EWCE-311

Structural Analysis and Design-I

Determinate truss analysis

Lecture 2

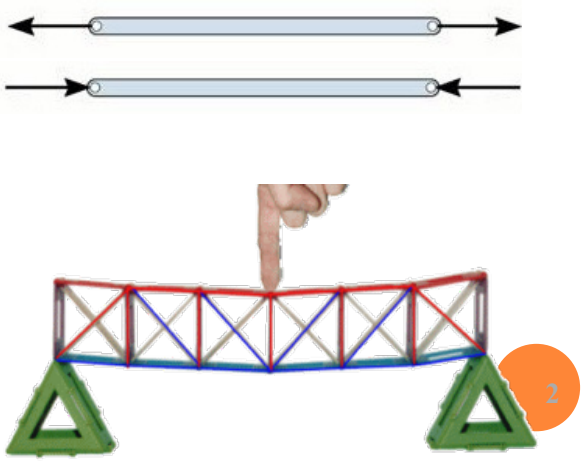
Instructor: Prof Dr. Noor M S Hasan

Analysis of Truss

Truss: is a structure composed of slender members (two-force members) joined together at their end points to support stationary or moving load.

- Each member of a truss is usually of uniform cross section along its length.



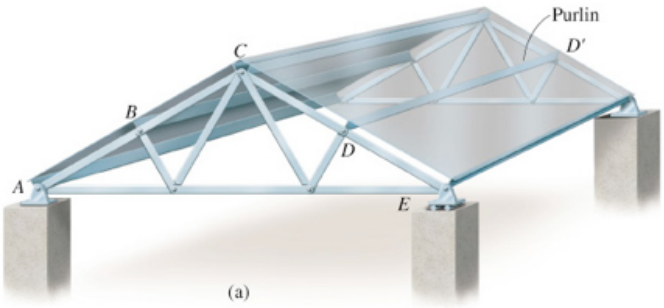


Types of Truss:

Types of Trusses

(a)Plane Truss

(b)Space Truss

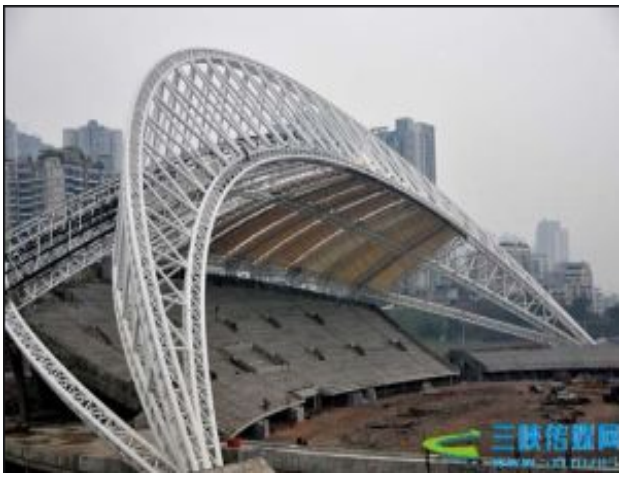


Analysis of Truss:

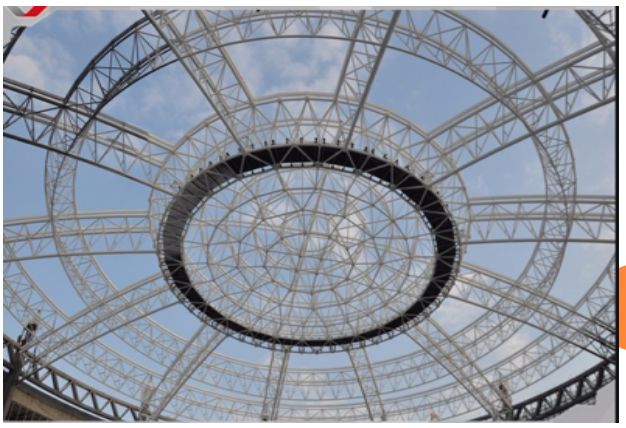
Analysis of Truss:

Examples of Truss Structures:









A Quick Question

What are the major advantages of a truss structures?

Lighter

Faster construction

Suitable for long span

Determinacy & Stability in Truss

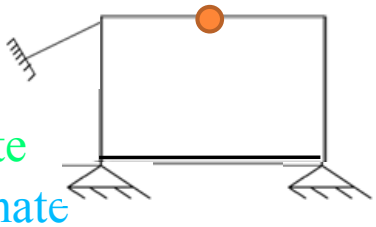
If

m=number of bars(members)
r = number of reactions
j = number of joints



Then:

m+r < 2j ----> unstable
m+r =2j -----> stable and determinate
m+r > 2j ------> stable and indeterminate



Also the horizontal/vertical restrain of the truss need to consider in case of stability

What will happen if there is an internal hinge in the truss??

Analysis of Truss

There are four main assumptions made in the analysis of truss

➢Truss members are connected together at their ends only.

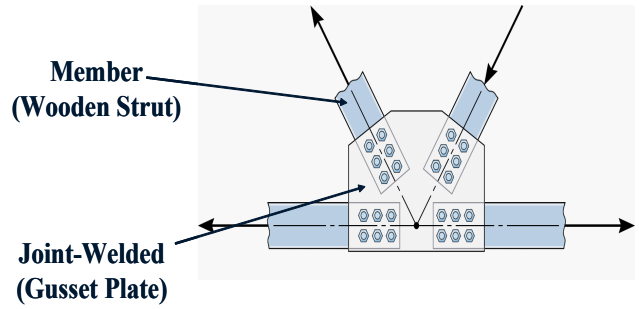
➢Truss are connected together by frictionless pins.

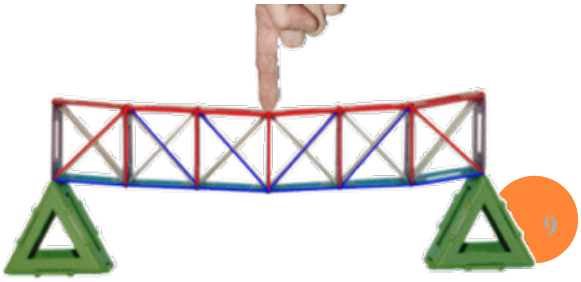
➢The truss structure is loaded only at the joints.

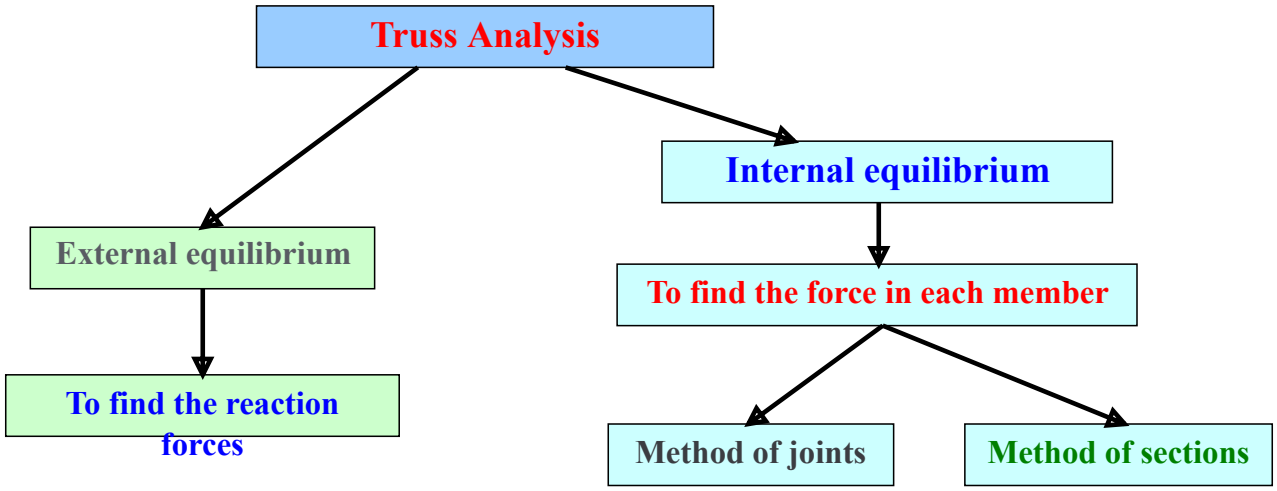
➢The weights of the members may be neglected

Joints: are usually formed by bolting or welding the members to a common plate, called a gusset plate, or simply passing a large bolt through each member.

➢ ***Joints*** are modeled by smooth pin connections.



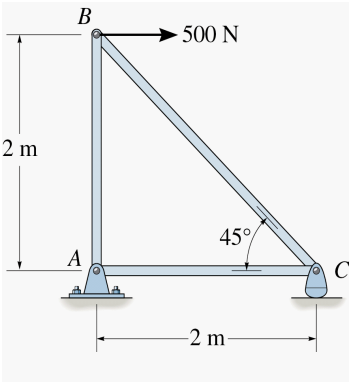


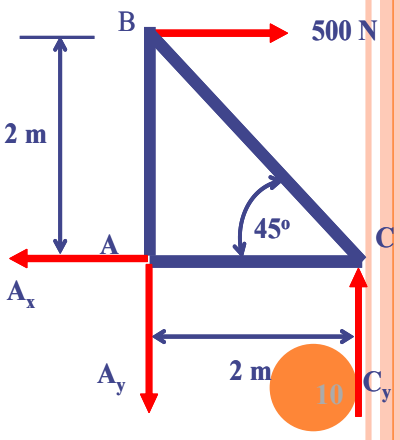


External Equilibrium: to find the ***reaction forces***, follow the below steps:

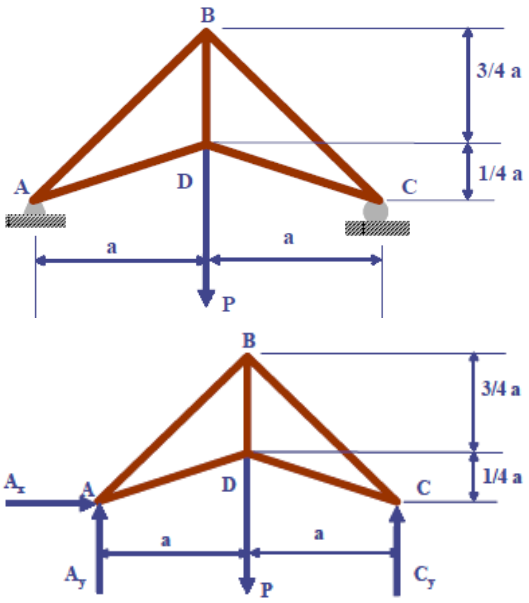
Draw the free-body diagram (***FBD***) for the entire truss system.

Determine the ***reactions***. Using the equations of (2 D) which states:





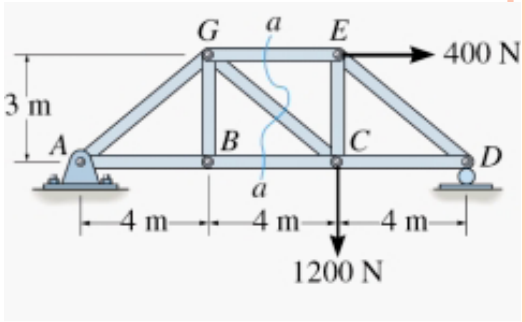
Method of Joints: to find the ***forces*** in any *member*, choose a ***joint***, to which that member is connected, and follow the below steps:

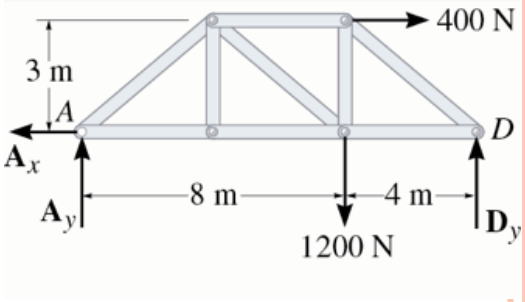


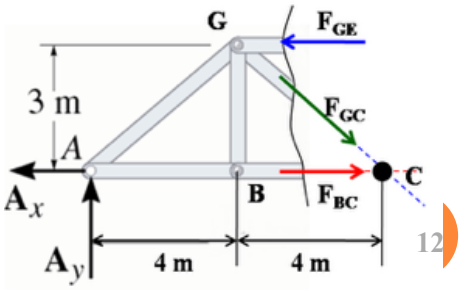
1. Draw the ***FBD*** for the entire truss system.
2. Determine the ***reactions***. Using the equations of (2 D) which states:
3. Choose the joint, and draw ***FBD*** of a ***joint*** with at least ***one known force*** and at most *two unknown forces*.
4. Using the equation of (**2 D**) which states:
5. The *internal forces* are determined.
6. Choose another ***joint***.

Method of section (Internal equilibrium): to find the ***forces*** in any *member*, choose a ***section***, to which that *member* is appeared as an internal force, and follow the below steps:

1. Draw the ***FBD*** for the entire truss system.
2. Determine the ***reactions***. Using the equations of (**2 D**) which states:
3. Choose the ***section***, and draw ***FBD*** of that *section,* shows how the forces replace the sectioned members.
4. Using the equation of (**2 D**) which states:
5. The *internal forces* are determined.
6. Choose another *section* or *joint*.





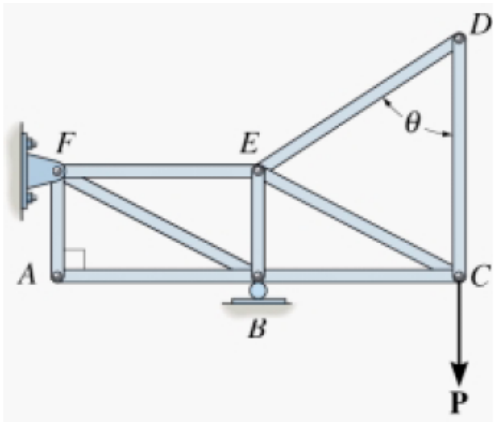


Analysis of trusses (Zero-force members):

Analysis of trusses system is simplified if one can identify those members that support no loads. We call these zero-force members.

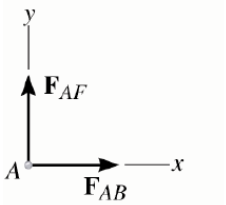
Examples to follow:

If ***two members*** form a *truss joint* and there is *no external load* or *support reaction* at that joint then *those members* are ***zero-force members***.



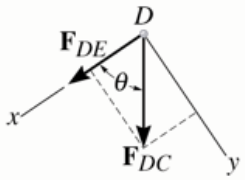
Joints D and A in the following figure are the joints with no external load or support reaction, so:

FAF = FAB = FDE = FDC = 0.



$\stackrel{+}{\rightarrow} \Sigma F_x = 0;\ F_{AB} = 0$

$+\uparrow \Sigma F_y = 0;\ F_{AF} = 0$



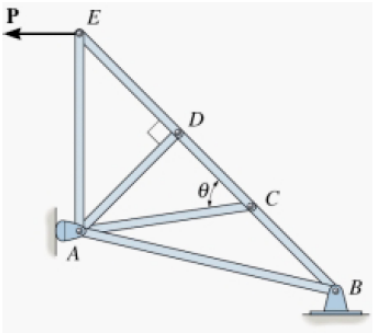
$+\searrow \Sigma F_y = 0;\ F_{DC} \sin\theta = 0;\ F_{DC} = 0 \text{ since } \sin\theta \neq 0$

$+\swarrow \Sigma F_x = 0;\ F_{DE} + 0 = 0;\ F_{DE} = 0$

Analysis of trusses (Zero-force members):

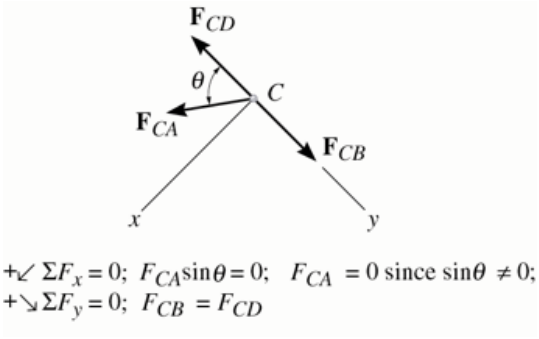
Examples to follow:

If ***three members*** form a truss **joint** and there is *no external load* or *support reaction* at that *joint* and *two of those members* are ***collinear*** then the *third member* is a ***zero-force member***.

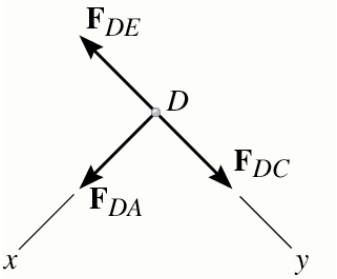


In the following figure, AC and AD are zero-force members, because Joints D and A in the following figure are the joints with three members, there is no external load or support reaction, so:

FCA = FDA = 0

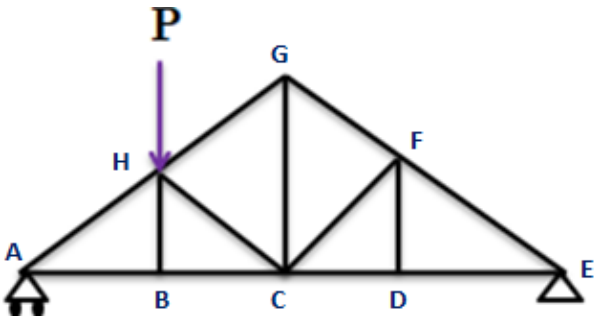


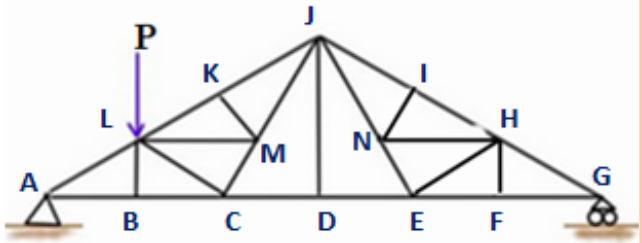
$+\swarrow \Sigma F_x = 0;\ F_{CA}\sin\theta = 0;\quad F_{CA} = 0$ since $\sin\theta \neq 0;$
$+\searrow \Sigma F_y = 0;\ F_{CB} = F_{CD}$

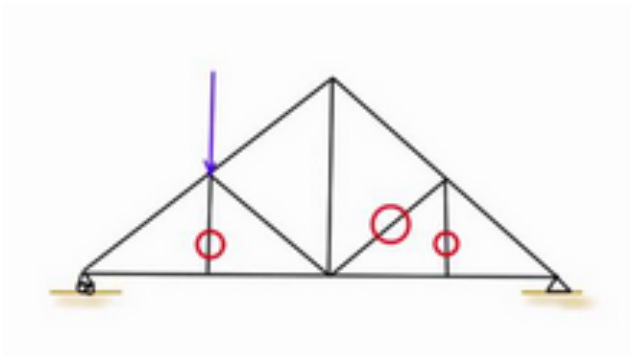


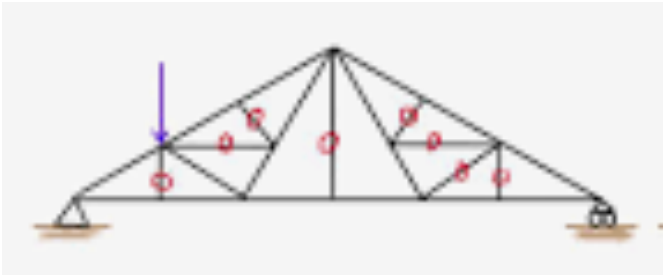
$+\swarrow \Sigma F_x = 0;\ F_{DA} = 0$
$+\searrow \Sigma F_y = 0;\ F_{DC} = F_{DE}$

How many zero-force members in the following truss?

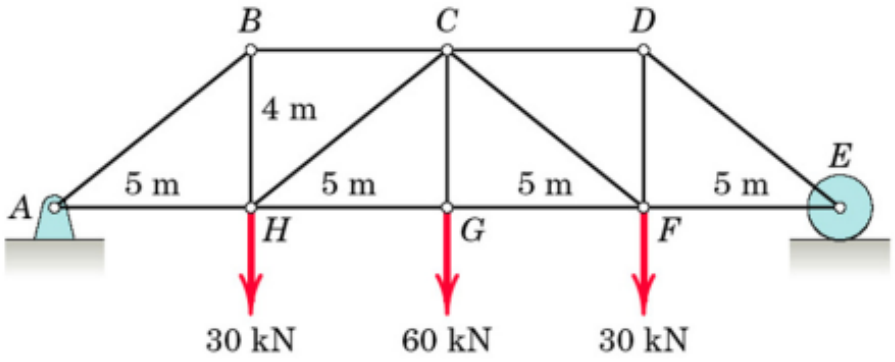








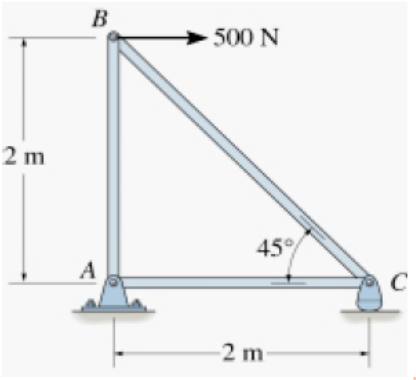
Analysis of Truss



Is the truss statically determinant externally?	Yes	Reaction = Known equations
Is the truss statically determinant internally?	Yes	m+r = 2j (16=16)
Are there any Zero Force Members in the truss?	No	

EXAMPLES of Trusses:

Example 1: Determine the support reactions in the joints of the following truss. Calculate the force in member (BA & BC.)



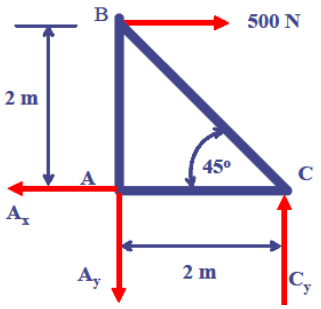
Solution

1. Draw FBD of entire truss and solve for support reactions:

$$\sum F_x = 0 \quad \Big| \quad \sum F_y = 0 \quad \Big| \quad \sum M_A = 0$$

$$500 - A_x = 0 \quad \Big| \quad C_y - A_y = 0 \quad \Big| \quad -500(2) + C_y(2) = 0$$

$$A_x = 500 \text{ N} \quad \Big| \quad A_y = C_y \quad \Big| \quad A_y = C_y = 500 \text{ N}$$



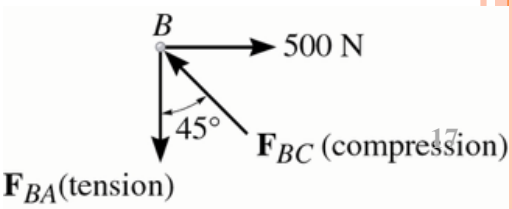
2. Draw FBD of a joint with at least one known force and at most two unknown forces. We choose joint B.

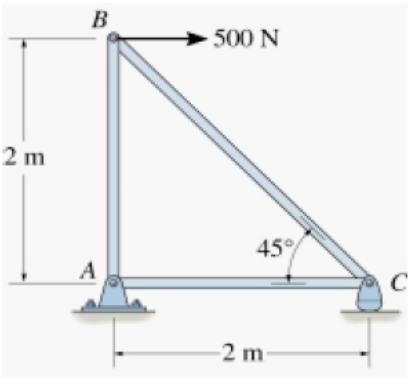
➢ Assume BC is in compression.

$$\sum F_x = 0 \qquad \sum F_y = 0$$

$$500 - F_{BC} \sin 45^\circ = 0 \qquad F_{BC} \cos 45^\circ - F_{BA} = 0$$

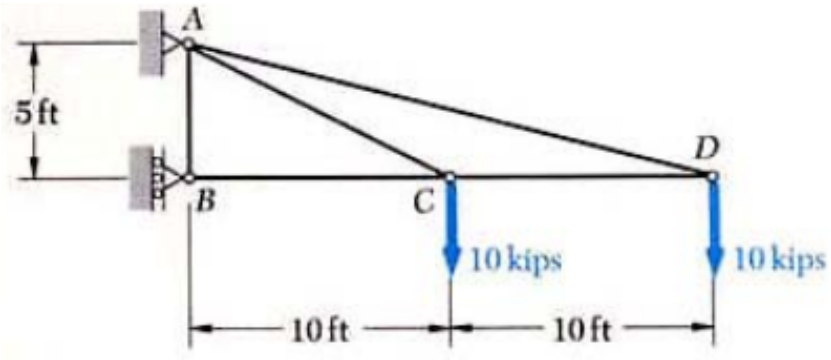
$$F_{BC} = 707.1 \text{ N (C)} \qquad F_{BA} = 500 \text{ N (T)}$$



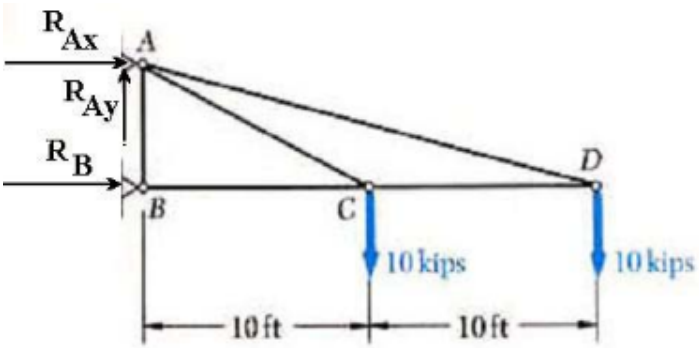

B
500 N
2 m
45°
A
C
2 m

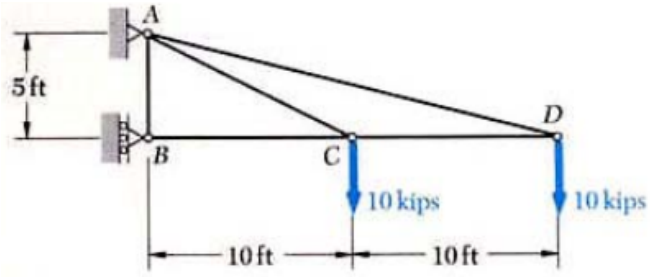
EXAMPLES of Trusses:

Determine the loads in each of the members by using the method of joints:



Solution Draw the free-body diagram. The summation of forces and moment about B result in




A
5 ft
B
C
D
10 kips
10 kips
10 ft
10 ft

EXAMPLES of Trusses:

$$\sum F_x = 0 = R_{Ax} + R_B$$

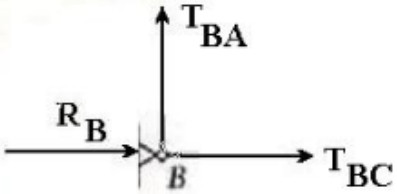
$$\sum F_y = 0 = R_{Ay} - 10 \text{ kips} - 10 \text{ kips} \Rightarrow R_{Ay} = 20 \text{ kips}$$

$$\sum M_A = 0 = R_B(5 \text{ ft}) - 10 \text{ kips}(10 \text{ ft}) - 10 \text{ kips}(20 \text{ ft})$$

$$\Rightarrow R_B = 60 \text{ kips}$$

$$\Rightarrow R_{Ax} = -60 \text{ kips}$$

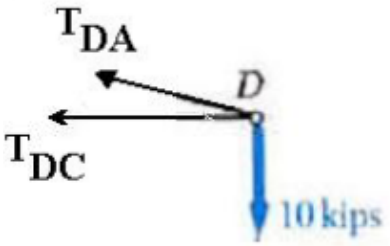
Look at Joint B



$$\sum F_x = 0 = T_{BC} + R_B = T_{BC} + 60 \text{ kips} \Rightarrow T_{BC} = -60 \text{ kips}$$

$$\sum F_y = 0 = T_{BA} \Rightarrow T_{BA} = 0 \text{ kips}$$

Look at Joint D and find the angle



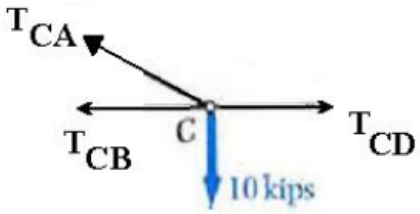
$$\alpha = \tan^{-1}\left(\frac{5\text{ ft.}}{20\text{ ft.}}\right) = 14.04^\circ$$

$$\sum F_x = 0 = -T_{DC} - T_{DA}\cos\alpha$$

$$\sum F_y = 0 = T_{DA}\sin\alpha - 10\text{ kips} \Rightarrow T_{DA} = 41.231\text{ kips}$$

$$T_{DC} = -40\text{ kips}$$

Look at Joint C and find the angle



$$\beta = \tan^{-1}\left(\frac{5\text{ ft.}}{10\text{ ft.}}\right) = 26.565^\circ$$

$$\sum F_y = 0 = T_{CA}\sin\beta - 10\text{ kips} \Rightarrow T_{CA} = 22.361$$

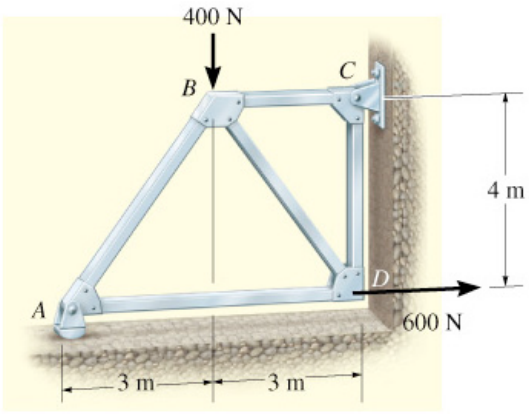
$$\sum F_x = 0 = T_{CD} - T_{CA}\cos\beta - T_{CB}$$

$$= (-40\text{ kips}) - (22.361\text{ kips})\cos(26.565^\circ) - (-60\text{ kips})$$

$$= 0$$

Analysis of trusses by the ***Method of Joints***, used to find the force in each and every member of the truss.

Example: Determine the force in each member of the truss shown below:

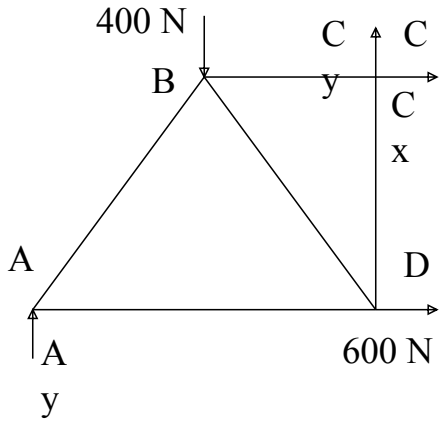


(a)

Procedure:

- Draw a FBD of the entire truss and determine the support reactions
- Draw a FBD of each joint and write equations for the unknown member forces.
- Solve the joint equations, report if each member is in tension or compression

Step 1: Determine the reaction forces on the entire truss:



$\sum Fx = Cx + 600\ N = 0$

$\sum Fy = Ay + Cy - 400\ N = 0$

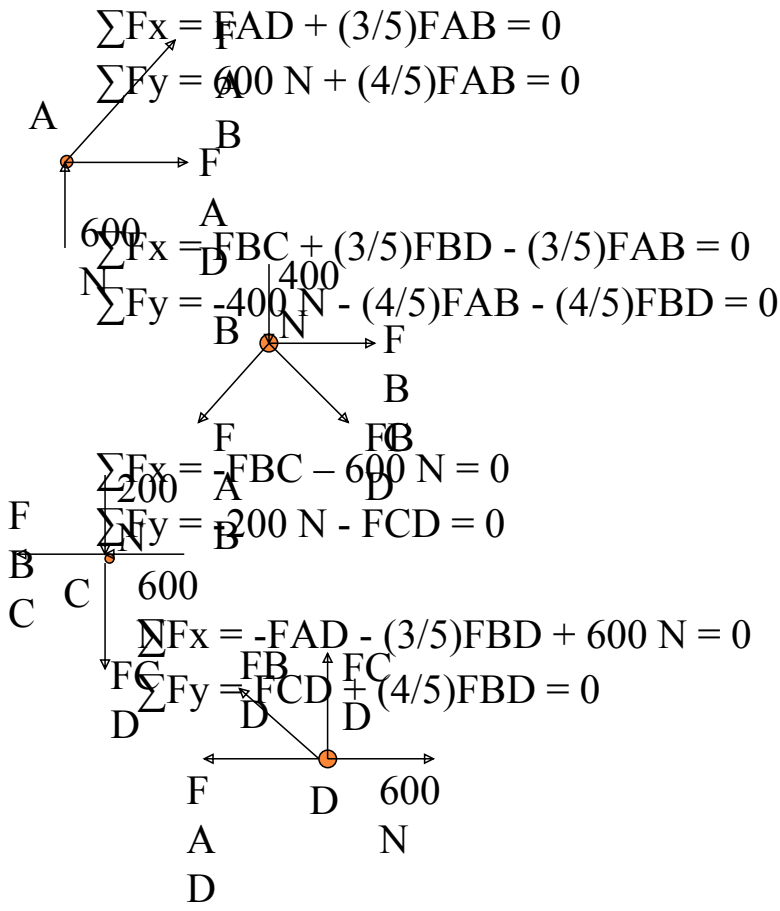
$\sum MC = (400\ N)(3\ m) - Ay\ (6\ m) + (600\ N)(4\ m) = 0$

$Cx = 600\ N \leftarrow$

$Cy = 200\ N \downarrow$

$Ay = 600\ N \uparrow$

Step 2: Draw a FBD of each joint, assume all members in tension, write and solve the joint equilibrium equations



$\sum Fx = FAD + (3/5)FAB = 0$

$\sum Fy = 600\ N + (4/5)FAB = 0$

$\sum Fx = FBC + (3/5)FBD - (3/5)FAB = 0$

$\sum Fy = -400\ N - (4/5)FAB - (4/5)FBD = 0$

$\sum Fx = -FBC - 600\ N = 0$

$\sum Fy = -200\ N - FCD = 0$

$\sum Fx = -FAD - (3/5)FBD + 600\ N = 0$

$\sum Fy = FCD + (4/5)FBD = 0$

Solution:

- FAB = 750 N (C)
- FAD = 450 N (T)
- FBC = 600 N (C)
- FBD = 250 N (T)
- FCD = 200 N (C)