EWCE 341

Principles of Soil Mechanics

Stresses in Soils

What we are going to discuss today?

-Stresses in soils

-Horizontal stress

-Measurement of horizontal stress

(in-situ & lateral earth pressure)

-Measurement of Geostatic stress and Induced Stress)

-Boussinesq's Equation (Point and Line Load)

-Newmark's Solution (Area load)

-Chart solution

What is stress in soils ?

Stress:

- All civil Engineering projects impart loads on the ground that supports them.
- **Loads produces stresses in soils** ie **compressive stress**, **shear stress** and **even tensile stress**.

Horizontal Stress:

- How horizontal stresses are:

-**Direct result** of **applied external loads** such as the braking forces from the wheels of a large track.

-**Indirect result** of **applied vertical geostatic stresses and induced stresses.**

Importance:

- **Important** for many engineering analysis.
- **Design of Retaining Walls (RW)** depends on the horizontal stresses in the soils being retained.









How can you measure the horizontal stresses in the ground?

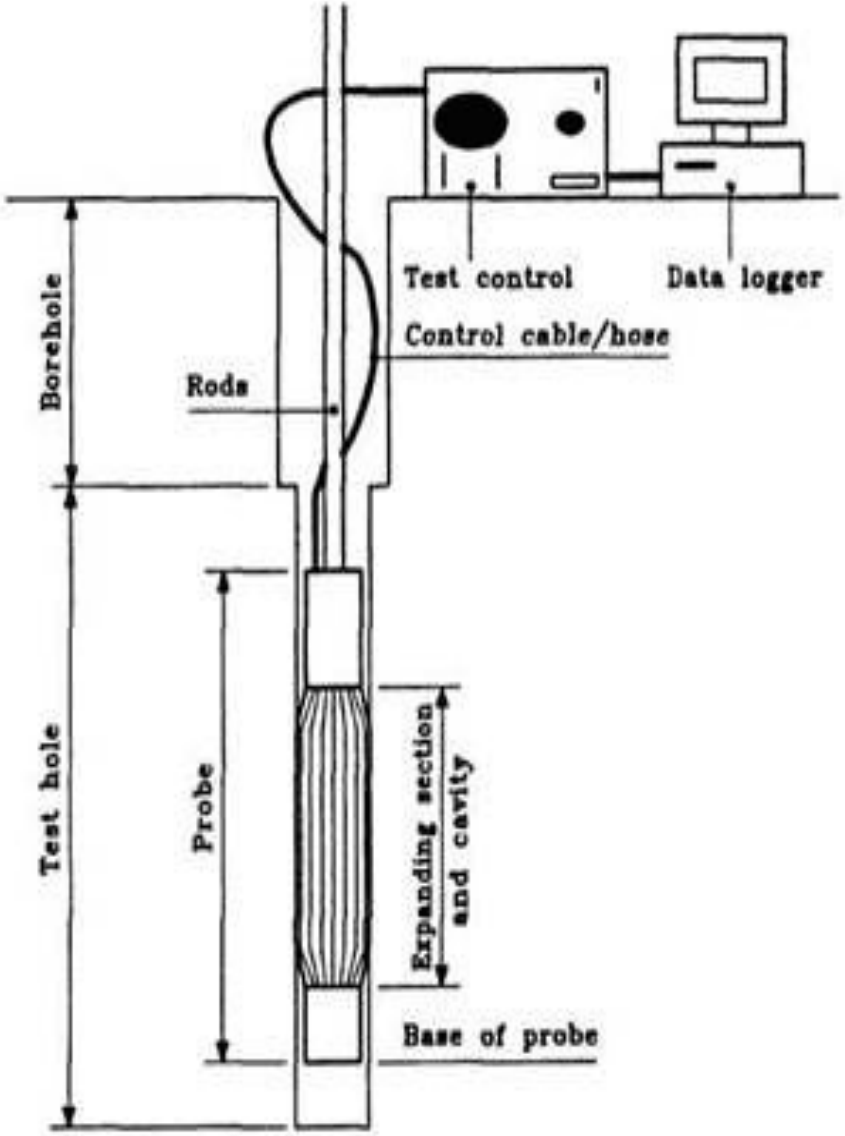
Measure of horizontal stress:

- **In-situ measurement** by:
  - -**PMT**: Pressure Meter Test
  - -**DMT**: Dilatometer Test
- **Estimated from the vertical stress**

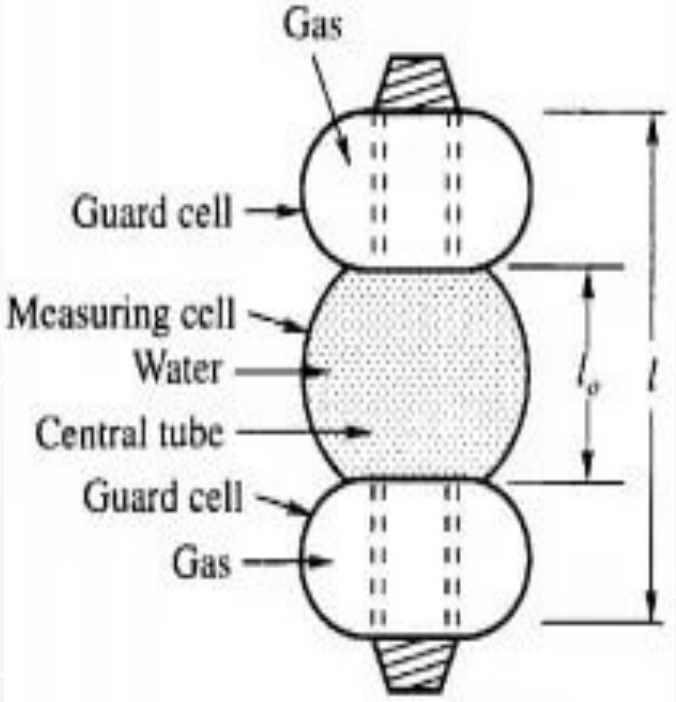
using of lateral earth pressure.

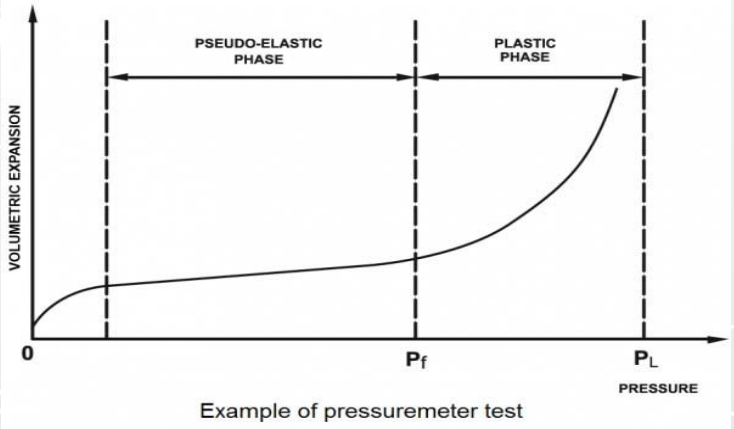
If **ground is horizontal** then $\sigma_x = \sigma_y$ for three dimensional analyses.





Pressure meter

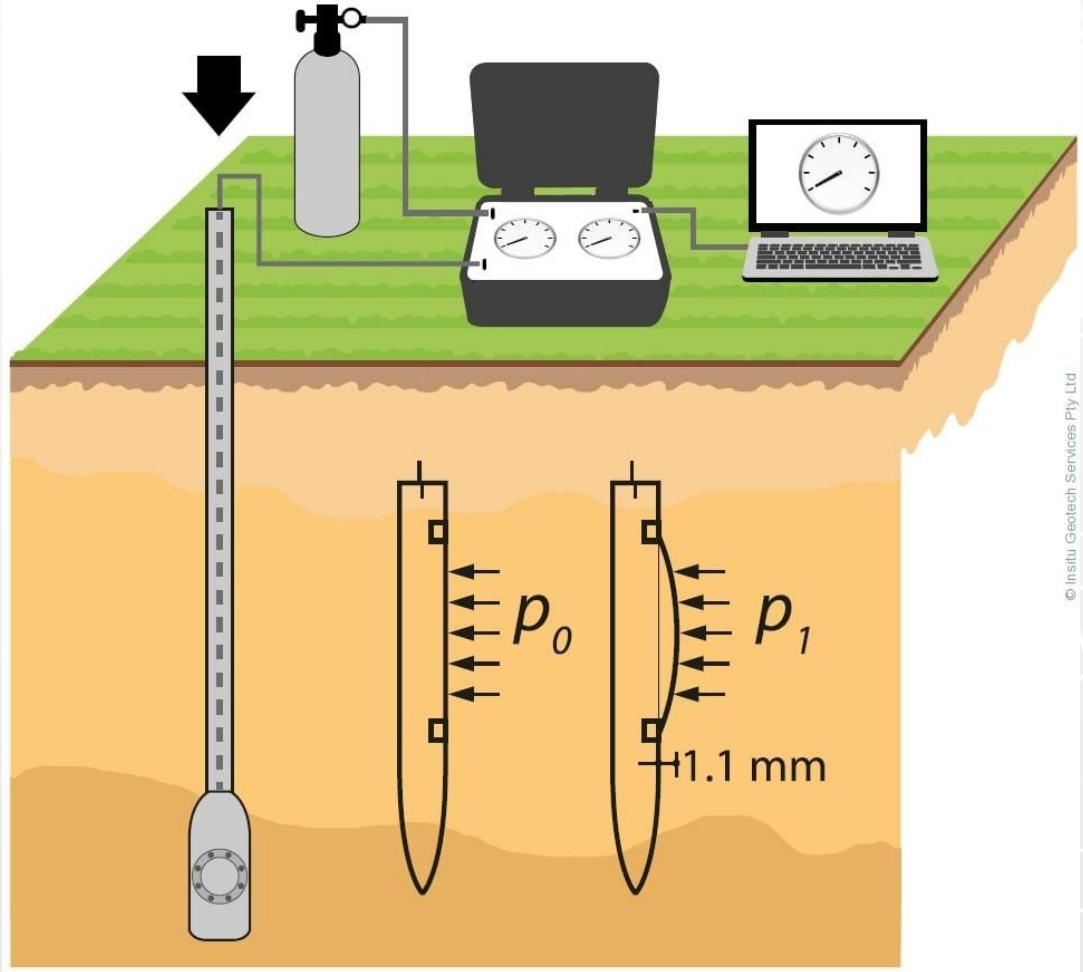


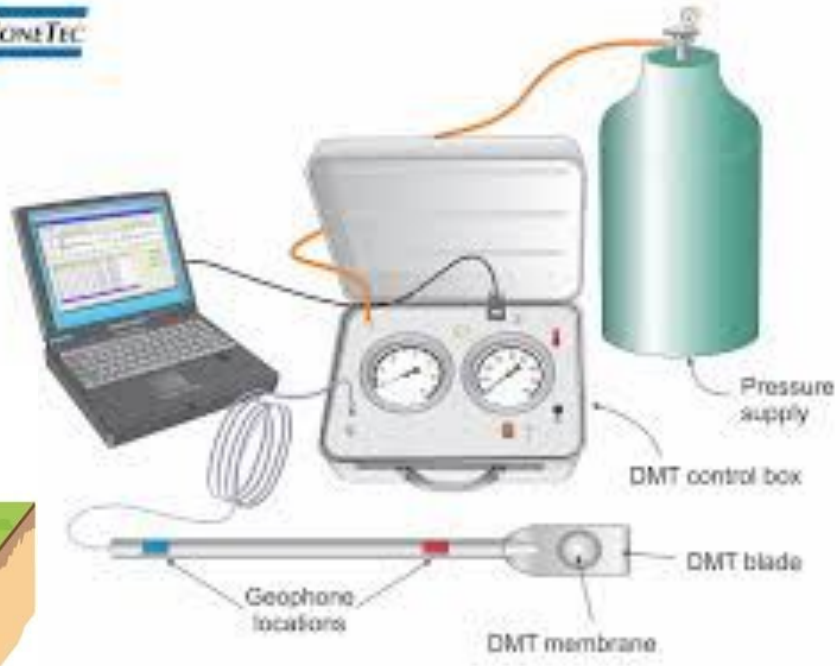


Example of pressuremeter test



Dilatometer Test





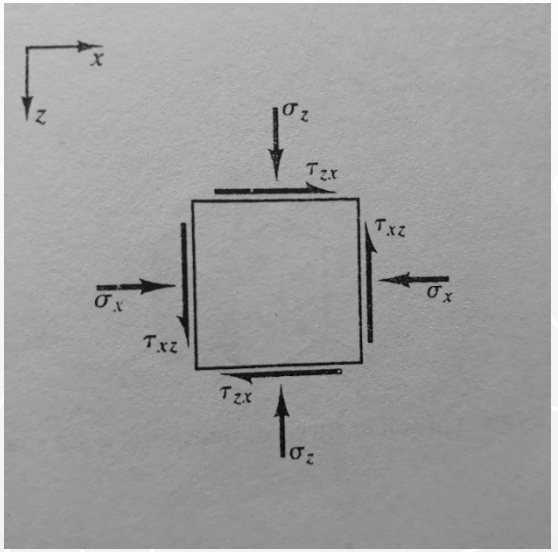
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Shear stress:

- If the **ground surface is horizontal**, the **geostatic shear stresses** on **horizontal and vertical planes** are all equal to zero.

ie $\tau_{xz} = \tau_{zx} = 0$



- Shear stresses may be present on **other planes**.
- Shear stresses conditions are more complex if ground surface is inclined.

Induced Stresses:

- Civil Engineering projects often **induce external loads on to the ground**, thus producing induced stresses.
- **External loads includes**, structural foundations, vehicles, stock piles and many others.
- Resulting induced stresses are significant because these **can be the sources of excessive settlement**, **shear failure** and **other problems**.

Measurement of Induced Stresses

Load System:

- **Application of Point Loads**
- Application of Line Loads.
- Application of Area Loads.

Solution System:

- **Analytical solutions**
- Numerical solutions
- **Charts Solution**

Measurement of induce stresses:

Boussinesq's Method:

- French Mathematician Joseph **Boussinesq developed** method of computing induced stresses in an ***infinite elastic half-space*** due to an applied external load.

Elastic Half-space

- Each **discrete particle of soil**, in contact with another, **will deform when loaded**, depending on the force that is "driving" contact between the two particles.
- **Elastic half-space** is the volume in which this deformation takes place.

Soil is a little more complicated because as **each particle is loaded**, it has two choices

- **to move into an unoccupied space, or**
- **deform in contact with another particle**.

Deform in contact with another particle:

- **If soil deforms**, it does so in the **"half space"** defined by the **contact area that increases** with deformation until the stress stabilizes.
- This **part of soil movement is recoverable** and is known as **"elastic deformation"** or "**elastic settlement".**

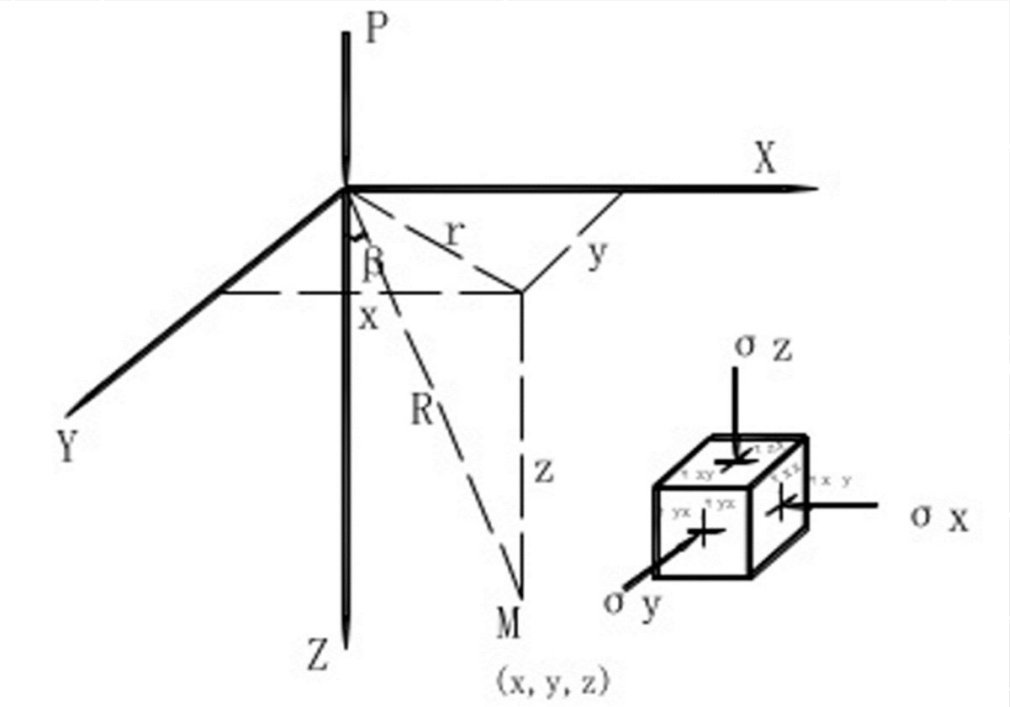
Move into an unoccupied space:

- It is **re-orientation of the soil grains** which is **generally not recoverable** and results in **permanent or plastic deformation** of the soil mass.

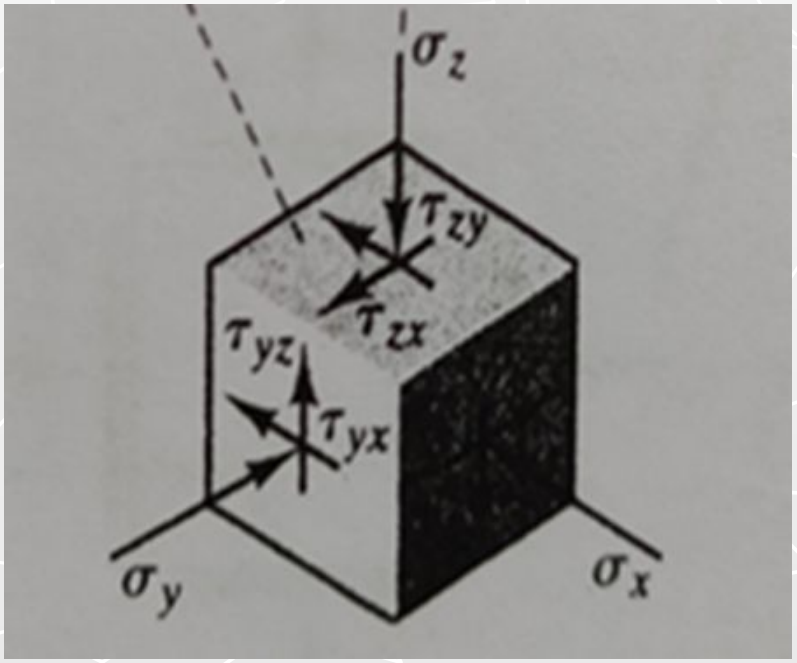
- Bousinesq solve the problem **where a point load (vertical load) P, is perpendicular to this plane** (ground surface). Figure below.

- According to his solution, such a load will induce the **following stresses at a point in the ground** shown below.

- For three dimensional analyses in the x-y-z plane, we only require $\triangle \sigma_x$, $\triangle \sigma_y$, $\triangle \sigma_z$ and τ_{zx}, τ_{zy}; τ_{yz}, τ_{yx}; τ_{xz}, τ_{xy}.



Stresses due to concentrated load



$$\Delta\sigma_x = \frac{P}{2\pi}\left[\frac{3x_f^2 z_f}{R^5} - (1-2\nu)\left(\frac{x_f^2 - y_f^2}{Rr^2(R+z_f)} + \frac{y_f^2 z_f}{R^3 r^2}\right)\right]$$

$$\Delta\sigma_y = \frac{P}{2\pi}\left[\frac{3y_f^2 z_f}{R^5} - (1-2\nu)\left(\frac{y_f^2 - x_f^2}{Rr^2(R+z_f)} + \frac{x_f^2 z_f}{R^3 r^2}\right)\right]$$

$$\Delta\sigma_z = \frac{3Pz_f^3}{2\pi R^5}$$

$$\Delta\tau_{zx} = -\Delta\tau_{xz} = \frac{3Pz_f^2 x_f}{2\pi R^5}$$

$$\Delta\tau_{yx} = -\Delta\tau_{xy} = \frac{P}{2\pi}\left[\frac{3x_f y_f z_f}{R^5} - (1-2\nu)\left(\frac{(2R+2)x_f y_f)}{(R+z_f^2)R^3}\right)\right]$$

$$\Delta\tau_{yz} = -\Delta\tau_{zy} = \frac{3Pz_f^2 y_f}{2\pi R^5}$$

$$R = \sqrt{x_f^2 + y_f^2 + z_f^2}$$

$$r = \sqrt{x_f^2 + y_f^2}$$

Application of Line Load:

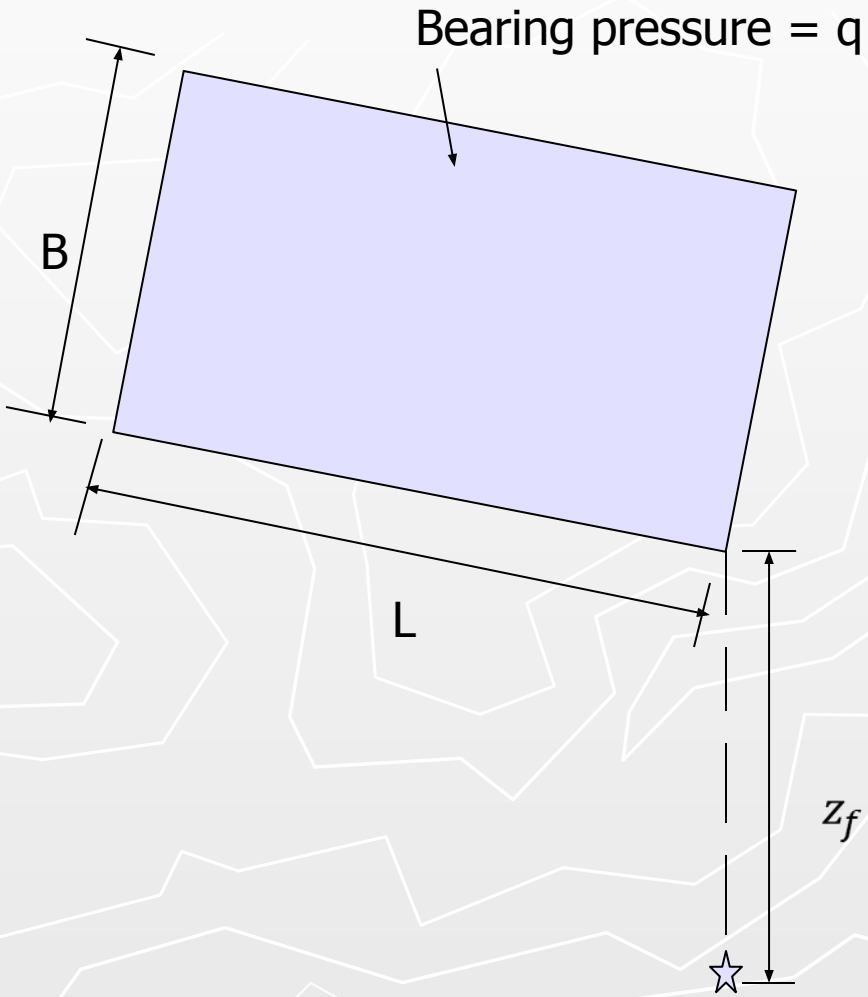
A line load of infinite length parallel to y axis,

$$\sigma_z = \frac{2\, z_f^3 \frac{P}{b}}{\pi\left(x_f^2 + z_f^2\right)^2}$$

P=vertical load

b= unit length along the line

Newmark's Solution for Induced Vertical Stresses beneath the corner of a rectangular Loaded area.



Equation for analytical solution for **vertical induced stress** at depth z beneath the corner of a loaded Rectangle.

If $B^2 + L^2 + z_t^2 < B^2L^2/z_t^2$,

$$I_\sigma = \frac{1}{4\pi}\left[\left(\frac{2BLz_t\sqrt{B^2 + L^2 + z_t^2}}{z_t^2(B^2 + L^2 + z_t^2) + B^2L^2}\right)\left(\frac{B^2 + L^2 + 2z_t^2}{B^2 + L^2 + z_t^2}\right)\right.$$

$$\left. + \pi - \sin^{-1}\frac{2BLz_t\sqrt{B^2 + L^2 + z_t^2}}{z_t^2(B^2 + L^2 + z_t^2) + B^2L^2}\right]$$

Otherwise,

$$I_\sigma = \frac{1}{4\pi}\left[\left(\frac{2BLz_t\sqrt{B^2 + L^2 + z_t^2}}{z_t^2(B^2 + L^2 + z_t^2) + B^2L^2}\right)\left(\frac{B^2 + L^2 + 2z_t^2}{B^2 + L^2 + z_t^2}\right)\right.$$

$$\left. + \sin^{-1}\frac{2BLz_t\sqrt{B^2 + L^2 + z_t^2}}{z_t^2(B^2 + L^2 + z_t^2) + B^2L^2}\right]$$

Equation for analytical solution for **vertical induced stress** at depth z beneath the corner of a loaded Rectangle.

where:

I_σ = influence factor for vertical stress at a point beneath the corner of the loaded rectangle

B = width of loaded rectangle

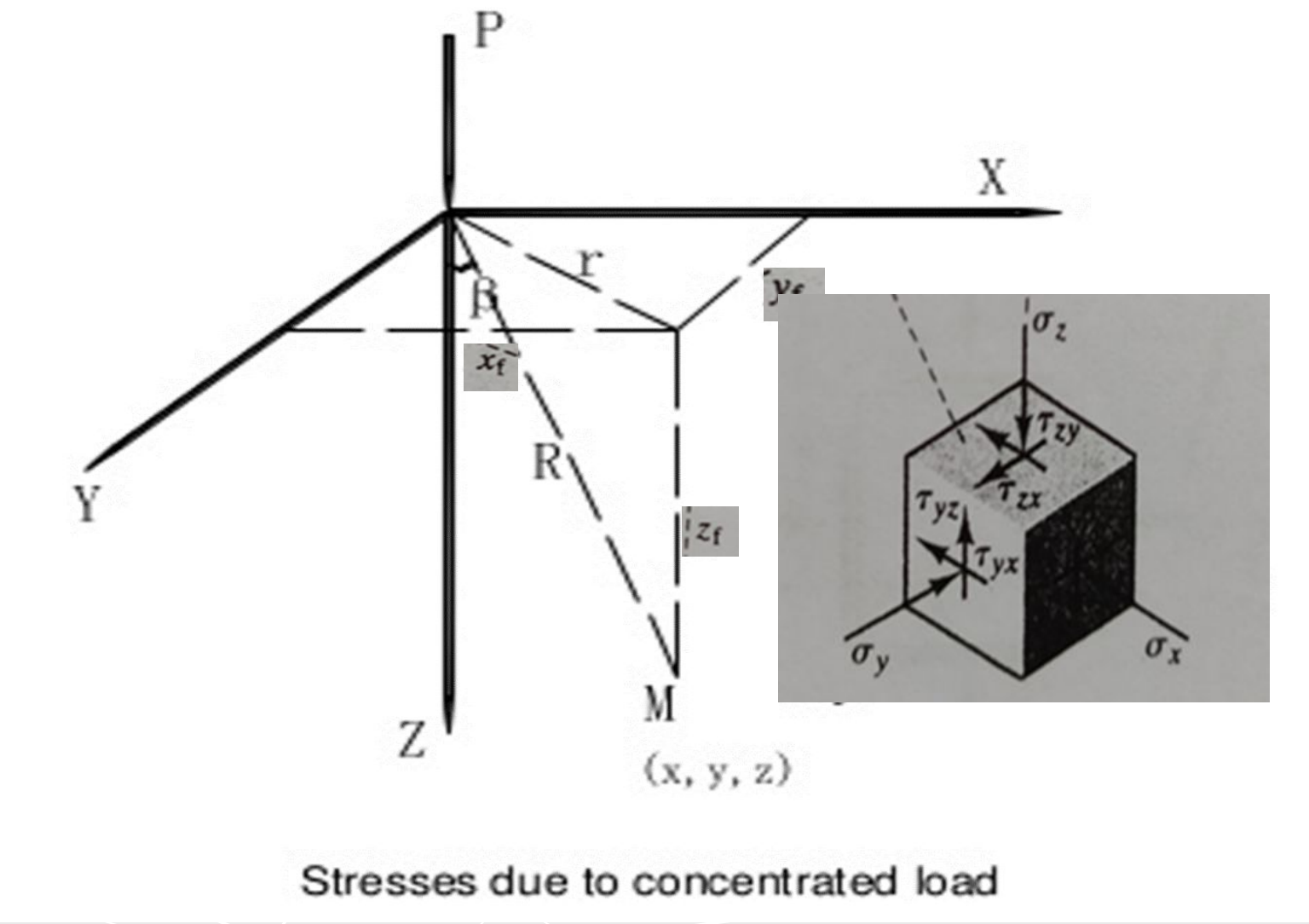
L = length of loaded rectangle

z_f = vertical distance from loaded rectangle to the point (always > 0)

Notes:

1. The $\sin^{-1}$ term must be expressed in radians.
2. Newmark's solution is often presented as a single equation with a $\tan^{-1}$ term, but that equation is incorrect when $B^2 + L^2 + z_f^2 < B^2L^2/z_f^2$.
3. It is customary to use B as the shorter dimension and L as the longer dimension.

Ex. 1: The dimension in Figure below are x_f=10.0 ft, y_f=0.0 and z_f=15.0 ft. The load P is **132 K,** and the soil is a partially saturated clay. Compute the induced stresses $\triangle \sigma_x$, $\triangle \sigma_z$ and τ_{zx} in the soil element.



Stresses due to concentrated load

Fig : Element of soil in three-dimensional x-y-z space/plane.

Solution (Ex. 1):

- For partially saturated clay, assume poison's ratio ν=0.35

$$R = \sqrt{(x_f^2 + y_f^2 + z_f^2)}$$

$$= \sqrt{(10.0^2 + 0.0^2 + 15.0^2)}$$

$$= 18.0 \text{ ft}$$

$$r = \sqrt{(x_f^2 + y_f^2)}$$

$$= \sqrt{(10.0^2 + 0.0^2)}$$

$$= 10.0 \text{ ft}$$

Solution – Ex.1 (cont..):

$$\Delta\sigma_x = \frac{P}{2\pi}\left[\frac{3x_f^2 z_f}{R^5} - (1-2\nu)\left(\frac{x_f^2 - y_f^2}{Rr^2(R+z_f)} + \frac{y_f^2 z_f}{R^3 r^2}\right)\right]$$

Putting the values, P=132000 lb, other value, we get,

$\Delta\sigma_x$= 183.75 lb/ft^2 Ans. (ν=0.30)

$$\Delta\sigma_z = \frac{3Pz_f^3}{2\pi R^5} = \frac{3(132{,}000)(15.0)^3}{2\pi(18.0)^5} = 115.05 \text{ lb/}ft^2 \text{ Ans.}$$

$$\Delta\tau_{zx} = \frac{3Pz_f^2 x_f}{2\pi R^5} = \frac{3(132{,}000)(15.0)^2(10.0)}{2\pi(18.0)^5} = 75.0 \text{ lb/}ft^2 \text{ Ans.}$$

Chart Solutions

Pressure/Stress bulbs/Curves:

- Curves which **connect the points of equal induced stresses** are sometimes called **pressure bulbs or stress bulbs.**

- **Charts provide a visual sense** of how the stresses are distributed.

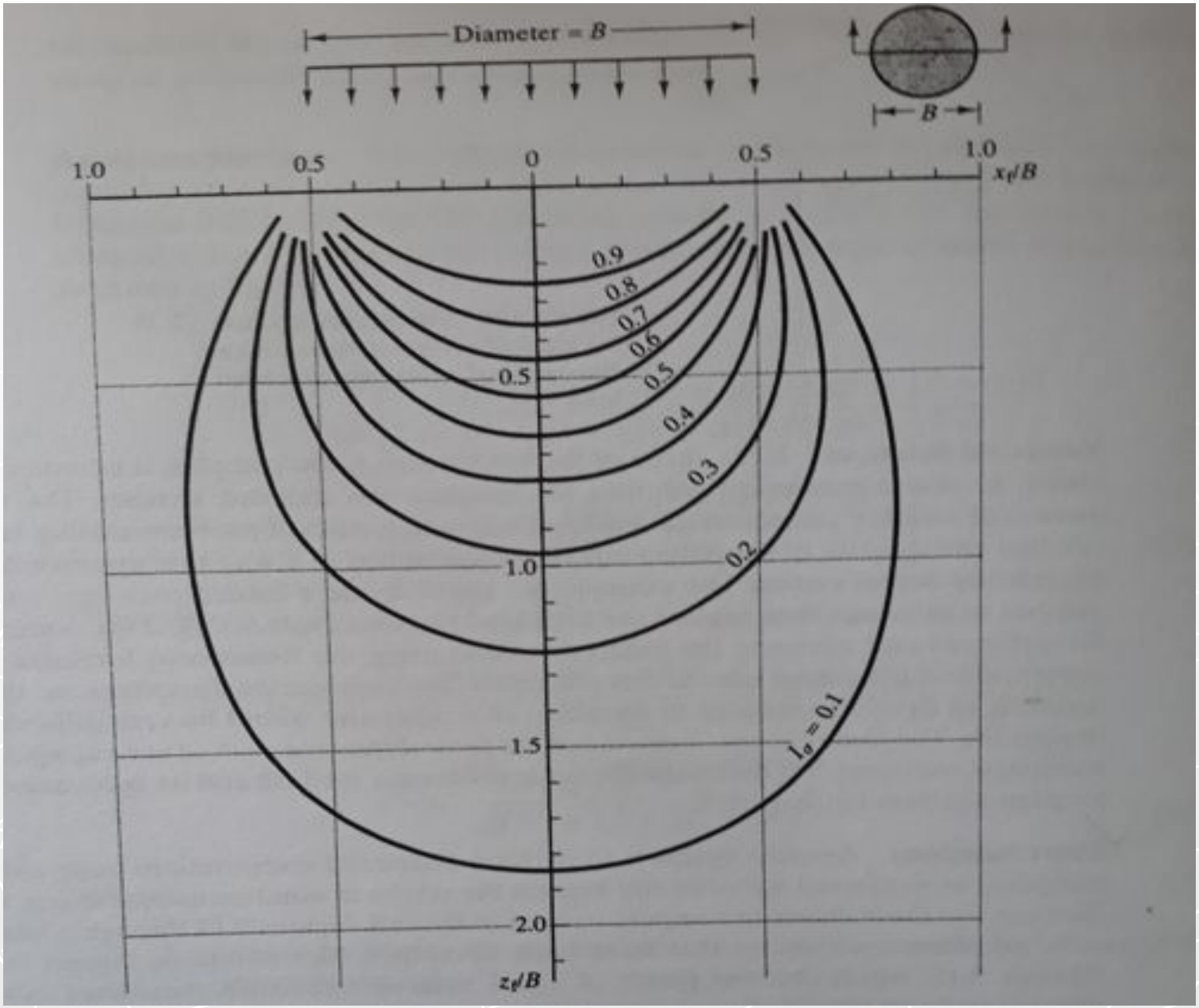


Fig 1: Influence factor for induced vertical stress under **circular loaded area**, as per Boussinesq.

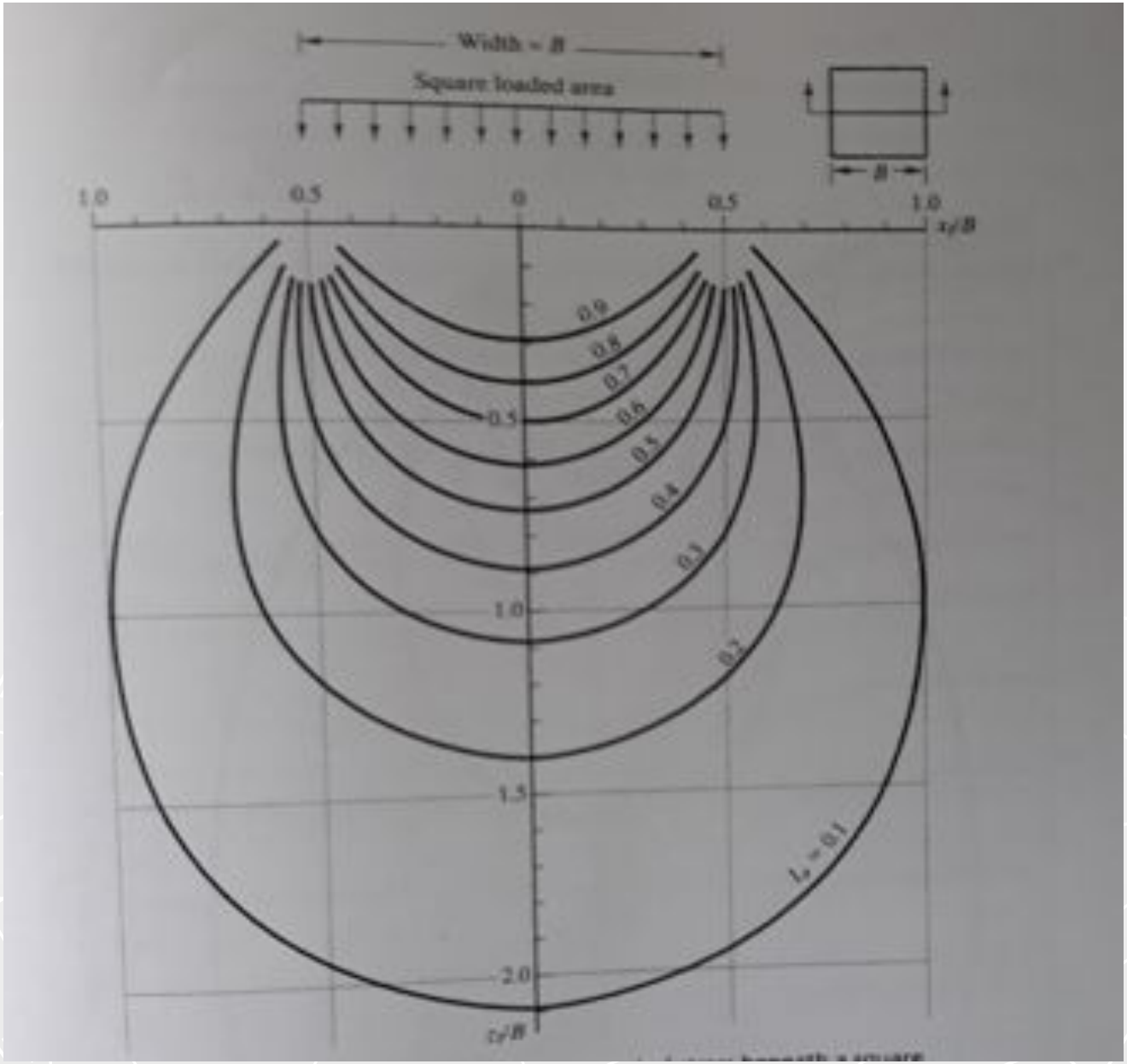


Fig 2: Influence factor for induced vertical stress under square loaded area, as per Boussinesq.

(of a line or solid) limited in one direction and stretching to infinity in the other.

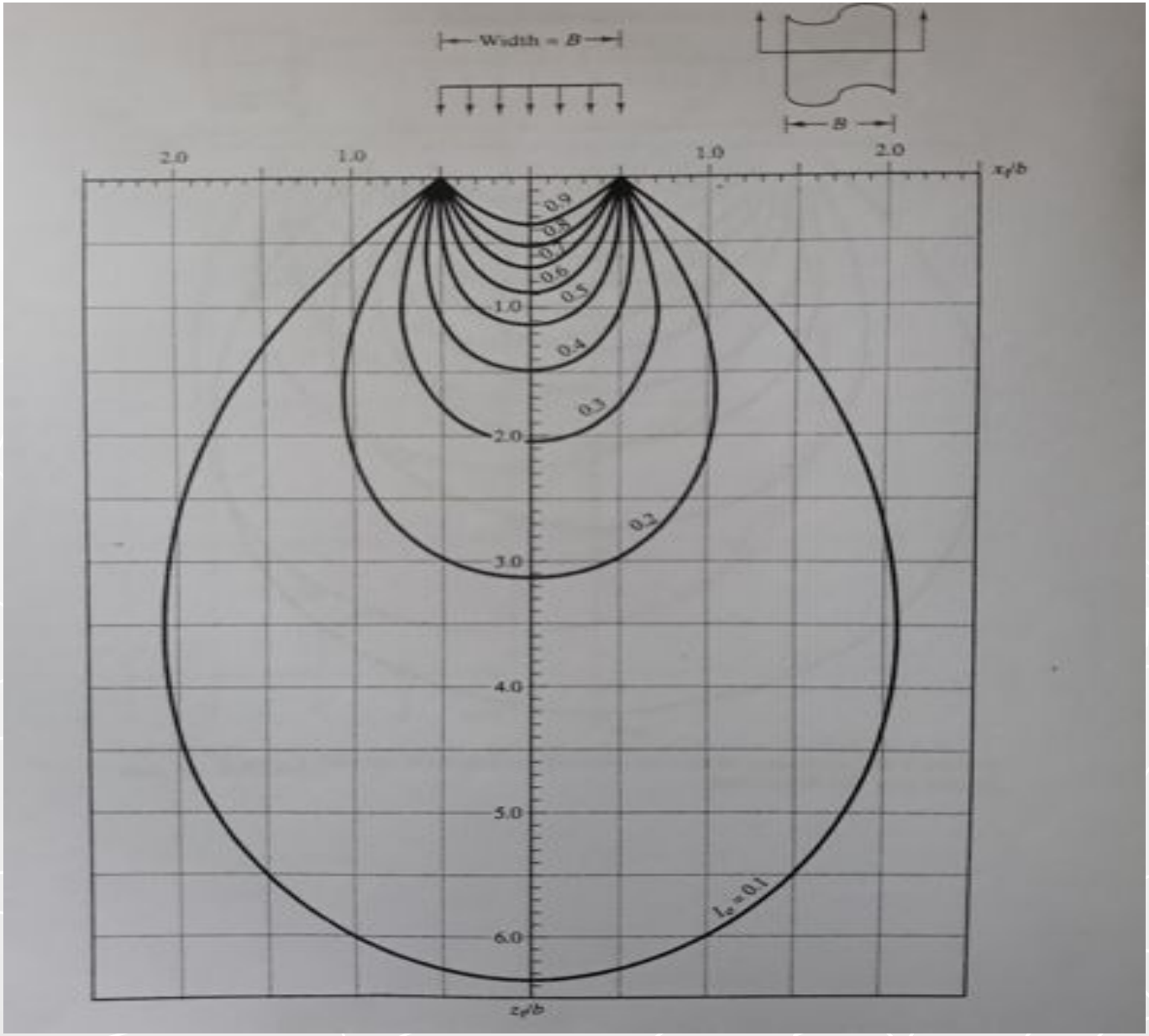
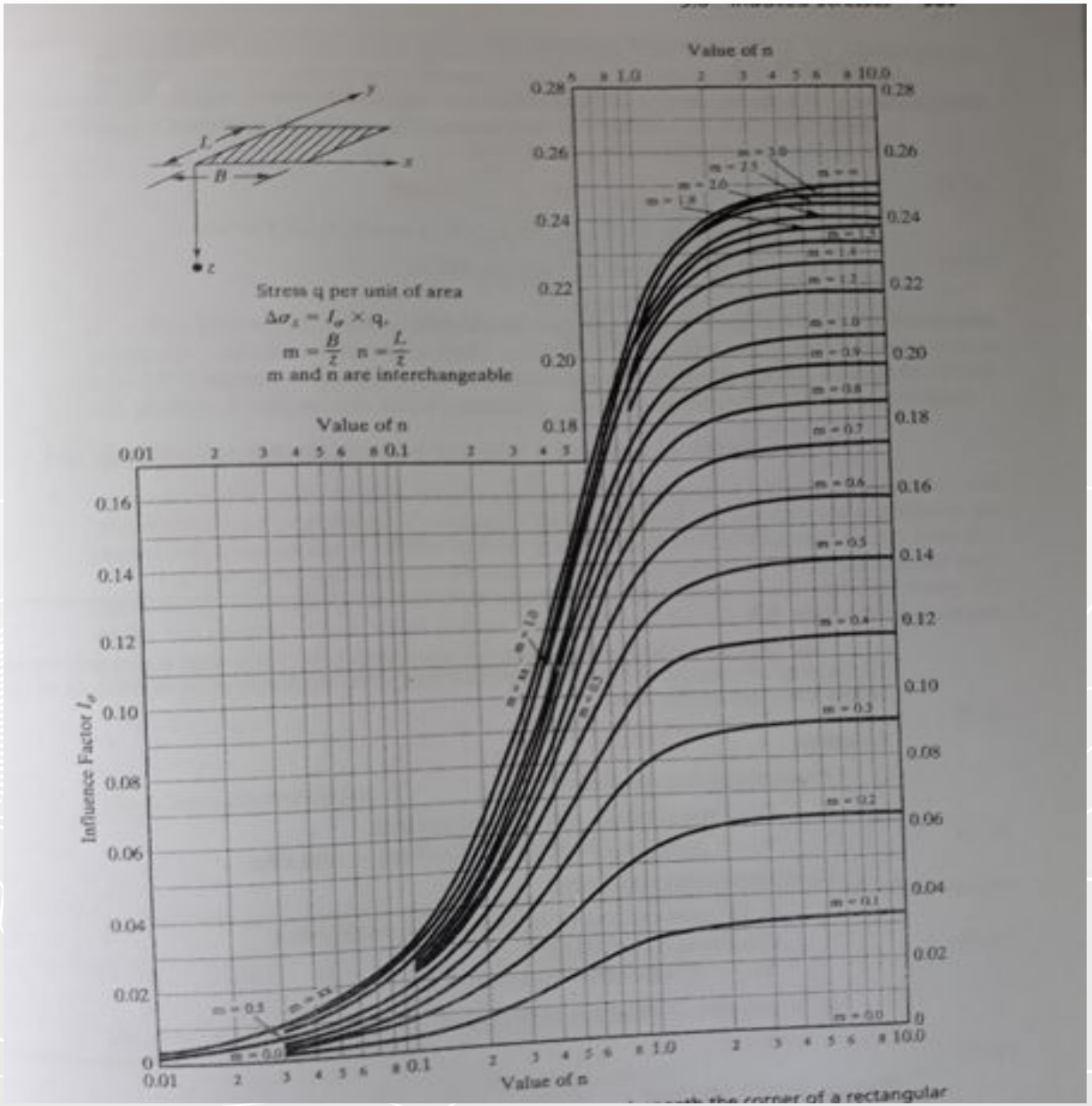


Fig 3: Influence factor for induced vertical stress under **semi infinite strip load**, as per Boussinesq.

Fig 3: Influence factor for induced vertical stress **beneath the corner of a rectangular loaded area**.



Ex. 2: Compute $\triangle \sigma_z$ at a depth of **10.0m** below the edge of a **25.0m** diameter of footing of **water tank**. The tank, its foundation and it contents have a **total mass of $6.1X10^6$ kg,** which is uniformly distributed across its base.

Solution (Ex.2):

$$W = mg = (6.1 \times 10^6 \text{ kg})(0.00981) = 59{,}841 \text{ kN}$$

$$A = \pi r^2 = \pi (12.5)^2 = 491\ m^2$$

$$q = \frac{P}{A} = \frac{59{,}841 kN}{491 m^2} = 122 \text{ kPa}$$

$$\frac{z_f}{B} = \frac{10m}{25m} = 0.40$$

From Fig. of Ex. 2: at $\frac{z_f}{B} = \frac{10}{25} = 0.40$ and beneath the edge of the tank, $\frac{x_f}{B} = \frac{12.5}{25} = 0.50$;

$$I_\sigma = 0.40$$

$$\Delta \sigma_z = I_\sigma \times q = 0.40 \times (122 \text{ kPa}) = 49 \text{ kPa Ans.}$$

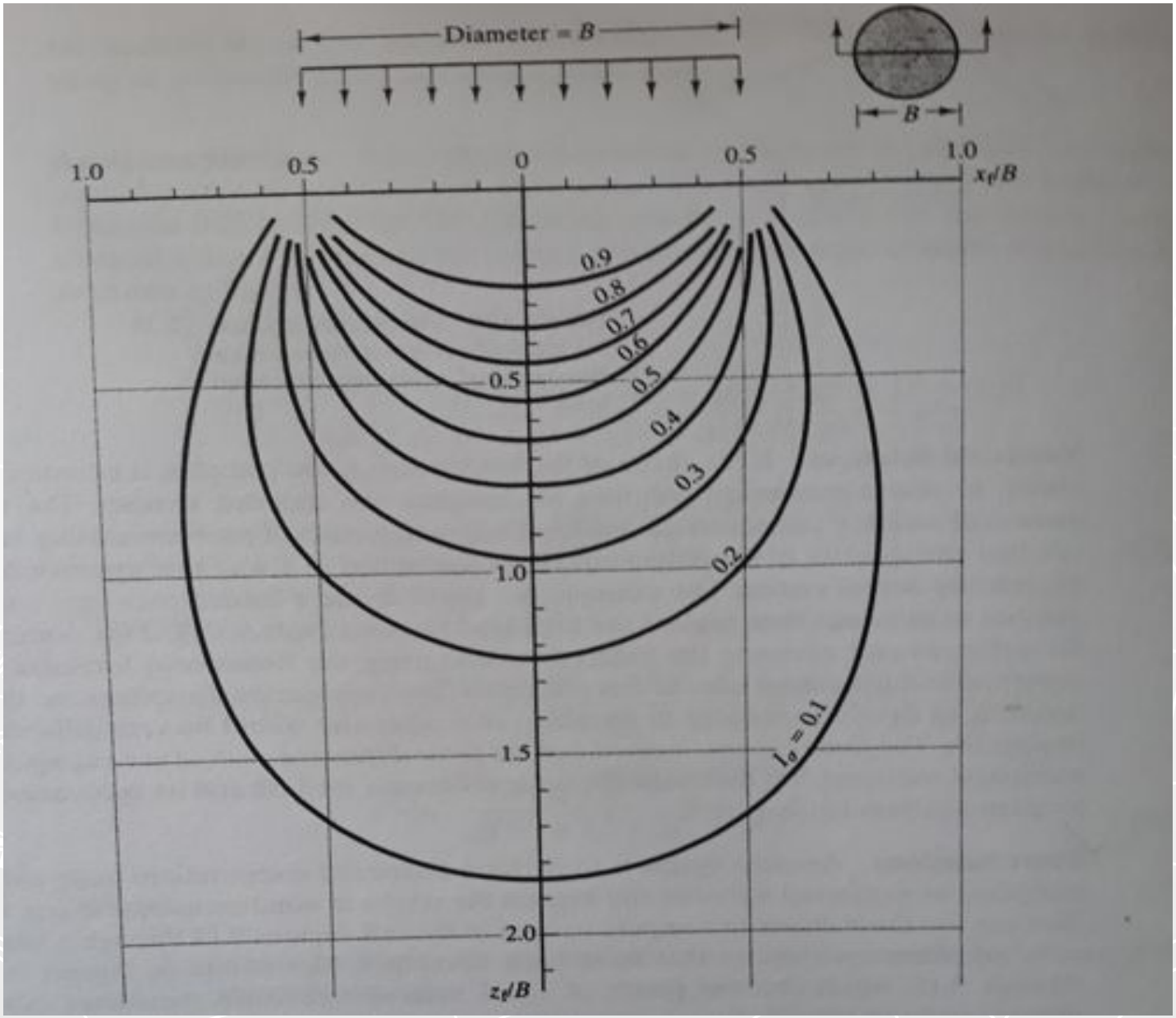


Fig of Ex. 2: Influence factor for induced vertical stress under circular loaded area, per Boussinesq.

Ex. 3: Compute $\triangle \sigma_z$ at a depth **of 10.0m** below the **edge of a 25.0m** diameter water tank. The tank, its foundation and it contents have a total mass of $6.1X10^6$ kg, which is uniformly distributed across its base. The tank is underlain by a soil that has a unit weight of **18.0 kN/m^3**. Compute the **preconstruction** and **post construction** σ_z at a depth of 10.0m below the edge of the tank.

Solution (Ex. 3):

Preconstruction Condition:

Only the geostatic stresses are present,

$\sigma_z = \sum \boldsymbol{\gamma H} = (18.0\text{kN}/m^3)\text{x } (10.0\text{m})=180 \text{ kPa}$

Post-construction Condition:

Both geostatic and induced stresses are present,

- **Geostatic stress σ_z**= 180 kPa

Solution Ex.3: (cont..)

- **Induced stress**:

$$W = mg = (6.1 \times 10^6 \text{ kg})(0.00981) = 59{,}841 \text{ kN}$$

$$A = \pi r^2 = \pi (12.5)^2 = 491\ m^2$$

$$q = \frac{P}{A} = \frac{59{,}841 kN}{491 m^2} = 122 \text{ kPa}$$

$$\frac{z_f}{B} = \frac{10m}{25m} = 0.40$$

From Fig. of Ex. 2: at $\frac{z_f}{B} = 0.40$ and beneath the edge of the tank, $\frac{x_f}{B} = \frac{12.5}{25} = 0.50$;

$$I_\sigma = 0.40$$

$$\Delta \sigma_z = I_\sigma \times q = 0.40 \times (122 \text{ kPa}) = 49 \text{ kPa Ans.}$$

Thereby, Post-construction stress $\sigma_z = 180 \text{ kPa} + 49 \text{ kPa} = 229 \text{ kPa}$ Ans.

Westergaard's Method

- Westergaard, Harold M (1938), a **Danish structural Engineer**, solved the same problem, Boussinesq addressed but with slightly different assumptions.

- **Instead of using a perfectly elastic material**, he assumed one that **contained closely spaced horizontal reinforcement members** of **infinitesimal thickness**.

- In such cases, **the horizontal strain zero at all points**.

- It is a **precise representation of layered soils** where some layers are much stiffer than others.

Based on Westergaard's method, Terzaghi presented the following formula for σ_z, due to vertical point load P.

$$\Delta\sigma_z = \frac{PC}{2\pi z_f^2}\left[\frac{1}{C^2 + (r/z_f)^2}\right]^{1.5}$$

$$C = \sqrt{\frac{1-2\nu}{2(1-\nu)}}$$

- Westergaard's solution produces σ_z values **equal to or less** than the Boussinesq values.
- **As ν increases, the computed stress becomes smaller**, **eventually reaching zero at ν=0.5.**
- Some Geo Tech Engineers prefer Westergaard, at least for certain soil.
- Boussinesq is more commonly used and appropriate for most problems.

Approximate method:

- Sometimes it is useful to use such methods of computing stresses in soils.

- **Useful when an quick answer is needed** or **when a computer is not available.**

- Following formulas compute the **influence factor** I_σ for induced vertical stress beneath the **center of an area load.**
- They **produce values that are within 5% of Boussinesq values,** which is more than sufficient for virtually all practical problems.

For circular loaded areas (Poulos and Davis, 1974),

$$I_\sigma = 1 - \left(\frac{1}{1 + (B/2z_f)^2}\right)^{1.5}$$

For square loaded areas,

$$I_\sigma = 1 - \left(\frac{1}{1 + (B/2z_f)^2}\right)^{1.76}$$

For continuous loaded areas (also known as *strip loads*) of width B and infinite length,

$$I_\sigma = 1 - \left(\frac{1}{1 + (B/2z_f)^{1.38}}\right)^{2.60}$$

For rectangular loaded areas of width B and length L,

$$I_\sigma = 1 - \left(\frac{1}{1 + (B/2z_f)^{1.38+0.62B/L}}\right)^{2.60-0.84B/L}$$

where:

I_σ = influence factor for vertical stress beneath the center of a loaded area

z_f = depth from bottom of loaded area to point

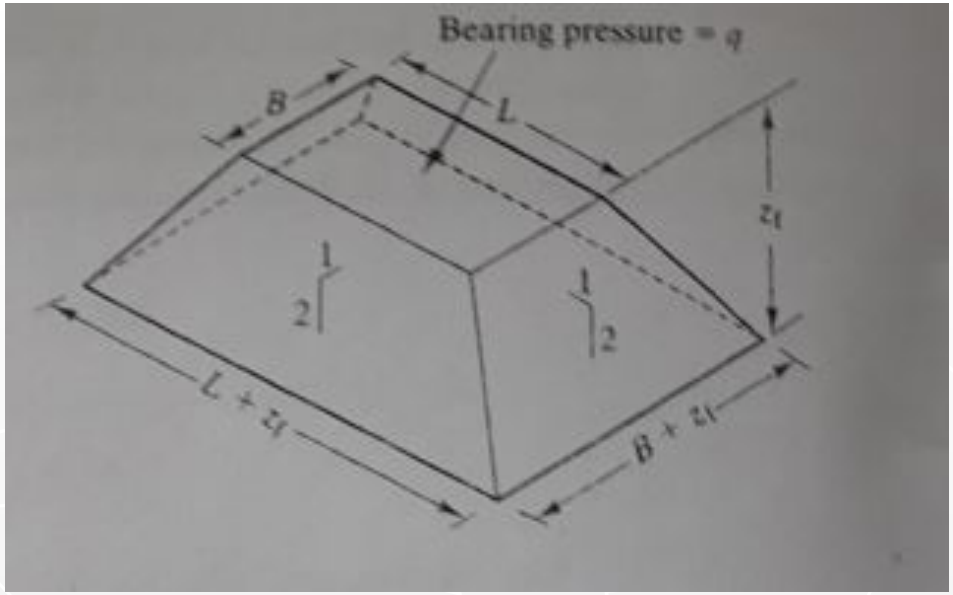


Fig.: Use of 1:2 method to compute the average $\triangle \sigma_z$ at a specified depth below the loaded area.

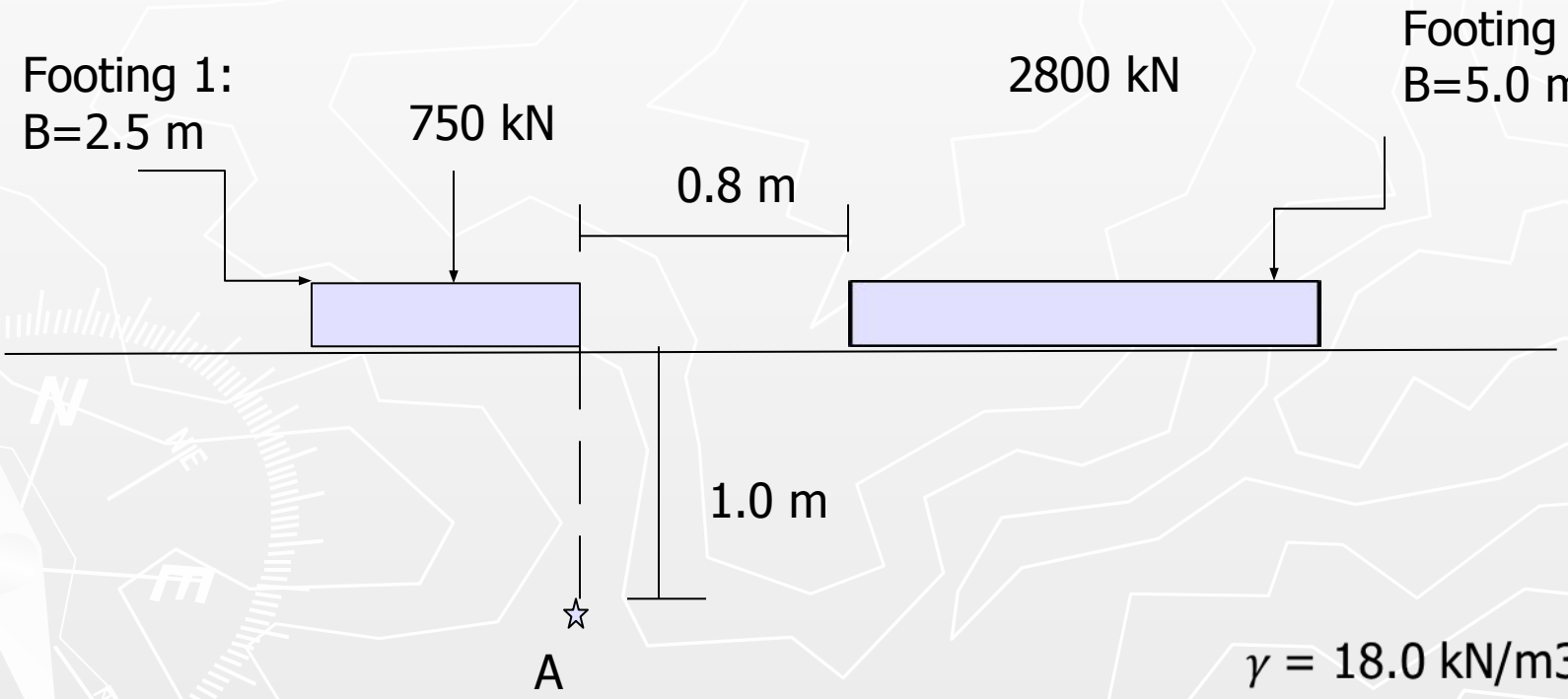
When applied to a rectangular loaded area of $B \times L$, the 1:2 method produces the following formula for the influence factor for vertical stress, I_σ, at a depth z_f:

$$I_\sigma = \frac{BL}{(B + z_f)(L + z_f)}$$

How to compute Stresses in soils

- Boussinesq's Method
- New Mark's Solution Method
- Westergaard's Method
- Approximate Method
- Chart Solutions

Ex. 4: Two square footings **carry vertical loads** as shown in the following figure. Compute the **magnitude of σ_z** at Point A under the edge of the smaller footing, considering both geostatic and induced stress.



Footing: 1

Footing: 2

Solution (Ex.4):

- Geostatic stress:

$$\sigma_z = \sum \gamma H = (18.0\ \text{kN}/m^2)(1.0\text{m}) = 18\text{kPa}$$

- Bearing Pressures:

Footing 1: $q_1 = \frac{P}{A} = \frac{750kN}{(2.5\ m)^2} = 120\text{kPa}$

Footing 2: $q_2 = \frac{P}{A} = \frac{2800kN}{(5.0\ m)^2} = 112\text{kPa}$

Solution (Ex.4) (cont..):

- Relative z and x coordinates for point of interest:

Footing 1:

$$\frac{z_f}{B} = \frac{1.0m}{2.5m} = 0.40$$

From the figure, beneath the edge of footing $\frac{x_f}{B} = \frac{1.25}{2.5} = 0.50$

Therefore, $I_\sigma = 0.45$

$$\Delta \sigma_{z_1} = I_\sigma \, q_1 = 0.45 \text{ x } (120.0 \text{ kPa}) = 54 \text{ kPa}$$

Footing 2:

$$\frac{z_f}{B} = \frac{1.0m}{5.0m} = 0.20$$

From the figure, beneath the edge of footing $\frac{x_f}{B} = \frac{2.5+0.8}{5} = 0.66$

Solution (Ex.4):

Therefore, $I_\sigma = 0.15$

$$\Delta \sigma_{z_2} = I_\sigma \, q_1 = 0.15 \text{ x } (112.0 \text{ kPa}) = 16.8 \text{ kPa}$$

$$\Delta \sigma_{z_{final}} = \sigma_z + \Delta \sigma_{z1} + \Delta \sigma_{z2}$$

$$= 18 \text{ kPa} + 54 \text{ kPa} + 16.8 \text{ kPa}$$

$$= 88.8 \text{ kPa} \quad \text{Ans.}$$

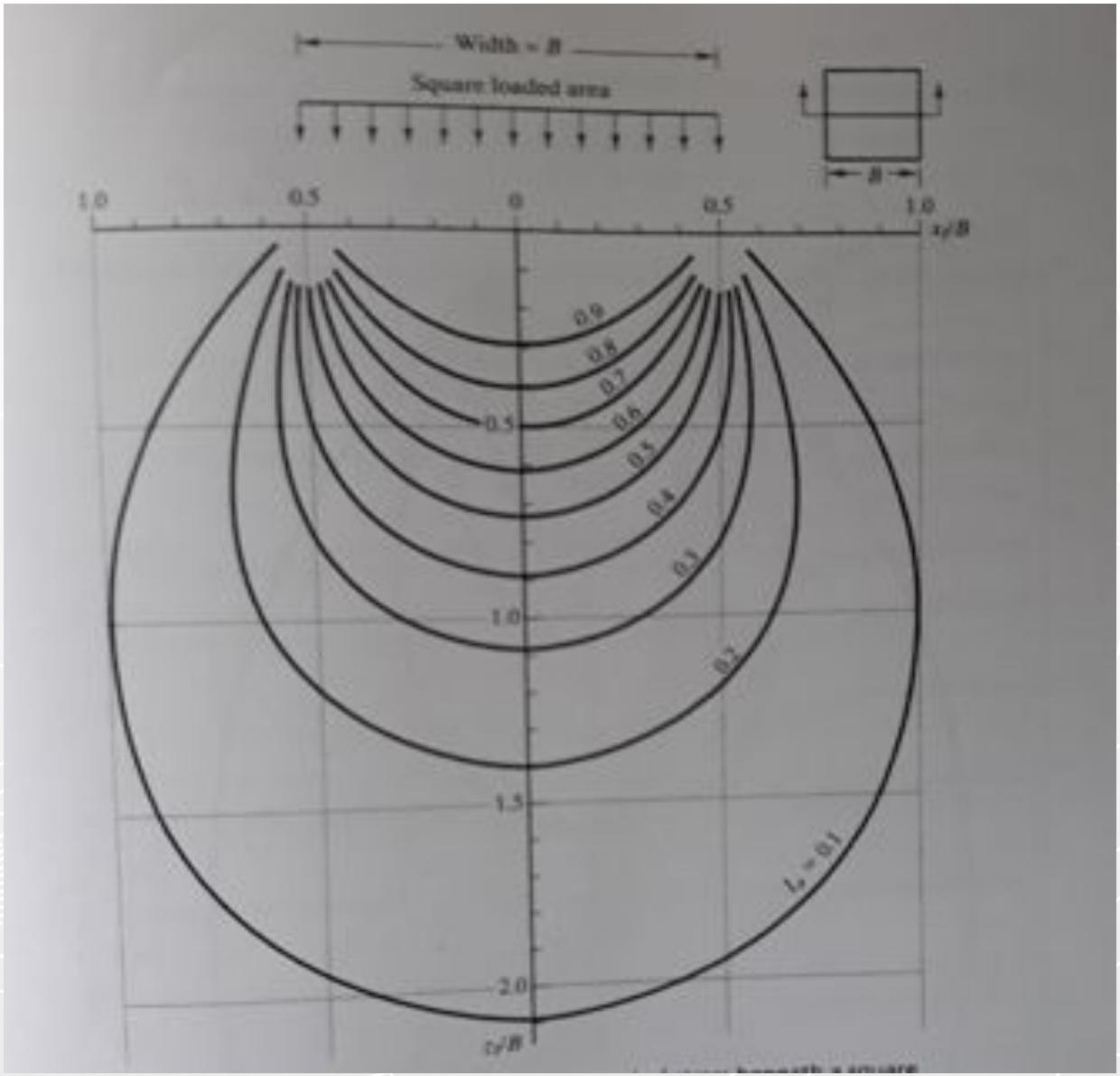
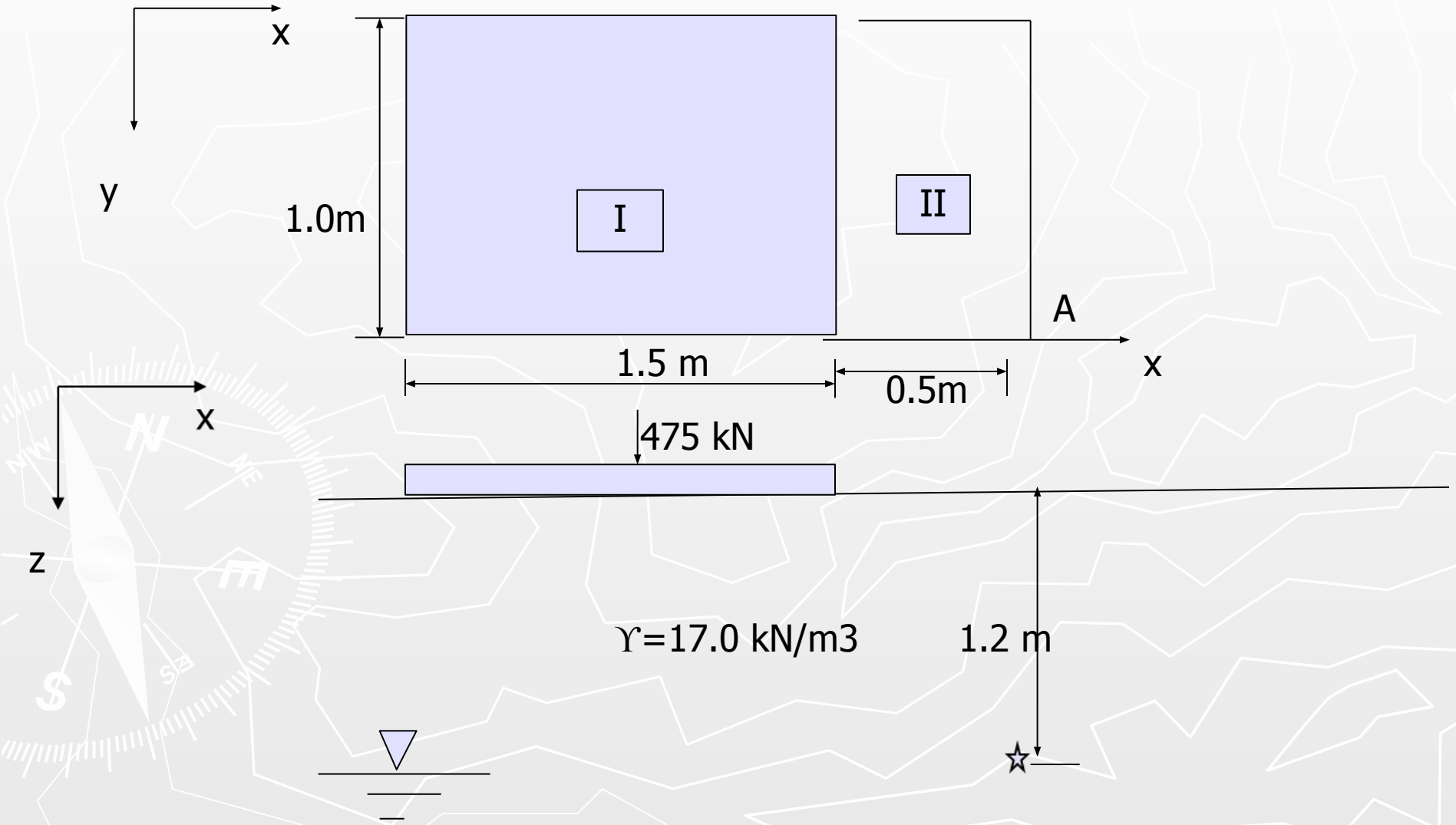


Figure of Ex. 4: Influence factor for induced vertical stress under square loaded area, per Boussinesq.

Ex. 5: The **1.0m X 1.5m** footing shown in the following figure, supports a **vertical load of 475 kN**. Compute the magnitude of σ_z at point A, considering both **geostatic and induced stresses**.



Solutions of Ex. 5:

- **Geostatic stress**:

$$\sigma_z = \sum \gamma H = (17.0\ \text{kN}/m^2)(1.2\text{m}) = 20.4\ \text{kPa}$$

Induced stress:

- Equations below used to compute σ_z, below the corner of a loaded rectangle, but point A is not beneath the corner.
- Stress can be computed beneath a **1st fictitious footing** (I + II), which is 1.0m wide and (1.5m+0.5m=) 2.0m long.
- **2nd fictitious footing II**, which is 0.5m wide and 1.0m long.
- **Both have a corner over point A.**
- Have the **same bearing pressure as the real footing**.

Solutions of Ex. 5 (cont..)

- **Bearing pressures,**

$$q = \frac{P}{A} = \frac{475\ kN}{(1.0m)(1.5m)} = 317\ \text{kPa}$$

- **Footing (I+II):**

B=1.0. L=2.0, z_f= 1.2, $B^2 + L^2 + z_f^2$=6.44 > B^2L^2/z_f^2=3.34

Hence, equation 2 to be used.

Therefore, I_σ=**0.182**

σ_z =(0.182)(317)= **57.7kPa**

- **Footing (II):**

B=0.5. L=1.0, z_f= 1.2, $B^2 + L^2 + z_f^2$ =2.69> B^2L^2/z_f^2=0.02

Hence, equation 2 to be used.

Therefore, I_σ=**0.098**

σ_z =(0.098)(317)= **31.1 kPa**

Hence, $\Delta\ \sigma_z$ = **57.7 – 31.1 = 26.6 kPa**

Combined results σ_z= **20.4+ 26.6** = 47.0 kPa Ans.

Thank you