Truss Analysis Calculations

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1. Truss Geometry

The overall configuration of the 2-dimensional truss is shown in Figure 1. The specific node and member configurations are also summarized in Table 1 and Table 2 below.

The total span of the truss is 30 m and overall height of the truss is 8 m.

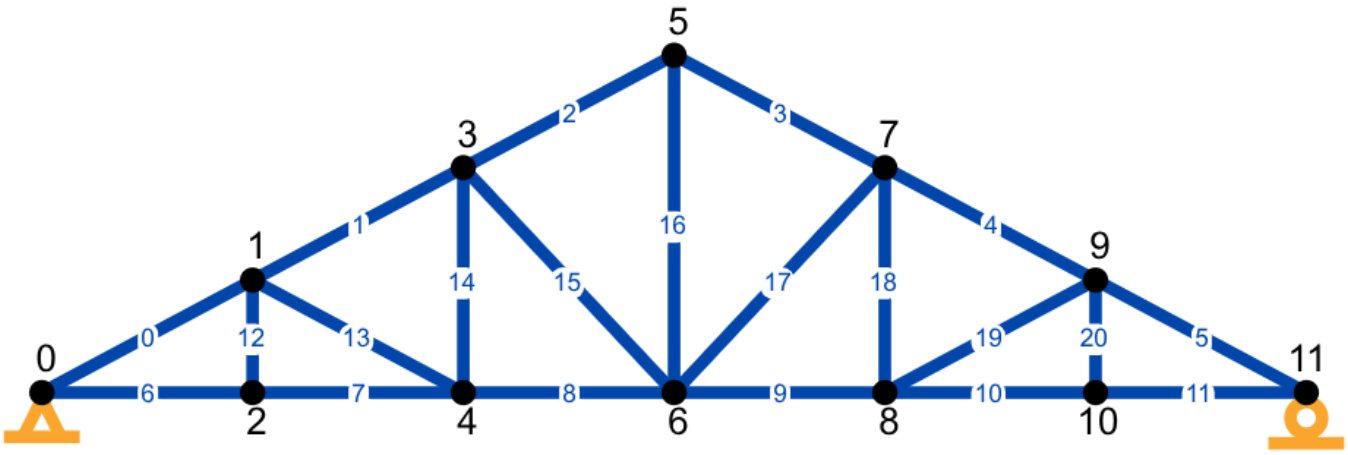


Figure 1: Truss global configuration

Node ID	X-Position (m)	Y-Position (m)	Fixity (if not free)
0	0	0	pin
1	5	2.67	--
2	5	0	--
3	10	5.33	--
4	10	0	--
5	15	8	--
6	15	0	--
7	20	5.33	--
8	20	0	--
9	25	2.67	--
10	25	0	--
11	30	0	roller

Table 1: Structure node geometry

Member ID	Start -> End Node	Length (m)
0	0 → 1	5.667
1	1 → 3	5.667

Member ID	Start -> End Node	Length (m)
2	3 → 5	5.667
3	5 → 7	5.667
4	7 → 9	5.667
5	9 → 11	5.667
6	0 → 2	5
7	2 → 4	5
8	4 → 6	5
9	6 → 8	5
10	8 → 10	5
11	10 → 11	5
12	1 → 2	2.667
13	1 → 4	5.667
14	3 → 4	5.333
15	3 → 6	7.311
16	5 → 6	8
17	6 → 7	7.311
18	7 → 8	5.333
19	8 → 9	5.667
20	9 → 10	2.667

Table 2: Structure member geometry

2. Applied Loading to Nodes

The loads applied to this truss structure are represented in Figure 2 and summarized in detail below in Table 3. Note that if a node is omitted from Table 3, no loads have been applied to it.

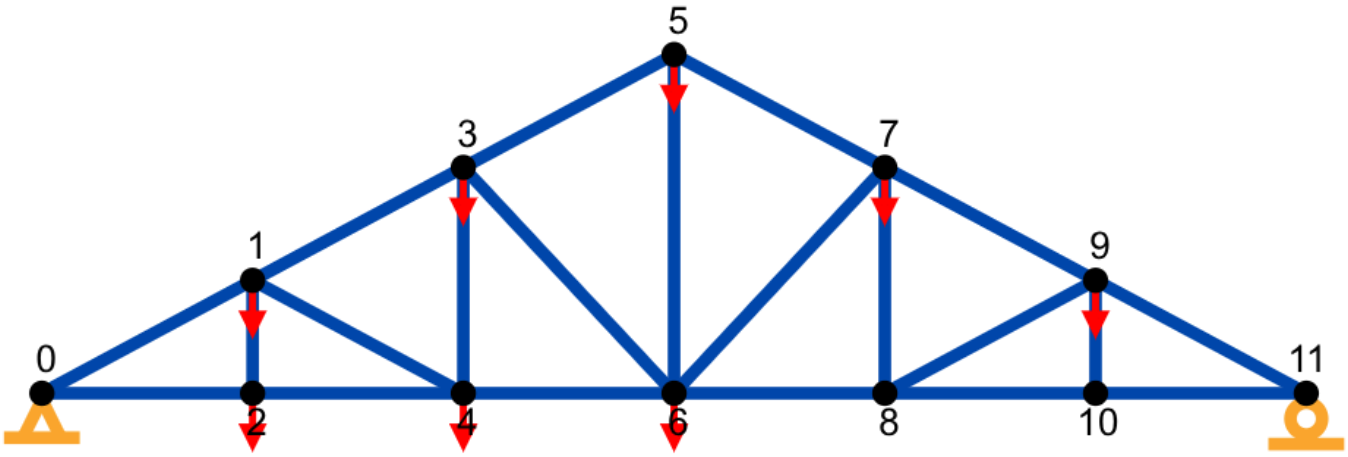


Figure 2: Graphical representation of loads applied to the structure (arrow length not to scale)

Node ID	Fx (kN)	Fy (kN)
1	0	-1
2	0	-5
3	0	-1
4	0	-5
5	0	-1
6	0	-5
7	0	-1
9	0	-1

Table 3: Applied loading to nodes

3. Truss Analysis Using the Direct Stiffness Method

With the truss geometry and loading defined above, the member forces and deflections are calculated using the direct stiffness method. It is assumed that all members behave elastically and have sufficient strength at connections to transfer the required load to the member.

3.1 Member Stiffness Matrix

First, each member stiffness matrix is composed in the global coordinate system. For truss analysis, it is assumed that both ends of the member are rotationally unconstrained so that each member will only be loaded axially. The member stiffness matrix in the global coordinate system will be a 4x4 matrix for a 2-dimensional truss. Each member will be defined as follows:

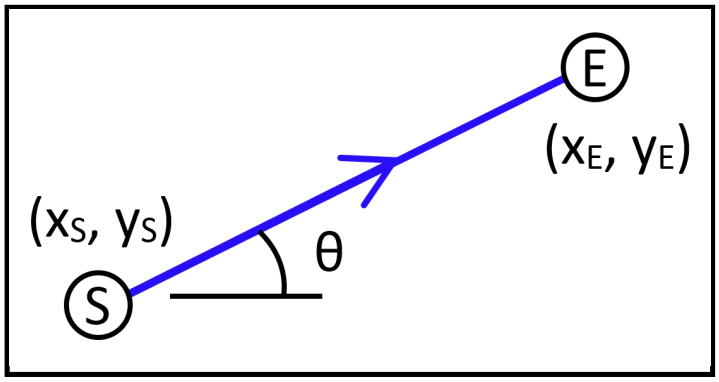


Figure 3: General member geometry definition

Ⓢ → Member starting node

Ⓔ → Member ending node

θ → Member rotation angle from horizontal

↗ → Member direction

Having member properties:

L → Member length

A → Member cross-sectional area

E → Member material modulus of elasticity

In this analysis, A and E have been set to the following values:

Member ID	Cross-sectional Area (mm^2)	Elastic Modulus (MPa)
0	5	29000
1	5	29000
2	5	29000
3	5	29000
4	5	29000
5	5	29000
6	5	29000
7	5	29000
8	5	29000
9	5	29000
10	5	29000
11	5	29000

Member ID	Cross-sectional Area (mm^2)	Elastic Modulus (MPa)
12	5	29000
13	5	29000
14	5	29000
15	5	29000
16	5	29000
17	5	29000
18	5	29000
19	5	29000
20	5	29000

For simplicity in this general example, the following constants are calculated:

$$c=cos\theta$$
$$s=sin\theta$$

And a stiffness matrix is assembled for each member using the following equation:

$$k_i = \frac{AE}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

For example, the stiffness matrix for member 0 is:

$$k_0 = \frac{5\,mm^2 * 29000\,MPa}{5.67m}\begin{bmatrix} 0.779 & 0.415 & -0.779 & -0.415 \\ 0.415 & 0.221 & -0.415 & -0.221 \\ -0.779 & -0.415 & 0.779 & 0.415 \\ -0.415 & -0.221 & 0.415 & 0.221 \end{bmatrix}$$

3.2 Global Structure Stiffness Matrix

All of the member stiffness matrices will be combined to form the global structure stiffness matrix, K, by grouping each nodal degree of freedom and summing the attached member stiffness matrix elements. For this 2-dimensional truss with 12 nodes, the global stiffness matrix will be 24x24.

This operation yields the following structural stiffness matrix for the above defined truss:

48.9	10.6	-19.9	-10.6	-29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10.6	5.67	-10.6	-5.67	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-19.9	-10.6	59.8	10.6	0	0	-19.9	-10.6	-19.9	10.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-10.6	-5.67	10.6	71.4	0	-54.4	-10.6	-5.67	10.6	-5.67	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-29	0	0	0	58	0	0	0	-29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-54.4	0	54.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-19.9	-10.6	0	0	49.1	11.4	0	0	-19.9	-10.6	-9.28	9.9	0	0	0	0	0	0	0	0	0	0
0	0	-10.6	-5.67	0	0	11.4	49.1	0	-27.2	-10.6	-5.67	9.9	-10.6	0	0	0	0	0	0	0	0	0	0
0	0	-19.9	10.6	-29	0	0	0	77.9	-10.6	0	0	-29	0	0	0	0	0	0	0	0	0	0	0
0	0	10.6	-5.67	0	0	0	-27.2	-10.6	32.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-19.9	-10.6	0	0	39.8	0	0	0	-19.9	10.6	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-10.6	-5.67	0	0	0	29.5	0	-18.1	10.6	-5.67	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-9.28	9.9	-29	0	0	0	76.6	1.78e-15	-9.28	-9.9	-29	0	0	0	0	0	0	0
0	0	0	0	0	0	9.9	-10.6	0	0	0	-18.1	0	39.2	-9.9	-10.6	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-19.9	10.6	-9.28	-9.9	49.1	-11.4	0	0	-19.9	10.6	0	0	0	0
0	0	0	0	0	0	0	0	0	0	10.6	-5.67	-9.9	-10.6	-11.4	49.1	0	-27.2	10.6	-5.67	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-29	0	0	0	77.9	10.6	-19.9	-10.6	-29	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-27.2	10.6	32.9	-10.6	-5.67	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-19.9	10.6	-19.9	-10.6	59.8	-10.6	0	0	-19.9	10
0	0	0	0	0	0	0	0	0	0	0	0	0	0	10.6	-5.67	-10.6	-5.67	-10.6	71.4	0	-54.4	10.6	-5.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-29	0	0	0	58	0	-29	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-54.4	0	54.4	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-19.9	10.6	-29	0	48.9	-10
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10.6	-5.67	0	0	-10.6	5.

Structure Stiffness Matrix, K

3.3 Reduced Structure Stiffness Matrix

With the reactions at the structure supports being unknown, the structure stiffness matrix is reduced by removing the rows and columns which correspond to the node support directions, resulting in the reduced structure stiffness matrix, K_R :

59.8	10.6	0	0	-19.9	-10.6	-19.9	10.6	0	0	0	0	0	0	0	0	0	0	0	0	0
10.6	71.4	0	-54.4	-10.6	-5.67	10.6	-5.67	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	58	0	0	0	-29	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-54.4	0	54.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-19.9	-10.6	0	0	49.1	11.4	0	0	-19.9	-10.6	-9.28	9.9	0	0	0	0	0	0	0	0	0
-10.6	-5.67	0	0	11.4	49.1	0	-27.2	-10.6	-5.67	9.9	-10.6	0	0	0	0	0	0	0	0	0
-19.9	10.6	-29	0	0	0	77.9	-10.6	0	0	-29	0	0	0	0	0	0	0	0	0	0
10.6	-5.67	0	0	0	-27.2	-10.6	32.9	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-19.9	-10.6	0	0	39.8	0	0	0	-19.9	10.6	0	0	0	0	0	0	0
0	0	0	0	-10.6	-5.67	0	0	0	29.5	0	-18.1	10.6	-5.67	0	0	0	0	0	0	0
0	0	0	0	-9.28	9.9	-29	0	0	0	76.6	1.78e-15	-9.28	-9.9	-29	0	0	0	0	0	0
0	0	0	0	9.9	-10.6	0	0	0	-18.1	0	39.2	-9.9	-10.6	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	-19.9	10.6	-9.28	-9.9	49.1	-11.4	0	0	-19.9	10.6	0	0	0
0	0	0	0	0	0	0	0	10.6	-5.67	-9.9	-10.6	-11.4	49.1	0	-27.2	10.6	-5.67	0	0	0
0	0	0	0	0	0	0	0	0	0	-29	0	0	0	77.9	10.6	-19.9	-10.6	-29	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-27.2	10.6	32.9	-10.6	-5.67	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-19.9	10.6	-19.9	-10.6	59.8	-10.6	0	0	-19.9
0	0	0	0	0	0	0	0	0	0	0	0	10.6	-5.67	-10.6	-5.67	-10.6	71.4	0	-54.4	10.6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-29	0	0	0	58	0	-29
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-54.4	0	54.4	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-19.9	10.6	-29	0	48.9

Reduced Structure Stiffness Matrix

3.4 Reduced structure force matrix

Given the loads applied to the structure, as described in Table 3, the global force matrix, Q, is assembled to match the dimensional size of the reduced structure stiffness matrix. Each node degree of freedom for the structure will match between the structure force and structure stiffness matrices. Since the reactions at the constrained nodes are unknown until the analysis is completed, the node support direction forces are removed from the global structure force matrix to create the reduced structure load matrix, Q_R :

$$\begin{bmatrix} 0 & -1 & 0 & -5 & 0 & -1 & 0 & -5 & 0 & -1 & 0 & -5 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

Structural Load Matrix (reduced)

3.5 Analysis for global displacements

The unknown nodal displacements are calculated by inverting the reduced stiffness matrix and multiplying it with the reduced structure force matrix: $K_R^{-1} \cdot Q_R$

Then, the known support displacements of 0 are added to compose the global stiffness matrix, D.

The resulting displacement at each node along with known support displacements are given below:

Node ID	Δx (m)	Δy (m)
0	0	0
1	2.76	-7.38
2	0.808	-7.47
3	2.57	-8.7
4	1.62	-9
5	1.47	-7.79
6	2.23	-8.45
7	1.05	-7.44
8	2.68	-7.46
9	1.51	-5.35
10	3.17	-5.35
11	3.65	0

Table 4: Structure node displacements derived from global stiffness matrix

3.6 Calculate member axial demands

Using the relative displacements of each member's start and end nodes along with a transformed stiffness matrix, the axial demand on a member, q, is calculated as follows:

$$q_i = \frac{AE}{L} \begin{bmatrix} -c & -s & c & s \end{bmatrix} \begin{bmatrix} \Delta_{Sx} \\ \Delta_{Sy} \\ \Delta_{Ex} \\ \Delta_{Ey} \end{bmatrix}$$

Where Δ_{Sx} is the displacement of the starting node in the x-direction for member i.

The member axial demands for the truss described above are displayed in Figure 4 and summarized in detail in Table 5 along with the member's length. Tensile axial loads are represented as negative forces, and compression axial demands are represented as positive forces.

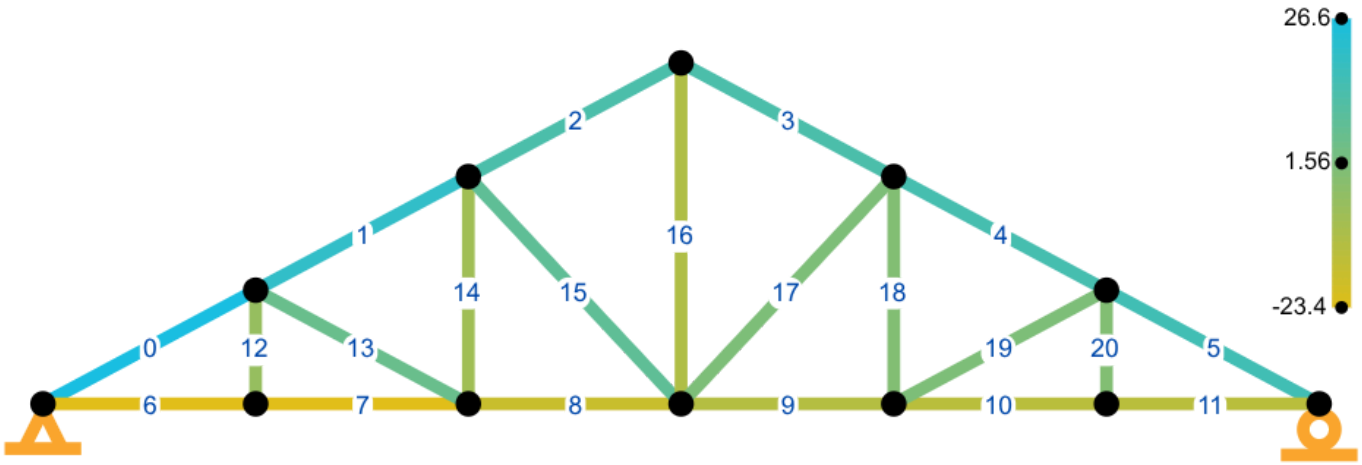


Figure 4: Structure member loading (kN)

Member ID	Length (m)	Axial Demand (kN)
0	5.667	26.56
1	5.667	20.19
2	5.667	13.81
3	5.667	13.81
4	5.667	14.88
5	5.667	15.94
6	5	-23.44
7	5	-23.44
8	5	-17.81
9	5	-13.13
10	5	-14.06
11	5	-14.06
12	2.667	-5
13	5.667	6.375
14	5.333	-8
15	7.311	8.224
16	8	-12
17	7.311	1.371
18	5.333	-0.5
19	5.667	1.062
20	2.667	0

Table 5: Structure member demand summary (+Compression/-Tension)

3.7 Calculate support reactions

First, the unknown values in the global force matrix, Q, is assembled by multiplying the global stiffness matrix by the global displacement matrix: $Q = K \cdot D$

The total force at the truss supports are then found by removing all of the free degrees of freedom (reduced structural load matrix, Q_R) from the force matrix so that only the supported degrees of freedom remain.

Finally, to calculate the supports, any loads applied to the supports are subtracted out of this reduced force matrix yielding the following support reactions:

Node ID	Rx (kN)	Ry (kN)
0	0	12.5
11	0	7.5

Table 6: Structure support reaction summary