EWCE-311

STRUCTURAL ANALYSIS AND DESIGN-I

Suspension cable analysis

Lecture 9 & 10

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Suspension Cable Structure

Introduction

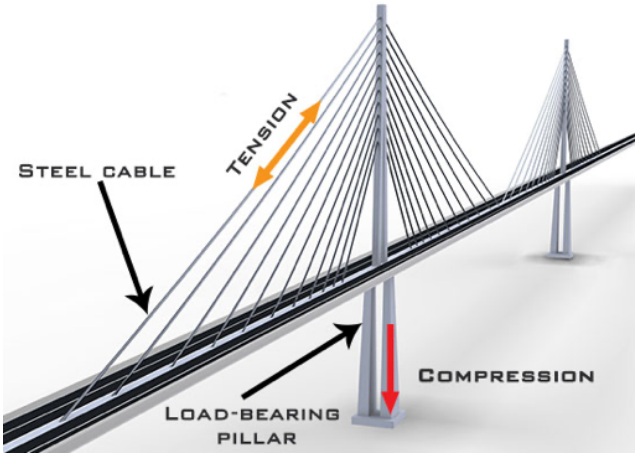
Cables carry applied loads & **develop mostly tensile stresses.**

In cables, the loads applied through hangers is considered to be a uniformly distributed load.

Cables near the end supporting structures experience bending moments and shear forces.

Cable type structures - Suspension roof, suspension bridges, cable cars, guy-lines, transmission lines, etc.

Suspension Cable Structure









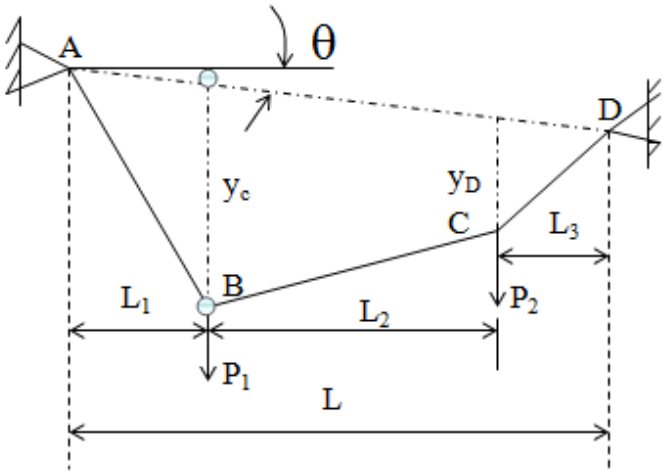
Suspension Cable Structure

Assumptions

- Cable is **flexible and in-extensible**; hence does not resist any bending moment or shear force (this is not always true - e.g., fatigue of cables); **self weight of cable neglected** when external loads act on the cable
- Since only axial tensile forces are carried by the cable, the **force in the cable is tangential to the cable profile**
- **Since it is in-extensible, the length is always constant**; as a consequence of the cable profile not changing its length and form, it is **assumed to be a rigid body during analysis**
- **Even when a moving load is acting on the cable**, the **load is assumed to be uniformly distributed** over the cable (since the cable profile is not assumed to change)

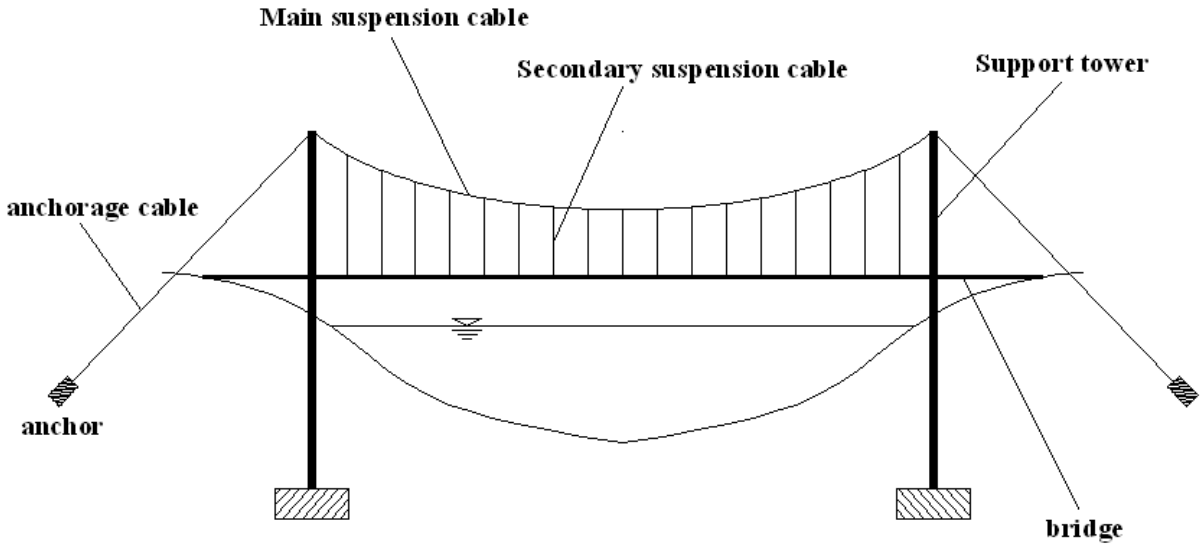
Cables subjected to concentrated loads

When the weight of the cable is neglected in analysis and is subjected to only concentrated loads, the **cable takes the form of several straight line segments; the shape is called as funicular polygon.** Consider for instance the cable shown below.



Suspension Cable Structure

Cables are often used in engineering structures for support and to transmit loads from one member to another. When used to support suspension bridges, cables form the main load-carrying element in the structure.



Note: Cable can only carrying tensile force.

Parabolic equation of Cable

$$y = kx(L - x)$$

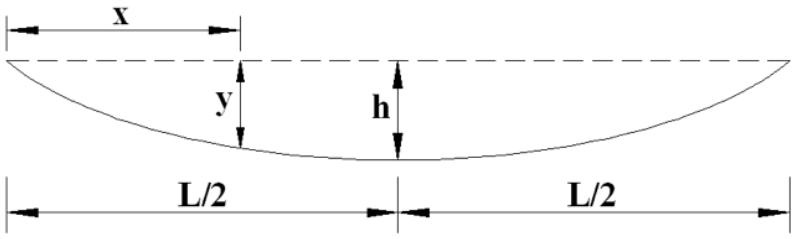
$$x = \frac{L}{2} \qquad y = h$$

$$h = k\frac{L}{2}(L - \frac{L}{2})$$

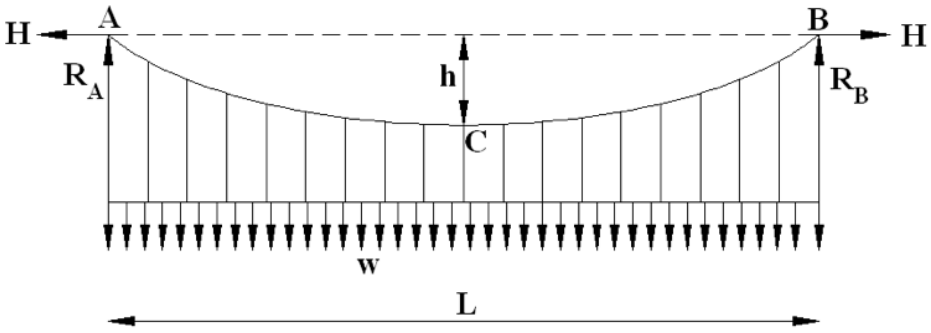
$$h = k\frac{L^2}{4}$$

$$k = \frac{4h}{L^2}$$

$$y = \frac{4hx(L - x)}{L^2}$$



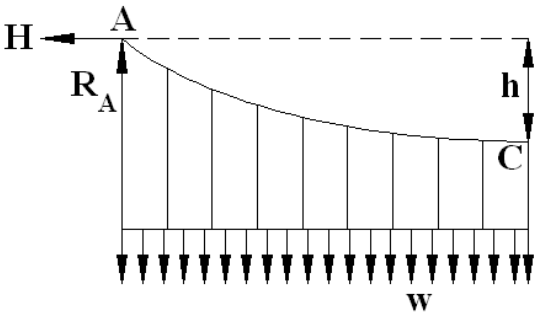
CABLE SUPPORTED AT SAME LEVEL



$$\sum F_y = 0:$$

$$R_A = R_B = \frac{wL}{2}$$

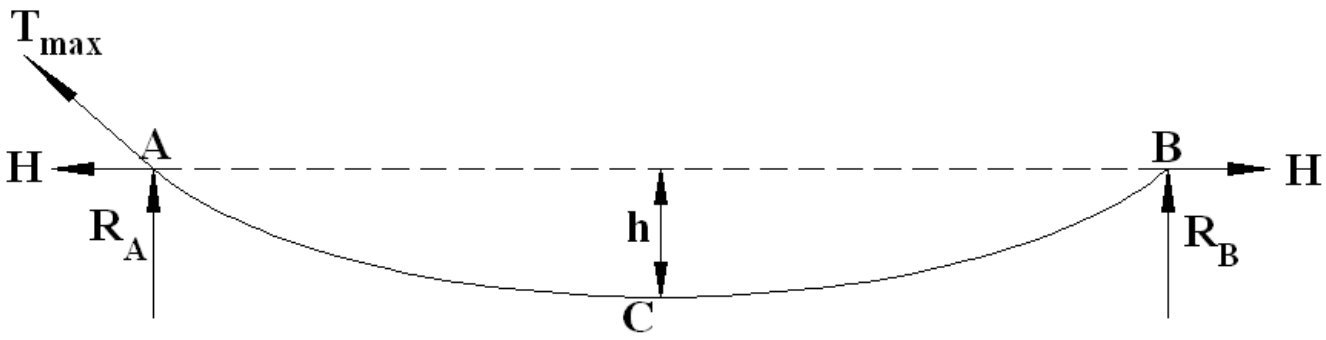
Consider free body diagram at left hand side of C:



$$\sum M_C = 0:$$

$$Hh + \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) - R_A\left(\frac{L}{2}\right) = 0$$

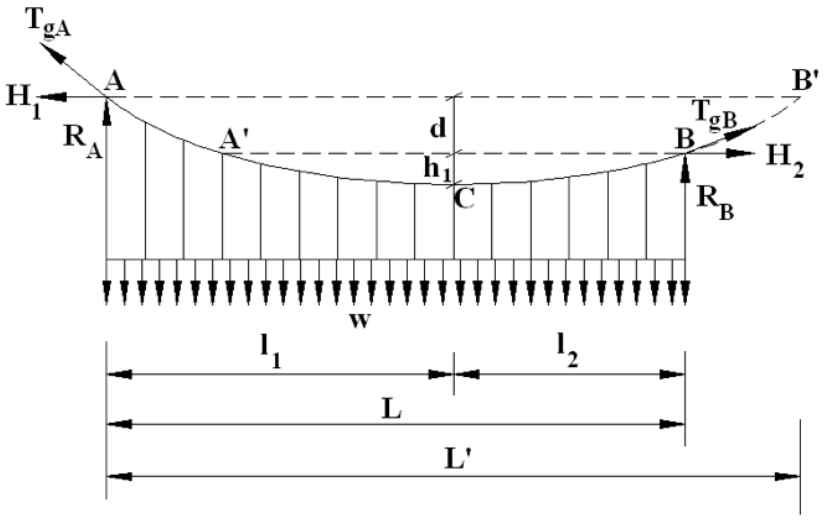
$$H = \frac{wL^2}{8h}$$



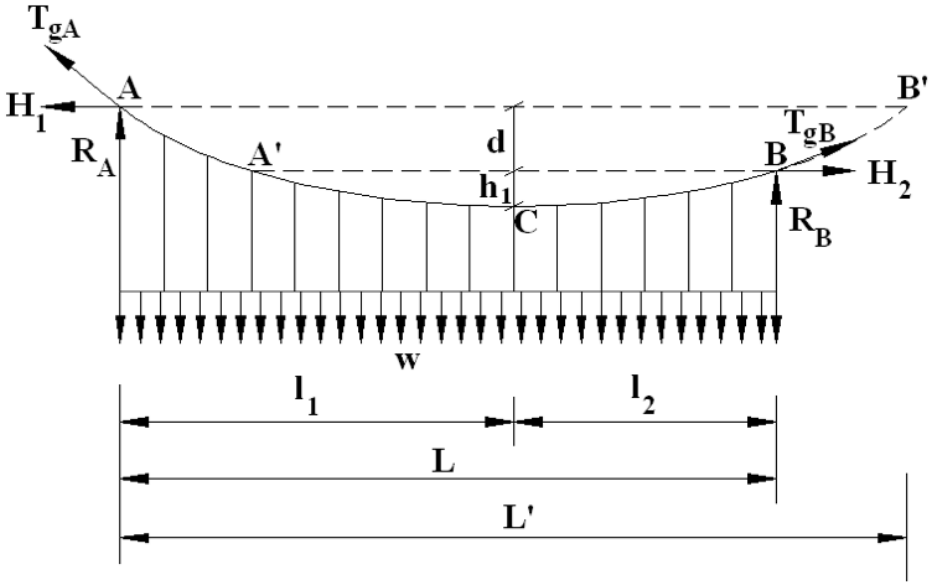
Maximum tensile force in the cable,

$$T_{\max} = \sqrt{R_A^2 + H^2}$$

CABLE SUPPORTED AT DIFFERENT LEVEL

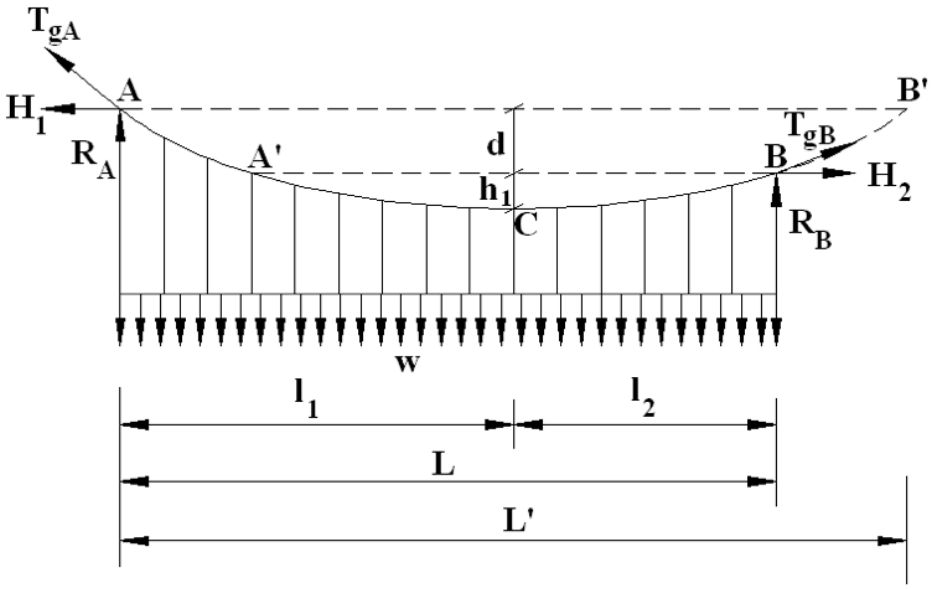


CABLE SUPPORTED AT DIFFERENT LEVEL



Consider cable ACB':

$$H_1 = \frac{wL'^2}{8(h_1 + d)} = \frac{w(2l_1)^2}{8(h_1 + d)} = \frac{wl_1^2}{2(h_1 + d)}$$



Consider cable A'CB:

$$H_2 = \frac{w(2l_2)^2}{8h_1} = \frac{wl_2^2}{2h_1}$$

But H1 = H2 = H,
then

$$\frac{wl_1^2}{2(h_1+d)} = \frac{wl_2^2}{2h_1}$$

$$\frac{l_1}{l_2} = \sqrt{\frac{h_1+d}{h_1}}$$ ← Lowest position equation

Vertical Reactions RA and RB

$$\sum M_B = 0: \qquad R_A = \frac{wL}{2} + \frac{Hd}{L}$$

$$\sum M_A = 0: \qquad R_B = \frac{wL}{2} - \frac{Hd}{L}$$

Tensile force in the cable at *A* and *B*

$$T_{gA} = \sqrt{R_A^2 + H^2} \qquad T_{gB} = \sqrt{R_B^2 + H^2}$$

CABLE SUPPORT

Two types – Driven Smooth Pulley Support, Roller Support

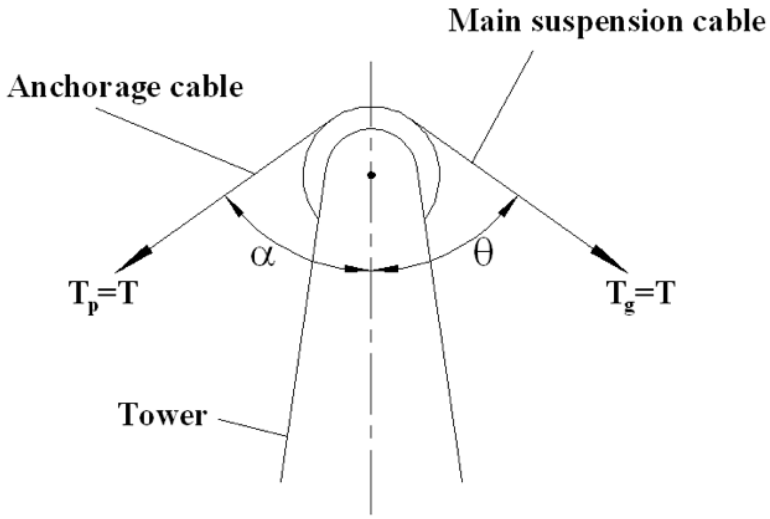
Smooth Pulley:

Tp = Tg =T

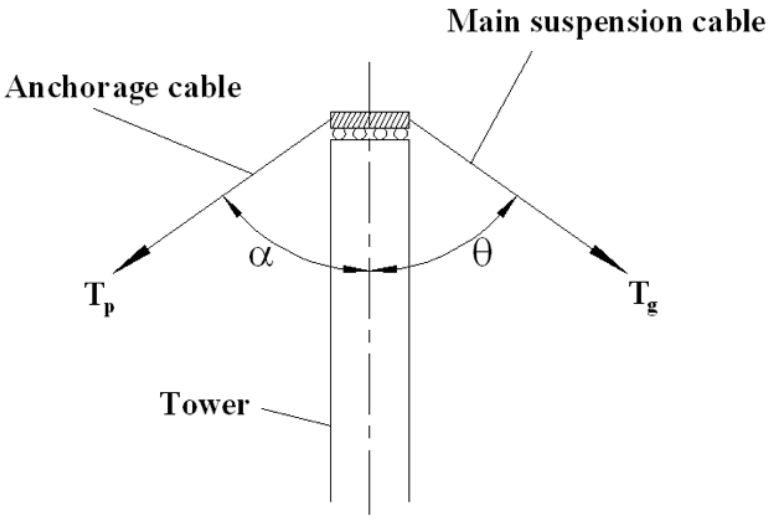
Reactions from tower (*menara*):

$$R_m = T\left(\cos\alpha + \cos\theta\right)$$

$$H_m = T\left(\sin\alpha - \sin\theta\right)$$



Roller Support:



Note: $T_p \neq T_g$

To ensure the roller not moves laterally:

$$T_p \sin\alpha = T_g \sin\theta$$

Equilibrium Vertically:

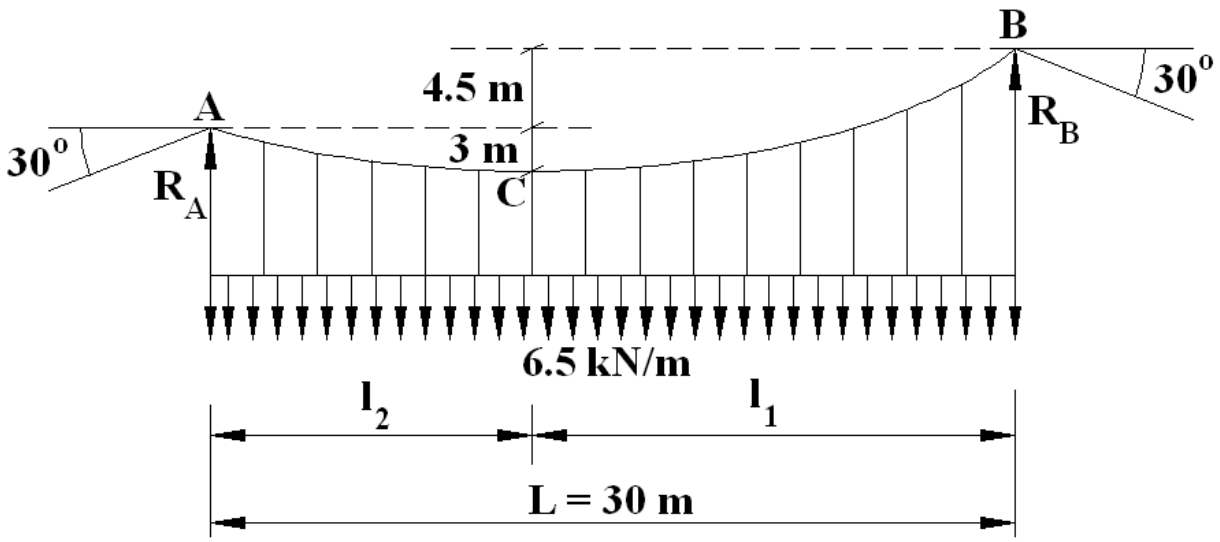
$$R_m = T_p \cos\alpha + T_g \cos\theta$$

EXAMPLE 1

Figure below shows a cable structure supported by frictionless roller which is located on top of tower. Determine:

(a) Maximum and minimum tensile force exerted by the cable

(b) Vertical reaction force exerted by tower at *A* and *B*



SOLUTION

Find *l*1 and *l*2

$$\frac{l_1}{l_2} = \sqrt{\frac{3+4.5}{3}} = 1.581 \qquad (a)$$

$$l_1 + l_2 = 30 \qquad (b)$$

Solving equations (a) and (b) give *l*1 = 18.376 m and *l*2 = 11.623 m

Find H

$$H = \frac{wL^2}{8h} = \frac{w(2l_2)^2}{8(3)} = \frac{6.5(2\times 11.623)^2}{24} = 146.35\,kN$$

or

$$H = \frac{wL^2}{8h} = \frac{w(2l_1)^2}{8(3+4.5)} = \frac{6.5(2\times 18.376)^2}{8(3+4.5)} = 146.33\,kN$$

Take H = 146.35 kN

Find RA and RB in the cable

$$\sum M_B = 0: \qquad R_A = \frac{wL}{2} - \frac{Hd}{L} = \frac{6.5(30)}{2} - \frac{146.35(4.5)}{30} = 75.5\,kN$$

$$\sum M_A = 0: \qquad R_B = \frac{wL}{2} + \frac{Hd}{L} = \frac{6.5(30)}{2} + \frac{146.35(4.5)}{30} = 119.5\,kN$$

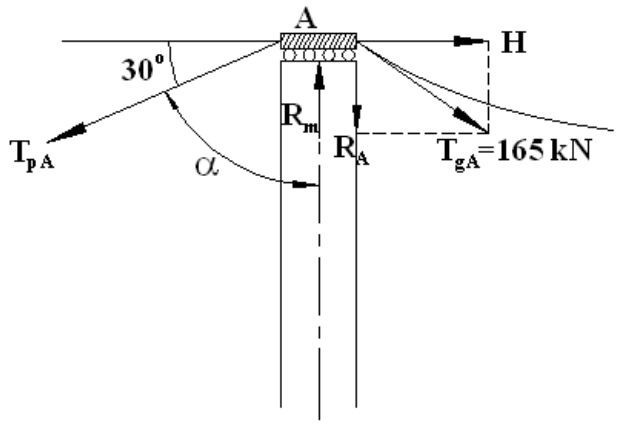
Find tensile force exerted by the cable (at A and B)

$$T_{gA} = \sqrt{R_A^2 + H^2} = \sqrt{75.5^2 + 146.35^2} = 165\,kN$$

$$T_{gB} = \sqrt{R_B^2 + H^2} = \sqrt{119.5^2 + 146.35^2} = 189\,kN$$

Maximum tensile force occurred at *B*, minimum tensile force occurred at *C* (lowest point in the cable) (=*H*=146.35 kN)

(b) Reaction forces exerted by the tower



(a) Roller support at A

$$\sum F_x = 0: \quad T_{pA}\sin\alpha = H$$

$$T_{pA}\sin\left(90^\circ - 30^\circ\right) = 146.35$$

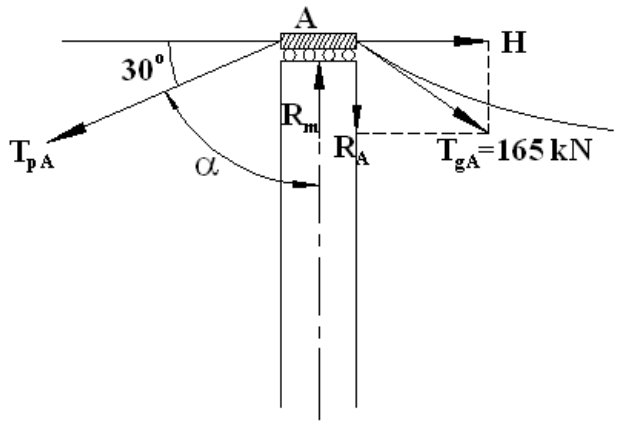
$$T_{pA} = 169\,kN$$

$$\sum F_y = 0: \quad R_m = T_{pA}\cos\alpha + R_A$$

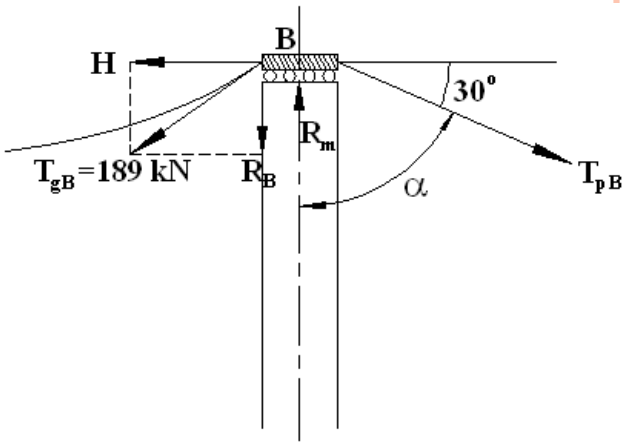
$$= 169\cos 60^\circ + 75.5$$

$$= 160\,kN$$

(b) Reaction forces exerted by the tower



(a) Roller support at A



(a) Roller support at B

$$\sum F_x = 0: \quad T_{pA}\sin\alpha = H$$

$$T_{pA}\sin\left(90^\circ - 30^\circ\right) = 146.35$$

$$T_{pA} = 169\,kN$$

$$\sum F_y = 0: \quad R_m = T_{pA}\cos\alpha + R_A$$

$$= 169\cos 60^\circ + 75.5$$

$$= 160\,kN$$

$$\sum F_x = 0: \quad T_{pB}\sin\alpha = H$$

$$T_{pB}\sin\left(90^\circ - 30^\circ\right) = 146.35$$

$$T_{pB} = 169\,kN$$

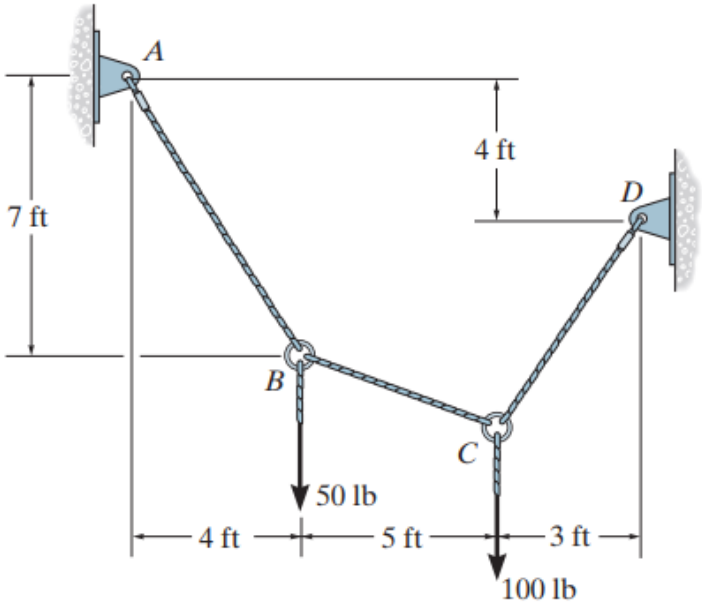
$$\sum F_y = 0: \quad R_m = T_{pB}\cos\alpha + R_B$$

$$= 169\cos 60^\circ + 119.5$$

$$= 204\,kN$$

Cable under concentrated loads

Example 2: Determine the tension in each segment of the cable and the cable's total length

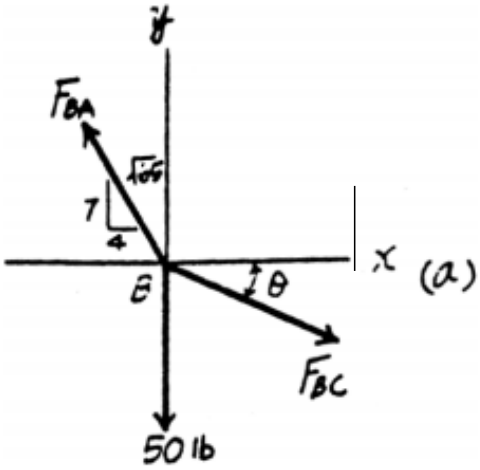


Equations of Equilibrium: Applying method of joints, we have

Joint *B*:

$$\stackrel{+}{\rightarrow} \sum F_x = 0; \qquad F_{BC}\cos\theta - F_{BA}\left(\frac{4}{\sqrt{65}}\right) = 0 \qquad [1]$$

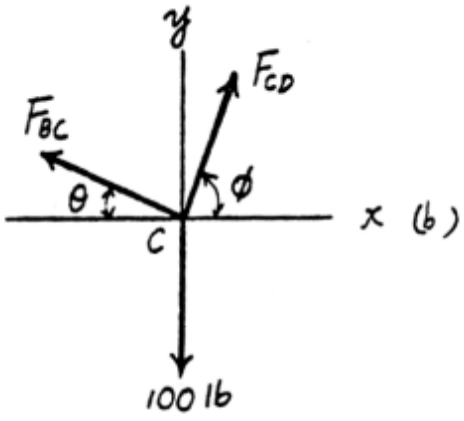
$$+\uparrow \sum F_y = 0; \qquad F_{BA}\left(\frac{7}{\sqrt{65}}\right) - F_{BC}\sin\theta - 50 = 0 \qquad [2]$$



Joint *C*:

$$\overset{+}{\rightarrow} \sum F_x = 0; \qquad F_{CD}\cos\phi - F_{BC}\cos\theta = 0 \qquad [3]$$

$$+\uparrow \sum F_y = 0; \qquad F_{BC}\sin\theta + F_{CD}\sin\phi - 100 = 0 \qquad [4]$$



Geometry:

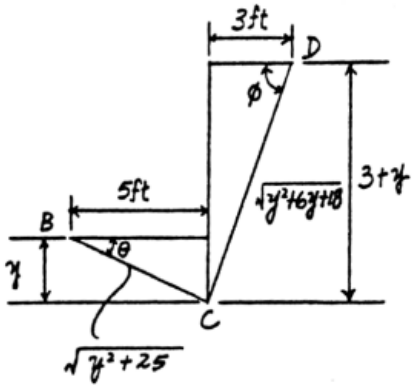
$$\sin\theta = \frac{y}{\sqrt{y^2 + 25}} \qquad \cos\theta = \frac{5}{\sqrt{y^2 + 25}}$$

$$\sin\phi = \frac{3 + y}{\sqrt{y^2 + 6y + 18}} \qquad \cos\phi = \frac{3}{\sqrt{y^2 + 6y + 18}}$$

Substitute the above results into Eqs. [1], [2], [3] and [4] and solve. We have

$F_{BC} = 46.7$ lb $\qquad F_{BA} = 83.0$ lb $\qquad F_{CD} = 88.1$ lb

$y = 2.679$ ft



The total length of the cable is

$$l = \sqrt{7^2 + 4^2} + \sqrt{5^2 + 2.679^2} + \sqrt{3^2 + (2.679 + 3)^2}$$

$$= 20.2 \text{ ft}$$

Example: Cable *ABCD* supports the loading shown. Determine the maximum tension in the cable and the sag of point *B*.

