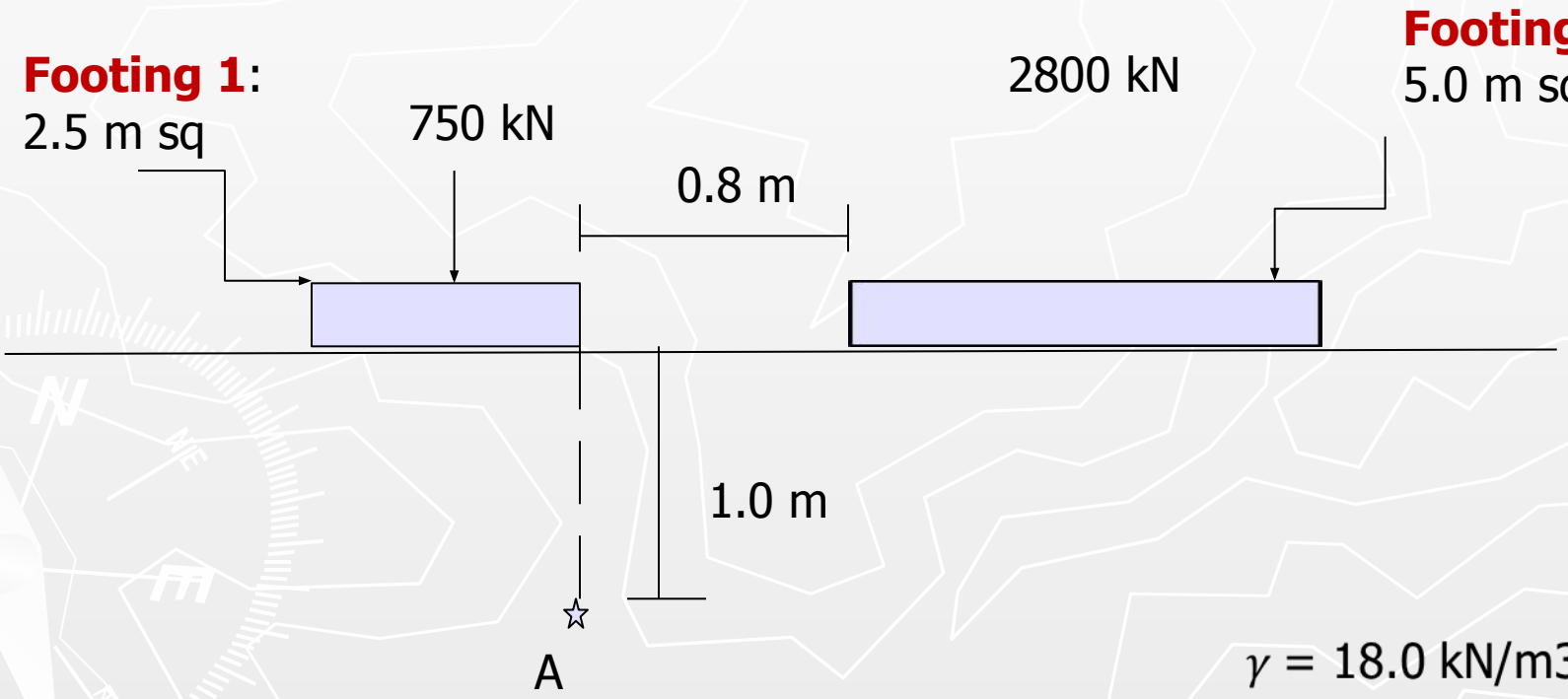
EWCE 341

Principles of Soil Mechanics

How to compute Stresses in soils

- Boussinesq's Method
- New Mark's Solution Method
- Westergaard's Method
- Approximate Method
- Chart Solutions

Ex. 4: Two **square footings** **carry vertical loads** as shown in the following figure. Compute the **magnitude of σ_z** at Point A under the edge of the smaller footing, considering both geostatic and induced stress.



Footing: 1

Footing: 2

Solution (Ex.4):

- **Geostatic stress**:

$$\sigma_z = \sum \gamma H = (18.0\ kN/m^2)(1.0m) = 18kPa$$

- **Bearing Pressures**:

Footing 1: $q_1 = \frac{P}{A} = \frac{750kN}{(2.5\ m)^2} = 120kPa$

Footing 2: $q_2 = \frac{P}{A} = \frac{2800kN}{(5.0\ m)^2} = 112kPa$

Solution (Ex.4) (cont..):

- Relative z and x coordinates for point of interest:

Footing 1:

$$\frac{z_f}{B} = \frac{1.0m}{2.5m} = 0.40$$

From the figure, beneath the edge of footing $\frac{x_f}{B} = \frac{1.25}{2.5} = 0.50$

Therefore, $I_\sigma = 0.45$

$$\Delta \sigma_{z_1} = I_\sigma \, q_1 = 0.45 \text{ x } (120.0 \text{ kPa}) = 54 \text{ kPa}$$

Footing 2:

$$\frac{z_f}{B} = \frac{1.0m}{5.0m} = 0.20$$

From the figure, beneath the edge of footing $\frac{x_f}{B} = \frac{2.5+0.8}{5} = 0.66$

Solution (Ex.4):

Therefore, $I_\sigma = 0.15$

$$\Delta \sigma_{z_2} = I_\sigma \, q_1 = 0.15 \text{ x } (112.0 \text{ kPa}) = 16.8 \text{ kPa}$$

$$\Delta \sigma_{z_{final}} = \sigma_z + \Delta \sigma_{z1} + \Delta \sigma_{z2}$$

$$= 18 \text{ kPa} + 54 \text{ kPa} + 16.8 \text{ kPa}$$

$$= 88.8 \text{ kPa} \quad \text{Ans.}$$

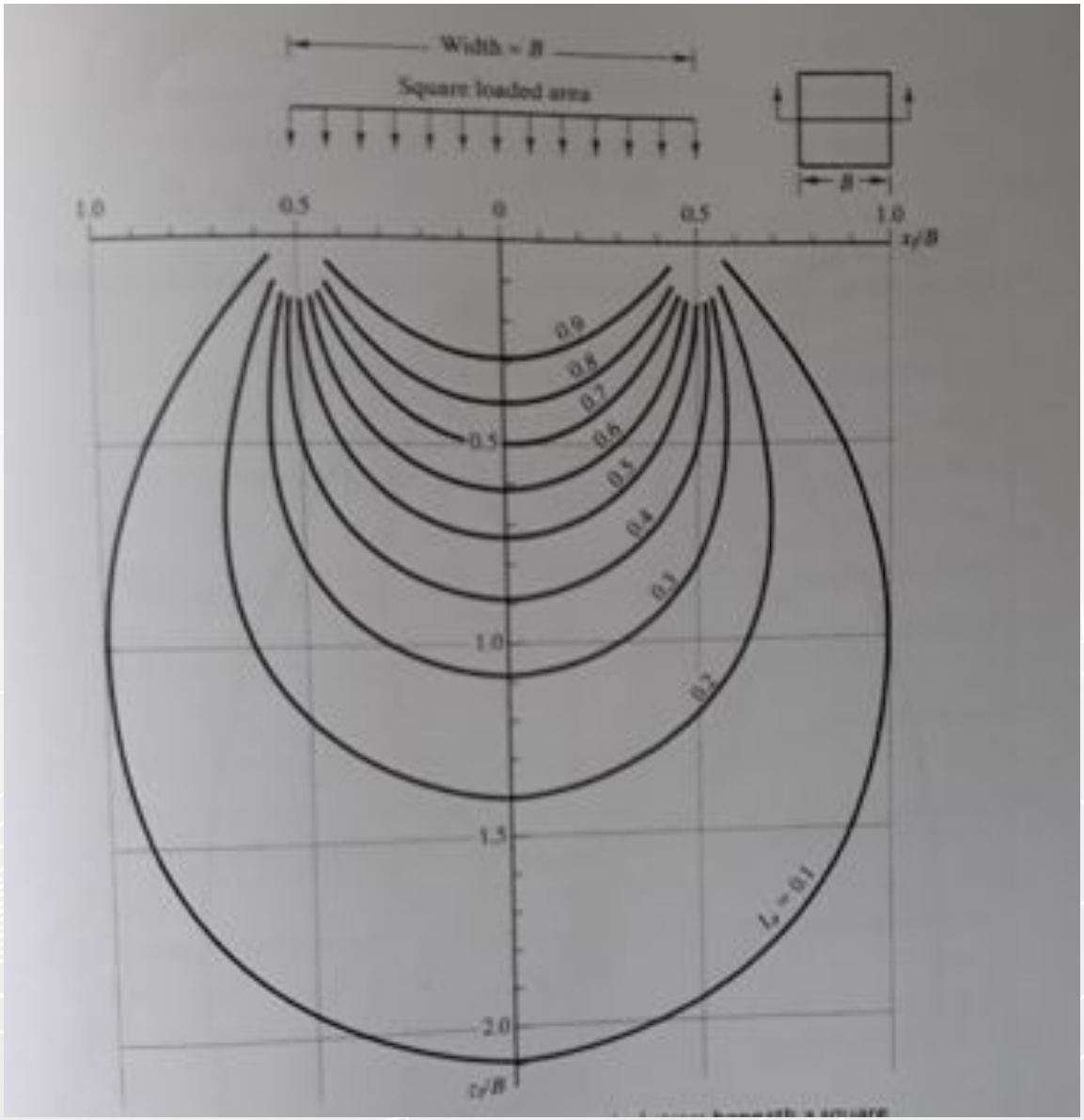
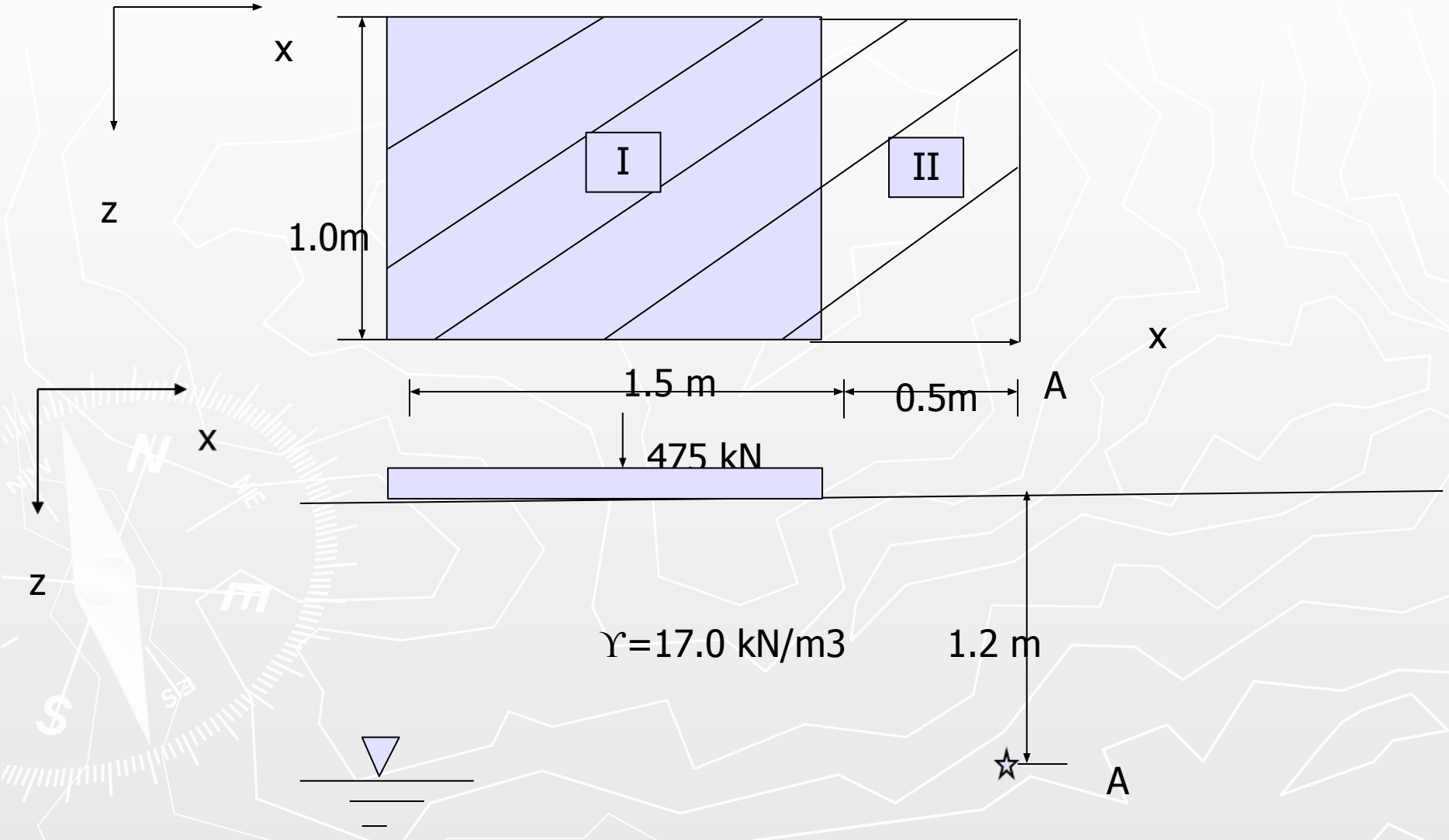


Figure of Ex. 4: Influence factor for induced vertical stress under square loaded area, per Boussinesq.

Ex. 5: The **1.0m X 1.5**m footing shown in the following figure, supports a **vertical load of 475 kN**. Compute the **magnitude of σ_z** at point A, considering both **geostatic and induced stresses**.



Solutions of Ex. 5:

- Geostatic stress:

$$\sigma_z = \sum \gamma H = (17.0\ \text{kN}/m^3)(1.2\text{m}) = 20.4\ \text{kPa}$$

Induced stress:

- Equations below used to compute σ_z, below the corner of a loaded rectangle, but point A is not beneath the corner.
- Stress can be computed beneath a **1st fictitious footing (I + II),** which is **1.0m wide** and (1.5m+0.5m=) **2.0m long**.
- **2nd fictitious footing II**, which is **0.5m wide** and **1.0m long.**
- Both have a corner over **point A.**
- Have the **same bearing pressure as the real footing**.

Solutions of Ex. 5 (cont..)

- Bearing pressures,

$$\mathbf{q} = \frac{P}{A} = \frac{475\ kN}{(1.0m)(1.5m)} = 317\ \text{kPa}$$

- **Footing (I+II):**

B=1.0. L=2.0, z_f= 1.2, $B^2 + L^2 + z_f^2$=6.44 > B^2L^2/z_f^2=3.34

Hence, **equation 2** to be used.

Therefore, I_σ=0.182

$$\sigma_z = (0.182)(317) = \mathbf{57.7kPa}$$

- **Footing (II):**

B=0.5. L=1.0, z_f= 1.2, $B^2 + L^2 + z_f^2$ =2.69> B^2L^2/z_f^2=0.02

Hence, **equation 2** to be used.

Therefore, I_σ=0.098

$$\sigma_z = (0.098)(317) = \mathbf{31.1\ kPa}$$

Hence, $\Delta\ \sigma_z$ = **57.7 – 31.1 = 26.6 kPa**

Combined results σ_z= **20.4+ 26.6** = 47.0 kPa Ans.

If $B^2 + L^2 + z_t^2 < B^2L^2/z_t^2$,

$$I_\sigma = \frac{1}{4\pi}\left[\left(\frac{2BLz_t\sqrt{B^2 + L^2 + z_t^2}}{z_t^2(B^2 + L^2 + z_t^2) + B^2L^2}\right)\left(\frac{B^2 + L^2 + 2z_t^2}{B^2 + L^2 + z_t^2}\right)\right.$$

$$\left. + \pi - \sin^{-1}\frac{2BLz_t\sqrt{B^2 + L^2 + z_t^2}}{z_t^2(B^2 + L^2 + z_t^2) + B^2L^2}\right]$$

Equation-1

Otherwise,

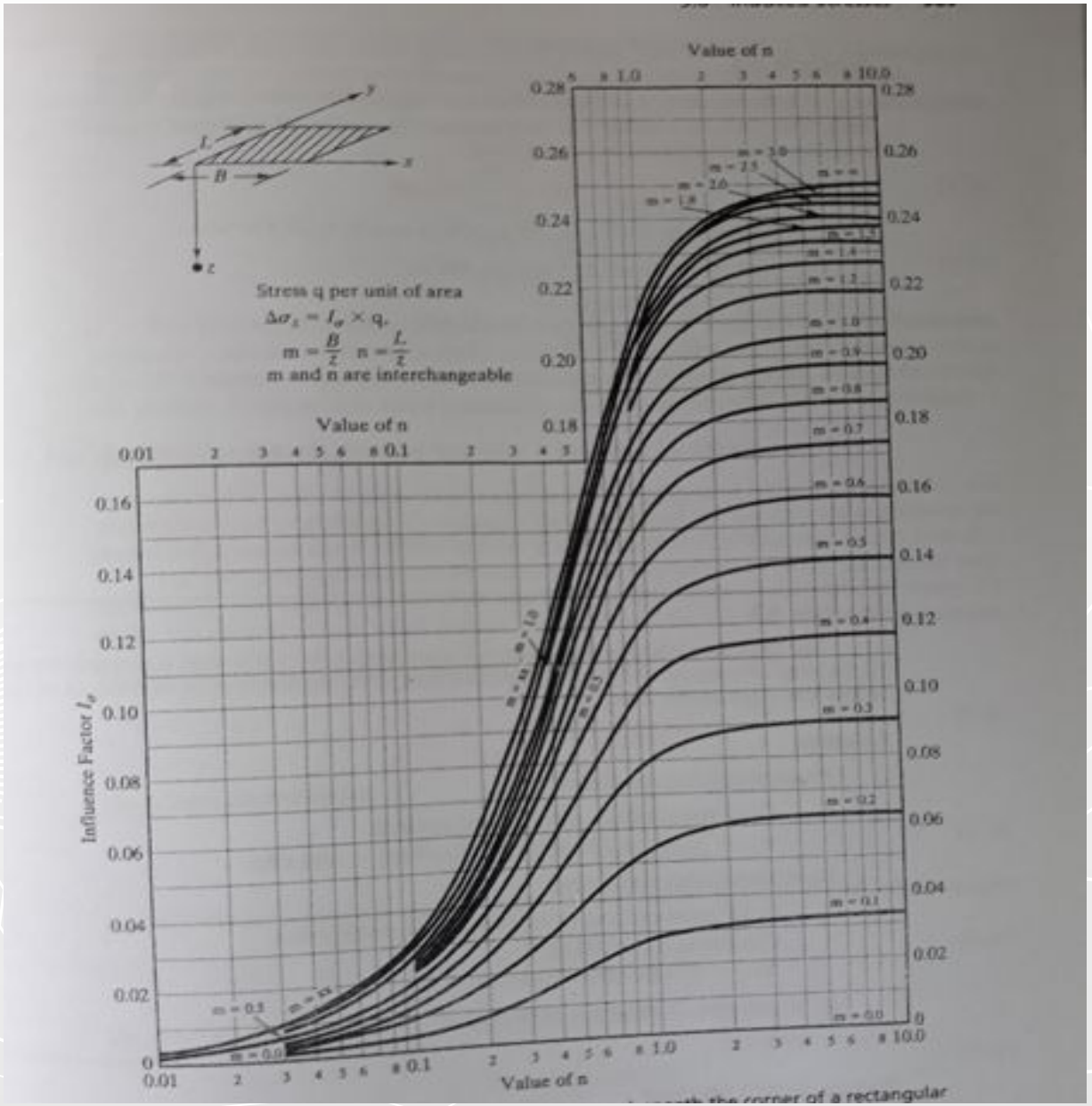
$$I_\sigma = \frac{1}{4\pi}\left[\left(\frac{2BLz_t\sqrt{B^2 + L^2 + z_t^2}}{z_t^2(B^2 + L^2 + z_t^2) + B^2L^2}\right)\left(\frac{B^2 + L^2 + 2z_t^2}{B^2 + L^2 + z_t^2}\right)\right.$$

$$\left. + \sin^{-1}\frac{2BLz_t\sqrt{B^2 + L^2 + z_t^2}}{z_t^2(B^2 + L^2 + z_t^2) + B^2L^2}\right]$$

Equation-2

Figure of Ex.5

Figure 3: **Influence factor for induced vertical stress beneath the corner of a rectangular loaded area**.



Stresses in soils:

- All civil Engineering projects impart loads on the ground that supports them.
- **Loads produces stresses in soils** ie **compressive stress**, **shear stress** and **even tensile stress**.

- **Compressive stresses** ie geostatic stresses and induced stresses developed in soils , **are carried partially by the solid particles and partially by the pore water**.

Total stress:

- **Sum of the stresses** which are carried by the two phases of soils ie solid and water.
- Total stress can be very useful if we **divide into two parts**.

What is effective stress in soils?

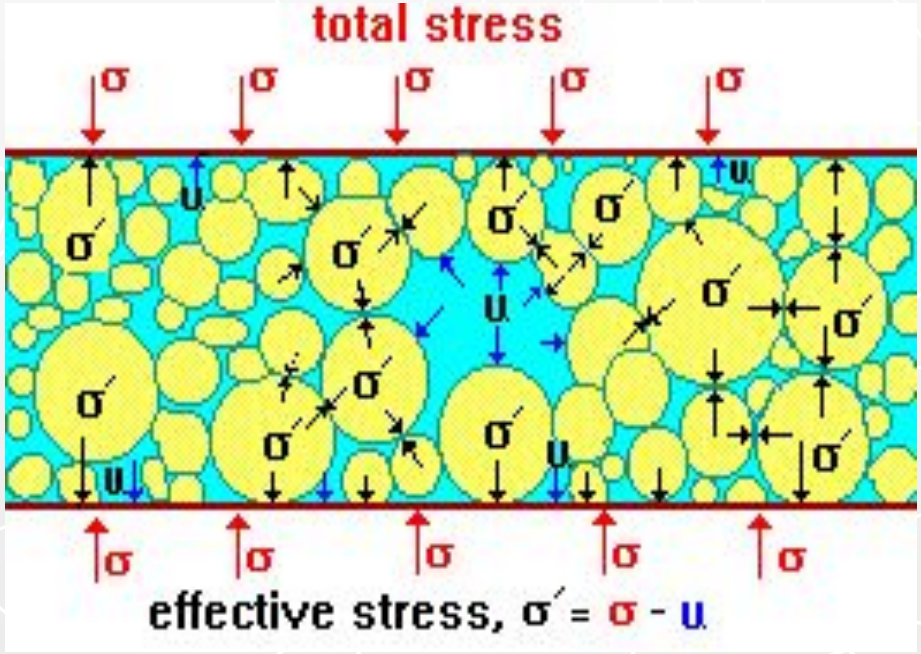
Total stress can be divided into two parts:

Effective stress (σ')

- Portion of stresses that are carried by the solid particles of soils.

Pore water pressure (u):

- Portion of stresses carried by the pore water.



Pore Water Pressures (cont..)

- Denoted by the symbol **u**.

$$\mathbf{u = \gamma_w z_w}$$ where, γ_w = unit weight of water,

z_w = depth of water in soils.

- Used for **finding the effective stress** beneath the soil.

- Difference between pore water stress and total stress is said to be effective stress.

- **Pore pressure in soil is divided into two categories:**
  - -Negative pore pressure and
  - -Positive pore pressure.

Positive pore pressure:

- **Z**one of soil that is below water table will be fully saturated.
- **Pressure in the pore is greater than atmospheric pressure.**
- Hence, it is said to be positive pore pressure.

Negative pore pressure:

•Above the water table, **soil may remain saturated or unsaturated and pressure in pore is lower than the atmospheric pressure**.

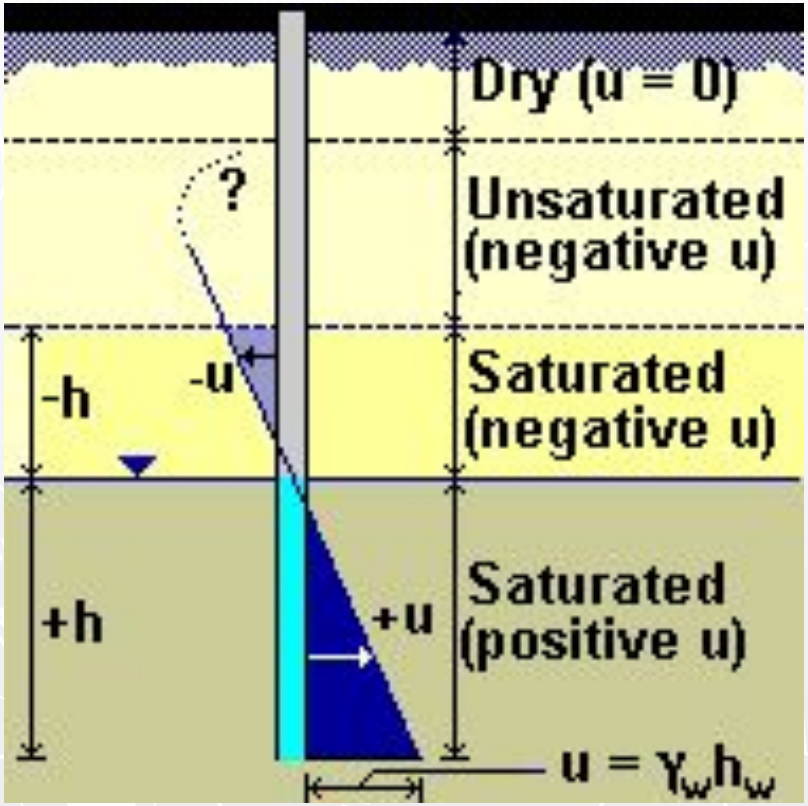
•Hence, it is said to be negative pore pressure.

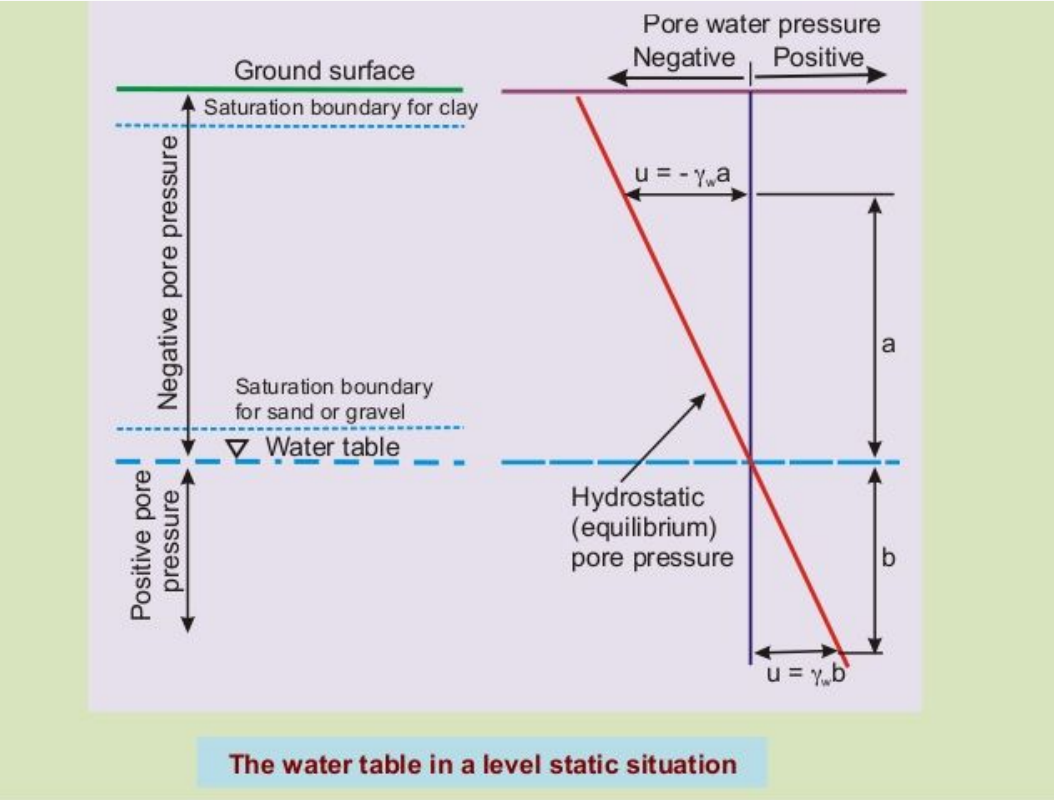
Below the water table, pore pressure is **positive;**

In dry soils and at the level of the water table, pore pressure is **zero;**

Above the water table, soil can remain saturated due to capillary rise but with **negative** pore pressure (suction).

Ideal Scenario for +ve and –ve Pore Pressure (u)





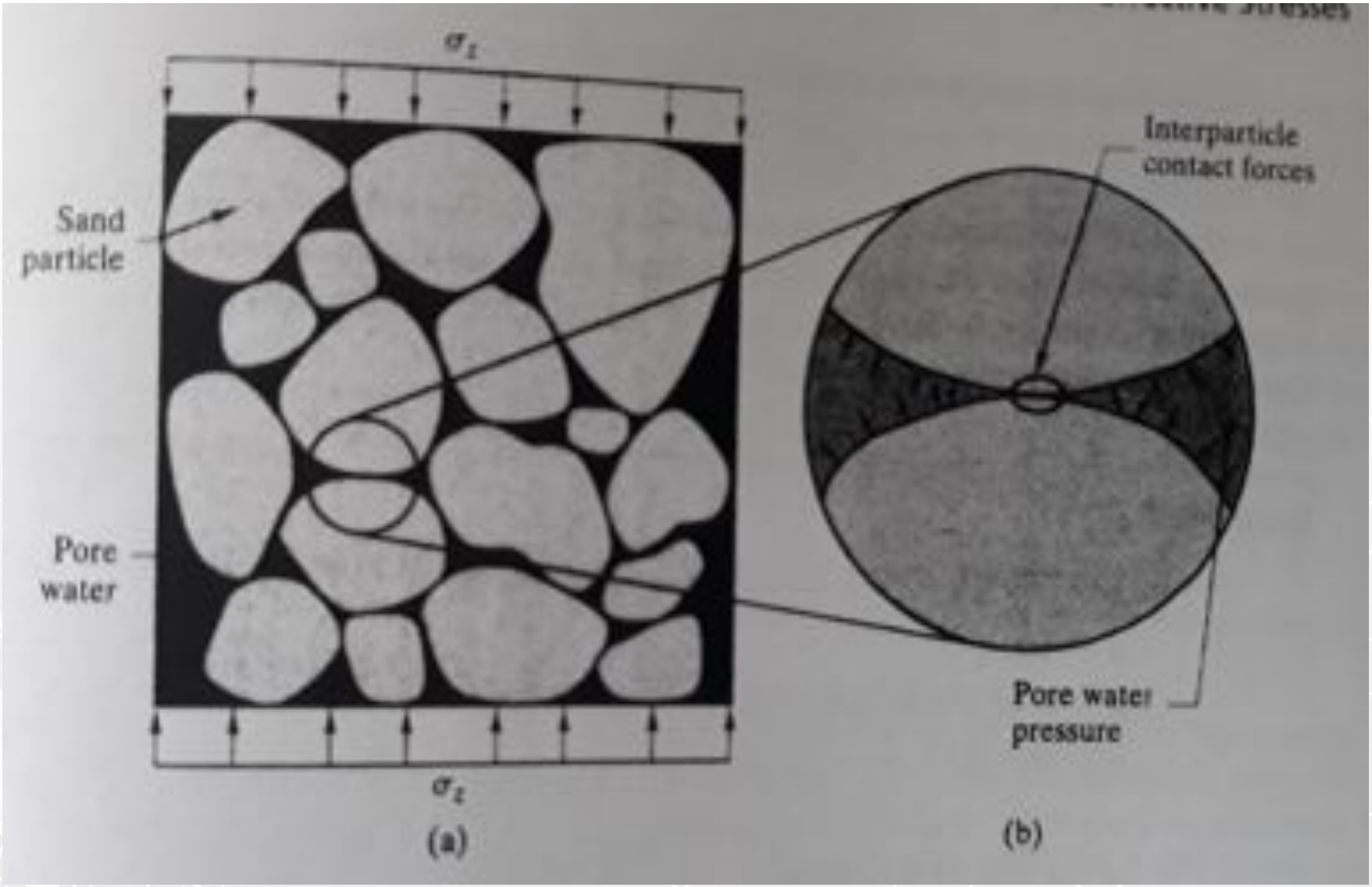


Figure: Division of total stress into effective stress and pore water pressure.

Determination of vertical Effective Stress

where,

σ'_z = vertical effective stress

σ_z = vertical total stress

u = pore water pressure

- If only **geostatic stresses** are present,

$$\sigma'_z = \sum \gamma H - u$$

If only **Induced stresses** are present,

$$\sigma'_z = \sum \Delta\sigma_z - u$$

- If both **geostatic and induced stress** are present,

$$\sigma'_z = \sum \gamma H + \sum \Delta\sigma_z - u$$

=Total stresses – Pore water pressure

Horizontal Effective stresses:

- Ratio of the horizontal to vertical effective stresses is defined as the ***coefficient of lateral earth pressure,* K.**

- For **geostatic stresses beneath a level ground surface**, is normally assumed that k in the x direction is equal to that in the y direction.

$$\mathbf{K} = \frac{\sigma'_x}{\sigma'_z}$$

- If ground surface is inclined, or if induced stresses are present, k may be different in x and y direction.

- Values of k varies about 0.3 to 3.0.

Stresses beneath the bodies of water:

Importance:

- Geotechnical engineers need to **compute stresses beneath the water** such as lakes, rivers or oceans.
- Might be necessary while **designing foundation for a bridge.**
- Might be necessary when evaluating the **stability of slopes that extends under water.**

Computation of Effective Stresses beneath the bodies of water:

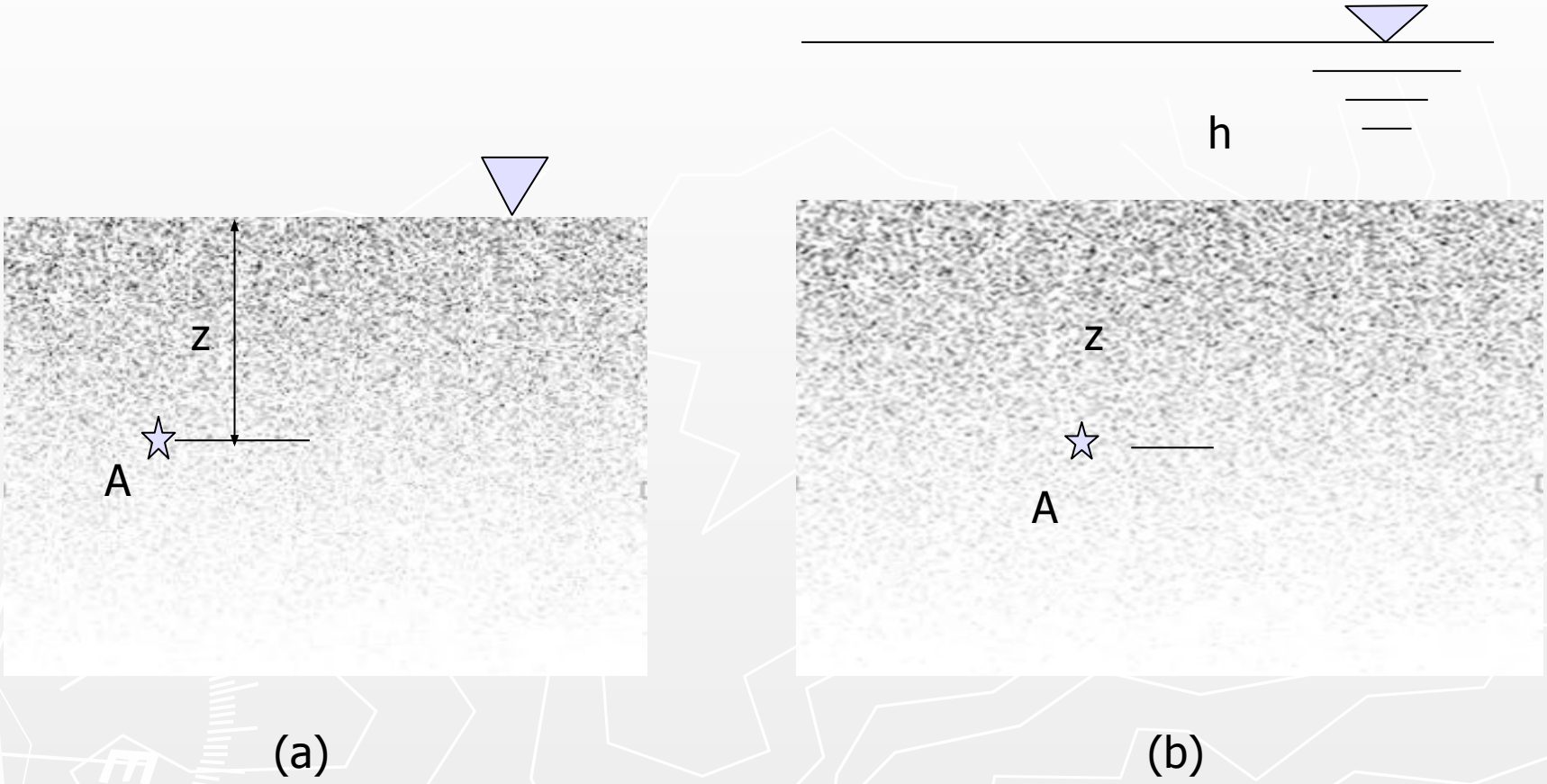


Figure: Computation of effective stress beneath bodies of water

Stresses beneath the bodies of water (cont..):

When GWT at ground surface, stress at Point A :

- **Fig. (a)**

$\sigma_z = \gamma z$

$u = \gamma_w z$

$\sigma'_z = \sigma_z - u = \gamma z - \gamma_w z = z(\gamma - \gamma_w) = z\gamma_b$

- **Fig. (b)**

$\sigma_z = \gamma z + \gamma_w h$

$u = \gamma_w (z + h)$

$\sigma'_z = \sigma_z - u = (\gamma z + \gamma_w h) - \gamma_w (z + h) = z(\gamma - \gamma_w) = z\gamma_b$

- **Water above the ground surface does not change the effective stress,** because it **contributes equally** to both the total stress and pore water pressure.

- We know, the buoyant unit, γ_b of a soil is equal to the total unit weight of soil less the unit weight of water.

$$\sigma'_z = z\,(\gamma - \gamma_w) = z\,(\gamma_{sat} - \gamma_w)$$

$$z\,\gamma_b = z\,\gamma_{sub}$$

Q: Explain with example.

Explain the stress conditions with negative pore water pressure in soils.

Stress conditions with negative pore water pressure or soil suction:

- Most geotechnical analyses **assume** that the pore water pressure above the GWT is zero. Therefore, effective stress is equal to the total stress.
- In reality, **soils above GWT generally have –ve pore water** pressures also known as soil suction, which means the effective stress is greater than the total stress.

Vadose Zone

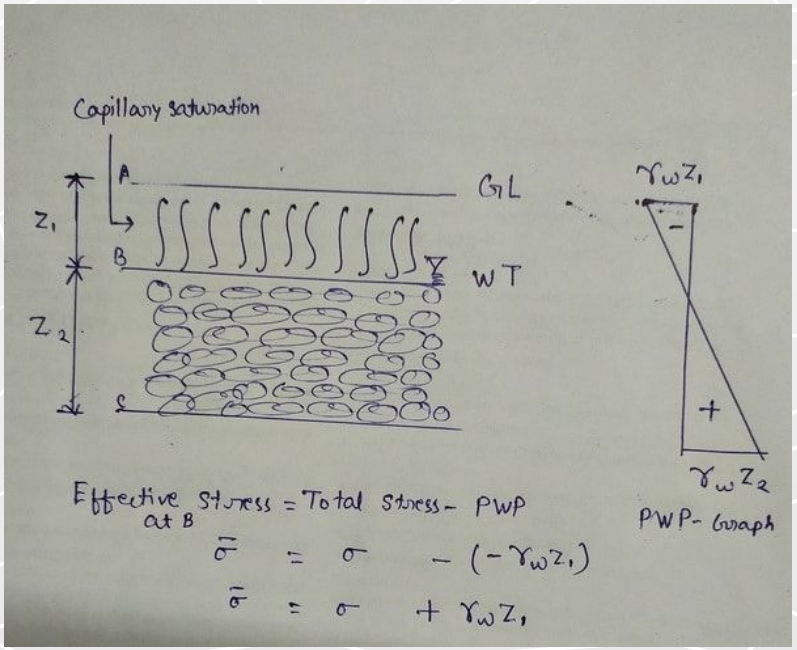
- In the figure, within the **vadose zone**, the soil remains saturated in the zone of capillary rise.

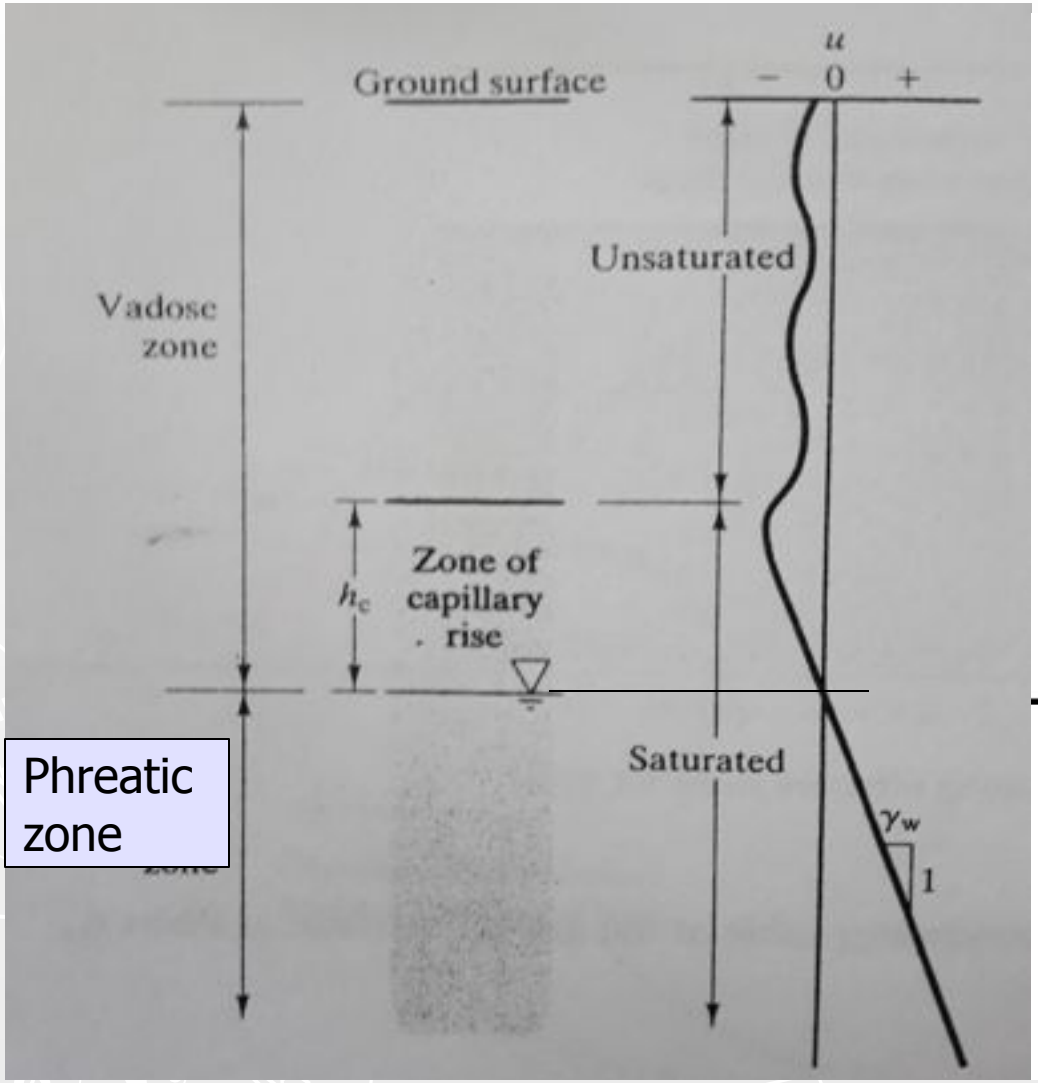
Zone of Capillary Rise:

- In the zone of capillary rise, the u may be computed by using the equation $u=\gamma_w z_w$, except z_w **is now negative**.

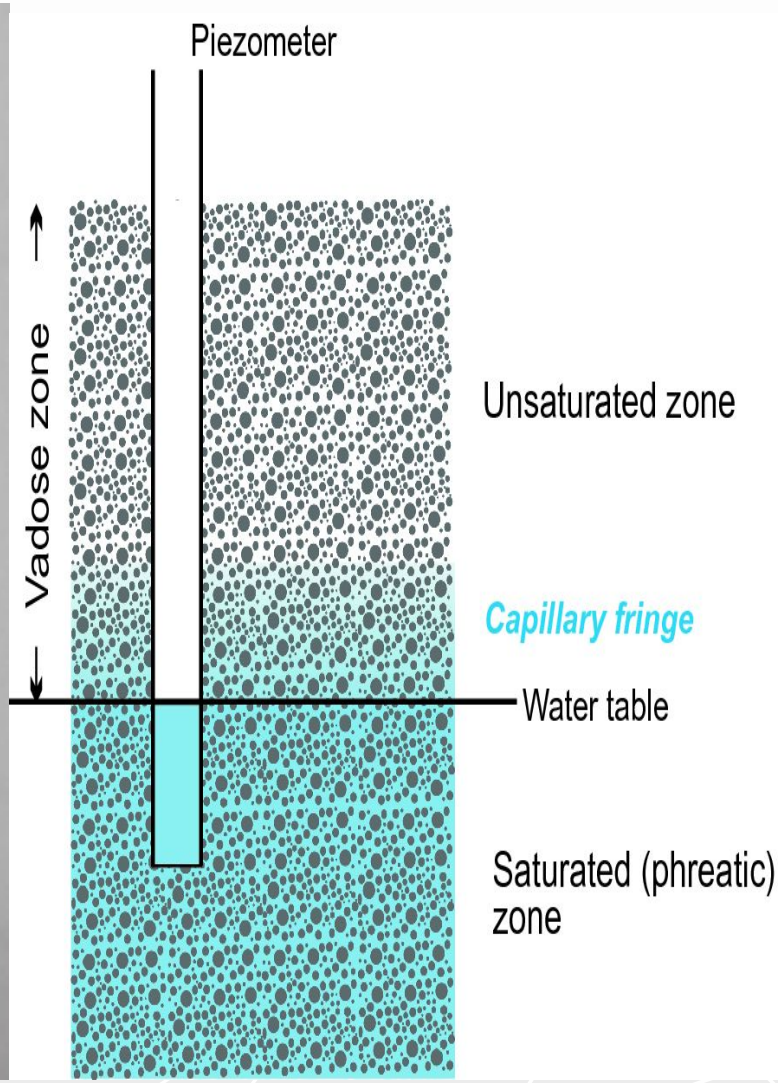
Capillary zone: Immediately above the water-table, into which **water may be drawn upwards as a consequence of capillary action** (intermolecular forces between the liquid and surrounding solid surfaces).

- This **produces a negative u** ie the pore water in the capillary zone may be visualized as a column of water held in tension by capillary forces.





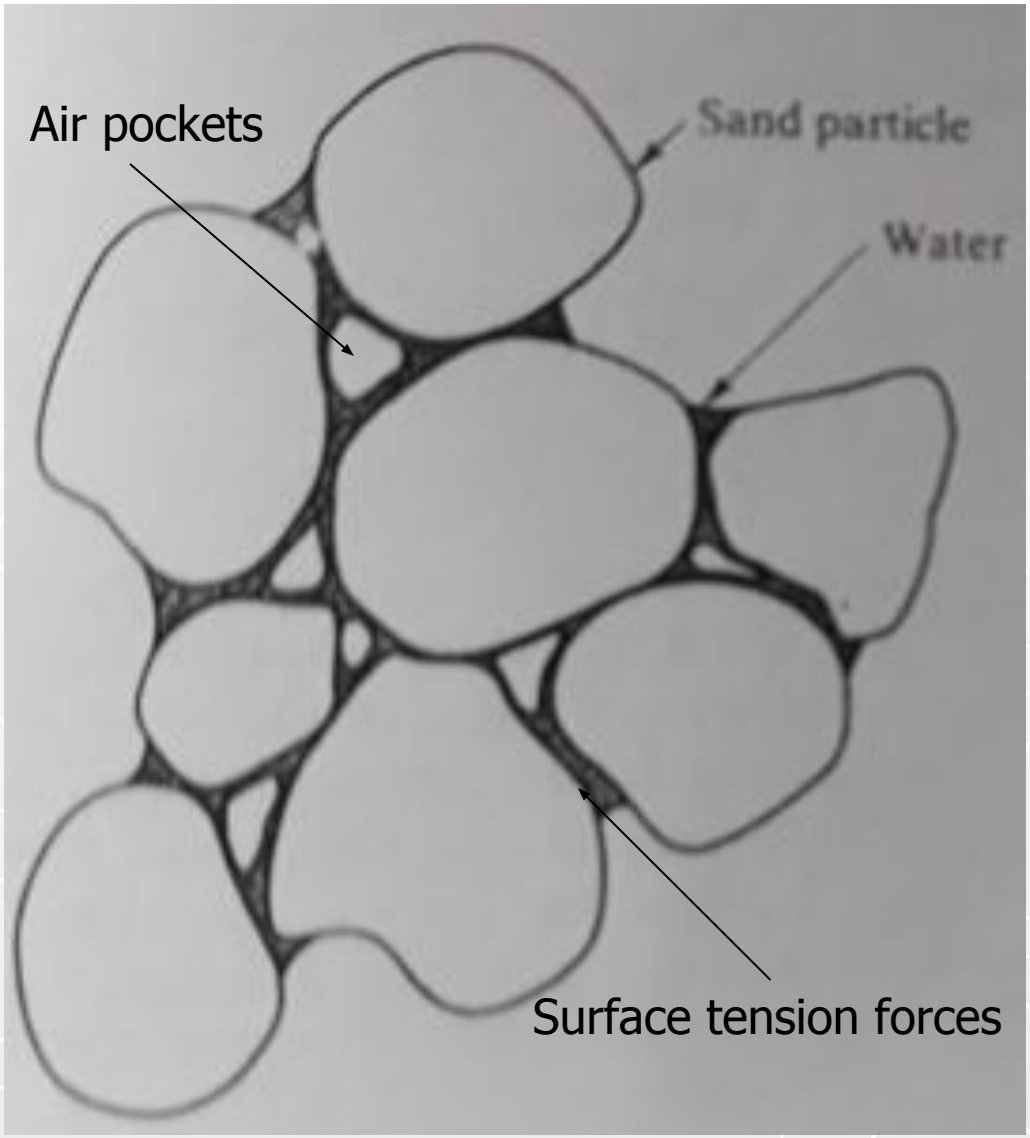
Phreatic zone



A plot of pore water pressure vs elevation.

Unsaturated zone:

- Above the Zone of Capillary Rise, the **degree of saturation falls well below 100% .**
- Pore water **collects into small capillary zones adjacent to particles contact points.**
- **Surface tension forces develop** between **this water and the particles,** creating tensile forces (-ve u) in the water.
- In the **air pockets, the pressure is atmospheric**.
- Negative pore water pressures can be measured in situ or in the laboratories.



Above the capillary zone, the pore water retreats to particle contact points.

Thank you