EWCE-311

STRUCTURAL ANALYSIS AND DESIGN-I

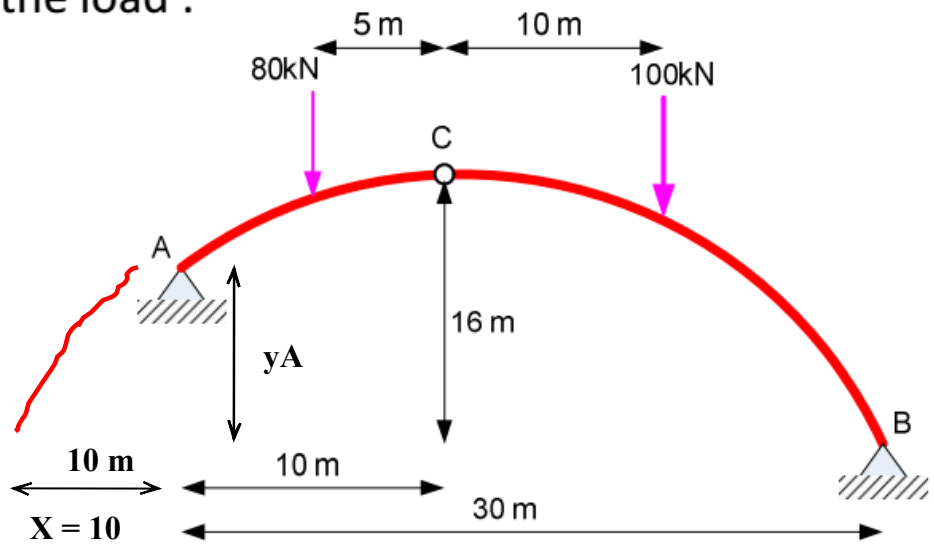
Practice examples on determinate arches

Lecture 6 & 7

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Arches

Determine the reactions at supports and bending moment under the load :



X = 10 m

L = 40 m

$$y_A = \frac{4hx}{L^2}(L-x)$$

$$y_A = \frac{4(16)(10)}{40^2}(40-10)$$

$$y_A = 12m$$

OR, assume x = 30 m

yA = (4*16*30)* (40-30)/402

= 12 m

Arches

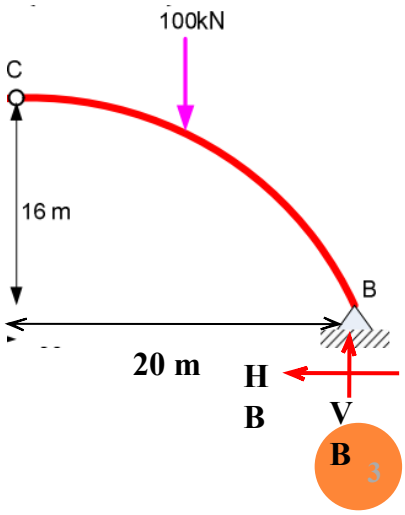
Consider the whole structure. Taking moment at A is equal zero

$$\sum M_A = 0$$

$$80(5) + 100(20) - V_B(30) + H_B(12) = 0$$

$$30V_B - 12H_B = 2400 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Consider RHS, taking moment at C is equal to zero



$$\sum M_C = 0$$

$$100(10) - V_B(20) + H_B(16) = 0$$

$$20V_B - 16H_B = 1000 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Arches

By solving Eq (1) & (2)

Resolving by using calculator

$V_B = 110kN$

$H_B = 75kN$

For whole structure

Apply static equation

$\sum F_y = 0$

$V_A + 110 - 80 - 100 = 0$

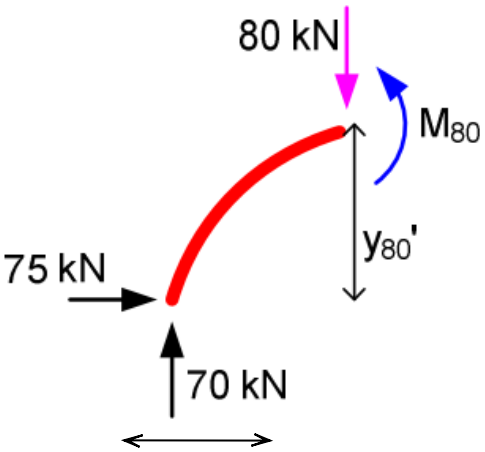
$V_A = 70kN$

$\sum F_x = 0$

$H_A = H_B = 75kN$

Arches

Consider LHS, taking moment under load 80kN



$$y_{80} = \frac{4hx}{L^2}(L-x)$$

$$= \frac{4(16)(15)}{40^2}(40-15)$$

$$= 15m$$

$$y'_{80} = 15-12 = 3m$$

yA = 12 m

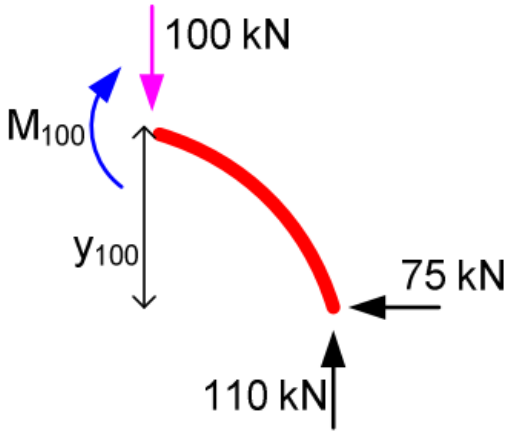
$$\sum M_{80} = 0$$

$$-M_{80} + 70(5) - 75(3) = 0$$

$$M_{80} = 125kNm$$

Arches

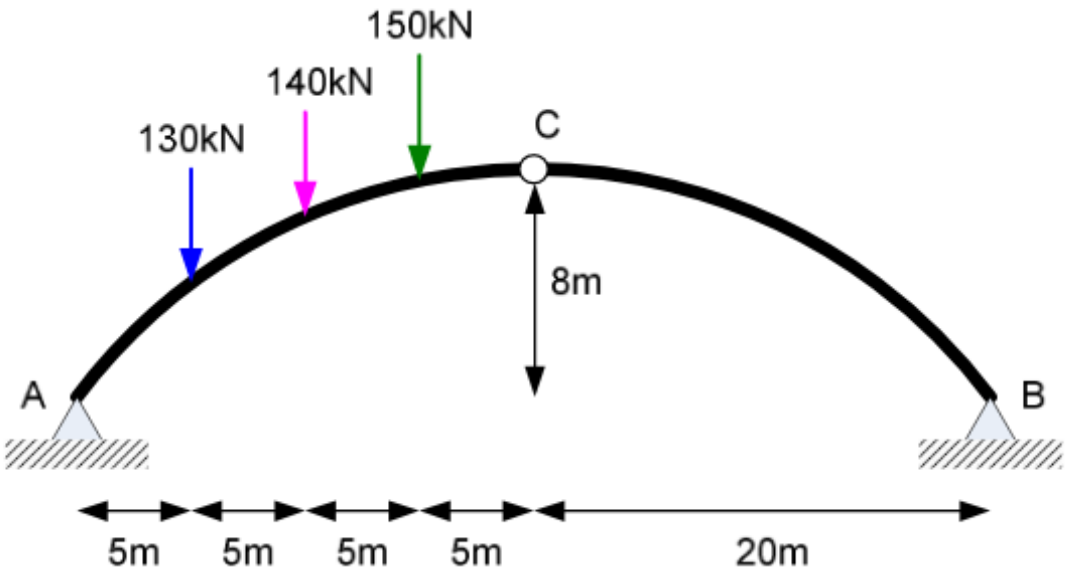
Consider RHS, taking moment under load 100kN



$$y_{100} = \frac{4hx}{L^2}(L-x)$$
$$= \frac{4(16)(10)}{40^2}(40-10)$$
$$= 12m$$

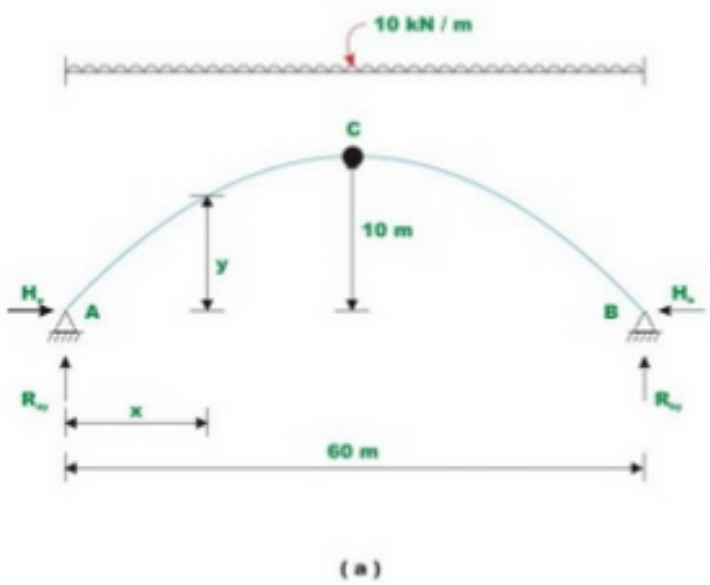
$$\sum M_{100} = 0$$
$$M_{100} + 75(12) - 110(10) = 0$$
$$M_{100} = 200kNm$$

Calculate the reaction at support A and B as shown in figure below. Also, determine the moment at the point where 140 kN is applied.

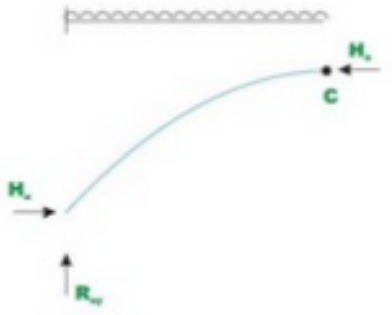


A three-hinged parabolic arch of uniform cross section has a span of 60 m and a rise of 10 m. It is subjected to uniformly distributed load of intensity 10 kN/m as shown in Fig. 32.6 Show that the bending moment is zero at any cross section of the arch.

Solution:



(a)



(a)

Free body diagram

Reactions:

Taking moment of all the forces about hinge A, yields

$$R_{ay} = R_{by} = \frac{10 \times 60}{2} = 300 \text{ kN} \quad (1)$$

Taking moment of forces left of hinge C about C, one gets

$$R_{ay} \times 30 - H_a \times 10 - 10 \times 30 \times \frac{30}{2} = 0$$

$$H_a = \frac{300 \times 30 - 10 \times 30 \times \left(\frac{30}{2}\right)}{10} \quad (2)$$

$$= 450 \text{ kN}$$

From $\sum Fx = 0$ one could write, $H_b = 450$ kN.

The shear force at the mid span is zero.

Bending moment

The bending moment at any section x from the left end is,

$$M_x = R_{ay}\,x - H_a\,y - 10\frac{x^2}{2}$$

The equation of the three-hinged parabolic arch is

$$y = \frac{2}{3}x - \frac{10}{30^2}x^2 \qquad \text{or} \qquad y_D = \frac{4hx}{L^2}(L-x)$$

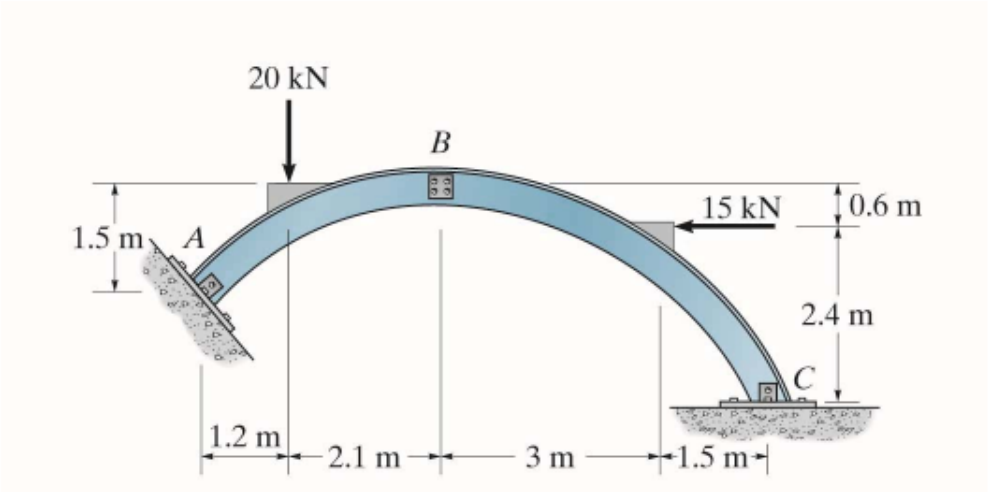
$$M_x = 300x - \left(\frac{2}{3}x - \frac{10}{30^2}x^2\right)450 - 5x^2$$

$$= 300x - 300x + 5x^2 - 5x^2 = 0$$

In other words a three hinged parabolic arch subjected to uniformly distributed load is not subjected to bending moment at any cross section. It supports the load in pure compression. Can you explain why the moment is zero at all points in a three-hinged parabolic arch?

Analysis of Three Hinged Arches:

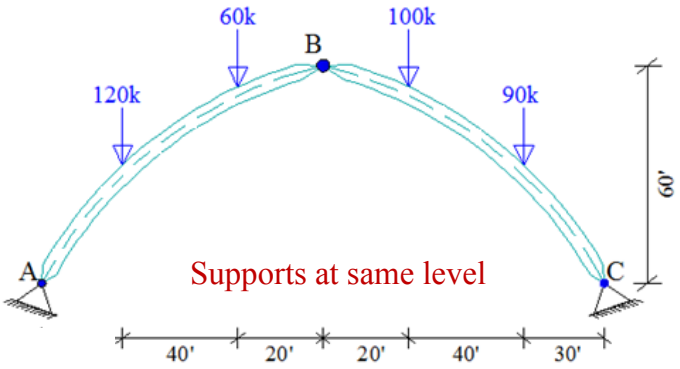
Determine the horizontal and vertical components of reaction at A, B, and C of the three-hinged arch. Assume A, B, and C are pin connected.

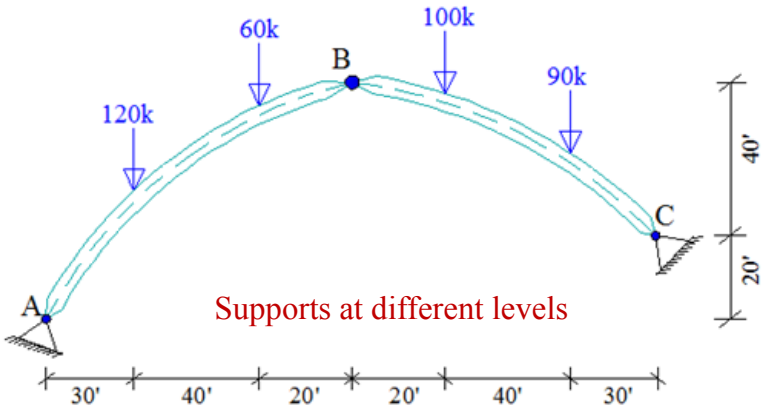


Analysis of Three Hinged Arches:

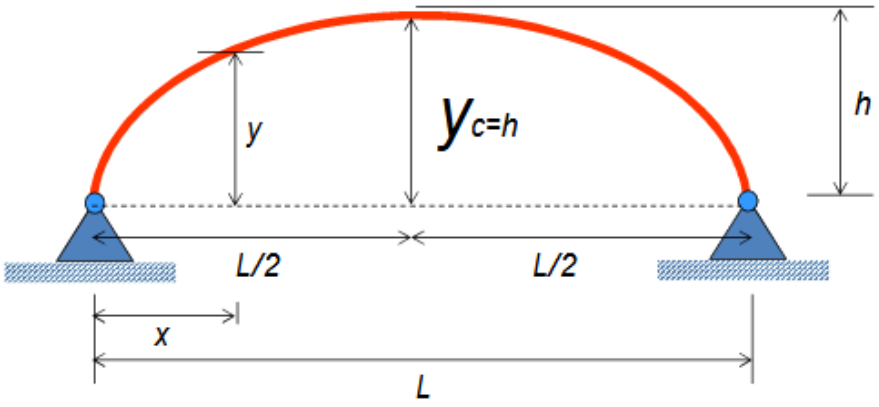
Find the support reactions:

- HA= 145.9k (→)
- VA= 190.6k (↑)
- HC= 145.9k (←)
- VC= 179.4k ( )





- HA= 175k (→)
- VA= 210k (↑)
- HC= 175k (←)
- VC= 160k (↑)



$y = kx(L - x)$ .....Parabolic equation

When $x = L/2$, $y = y_c$ sub into enq.

$y_c = k(L/2)(L - L/2) = kL^2/4$

Therefore $k = 4y_c/L^2 = 4h/L^2$ then sub into eqn.

$y = (4y_c/L^2)(Lx - x^2)$

$y = (4h/L^2)(Lx - x^2)$ simplify **$y = (4h/L^2)x(L - x)$**

At any point of the arch (parabolic)

$$\text{Slope} = \frac{dy}{dx}$$

$$\text{but} \quad y = \frac{4\,hx}{L^2}(L - x)$$

$$= \frac{4\,hxL}{L^2} - \frac{4\,hx^2}{L^2}$$

$$= \frac{4\,hx}{L} - \frac{4\,hx^2}{L^2}$$

$$\text{Slope} = \frac{dy}{dx} = \frac{4\,h}{L} - \frac{2(4\,h)\,x}{L^2}$$

$$= \frac{4\,h}{L} - \frac{8\,hx}{L^2}$$

$$= \frac{4\,h}{L^2}(L - 2\,x)$$

Where, $\theta = \tan^{-1}$ slope