EWCE-311

Structural Analysis And Design-I

Moving load analysis for reaction and shear

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Maximum ‘Support Reaction’ due to Wheel Loads

Consider the simply supported beam AB of length L, being subjected to the wheel load arrangement as shown in Fig. 1. The maximum reaction at support A will obviously be due to placement of one of the wheel loads directly on the support itself.

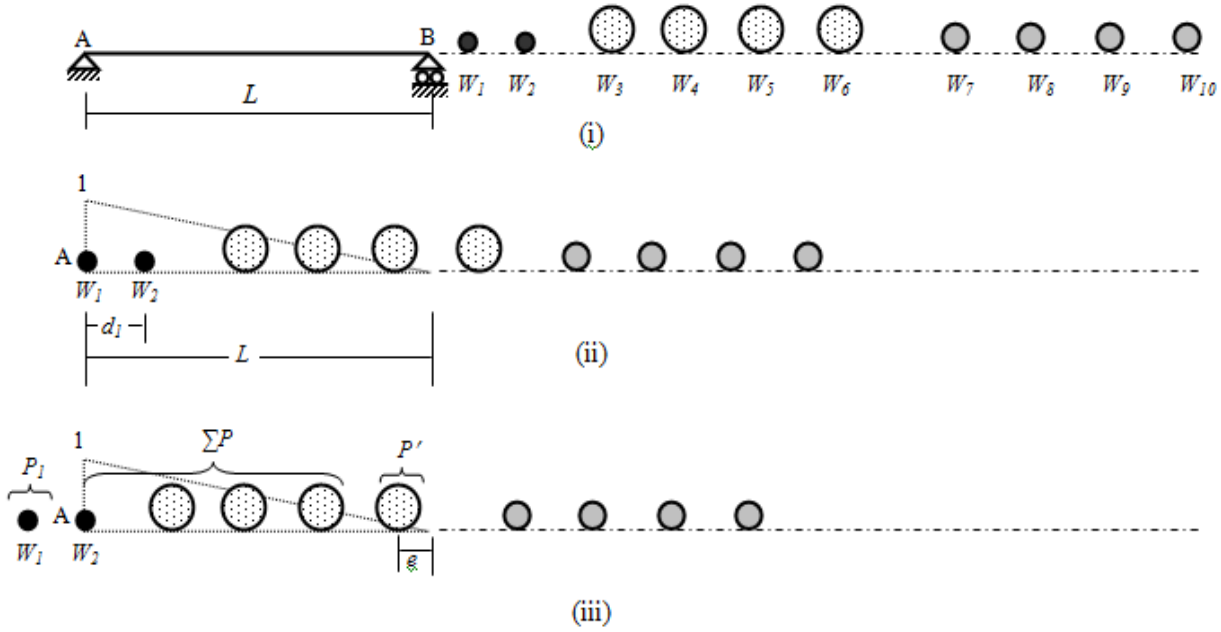


Fig. 1: Wheel Loads with ‘Reaction Type’ Influence Line

Maximum 'Support Reaction' due to Wheel Loads

- **For a series of Loads:**

Let, ΔR = Change in reaction due to moving from one load over the reaction to the next.

P_1 = The load which was over the reaction and is moved off the span.

d_1 = The distance between P_1 and the following wheel.

L = The length of the span.

$\sum P$ = The sum of all the loads which are on the span and stay on during the movement.

Then,

$$\Delta R = \frac{\sum P d_1}{L} - P_1$$

If a new load moves on the span during the movement and,

P' = a load which comes on during movement.

e = The distance which P' comes on the span.

$$\Delta R = \frac{\sum P d_1}{L} + \frac{P' e}{L} - P_1$$

If ΔR is positive, reaction increases and vice versa.

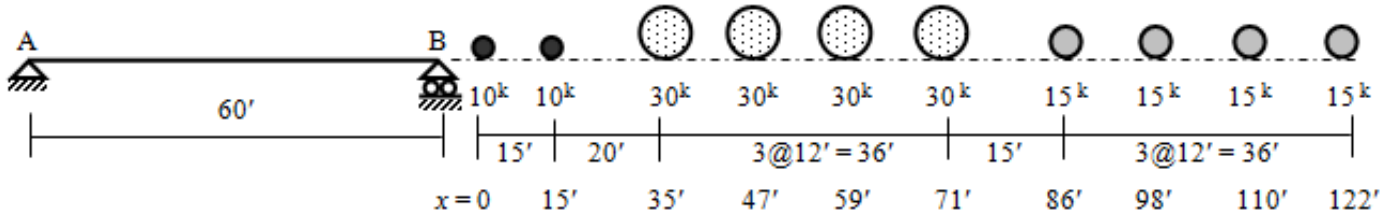
Considering the difference of support reaction at A (ΔR) between cases with wheel *W1* at A [(ii) in Fig. 1] and wheel *W2* at A [(iii) in Fig. 1], the increase in support reaction is due to the shift *d1* of load $\sum P$; i.e., an increase of ordinate by an amount *d1/L*. Moreover, there is an additional increase due to the new load $P\Delta$ moving a distance *e* within the influence line (ordinate increases *e/L*). However, since the load *P1* has moved out of the influence line; i.e., its ordinate decreases by 1, there is a further decrease of *P1* in the support reaction.

Therefore, the overall change of reaction between (ii) and (iii) is given by

$$\Delta R = \{(\sum P)\, d1 + P\Delta\, e\}/L - P1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

Since derivation of (1) is based on the shape of influence line, it is valid for all influence lines of similar shape.

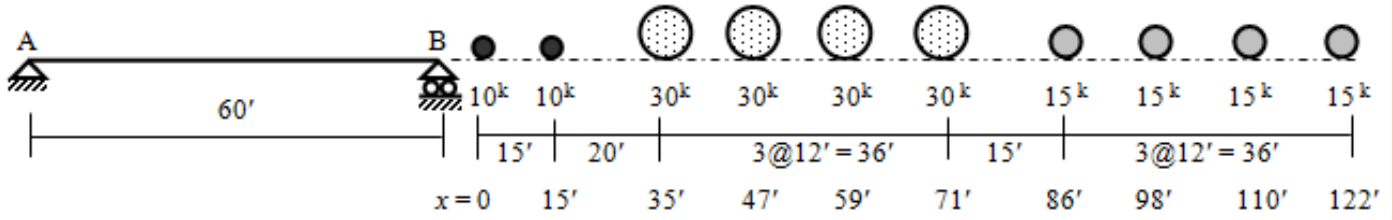
Calculate the maximum value of RA for the wheel load arrangement shown below.



Between *W1* and *W2*, $\sum P = 10 + 3 \times 30 = 100$ k, $d1 = 15$□, P□ $= 30$ k, $e = 4$□, $P1 = 10$ k

□□$R12 = \{100 \times 15 + 30 \times 4\}/60 - 10 = 17$ k

Calculate the maximum value of RA for the wheel load arrangement shown below.

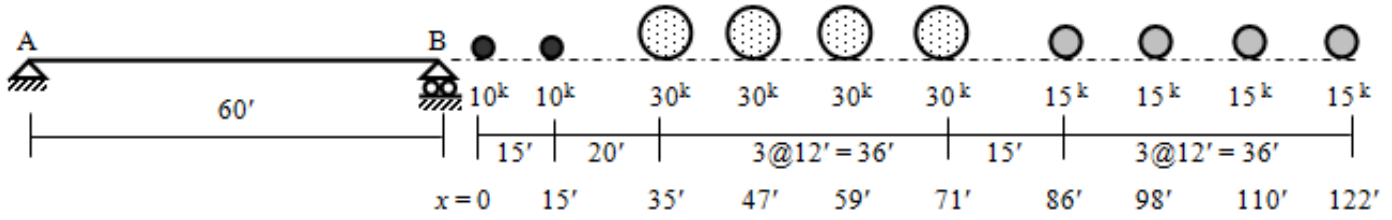


Between W2 and *W3*,

$\sum P = 4 \times 30 = 120$ k, $d1 = 20$□, P□ $= 15$ k, $e = 9$□, $P1 = 10$ k

□□$R23 = \{120 \times 20 + 15 \times 9\}/60 - 10 = 32.25$ k

Calculate the maximum value of RA for the wheel load arrangement shown below.

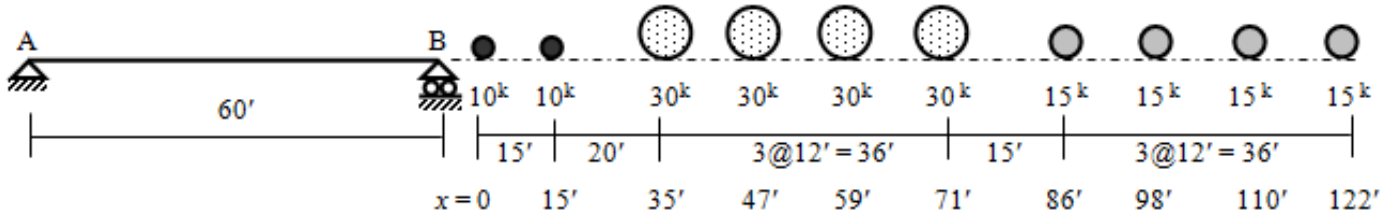


Between W3 and *W4,*

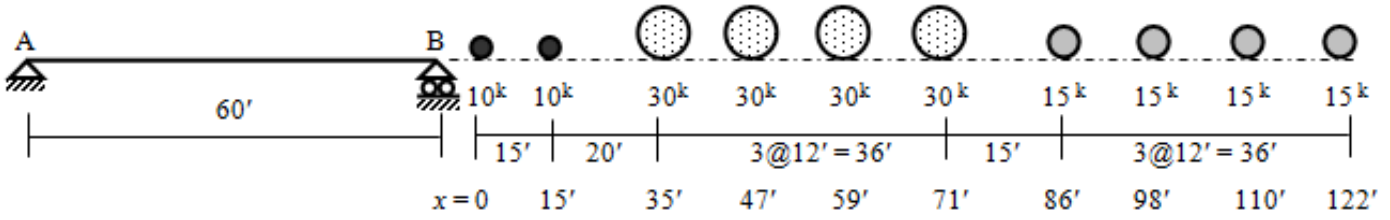
$\sum P = 3 \times 30 + 15 = 105$ k, $d1 = 12$□, P□ $= 15$ k, $e = 9$□, $P1 = 30$ k

□□ $R\ 34 = \{105 \times 12 + 15 \times 9\}/60 - 30 = -6.75$ k

Calculate the maximum value of RA for the wheel load arrangement shown below.



Calculate the maximum value of RA for the wheel load arrangement shown below.



RA is maximum when W3 is *at A* RA(Max) = *[30 × (60 + 48 + 36 + 24) + 15 × 9]/60 = 86.25 kips*

Maximum ‘Shear Force’ due to Wheel Loads

Using similar arguments as mentioned for ‘support reactions’, the change in shear force between successive wheel arrivals at a section is given by

$\square V = \{(\sum P)\, d1 + P\square\, e + P0\, e0\}/L - P1$ ……..(2)

where $\sum P$ = Load remaining on the influence line throughout the wheel movement, *d1* = Shift of the wheels, $P\square$ = New load moving a distance *e* within the influence line, *P1* = Load which shifted off the section, *and P0* = Load moving off the influence line from a distance *e0* inside.

These terms are illustrated in (i) and (ii) of Fig. 1, demonstrating transition from W2 to *W3* at a critical section.

Maximum ‘Shear Force’ due to Wheel Loads

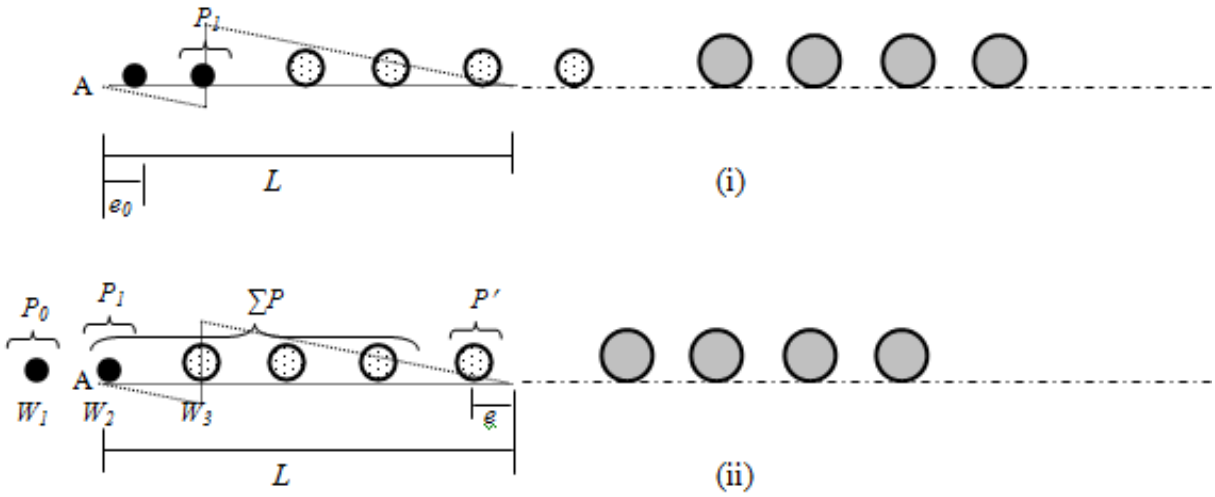
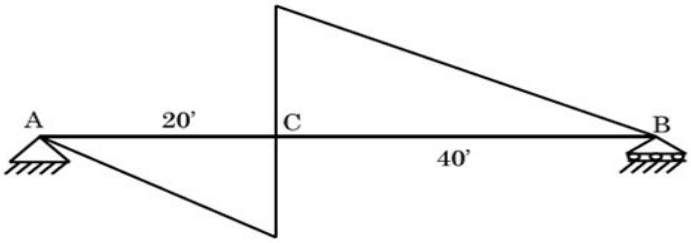
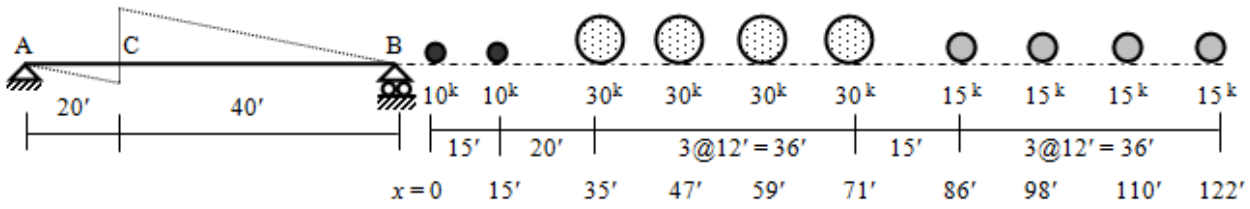


Fig. 1: Wheel Loads with ‘Shear Type’ Influence Line



Calculate the maximum value of VC for the wheel load arrangement shown below.

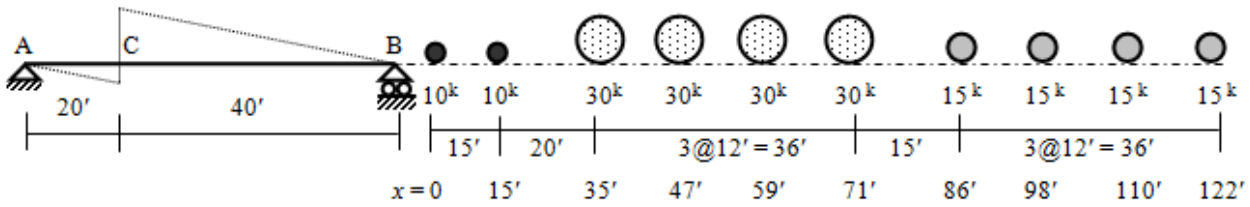


Between *W1* and *W2,*

$\sum P = 2 \times 10 + 30 = 50$ k, $d1 = 15$□, P□ $= 30$ k, $e = 8$□, $P0 = 0$, $e0 = 20$', $P1 = 10$ k

□□$V12 = \{50 \times 15 + 30 \times 8\}/60 - 10 = 6.5$ k

Calculate the maximum value of VC for the wheel load arrangement shown below.

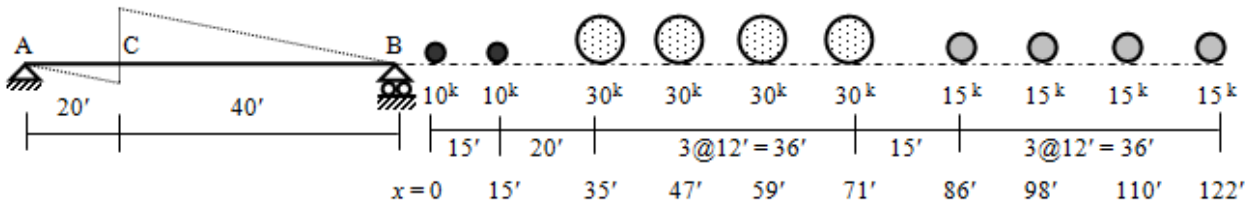


Between W2 and *W3,*

$\sum P = 10 + 3 \times 30 = 100$ k, $d1 = 20$□, P□ $= 30$ k, $e = 4$□, $P0 = 10$ k, $e0 = 5$□, $P1 = 10$ k

□□$V23 = \{100 \times 20 + 30 \times 4 + 10 \times 5\}/60 - 10 = 26.17$ k

Calculate the maximum value of VC for the wheel load arrangement shown below.

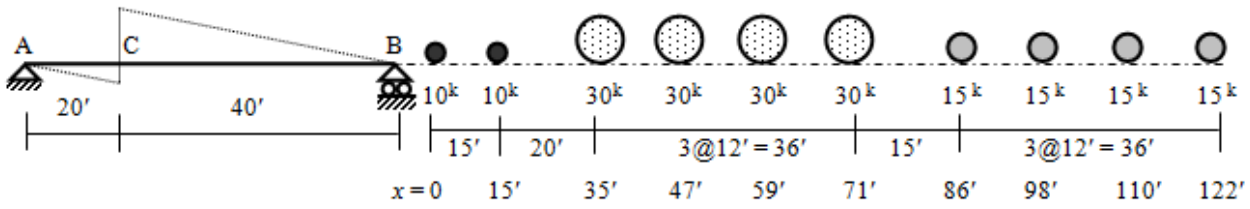


Between W3 and *W4,*

$\sum P = 4 \times 30 = 120$ k, $d1 = 12$□, P□ $= 15$ k, $e = 1$□, $P0 = 10$ k, $e0 = 0$, $P1 = 30$ k

□□$V\ 34 = \{120 \times 12 + 15 \times 1 + 10 \times 0\}/60 - 30 = -5.75$ k

Calculate the maximum value of VC for the wheel load arrangement shown below.



VC is maximum when W3 is *at C* VC(Max) = *[30 × (40 + 28 + 16 + 4) − 15 × 0]/60 = 44 kips*