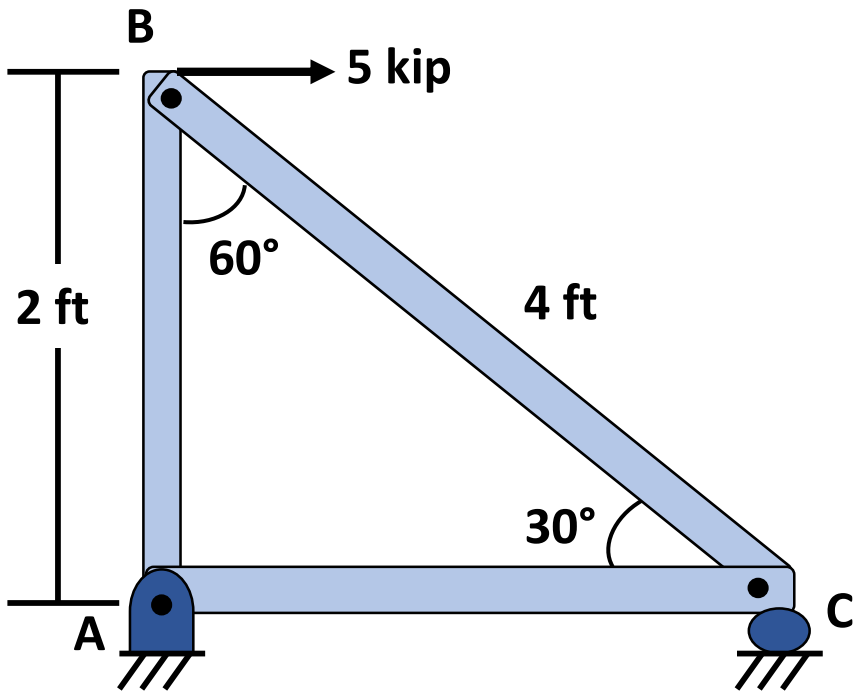
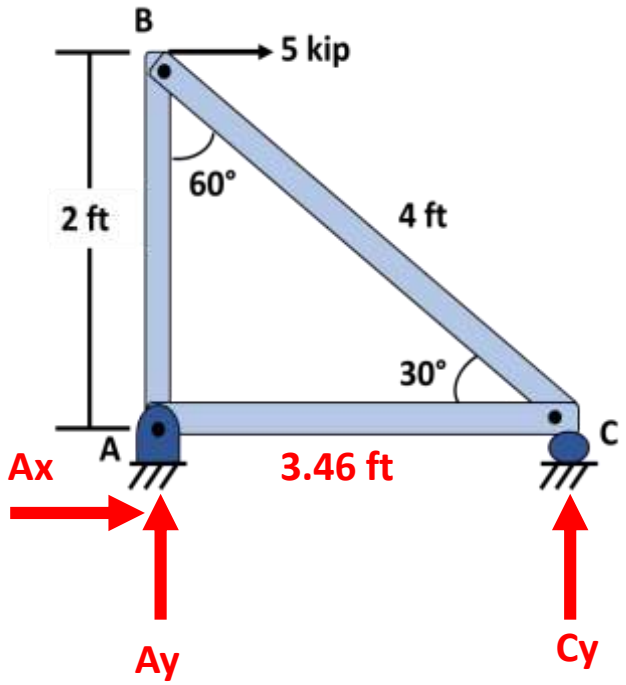
Deflections by Virtual Work Method

Example-1: Determine the horizontal displacement at point B. E=29000 ksi and A=0.2 in^2





$\sum M_A = 0$

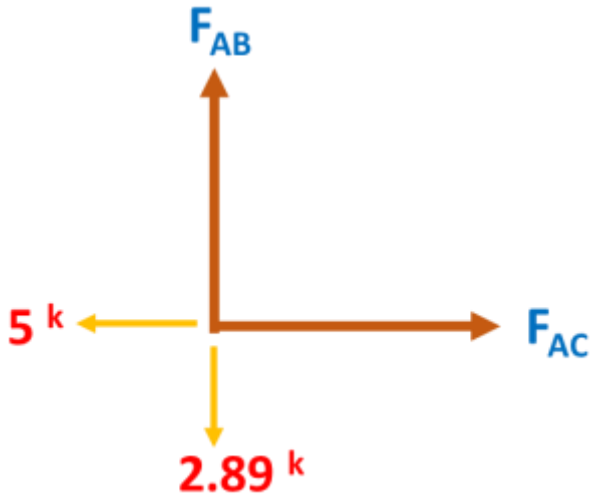
or, $-5 \times 2 + C_y \times 3.46 = 0$

$\therefore C_y = 2.89$ k

$A_y = -2.89$ k

$A_x = -5$ k

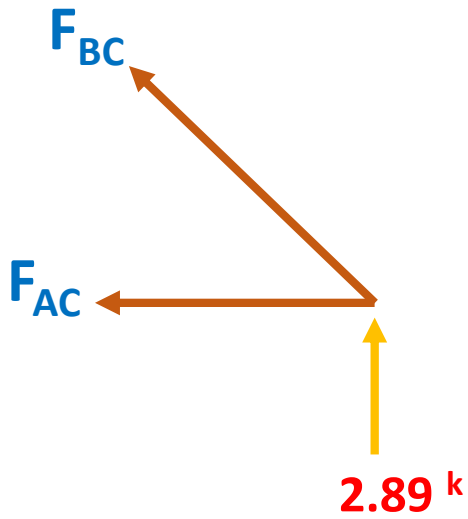
Joint A



$F_{AB} = 2.89$ k

$F_{AC} = 5$ k

Joint C



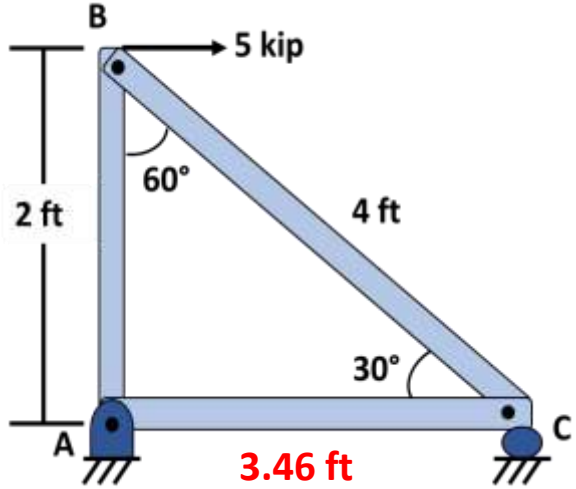
$\sum F_y = 0$

or, $2.89 + F_{BC} \sin 30° = 0$

$\therefore F_{BC} = -5.78$ k

$W_e = U_i$

$$\Rightarrow \frac{1}{2}P(\Delta_B) = \sum \int_0^{l_i} \frac{P_i^2}{2E_iA_i}\,dx = \sum \frac{P_i^2 L_i}{2E_iA_i}$$



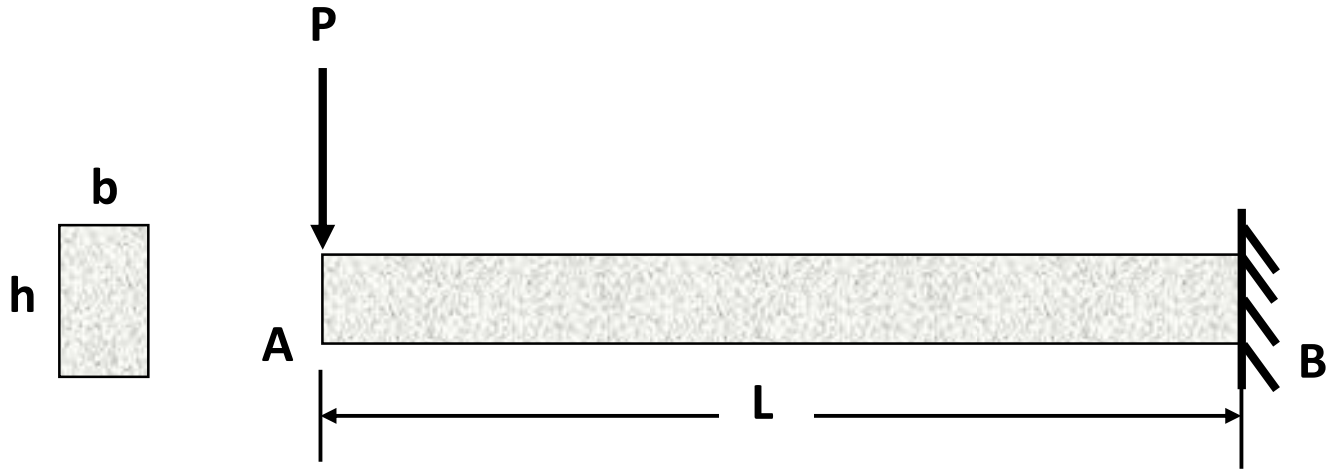
$$\Rightarrow \frac{1}{2} \times 5 \times \Delta_B = \frac{1}{2EA}\left(F_{AB}^2 L_{AB} + F_{BC}^2 L_{BC} + F_{AC}^2 L_{AC}\right)$$

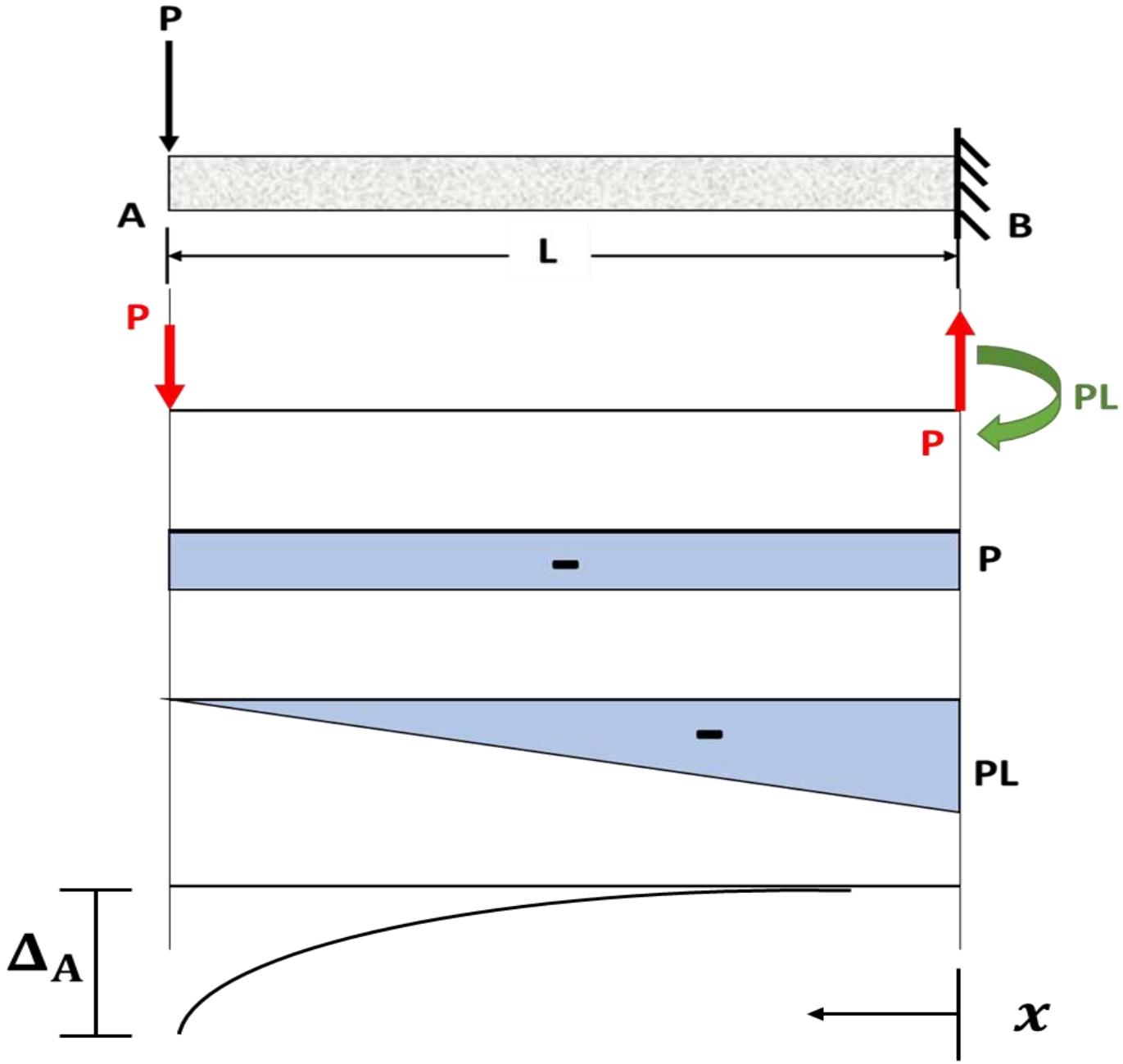
EA= 5800 kip

$$\Rightarrow 2.5\,\Delta_B = \frac{1}{2\times 5800}\left(2.89^2 \times 2 + (-5.78)^2 \times 4 + 5^2 \times 3.46\right)$$

$\therefore \Delta_B = 8.17 \times 10^{-3}$ ft = 0.098 in **(+ve→ same direction of the load)**

Example-2: Determine the vertical displacement at point A. The beam is made of a material that has a modulus of elasticity E and a shear modulus of elasticity G.




P
A
B
L
P
PL
P
P
PL
Δ_A
x

$$W_e = U_i$$

$$\text{or, } \frac{1}{2}P(\Delta_A) = \int_0^l \frac{f_s V^2}{2GA}\,dx + \int_0^l \frac{M^2}{2EI}\,dx$$

$$= \int_0^l \frac{1.2(-P)^2}{2GA}\,dx + \int_0^l \frac{(-Px)^2}{2EI}\,dx$$

$$= \left[\frac{1.2P^2x}{2GA}\right]_0^l + \frac{P^2}{2EI}\left[\frac{x^3}{3}\right]_0^l$$

$$\text{or, } \frac{1}{2}P(\Delta_A) = \frac{1.2P^2L}{2GA} + \frac{P^2L^3}{6EI}$$

fs=1.2(for rectangular section)

$$\therefore \Delta_A = \frac{1.2PL}{GA} + \frac{PL^3}{3EI}$$

Shear Moment

$$A = \text{bh}$$

$$\text{I} = \frac{\text{bh}^3}{12}$$

$$G = \frac{E}{2(1+\nu)}$$

If $l \gg$h $(\frac{l}{h} > 2.5)$→ shear is ignored **(Slender beam)**

If h ≈ l $(\frac{l}{h} < 2.5)$ → shear can't be ignored **(Deep beam)**

Shear is ignored if,

$$\Rightarrow \frac{6Pl}{5GA} << \frac{Pl^3}{3EI}$$

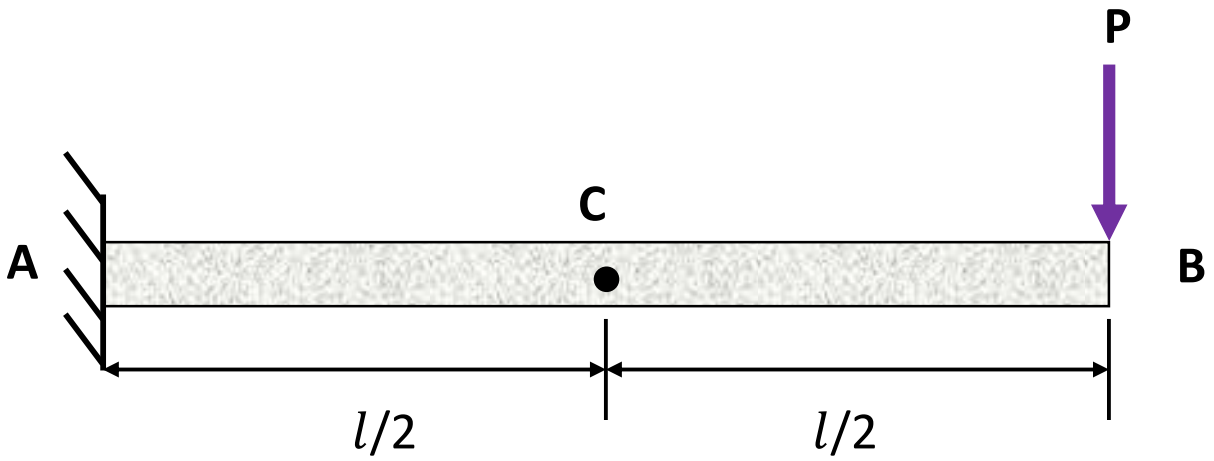
$$\Rightarrow \frac{6}{5Gbh} << \frac{l^2}{3E\frac{\text{bh}^3}{12}}$$

$$\Rightarrow \frac{l^2}{h^2} \gg \frac{E}{G} x \frac{3}{10}$$

In this case, all are slender beam so shear is ignored.

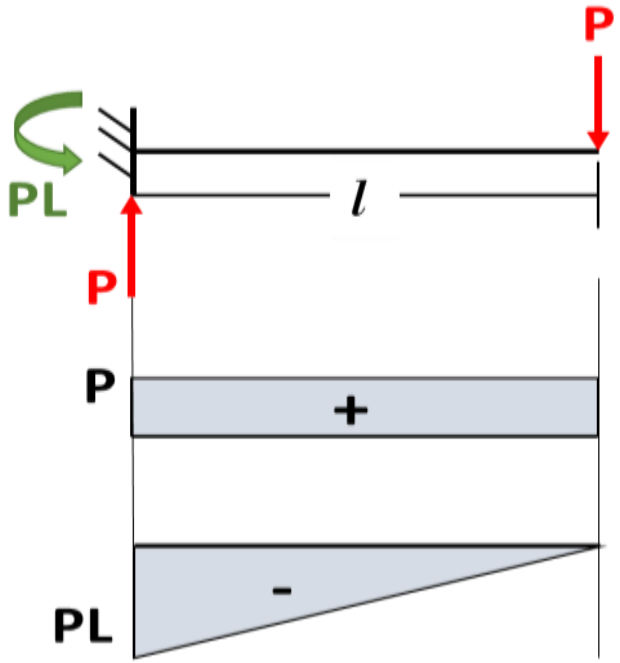
$$\therefore \Delta_A = \frac{Pl^3}{3EI}$$

Example-3: Calculate the rotation at point B and the deflection at C. EI is constant.

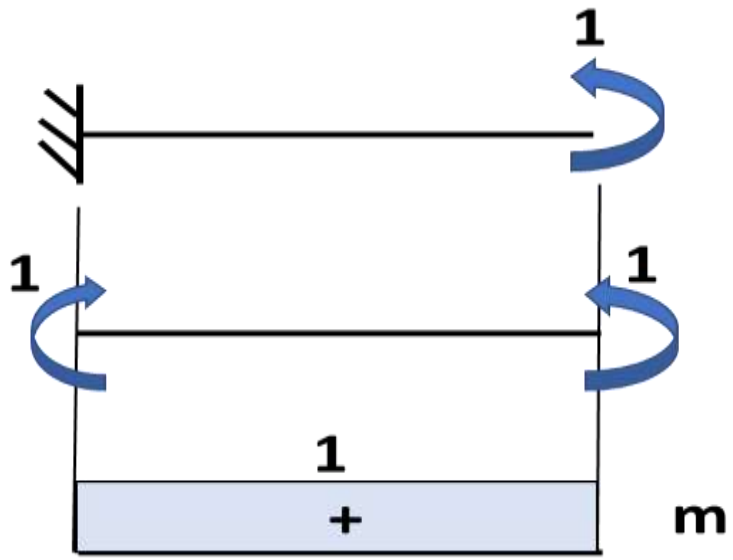


Rotation at B, θ_B

Real structure



Virtual structure

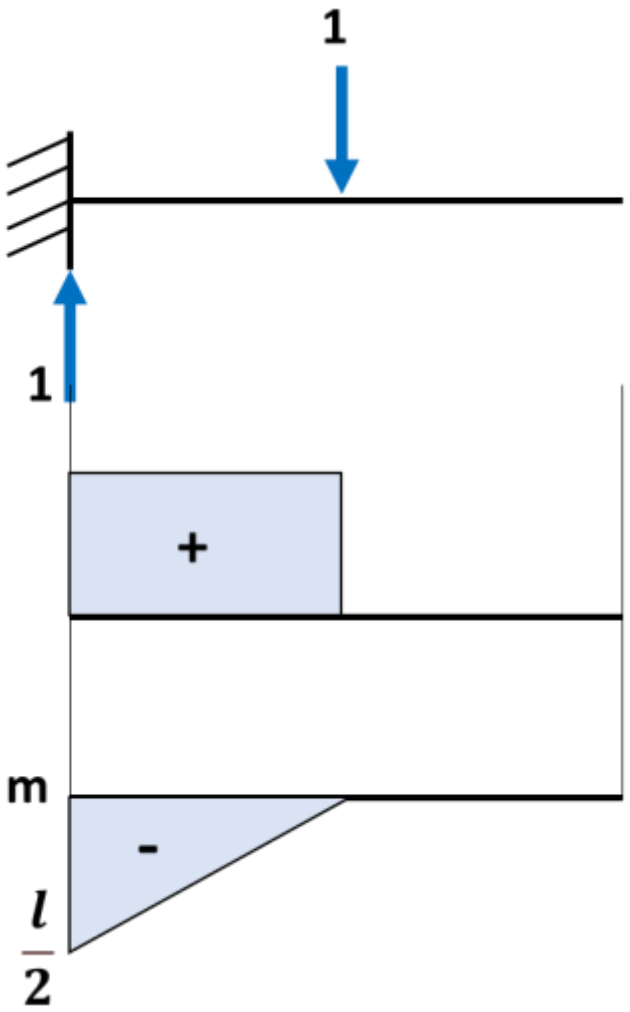


$$\theta_B = \int_0^l \frac{mM}{EI}\,dx = \frac{1}{EI}\left[\begin{array}{cc} 1 & Pl \\ + & - \end{array}\right]$$

$$= \frac{1}{EI}\left[(1 \times l)\left(\frac{1}{2} \times l \times -Pl\right)\right]$$

$$\boldsymbol{\theta_B = -\frac{Pl^3}{2EI}}$$

Deflection at C, Δ_C



$$\Delta_C = \int_0^l \frac{mM}{EI}\,dx$$

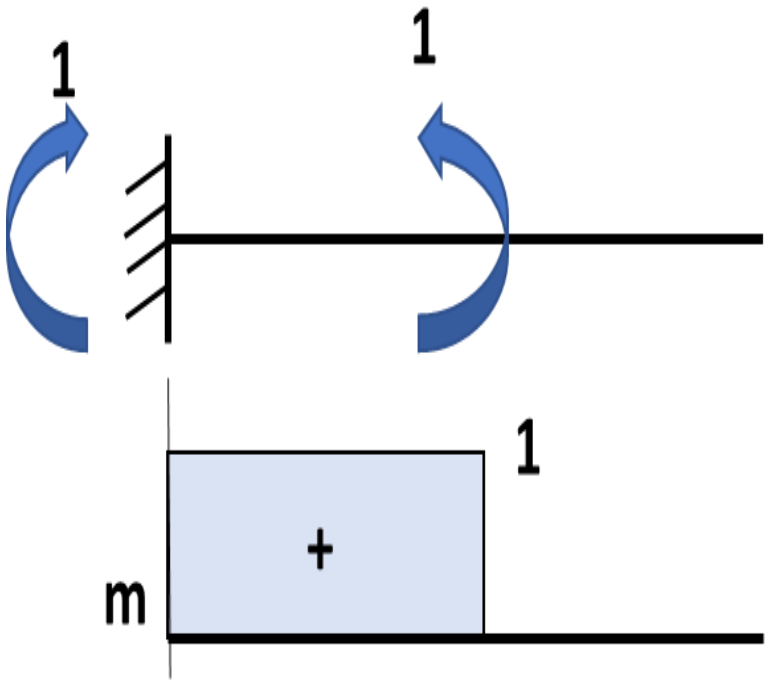
$$= \int_0^l \frac{\text{(m)}\quad\text{(M)}}{EI}\,dx$$

$$= \int_0^{l/2} \frac{\overset{l/2}{\text{(m)}}\quad\overset{-Pl}{\text{(M)}}}{EI}\,dx + \int_0^{l/2} \frac{\overset{0}{\text{(m)}}\quad\overset{-Pl/2}{\text{(M)}}}{EI}\,dx$$

$$= \frac{1}{EI}\left[\frac{1}{6}\left(-\frac{l}{2}\right)\left(-\frac{Pl}{2} + (-2Pl)\right)\frac{l}{2}\right]$$

$$= \frac{5Pl^3}{48EI}\,(+ve)$$

Rotation at C, θ_C



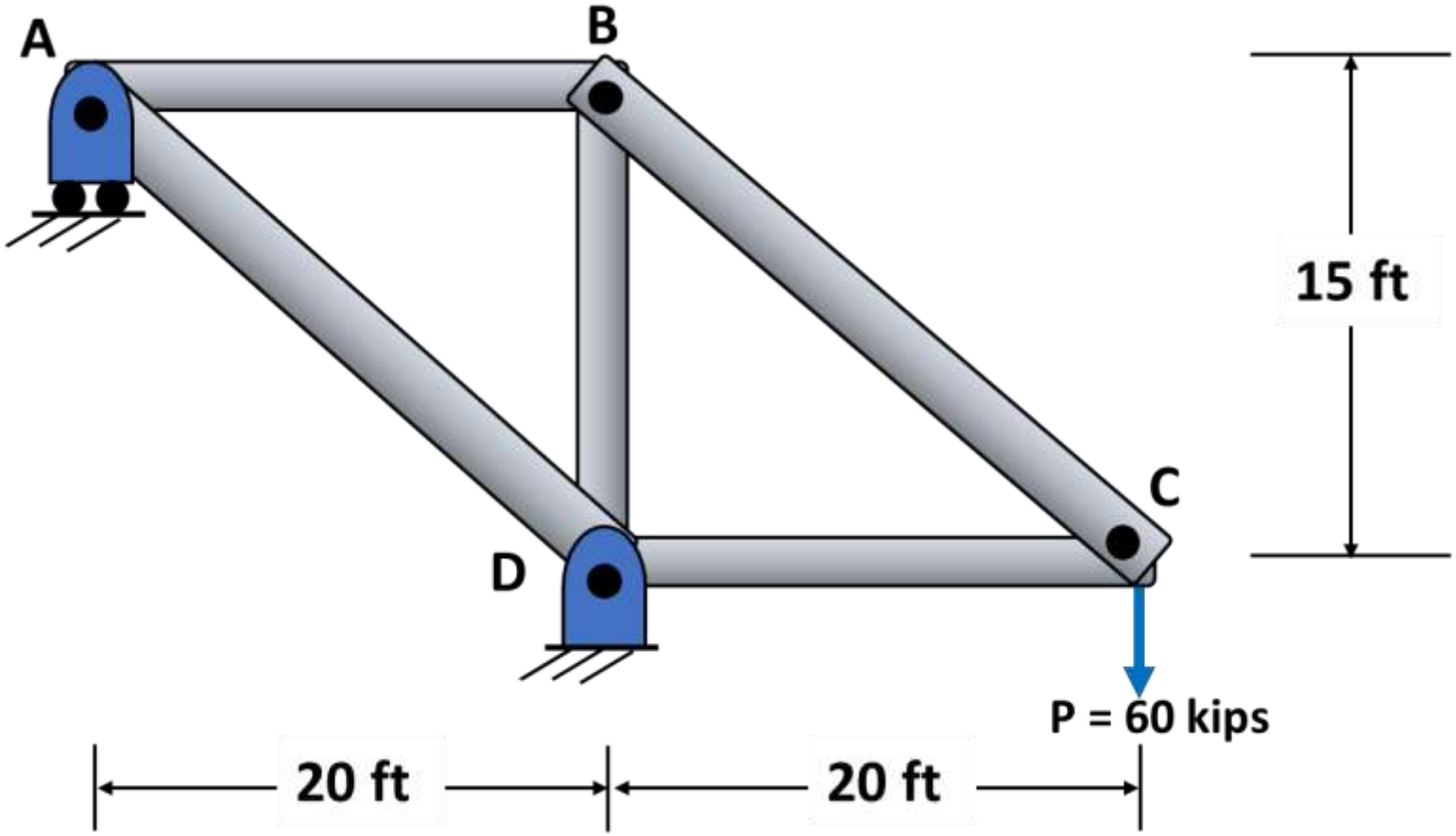
$$\theta_c = \int_0^l \frac{(+)\,1 \quad (-)\,Pl}{EI}\,dx$$

$$= \int_0^{l/2} \frac{(+)\,1 \quad (-)\,-Pl/2}{EI}\,dx + \int_0^{l/2} \frac{0 \quad (-)\,-Pl/2}{EI}\,dx$$

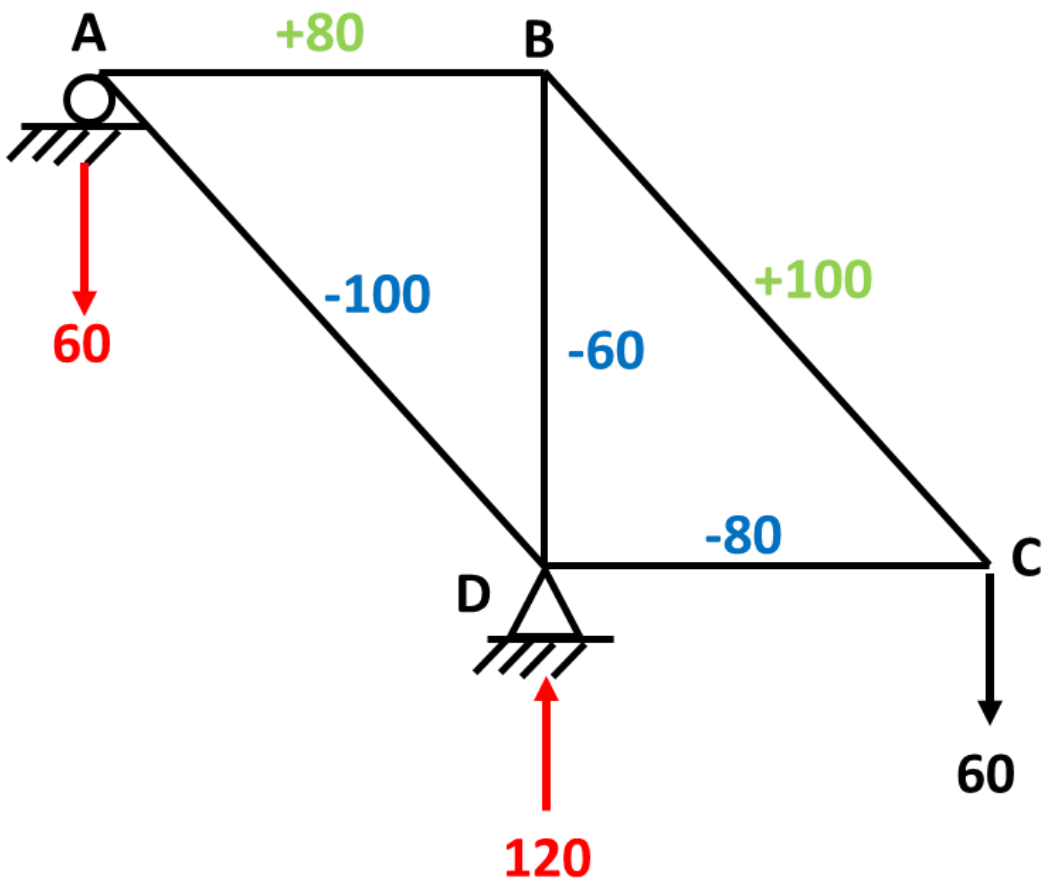
$$= \frac{1}{EI}\left[\frac{1}{2}(1)\left(-\frac{Pl}{2} - Pl\right)\left(\frac{l}{2}\right)\right]$$

$$= \frac{3Pl^2}{8EI}$$

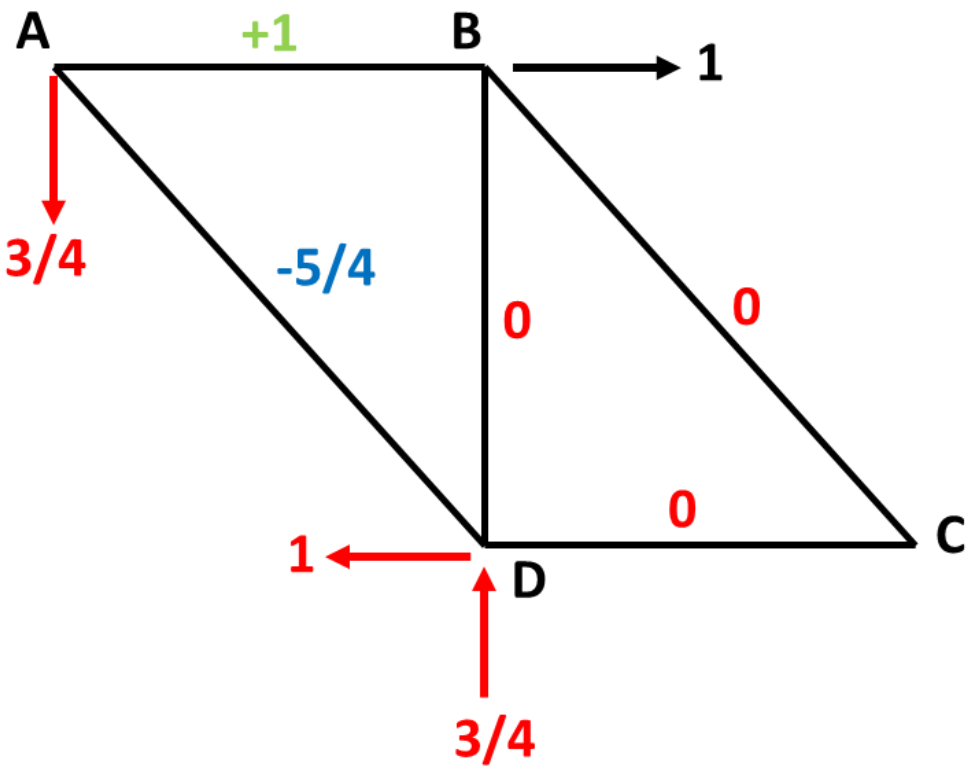
Example-4: Calculate the horizontal displacement at joint B. Members AD and BC has an area of 5 in^2 and all other members have an area of 4 in^2. E= 30,000 ksi.



Real Structure



Virtual Structure



$$(\Delta_B)_h = \sum_M \int_0^l \frac{Nn\,dx}{AE} = \sum_M \frac{NnL}{AE}$$

Member	E (ksi)	A (in²)	L (in)	N (k)	n (k)	$\frac{NnL}{AE}$
AB	30000	4	240	80	1	0.16
AD	30000	5	300	-100	-5/4	0.25
BD	30000	4	180	-60	0	0
BC	30000	5	300	100	0	0
CD	30000	4	240	-80	0	0

∑=0.41

$(\Delta_B)_h$ =0.41 in

Prob: 9-26. Determine the displacement and slope at point C of the cantilever beam. The moment of inertia of each segment is indicated in the figure. Take $E=29(10^3)$ ksi. Use the principle of virtual work**. (Hibbeler)**

