EWCE 341: Principles of Soil Mechanics

Permeability

What we discussed in previous class?

- What is Permeability?
- Important in a no. of soil engineering problems.
- GW Hydrology?
- Assessment of GWT
- Factors affecting permeability of soils

What we are going to learn today?

- Darcy's Law
- Determination of K?
- Problems

Darcy's Law:

- **Flow of free water through soil** is governed by the Darcy's Law.

- In 1856, Darcy demonstrated experimentally that for **laminar flow** in **homogenous soil,** the velocity of flow (v) is given by:

  $v = ki$ *where, k= coefficient permeability, i = hydraulic gradient*

  $i = \frac{h}{L}$, h= hydraulic head, L= length of the soil specimen

Velocity of flow v=**discharge velocity**.

Discharge is obtained by multiplying the velocity of flow by the total x-sectional area of soil (A) **normal to the direction flow**.

Q= $vA = kiA$ **Area *A* includes both solids and voids**.

Darcy's Law found valid for a **wide range of soil types from clay to coarse sands**.

Limitations:
For **clean gravels**, **accuracy of flow diminishes** as a **turbulent flow**.
For **clay**, with low hydraulic gradient, **accuracy diminishes,** as **flow rate is so small**.

Coefficient of Permeability:

- Defined by equation, $v=ki$

If $i = \frac{h}{L} = 1.0$ **then v = k**

Coefficient permeability **(k) is defined as the velocity of flow which would occur under unit hydraulic gradient,** expressed as velocity unit (cm/sec).

k of a homogeneous and isotropic soils depends on primarily:

- Viscosity of pore water (η)
- Void ratio (e) of the soil
- Size and shape of soil particles
- Degree saturation (S)

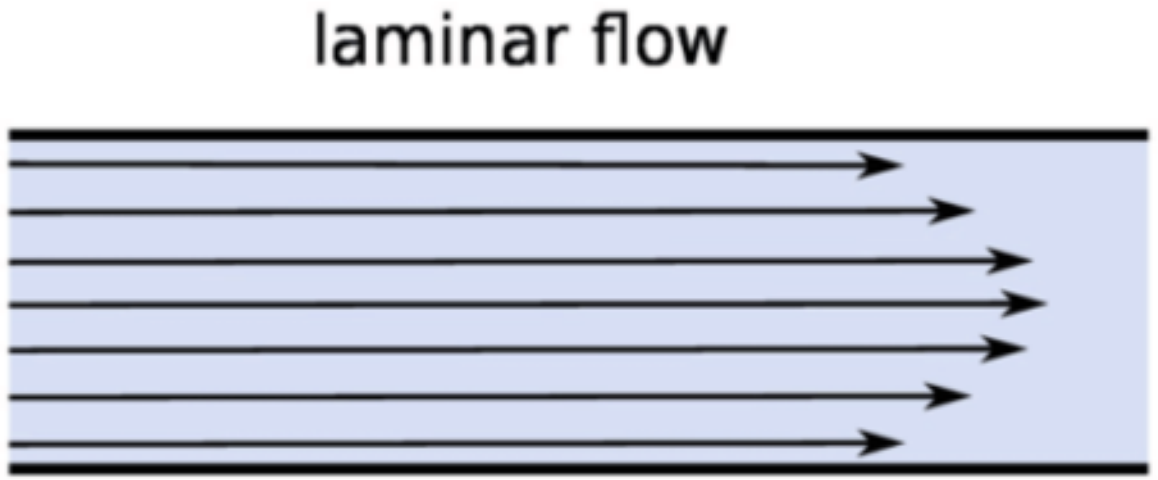
Some Typical Values of *k in Soils:*

Soil	*K* cm/sec	Degree of Permeability
Gravel	$> 10^{-1}$	Very High
Sandy gravel, clean sand, fine sand	$10^{-1} > k > 10^{-3}$	High to Medium
Sand, dirty sand, silty sand	$10^{-3} > k > 10^{-5}$	Low
Silt, silty clay	$10^{-5} > k > 10^{-7}$	Very Low
Clay	$< 10^{-7}$	Virtually Impermeable

GW Flow:

Laminar Flow

- **Laminar flow** occurs when a **fluid flows in parallel layers**, with no disruption between the layers.
- At **low velocities**, the fluid **tends to flow without lateral mixing**, and **adjacent layers slide past one another** like playing cards.
- In laminar flow, the **motion of the particles of the fluid is very orderly** with all particles moving in straight lines parallel.

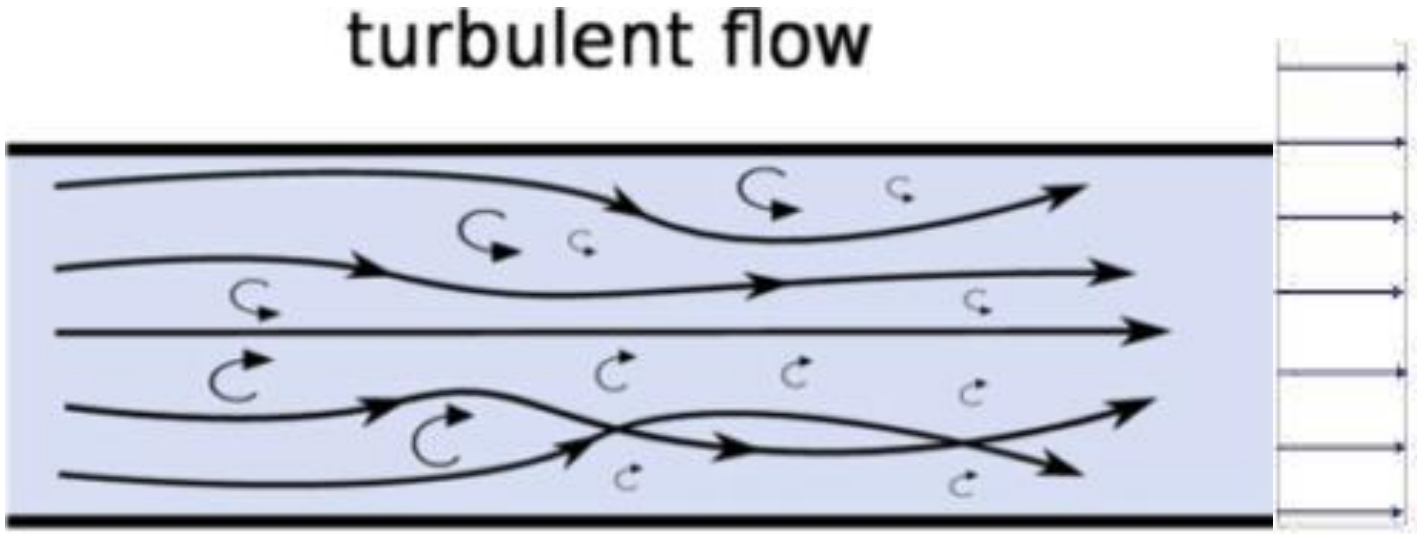


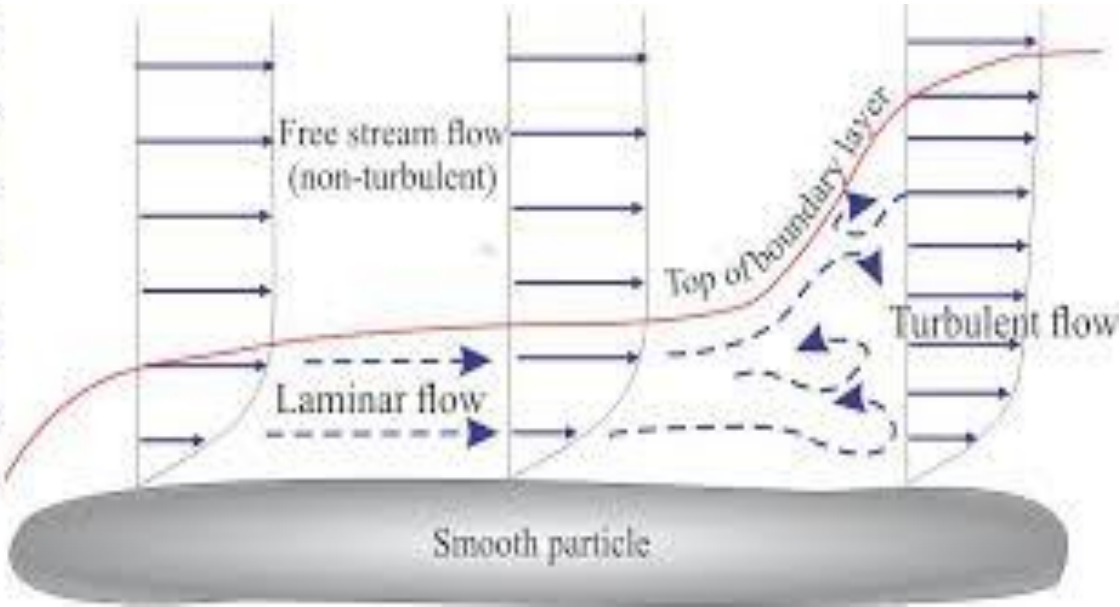
Turbulent Flow

- **Turbulent flow** is characterized by **chaotic property changes**.

includes **rapid variation** of **pressure** and flow **velocity** in space and time.

- Fluid **no longer travels in layers** and **mixing across**.
- Flows at **Reynolds numbers** **larger than 1** are typically (but not necessarily) **turbulent**, while those at low Reynolds numbers below 1 usually remain **laminar**.
- Flow in the range of Reynolds numbers in between and known as **transition**.



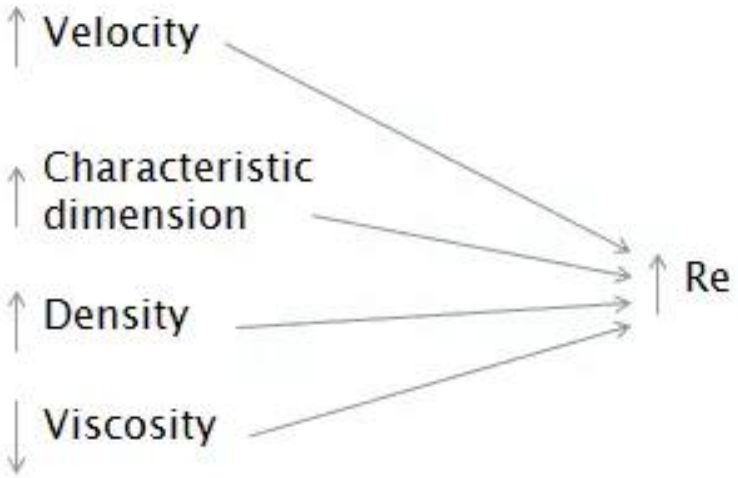


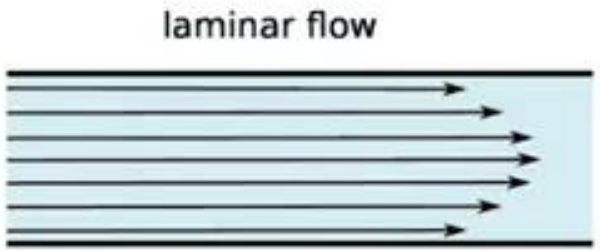
Reynolds Number

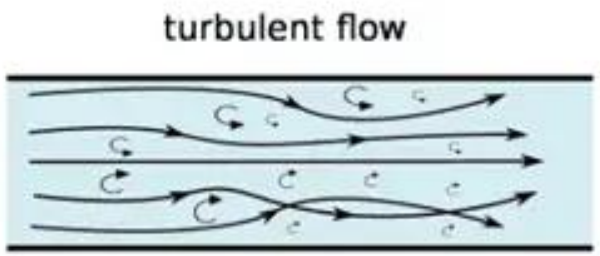
- Reynolds number, **referred to as Re**, is used to determine whether the fluid flow is laminar or turbulent.
- Although the Reynolds number **comprises** both static and kinetic properties of fluids, it is **specified as a flow property** since dynamic conditions are investigated.
- Reynolds number is **the ratio** of the inertial forces to the viscous forces.

$$\mathbf{Re} = \frac{inertial\ force}{viscous\ force}$$

$$Re = \frac{\text{inertia forces}}{\text{viscous forces}} = \frac{\rho \cdot u \cdot D}{\mu}$$







- Ratio helps to **categorize laminar flows from the turbulent ones.**
- $\mathbf{Re} = \frac{\rho V L}{\eta}$ or $\frac{\rho V D}{\eta}$

Reynolds Number through **pipe flow**

Flow Type	Reynolds Number range
Laminar	Upto Re-2300
Transition	2300 <Re<4000
Turbulent	Re> 4000

Reynolds Number – flow of water through soils

- For the flow of water through soil, the flow is laminar if Reynold's number **is less than 1**. In **fine-grained soil**, flow is laminar.

- In **coarse-grained soil**, the flow becomes turbulent when the size of particles is very large i.e. in gravel range

Validity of Darcy's Law

Darcy's Law is valid if the flow through soils is laminar flow:

- Flow of water through **soils** depends upon the **dimensions of interstices**, which in turn **depends on particles size**.
- **In fine-grained soils**, the **dimensions of interstices are very small** and flow is also generally laminar.
- In **very coarse-grained soils**, such as coarse gravel, flow may be turbulent.

For Flow of water through pipes, the flow is laminar when the Reynolds Number is less than 2300.

For flow of water through soils:

- Flow is **laminar** if the Reynolds Number is **less than unity**.
- **Characteristics length** of in the Reynolds number is taken as the average particle diameter (D).
- $\frac{\rho V D}{\eta} \leq 1$, maximum diameter of the particles for the flow to be laminar is about 0.50mm.
- Value of **Re $\leq$ 1** is however conservative.
- Flow **remains laminar** even up to **Re is 75**.

Validity of Darcy's law

- Darcy's law is **valid** for flow in **clays, silts and fine sand**, flow is laminar
- Darcy's Law **may not be applicable**, in **coarse sands, gravels and boulders**, flow is turbulent.
- In **extremely fine grained soils**, such as colloidal clay, **the interstices are very small,** Darcy's law is **not valid.**

Range of the validity of Darcy's law:

- **Difficult to predict** exact range of validity of Darcy's law.
- Best method is to **conduct experiments** and **determine the actual relationship** between velocity v and hydraulic gradient i.
- For Darcy's law to be valid, this **relationship ie. v and i to be approximately linear**.
- For coarse grained sands, gravels and boulders, v and i relationship is **non-linear.**

Determination of k

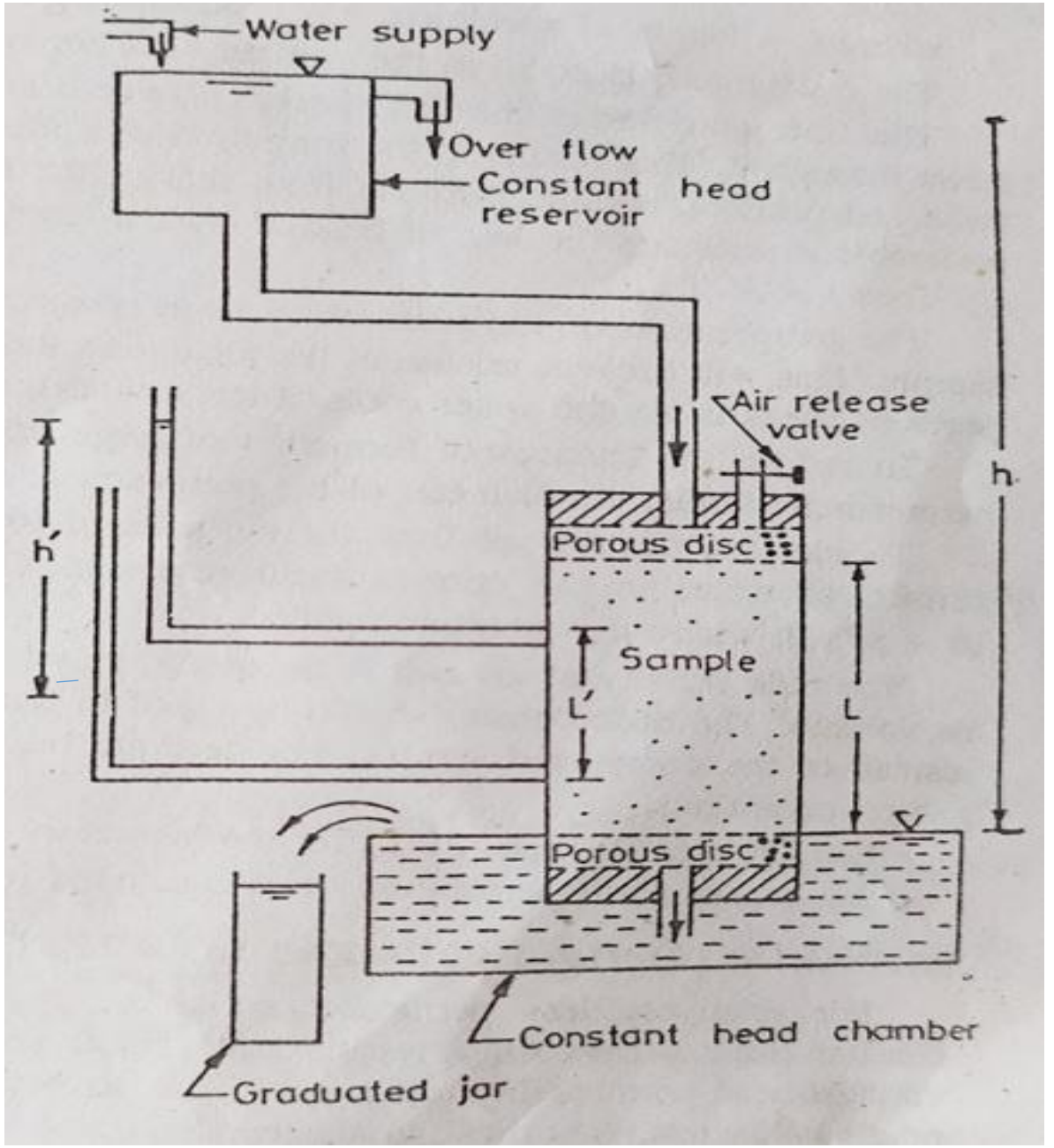
K of a soil sample can be determined by the following **Laboratory** methods:

- Constant Head Permeability Test
- Variable Permeability Test

Constant Head Permeability Test:

- Figure shows a schematic sketch.
- **Soil sample** is placed inside the mold between two porous disc.
- Soil sample is **saturated** and water is **allowed to flow out until a steady state is established.**
- Water level in the **constant-head reservoir** in which mold is placed is **kept constant**.

Constant Head Permeability Test (cont…):



- **Constant head chamber** is filled to the brim at the start of experiment.
- Water which **enters the chamber after flowing through the soil sample**, **spills over** the chamber and collected in a **graduated jar** over a period (time).
- **Head causing flow (h)** is equal to the difference in water levels between constant head reservoir and constant head chamber.
- A= x-sectional area of specimen A.
- Discharge, **q= kiA**

$$=k\frac{h}{L}A$$

$$\mathbf{k}=\frac{\boldsymbol{qL}}{\boldsymbol{Ah}}$$

- **Finer particles of the soil specimen have a tendency to migrate** towards the end faces when water flows through it.
- For more accurate results, **i=** $\frac{h'}{L'}$
- **Suitable for clean sand and clean gravel**.

Clean sand (USCS):

- **Less than 5%** of fines
- **SW, SP**.

Clean Gravel

- Less than 5% Fines
- GW, GP

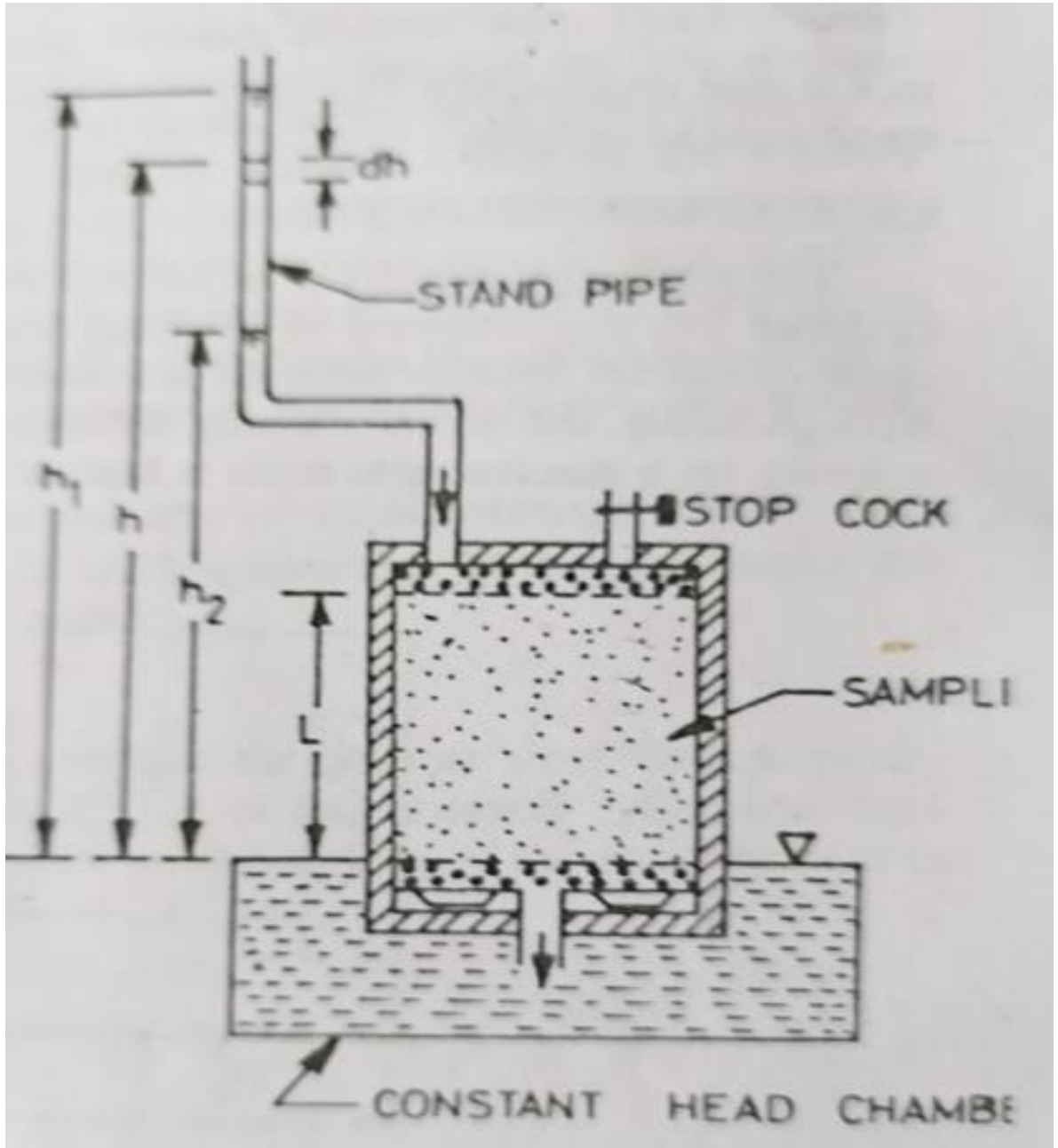
Variable Head Permeability Test:

- For **relatively less permeable soils**, **quantity water** collected in the graduated jar of the constant head permeability test is **very small** and **can not be measured** accurately.
- For such soils variable-head permeability test is used.

Test is started by **allowing water in the stand-pipe to flow through the sample** to the constant-head chamber from **which it over flows**.

- As water flows through the soil, the **water level in the stand-pipe falls**.
- Time required for the water level to fall from the known **initial head (h_1)** to a known **final head (h_2)** is determined.
- **Head is measured** with reference to the level of water in the constant head chamber.

Variable Head Permeability Test (cont…):



- Let us consider the **instant** (time), when **head is h**.
- For the infinitesimal **small time dt**, the **head falls dh**.
- Let the **discharge** through the sample be q.
- From the continuity of flow,

$$\mathbf{a.dh = - q.dt}$$

$$= -(\text{Ax k x i}) \text{ x dt}$$

$$= - \text{Ak} \frac{h}{L} \text{ x dt}$$

$$\frac{\boldsymbol{Ak\,dt}}{\boldsymbol{aL}} = -\frac{\boldsymbol{dh}}{\boldsymbol{h}}$$

Or, $\frac{Ak}{aL} \int_{t_1}^{t_2} dt = - \int_{h_1}^{h_2} \frac{dh}{h}$ or $\frac{Ak}{aL} (t_2 - t_1) = log_e \left(\frac{h_1}{h_2} \right)$

Or, $\mathbf{k = \frac{aL}{At} log_e \left(\frac{h_1}{h_2} \right) = \frac{2.30aL}{At} log_{10} \left(\frac{h_1}{h_2} \right)}$ where, $t_2 - t_1 = t$

- Rate of fall of water level in the stand pipe and rate of flow can be **adjusted** by **changing the area of the x-section of the stand pipe**.

- **Smaller diameter pipes are required for less pervious soils.**

- Variable permeameter is suitable for **fine sand and silt** with k = 10^{-2} to 10^{-5}mm/sec.

Ex. 1: A **constant head** permeability test has been performed with **3 standpipe tubes** (piezometers). The graduated cylinder collects **892ml** of water in **1120 seconds**. The dimensions are:

Soil sample **dia=1.8cm**

Elevation of water in **upper piezometer=181.0cm**

Elevation of water **in lower piezometer=161.0cm**

Piezometers **tips are spaced=16.7cm on center**

Compute the hydraulic gradient and coefficient of permeability of the soil sample.

Solution:

$$\mathbf{q} = \frac{\boldsymbol{Vol}}{\boldsymbol{t}} = \frac{892}{1120} = 0.796\ cm^3/\text{sec}$$

$$\mathbf{A} = \frac{\boldsymbol{\pi}}{\boldsymbol{4}}\boldsymbol{D^2} = \frac{\pi}{4}1.8^2 = 2.54\ cm^2$$

$$\mathbf{i} = \frac{\boldsymbol{\Delta l}}{\boldsymbol{\Delta h}} = \frac{161.6-181.0}{3x16.7} = 0.39$$

q=kiA

$$k = \frac{q}{iA} = \frac{0.796}{0.39x2.54} = 0.8\ \text{cm/sec}$$

Ex. 2: A constant head permeability test has been conducted on a **100mm diameter** 270mm tall **soil sample** in a permeameter. The upper and the lower reservoir elevation were 2010mm and 1671mm above the laboratory floor. The piezometers, whose **tips are spaced 200mm** apart, head readings of **1809mm** and **1578mm** and the graduated collected **910 ml of water** in 25 min 25 sec. Compute the permeability of the soil sample. Give your comments on the result.

Solution:

$$\mathbf{q} = \frac{\boldsymbol{Vol}}{\boldsymbol{t}} = \frac{910}{1525} = 0.597\ cm^3/\text{sec}$$

$$\mathbf{A} = \frac{\boldsymbol{\pi}}{\boldsymbol{4}}\boldsymbol{D^2} = \frac{\pi}{4}1^2 = 0.785\ cm^2$$

$$\mathbf{i} = \frac{\Delta \boldsymbol{l}}{\Delta \boldsymbol{h}} = \frac{1809-1578}{3x200} = 0.385$$

$$\mathbf{q = kiA}$$

$$k = \frac{q}{iA} = \frac{0.597}{0.385x0.785} = 1.98\ \text{cm/sec}$$

Gravel 1.0 -100 cm/sec

Fine sand: 0.01 – 0.001

Soil	*K* cm/sec	Degree of Permeability
Gravel	$> 10^{-1}$	Very High
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Ex. 3: A **3.2m** thick silty sand stratum **intersect of a reservoir** shown in the figure. This stratum has a hydraulic conductivity of **$4x10^{-2}$cm/sec** and extends along the entire **1000m** length of the reservoir. An observation well has been installed in this strata as shown. Compute the **seepage loss** **per month** from the reservoir through the stratum

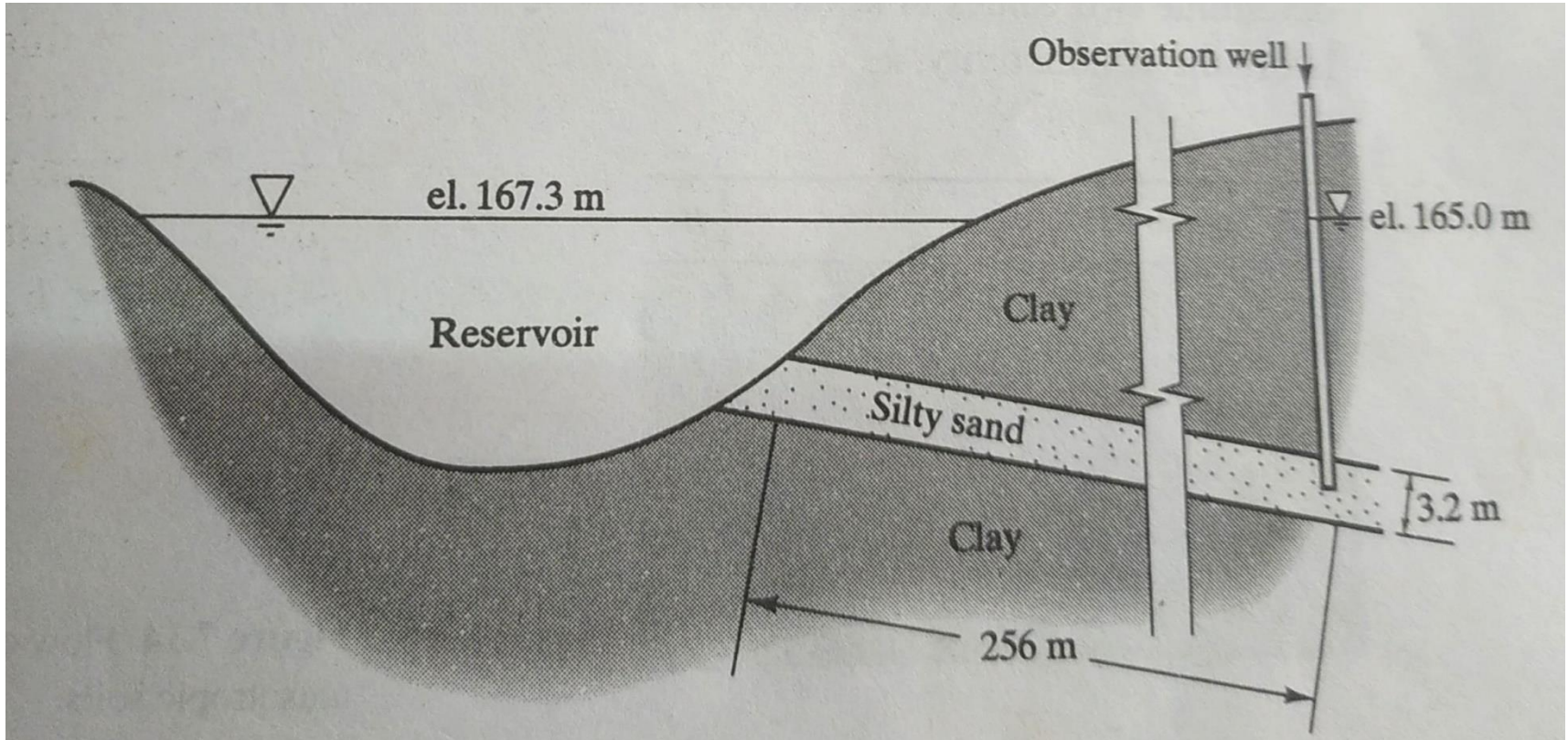


Figure 7.13 Cross-section through reservoir showing silty sand stratum. This cross-section is oriented parallel to the direction of flow.

Solution:

Observation well indicates a WL above the top of silty stratum, so it is an **confined aquifer**.

$$\mathbf{i}=\frac{\Delta h}{L}=\frac{167.3-165.0}{256}=0.0090$$

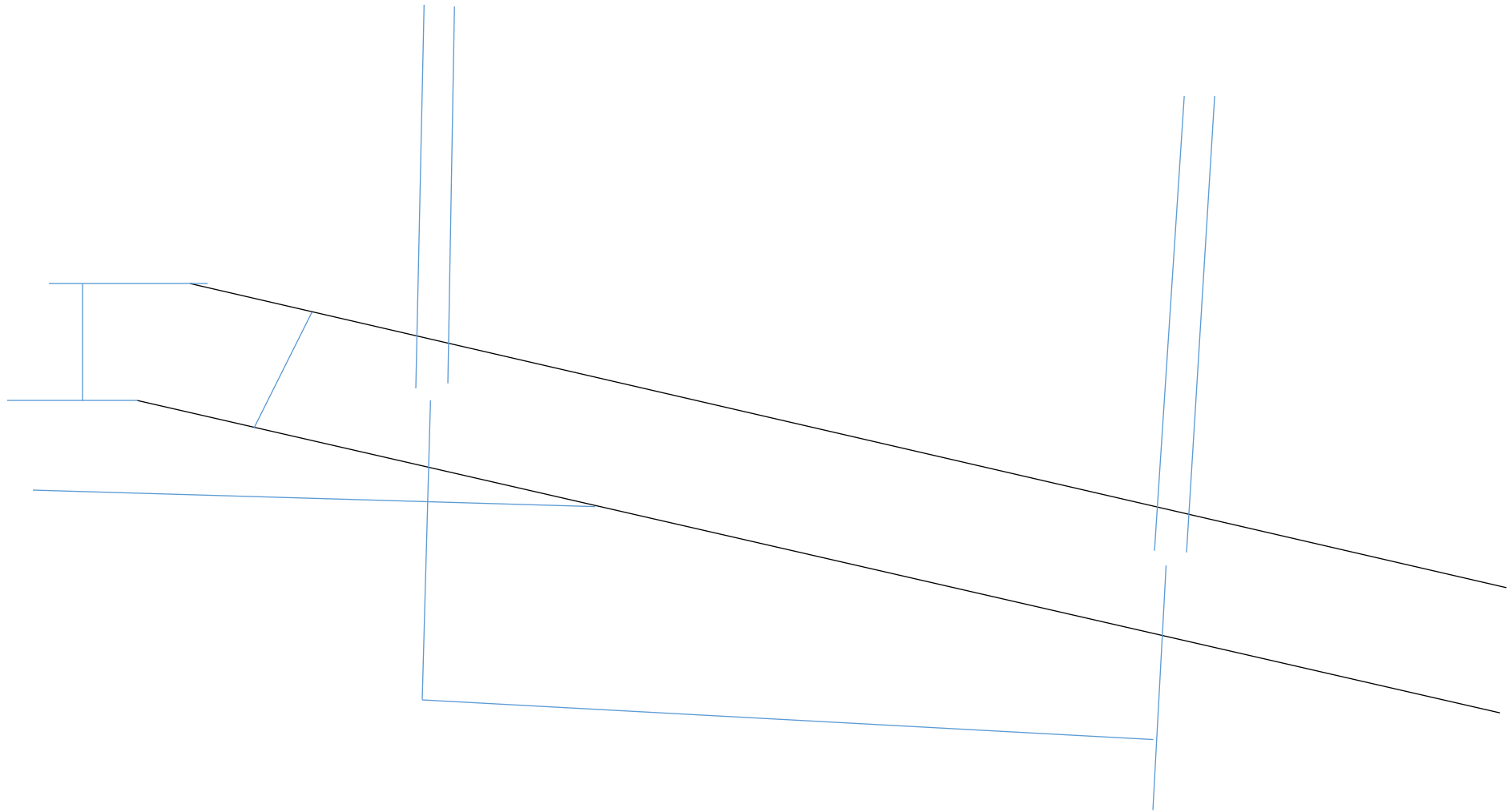
$$\mathbf{A=3.2x1000}=3200\ m^2$$

$$\mathbf{K=4x10^{-2}cm/sec}=4\text{x}\ \frac{3600x24x30}{100x100}=1036.8\ \text{m/month}$$

Q=kiA

$= 1036.8\text{x}0.0090\text{x}3200$

$=30{,}000\ m^3/\text{month}$



Thank You