EWCE 341

Principles of Soil Mechanics

Weight–Volume Relationship

Void Ratio and Porosity

Void ratio:

- Ratio of volumes of voids and solids may be expressed using the void ratio, e:

$$e = \frac{V_v}{V_s}$$

- **Densely packed soils** have a low void ratio.
- Typical values in the **field range from 0.1 to 2.5**

Porosity:

- Relative volume of voids to the volume of soil mass.

$$n = \frac{V_v}{V} \text{ x } 100\%$$

- Values ranges 9 to 70%.

Relation between porosity (n) and void ratio (e)

- We know, $\mathbf{n = \frac{V_v}{V} \times 100\%}$

Again $\mathbf{e = \frac{V_v}{V_s}}$, $V_v = e\ V_s$, and $V = V_v + V_s$

$$n = \frac{eV_s}{V_v + V_s} \times 100\%$$

$$= \frac{eV_s}{e\ V_s + V_s} \times 100\%$$

$$= \frac{V_s\,(e)}{V_s(e + 1)} \times 100\%$$

$$= \mathbf{\frac{e}{e + 1} \times 100\%}$$

Or, $ne + n = e$ or $e\,(1\text{-}n) = n$ or $\mathbf{e = \frac{n}{1 - n}}$

Relative Density

- Relative density is a **special weight-volume** parameter used in sandy and gravely soils.
- Defined as: $$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} \text{ x } 100\%$$

Where, D_r = Relative density

e = void ratio

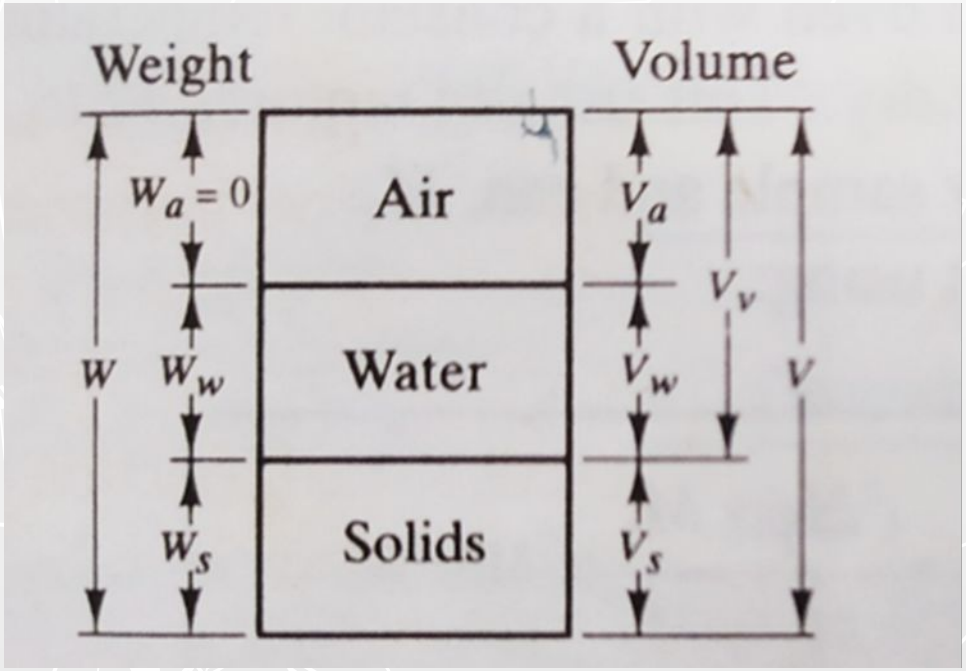
e_{min} = Minimum void ratio

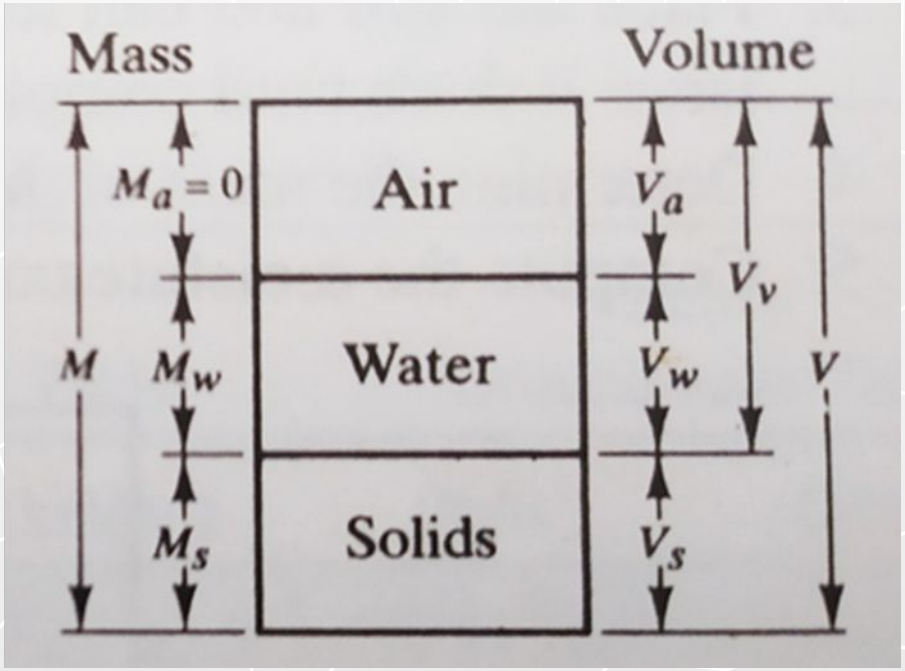
e_{max} = Maximum void ratio

- Values of e_{max} **and** e_{min} **represent the soil in very loose and very dense conditions** respectively and determined by standard laboratory test (ASTM D4253 and D4254).
- Loose soils have low values of D_r and while dense soils have high values, in theory **it varies from 0% to 100%.**

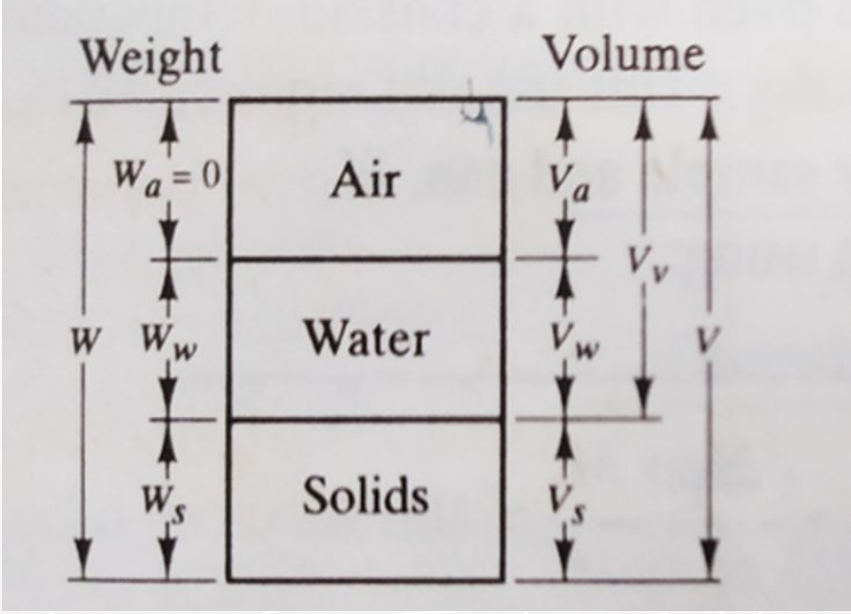
Derived Equation:

By combining definitions and the phase diagrams, we can derive a series of new weight-volume equations:





Prove that $S = \frac{w G_s}{e} \times 100\%$. (F-21)



Solution: Let $V_s = 1$

$e = \frac{V_v}{V_s}$ gives $V_v = e$

$S = \frac{V_w}{V_v} \times 100\%$

$= \frac{V_w}{e} \times 100\%$ gives $V_w = Se$

$\gamma_w = \frac{W_w}{V_w} = \frac{W_w}{Se}$ gives $W_w = \gamma_w Se$

$G_s = \frac{W_s}{V_s \gamma_w} = \frac{W_s}{(V_s = 1)\gamma_w}$ gives $W_s = G_s \gamma_w$

Now, $w = \frac{W_w}{W_s} \times 100\% = \frac{\gamma_w Se}{G_s \gamma_w} \times 100\%$ or $w = \frac{Se}{G_s} \times 100\%$

$G_s = \frac{Se}{w} \times 100\%$ or $S = \frac{w G_s}{e} \times 100\%$ proved.

Prove that $\gamma_d = \frac{G_s \gamma_w}{1+e}$ when water content is 100%

Solution: Let $V_s = 1$

$$\gamma_d = \frac{W_s}{V}$$

$$G_s = \frac{W_s}{V_s \gamma_w} = \frac{W_s}{(V_s = 1)\gamma_w} = \frac{W_s}{\gamma_w}$$

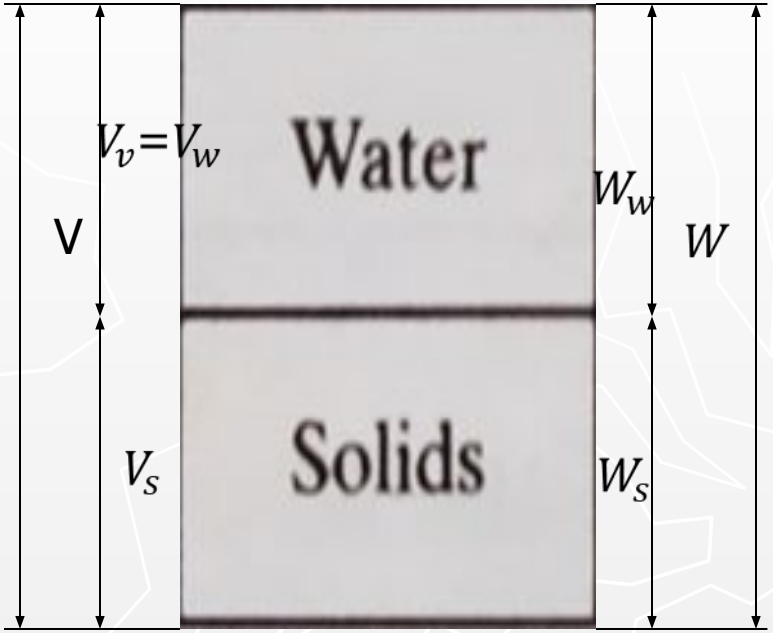
gives $W_s = G_s \gamma_w$

Again, $e = \frac{V_v}{V_s} = \frac{V_v}{(V_s = 1)} = V_v$ or $e = V_w$

Water content, $w = \frac{W_w}{W_s}$ gives $W_w = wW_s$

$$V = V_s + V_w = 1 + V_w \text{ or } V = 1 + V_v$$

$$= 1 + e$$



- $\gamma_d = \frac{W_s}{V} = \frac{G_s \gamma_w}{1+e}$ **Proved**

Again, $G_s = \frac{W_s}{V_s \gamma_w}$ **, Similarly** $G_w = \frac{W_w}{V_w \gamma_w}$

$$V_w = \frac{W_w}{G_w \gamma_w} = \frac{(W_w =) w W_s}{G_w \gamma_w} = \frac{w G_s \gamma_w}{G_w \gamma_w} = \mathbf{w} G_s \qquad (G_w = 1) \text{------- (1)}$$

- $\gamma_d = \frac{W_s}{V} = \frac{G_s \gamma_w}{1+V_w} = \frac{G_s \gamma_w}{1+\mathbf{w} G_s} = \frac{G_s \gamma_w}{1+\mathbf{w} G_s}$ **proved**

Again, $e = \frac{V_v}{V_s} = \frac{V_v}{(V_s=1)} = V_v = V_w$ **(from Figure)**

$e = V_w = \mathbf{w} G_s$ **---- (from 1)**

$\gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{G_s \gamma_w}{1+\mathbf{w} G_s}$ **Proved.**

Following equations may be used:

$$\gamma_d = \frac{\gamma}{1 + w}$$

$$\gamma_d = \frac{G_s \gamma_w}{1 + wG_s/S}$$

$$W_s = \frac{W}{1 + w}$$

$$M_s = \frac{M}{1 + w}$$

$$e = \frac{G_s \gamma_w}{\gamma_d} - 1$$

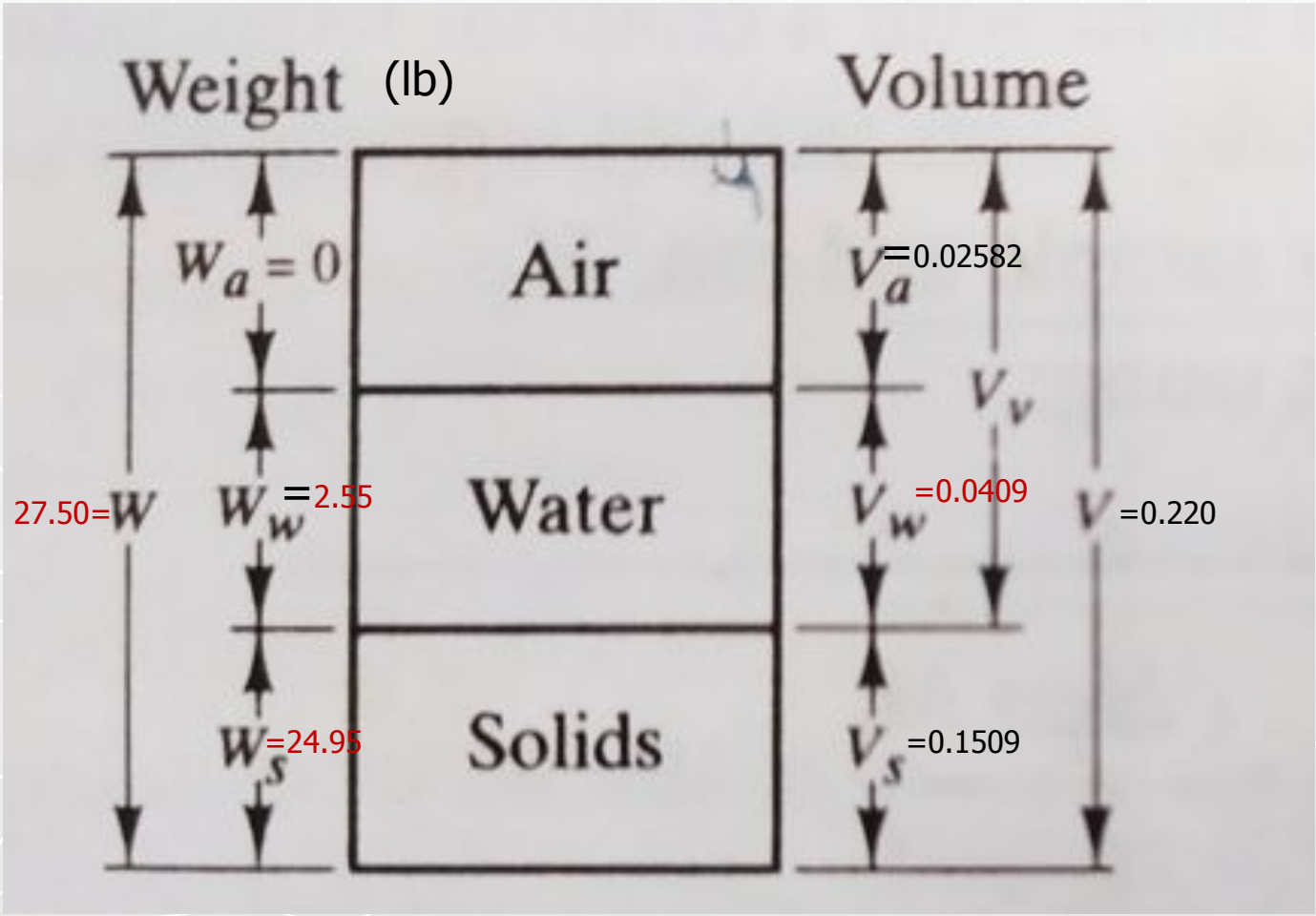
$$w = S\left[\frac{\gamma_w}{\gamma_d} - \frac{1}{G_s}\right] \times 100\%$$

$$S = \frac{w}{\frac{\gamma_w}{\gamma_d} - \frac{1}{G_s}} \times 100\%$$

Solving Weight – Volume problems


N
NW
NE
E
S
SE

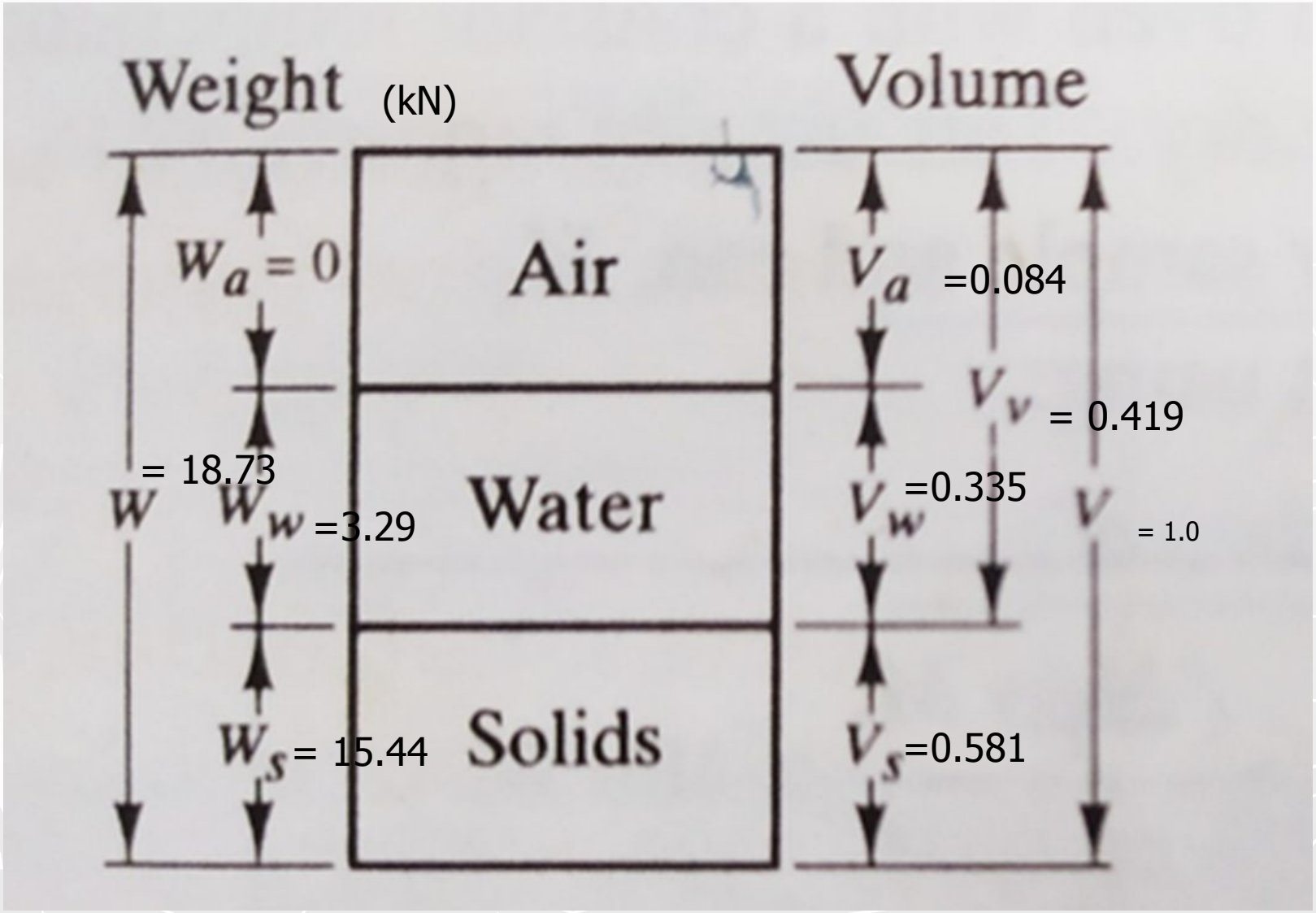

N
NW
NE
E
S
SE


Weight (lb)
Volume
$W_a = 0$
Air
V_a =0.02582
V_v
27.50=W
W_w =2.55
Water
V_w =0.0409
V =0.220
W_s =24.95
Solids
V_s =0.1509


N
NW
NE
E
S
SE


N
NW
NE
E
S
SE


N
NW
NE
E
S
SE


Weight (kN)
Volume
$W_a = 0$
Air
V_a =0.084
V_v = 0.419
W = 18.73
W_w =3.29
Water
V_w =0.335
V = 1.0
W_s = 15.44
Solids
V_s =0.581

Step-3: Determine dimensions in phase diagram.

$$\mathbf{n}=\frac{V_v}{V} \quad \text{or } 0.419=\frac{V_v}{1} \qquad \text{or } V_v=0.419\ m^3$$

$$V_s=\mathbf{V}-V_v \quad \text{or } V_s=1-0.419=0.581\ m^3$$

$$G_s=\frac{W_s}{V_s\gamma_w} \quad \text{or } 2.71=\frac{W_s}{(0.581m^3)(9.81\frac{kN}{m^3})} \qquad \text{or } W_s=15.44\ \text{kN}$$

$$w=\frac{W_w}{W_s}\ \mathbf{x100\%} \quad \text{or } 0.213=\frac{W_w}{15.44\ kN} \qquad \text{or } W_w=3.29\ \text{kN}$$

$$\mathbf{W}=W_s+W_w=15.44\ \text{kN}+3.29\text{kN} \quad \text{or W=18.73 kN}$$

$$\gamma_w=\frac{W_w}{V_w} \quad \text{or } 9.81\text{kN}/m^3=\frac{3.29kN}{V_w} \qquad \text{or } V_w=0.335\ m^3$$

$$V_a=V_v-V_w=0.419m^3-0.335m^3=0.084m^3$$

Step-4: Compute parameters.

$$\mathbf{S}=\frac{V_w}{V_v}\ \mathbf{x100\%}\ =\frac{0.335m^3}{0.335m^3+0.084m^3}\ \text{x}100\%\ =80.0\%$$

$$\boldsymbol{\gamma}=\frac{W}{V}\ =\frac{18.73kN}{1m^3}\ =18.73\ \text{kN}/m^3$$

Thanks