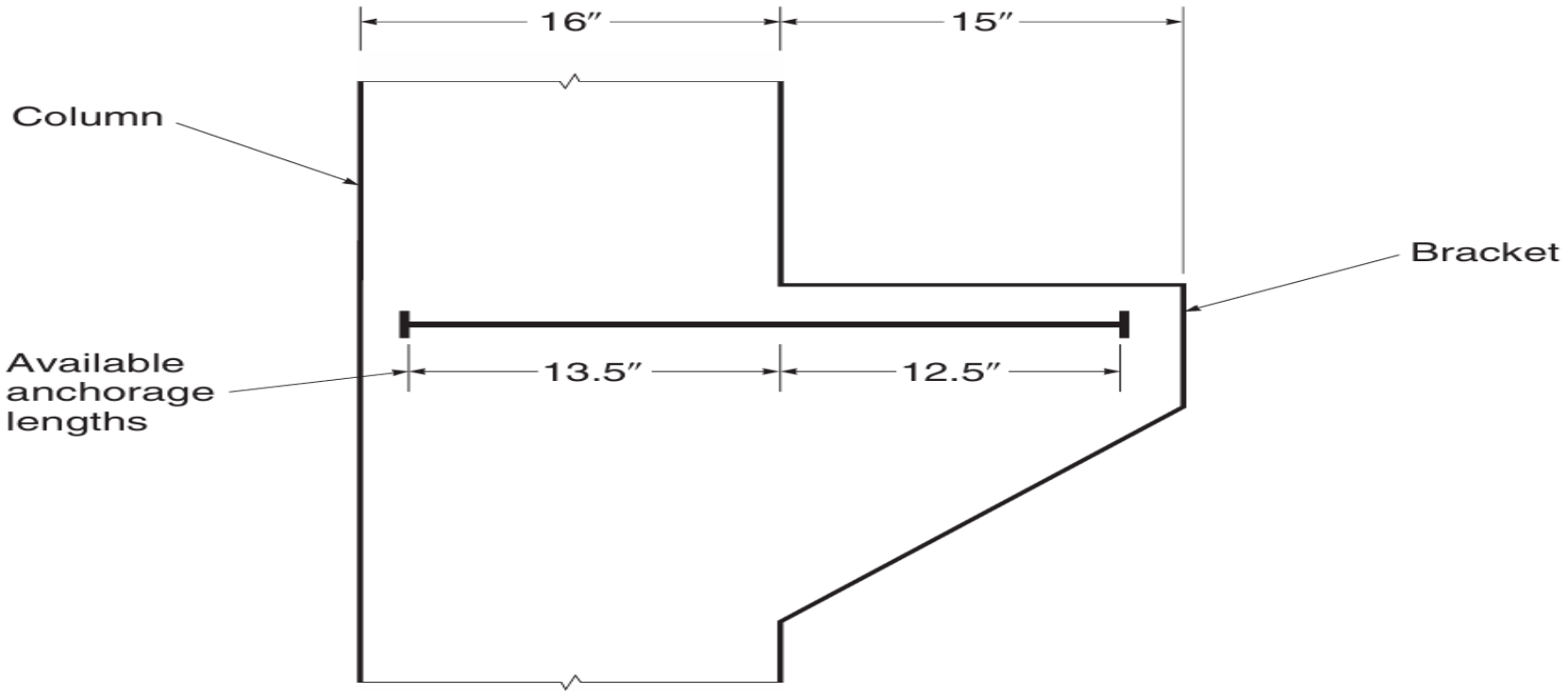
Unlike the development lengths of straight and hooked bars, the value of ℓ_{dt} may not be reduced by the factor A_s requires/A_s provided. Under any circumstances, ℓ_{dt} may not be less than 8 bar diameters or less than 6 in. Calculation of the development length ℓ_{dt} and the applicable modifying factors are summarized in Table 6.3.

EXAMPLE 6.3 **Development of headed deformed bars in tension.** Three No. 7 (No. 22) bars serve as top reinforcement for a bracket framing into a 16 × 16 in. column (Fig. 6.16). The bracket projects 15 in. from the column and is the same width as the column. The top cover to the center of the bars is 3 in., and the side cover to the center of the bars is 3.5 in. The bars are spaced laterally at 4.5 in. These dimensions are inadequate for straight development length or for standard hooks. Based on other reinforcement, cover requirements, and head thickness, total development lengths for headed bars of 13.5 in. in the column and 12.5 in. in the bracket are available. The reinforcing bars have $f_y = 60{,}000$ psi, and the concrete is normalweight with $f'_c = 5000$ psi. Determine if a bar with heads at both ends can be used in this application.

Bond, Anchorage and Development Length

SOLUTION. The minimum head size is $A_{brg} = 4A_b = 2.4$ in^2. The smaller available anchorage length in the bracket governs. The development length ℓ_{dt} calculated using Eq. (6.7) is

$$\ell_{dt} = \left(\frac{0.016 f_y \psi_e}{\sqrt{f'_c}}\right) d_b = \left(\frac{0.016 \times 60{,}000 \times 1.0}{\sqrt{5000}}\right) 0.875 = 11.9 \text{ in.}$$



which must be checked against the minimum values for ℓ_{dt}, which are

$$\ell_{dt} \geq 8d_b = 7 \text{ in.}$$

$$\ell_{dt} \geq 6 \text{ in.}$$

The value of ℓ_{dt} obtained using Eq. (6.7) governs and is less than the available anchorage lengths in both the column and the bracket. Thus, a bar with heads at both ends can be used. As shown in Fig. 6.16, the heads are located at the far faces of the column and the bracket, with a distance between the faces of the heads of 26 in.

Thank You