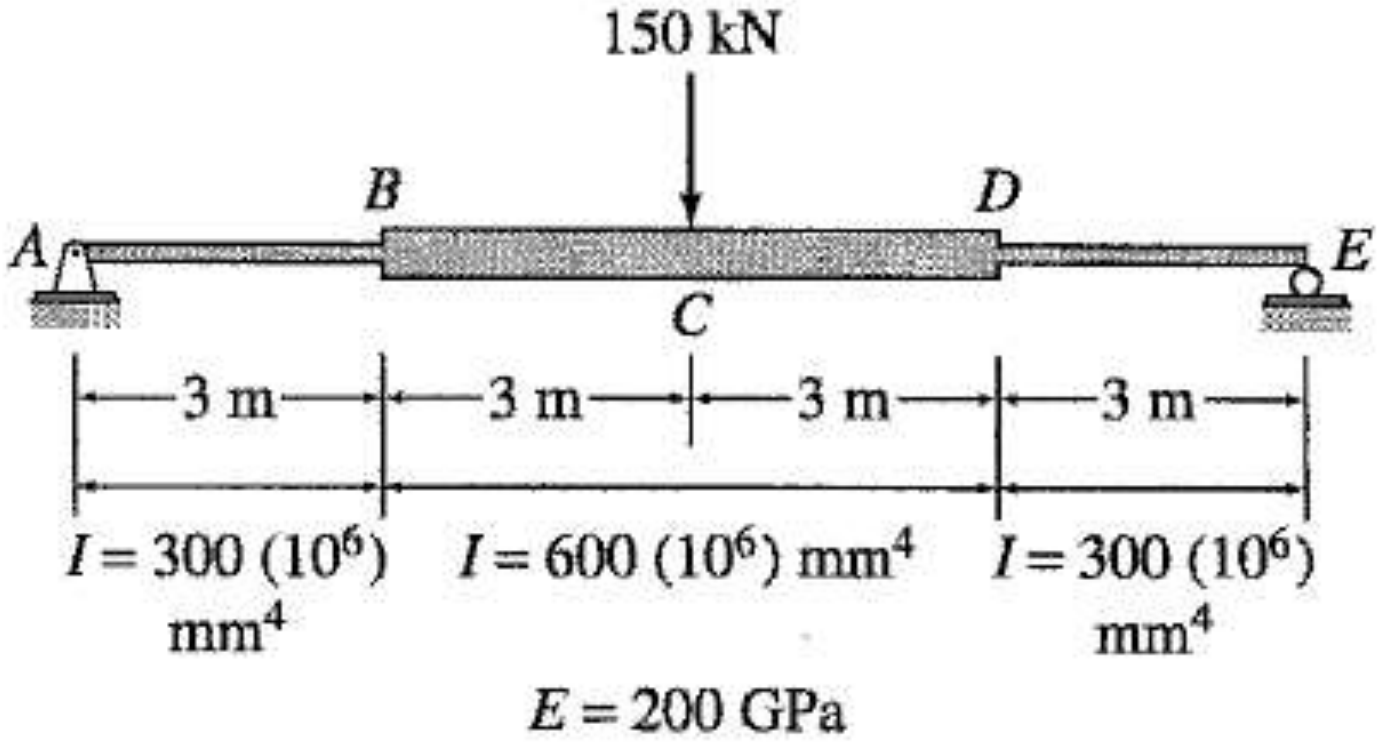
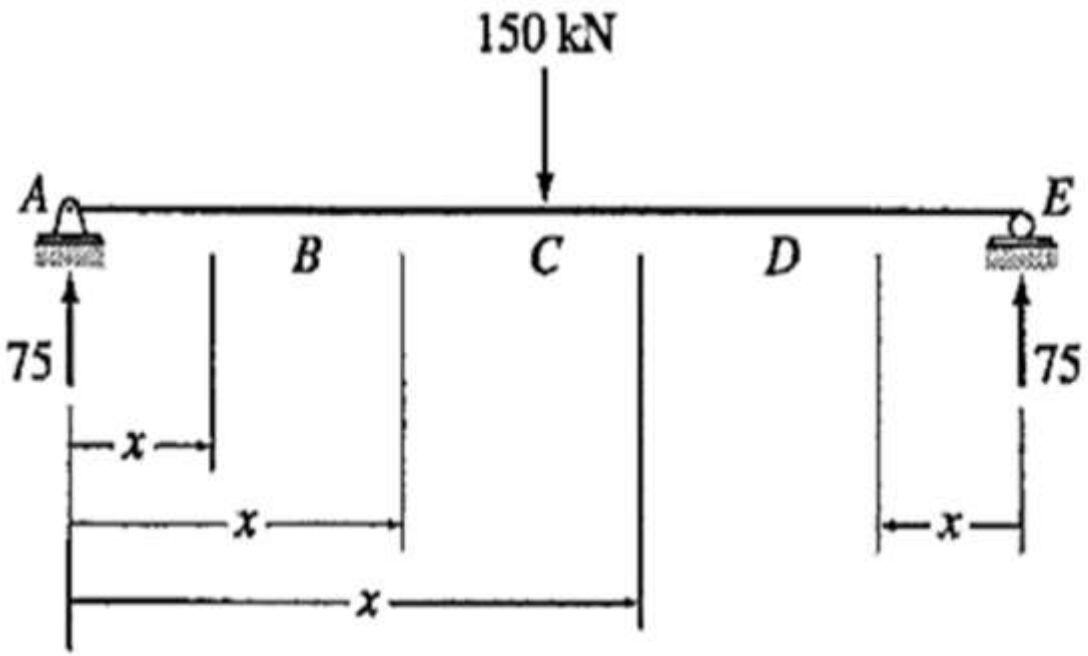
Problem: Determine the deflection at point D of the beam shown in the figure by the virtual work method.

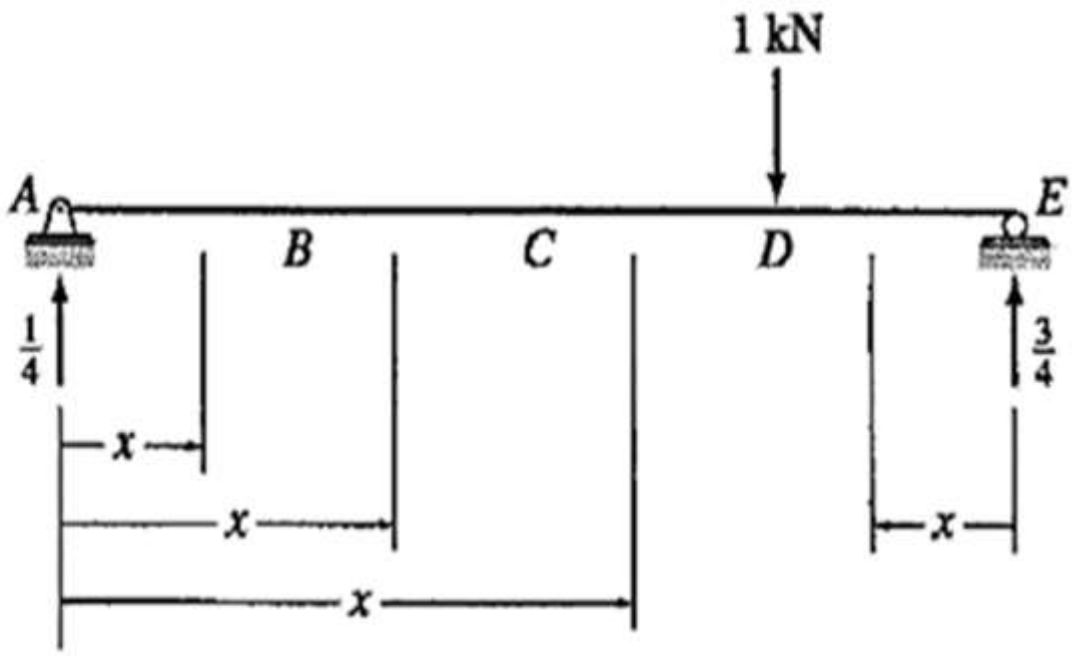


(a)

Solution



(b) Real System — M



(c) Virtual System — M_v

The x coordinates selected for determining the bending moment equations are shown in Fig. (b) and (c). Note that in any particular segment of the beam, the same x coordinate must be used to write both equations—that is, the equation for the real bending moment (M) and the equation for the virtual bending moment (M_v).

The real and virtual systems are shown in Fig. (b) and (c), respectively. It can be seen from Fig. (a) that the flexural rigidity EI of the beam changes abruptly at points B and D. Also, Fig. (b) and (c) indicates that the real and virtual loadings are discontinuous at points C and D, respectively. Consequently, the variation of the quantity $(\mathrm{M_v M/EI})$ will be discontinuous at points B; C, and D. Thus, the beam must be divided into four segments, AB; BC; CD, and DE; in each segment the quantity $(\mathrm{M_v M/EI})$ will be continuous and, therefore, can be integrated.

The deflection at D can now be computed by applying the virtual work expression:

$$1(\Delta_D) = \int_0^L \frac{M_v M}{EI} dx$$

$$1(\Delta_D) = \frac{1}{EI}\left[\int_0^3 \left(\frac{x}{4}\right)(75x)\,dx + \frac{1}{2}\int_3^6 \left(\frac{x}{4}\right)(75x)\,dx + \frac{1}{2}\int_6^9 \left(\frac{x}{4}\right)(-75x+900)\,dx + \int_0^3 \left(\frac{3}{4}x\right)(75x)\,dx\right]$$

$$(1\text{ kN})\Delta_D = \frac{2{,}193.75\text{ kN}^2\text{–m}^3}{EI}$$

Therefore,

$$\Delta_D = \frac{2{,}193.75\text{ kN–m}^3}{EI} = \frac{2{,}193.75}{200(300)} = 0.0366\text{ m} = 36.6\text{ mm}$$

$$\Delta_D = 36.6\text{ mm} \downarrow$$

The equations for M and Mv for the four segments of the beam, determined by using the method of sections. are tabulated in the table.

Segment	x Coordinate: Origin	x Coordinate: Limits (m)	EI ($I = 300 \times 10^6$ mm^4)	M (kN–m)	M_v (kN–m)
AB	A	0–3	EI	$75x$	$\frac{x}{4}$
BC	A	3–6	$2EI$	$75x$	$\frac{x}{4}$
CD	A	6–9	$2EI$	$75x - 150(x - 6)$	$\frac{x}{4}$
ED	E	0–3	EI	$75x$	$\frac{3}{4}x$