# EWCE 341

# Principles of Soil Mechanics

## Weight–Volume Relationship

# Void Ratio and Porosity

## Void ratio:

- Ratio of volumes of voids and solids may be expressed using the void ratio, e:

$$e = \frac{V_v}{V_s}$$

- **Densely packed soils** have a low void ratio.
- Typical values in the **field range from 0.1 to 2.5**

## Porosity:

- Relative volume of voids to the volume of soil mass.

$$n = \frac{V_v}{V} \text{ x } 100\%$$

- Values ranges 9 to 70%.

# Relation between porosity (n) and void ratio (e)

We know, $\mathbf{n = \frac{V_v}{V} \times 100\%}$

**Again e =** $\frac{V_v}{V_s}$, $V_v = e\, V_s$ , and $V = V_v + V_s$

$$n = \frac{eV_s}{V_v + V_s} \times 100\%$$

$$= \frac{eV_s}{e\, V_s + V_s} \times 100\%$$

$$= \frac{V_s\,(e)}{V_s(e + 1)} \times 100\%$$

$$= \mathbf{\frac{e}{e + 1} \times 100\%}$$

Or, ne + n = e or e (1-n) = n or $\mathbf{e = \frac{n}{1 - n}}$

# Relative Density

- Relative density is a **special weight-volume** parameter used in sandy and gravely soils.
- Defined as: $$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} \text{ x } 100\%$$

Where, $D_r$ = Relative density

e = void ratio

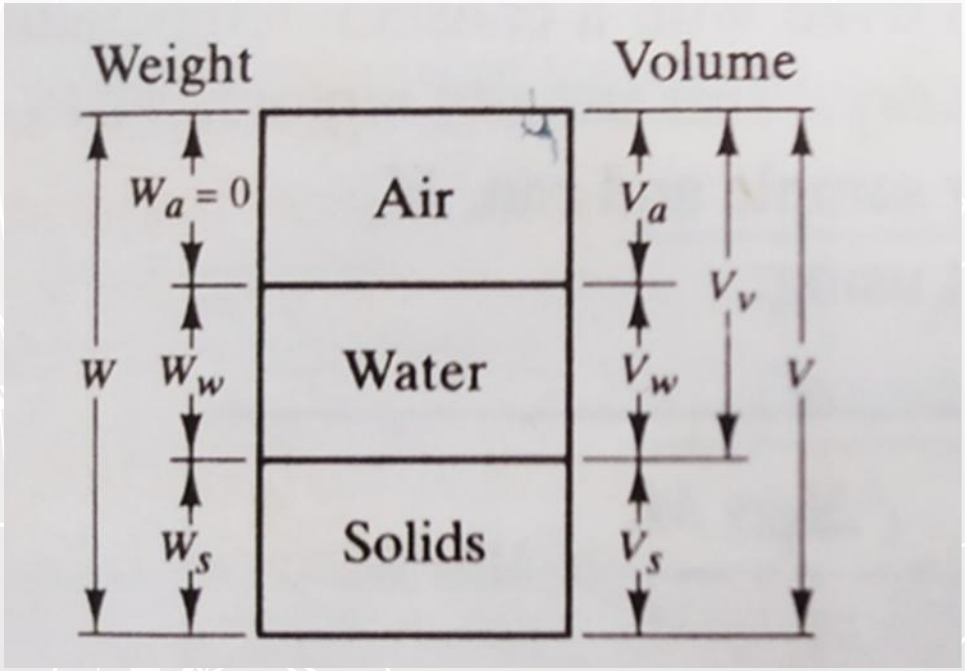
$e_{min}$ = Minimum void ratio

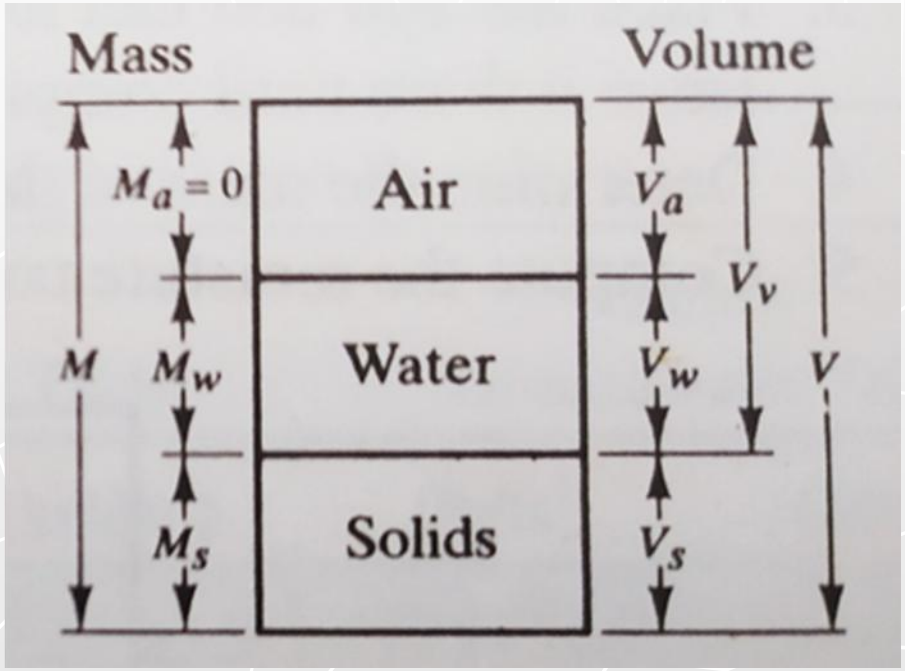
$e_{max}$ = Maximum void ratio

- Values of $e_{max}$ **and** $e_{min}$ **represent the soil in very loose and very dense conditions** respectively and determined by standard laboratory test (ASTM D4253 and D4254).
- Loose soils have low values of $D_r$ and while dense soils have high values, in theory **it varies from 0% to 100%.**

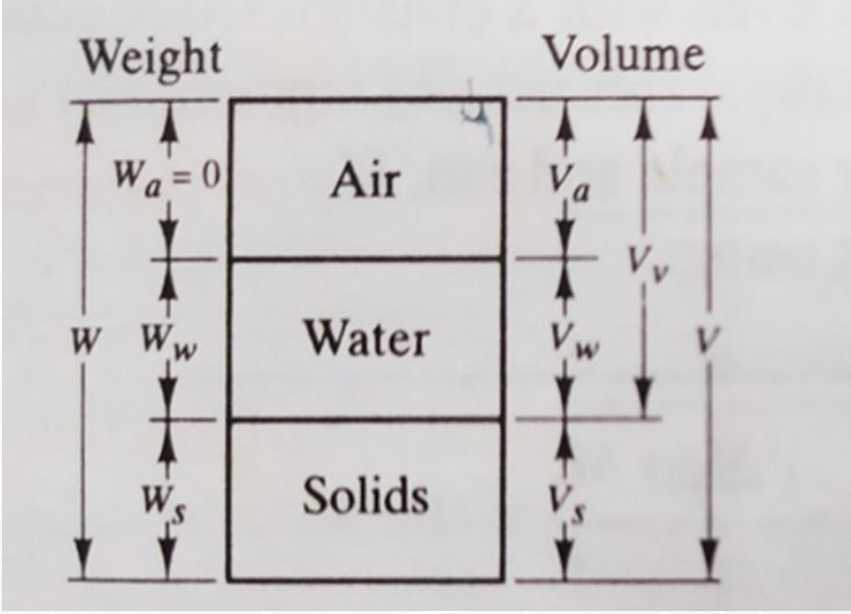
## **Derived Equation**:

By combining definitions and the phase diagrams, we can derive a series of new weight-volume equations:



**Prove that S= $\frac{w\,G_s}{e}$ x 100%. (F-21)**




Solution: Let $V_s$=1

$\mathbf{e = \frac{V_v}{V_s}}$ gives $V_v$=e

$\mathbf{S= \frac{V_w}{V_v}\ x\ 100\%}$

$=\frac{V_w}{e}$ x 100% gives $V_w$=Se

$\gamma_w = \frac{W_w}{V_w} = \frac{W_w}{Se}$ gives $W_w = \gamma_w Se$

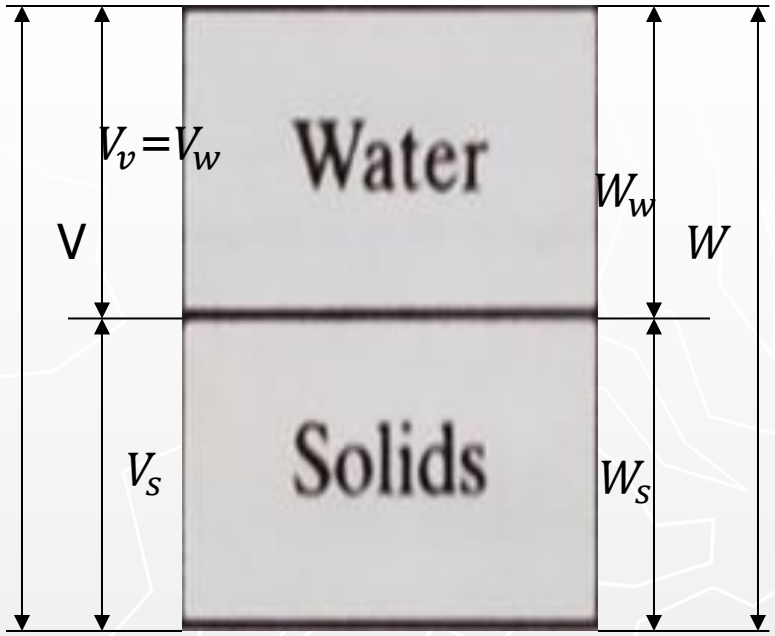
$G_s = \frac{W_s}{V_s \gamma_w} = \frac{W_s}{(V_s=1)\gamma_w}$ gives $W_s = G_s \gamma_w$

Now, $\mathbf{w= \frac{W_w}{W_s}\ x\ 100\%} = \frac{\gamma_w Se}{G_s \gamma_w}$ x 100% or $\mathbf{w= \frac{Se}{G_s}\ x\ 100\%}$

$G_s = \mathbf{\frac{Se}{w}\ x\ 100\%}$ or $\mathbf{S= \frac{wG_s}{e}\ x\ 100\%}$ proved.

Prove that $\gamma_d = \frac{G_s \gamma_w}{1+e}$ when water content is 100%

Weight




Solution: Let $V_s = 1$

$$\gamma_d = \frac{W_s}{V}$$

$$G_s = \frac{W_s}{V_s \gamma_w} = \frac{W_s}{(V_s = 1)\gamma_w} = \frac{W_s}{\gamma_w}$$

gives $W_s = G_s \gamma_w$

Again, $e = \frac{V_v}{V_s} = \frac{V_v}{(V_s = 1)} = V_v$ or $e = V_w$

Water content, $w = \frac{W_w}{W_s}$ gives $W_w = wW_s$

$V = V_s + V_w = 1 + V_w$ or $V = 1 + V_v$

$= 1 + e$

- $\gamma_d = \frac{W_s}{V} = \frac{G_s \gamma_w}{1+e}$ **Proved**

**Again,** $G_s = \frac{W_s}{V_s \gamma_w}$ **, Similarly** $G_w = \frac{W_w}{V_w \gamma_w}$

$$V_w = \frac{W_w}{G_w \gamma_w} = \frac{(W_w =) w W_s}{G_w \gamma_w} = \frac{w G_s \gamma_w}{G_w \gamma_w} = \mathbf{w} G_s \qquad (G_w = 1) \text{------ (1)}$$

- $\gamma_d = \frac{W_s}{V} = \frac{G_s \gamma_w}{1+V_w} = \frac{G_s \gamma_w}{1+\mathbf{w} G_s} = \frac{G_s \gamma_w}{1+\mathbf{w} G_s}$ **proved**

**Again,** $e = \frac{V_v}{V_s} = \frac{V_v}{(V_s = 1)} = V_v = V_w$ **(from Figure)**

$e = V_w = \mathbf{w} G_s$ **---- (from 1)**

$$\gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{G_s \gamma_w}{1+\mathbf{w} G_s}$$ **Proved.**

# Following equations may be used:

$$\gamma_d = \frac{\gamma}{1 + w}$$

$$\gamma_d = \frac{G_s \gamma_w}{1 + w G_s / S}$$

$$W_s = \frac{W}{1 + w}$$

$$M_s = \frac{M}{1 + w}$$

$$e = \frac{G_s \gamma_w}{\gamma_d} - 1$$

$$w = S\left[\frac{\gamma_w}{\gamma_d} - \frac{1}{G_s}\right] \times 100\%$$

$$S = \frac{w}{\frac{\gamma_w}{\gamma_d} - \frac{1}{G_s}} \times 100\%$$

# Solving Weight – Volume problems

Two methods:

- Using derived equations
- Using phase diagrams

## Using derived equations

# **A 27.50 lb** soil sample has a volume of **0.220 $ft^3$**, a moisture content of **10.2%**, and specific gravity of solids of **2.65.** Compute the unit weight, dry unit weight, degree of saturation, void ratio, and porosity.

Solutions:

Using derived equations:

$$\gamma = \frac{W}{V} = \frac{27.50\ lb}{0.220\ ft^3} = 125.0\ \text{lb}/ft^3$$

$$\gamma_d = \frac{\gamma}{1+w} = \frac{125.0\ lb/ft^3}{1+0.102} = 113.4\ \text{lb}/ft^3$$

$$S = \frac{w}{\frac{\gamma_w}{\gamma_d} - \frac{1}{G_s}} \text{ x } 100\% = \frac{0.102}{\frac{62.4\ lb/ft^3}{113.0\ lb/ft^3} - \frac{1}{2.65}} \text{ x } 100 = 59\%$$

$$e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{2.65\ x\ 62.4\ lb/ft^3}{113.0\ lb/ft^3} - 1 = 0.4582$$

$$n = \frac{e}{1+e} \text{ x } 100\% = \frac{0.458}{1+0.458} \text{ x } 100\% = 31.4\%$$

# **Using Phase Diagram** – Procedure:

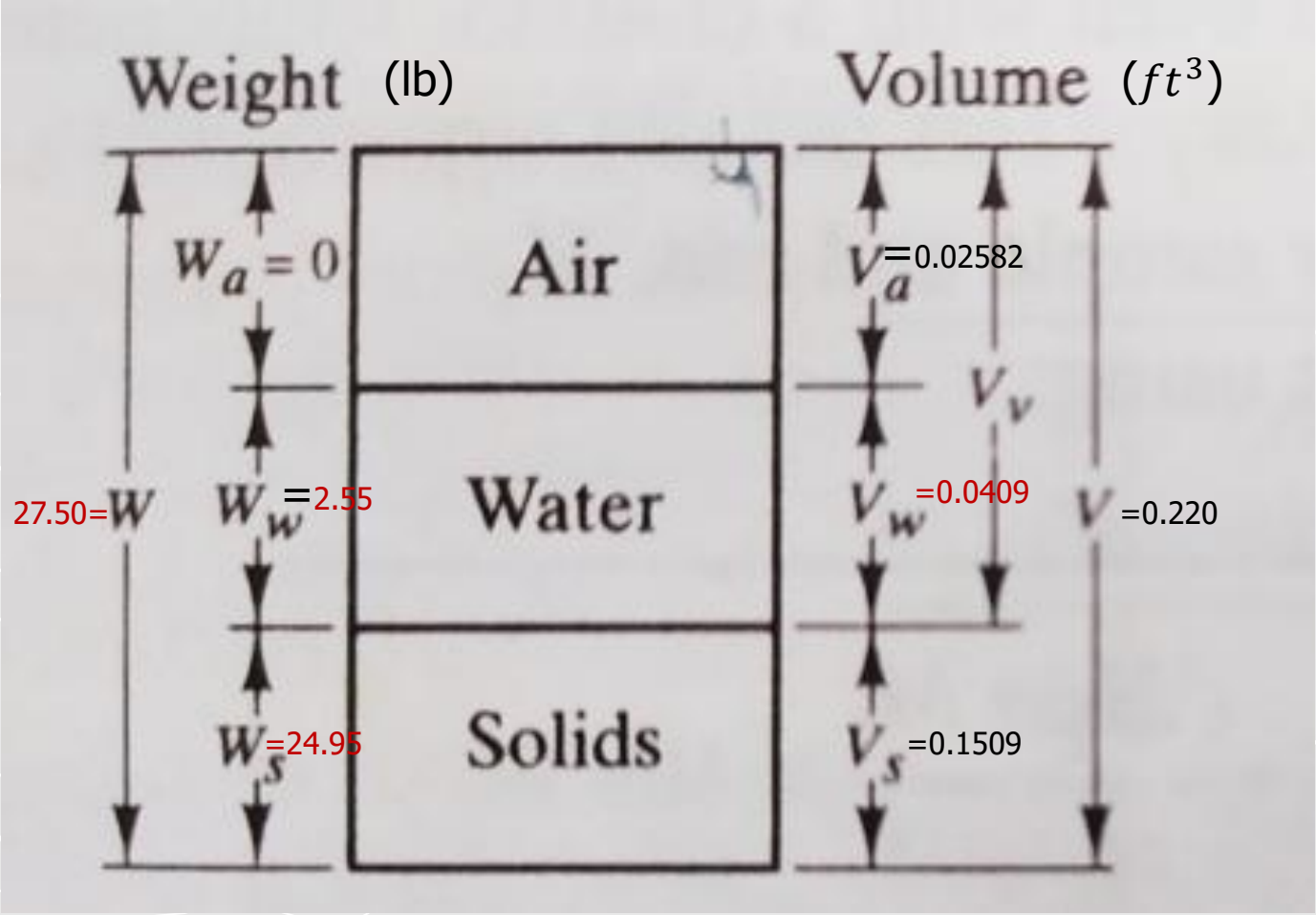
- Draw a phase diagram and annotate all of the dimensions presented in the problem statement. Remember, set $W_a$**=0**. If the soil is saturated, also set $V_a$**=0**.
- If **no dimensions** are given, a quantity may be assumed. For example V= 1 $m^3$ or 1 $ft^3$
- **Addition and subtraction** may be done to determine other dimensions.
- **Compute the required parameters** from these dimensions.
- $\gamma = \frac{W}{V}$ $\gamma_d = \frac{W_s}{V}$ $S=\frac{V_w}{V_v}x100\%$ $e=\frac{V_v}{V_s}$ $n=\frac{V_v}{V}x100\%$

Using Phase Diagram:

# A 27.50 lb soil sample has a volume of 0.220 $ft^3$, a moisture content of 10.2%, and specific gravity of solids of 2.65. Compute the unit weight, dry unit weight, degree of saturation, void ratio, and porosity.

**Solutions:**

Step-1:Draw and annotate a phase diagram.



- Step-2: Assume dimension - na
- Step-3: Determine dimensions in phase diagram:

$$W_s = \frac{W}{1+w} = \frac{27.50}{1+0.102} = 24.95 \text{ lb}$$

$$W_w = W - W_s = 27.50 - 24.95 = 2.55 \text{ lb}$$

$$G_s = \frac{W_s}{V_s \gamma_w} \text{ or } \quad 2.65 = \frac{24.95}{V_s\,(6.24\ lb/ft^3)} \text{ or } V_s = 0.1509 ft^3$$

$$\gamma_w = \frac{W_w}{V_w} = \frac{2.55\ lb}{V_w} \qquad \text{or} \qquad V_w = 0.0409\ ft^3$$

$$V_a = V - V_w - V_s = 0.220\ ft^3 - 0.0409\ ft^3 - 0.1509 ft^3 = 0.0282 ft^3$$

Step-4: Compute parameters

$$\gamma = \frac{W}{V} = \frac{27.50\ lb}{0.220\ ft^3} = 125.0\ \text{lb}/ft^3$$

$$n = \frac{V_v}{V} \text{ X } 100\% = \frac{0.0282ft^3 + 0.0409ft^3}{0.220ft^3} \text{ X } 100\% = 31.4\%$$

$$e = \frac{V_v}{V_s} = \frac{0.0409ft^3 + 0.0282ft^3}{0.1509ft^3} = 0.458$$

$$\gamma_d = \frac{W_s}{V} = \frac{24.95\ lb}{0.220ft^3} = 113.0\ \text{lb}/ft^3$$

$$S = \frac{V_w}{V_v} \text{ x } 100\% = \frac{0.0409ft^3}{0.0409ft^3 + 0.0282ft^3} \text{ x } 100\% = 59\%$$

# A certain soil has the following properties:

**$G_s$= 2.71**

**n=41.9%**

**w=21.3%**

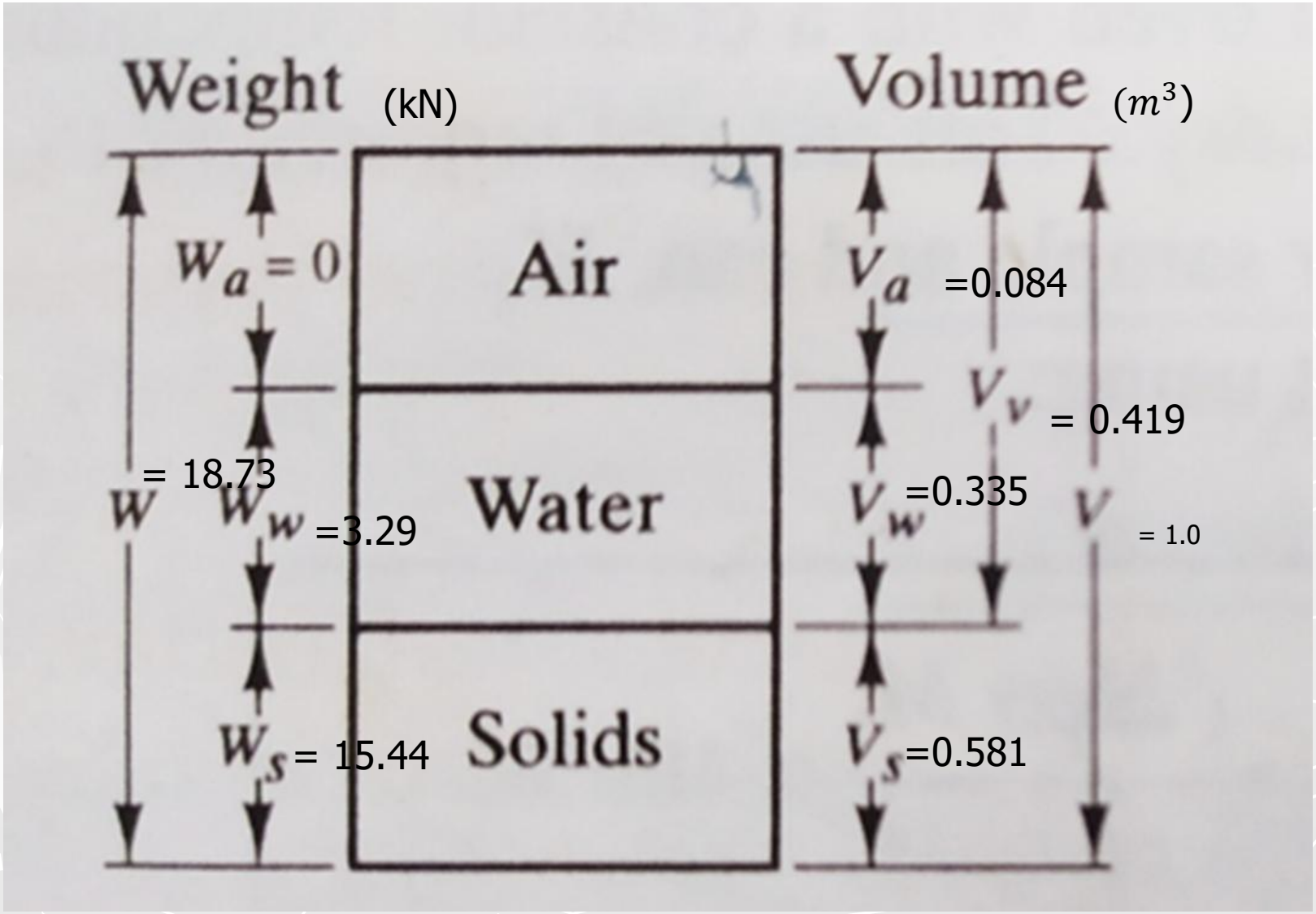
Find the degree of saturation S, and unit weight, $\gamma$, using phase diagrams.

Solution:

**Step-1**: Draw and annotate phase diagram.

**Step-2**: Assume dimensions.

- No volumes or weighs are stated.
- All data applies to entire soil strata.
- Assume V= 1 $m^3$


Weight (kN)
Volume $(m^3)$
$W_a = 0$
Air
$V_a$ =0.084
$V_v$ = 0.419
$W$ = 18.73
$W_w$ =3.29
Water
$V_w$ =0.335
$V$ = 1.0
$W_s$ = 15.44
Solids
$V_s$ =0.581

Step-3: Determine dimensions in phase diagram.

$$\mathbf{n}=\frac{V_v}{V} \quad \text{or } 0.419=\frac{V_v}{1} \qquad \text{or } V_v=0.419\ m^3$$

$$V_s=\mathbf{V}-V_v \quad \text{or } V_s=1-0.419=0.581\ m^3$$

$$G_s=\frac{W_s}{V_s\gamma_w} \quad \text{or } 2.71=\frac{W_s}{(0.581m^3)(9.81\frac{kN}{m^3})} \qquad \text{or } W_s=15.44\ \text{kN}$$

$$w=\frac{W_w}{W_s}\mathbf{x100\%} \quad \text{or } 0.213=\frac{W_w}{15.44\ kN} \qquad \text{or } W_w=3.29\ \text{kN}$$

$$\mathbf{W}=W_s+W_w=15.44\ \text{kN}+3.29\text{kN} \quad \text{or } W=18.73\ \text{kN}$$

$$\gamma_w=\frac{W_w}{V_w} \quad \text{or } 9.81\text{kN}/m^3=\frac{3.29kN}{V_w} \qquad \text{or } V_w=0.335\ m^3$$

$$V_a=V_v-V_w=0.419m^3-0.335m^3=0.084m^3$$

Step-4: Compute parameters.

$$\mathbf{S=\frac{V_w}{V_v}\ x100\%} = \frac{0.335m^3}{0.335m^3+0.084m^3}\ \text{x100\%} = 80.0\%$$

$$\boldsymbol{\gamma = \frac{W}{V}} = \frac{18.73kN}{1m^3} = 18.73\ \text{kN/}m^3$$

# Thanks