EWCE 341

Principles of Soil Mechanics

Stresses in Soils

What we learned in the last class?

- What is stress in soils ?
- Shear Failures in Soils.
- Sources of stresses in soils
- Computation of Vertical Geostatic Stress.

What we are going to learn today?

- What is stress in soils ?
- Simplifying assumptions for computation of stresses.
- Designation of stress and strain
- Elastic parameters of soils ie. E, G and ν
- Mohr Circle

What is stress in soils ?

Stress:

- All civil Engineering projects impart loads on the ground that supports them.
- **Loads produces stresses** in soils ie compressive stress, shear stress and even tensile stress.

Simplifying Assumptions

For **computing stresses, soils or rocks will have to have following assumptions:**

- It is **continuous material** or not a particulate materials.
- It is **homogenous** ie the relevant engineering properties are **same at all locations**. In stress analysis E, G and ν to be constant ie. there are no '**hard spots**' or '**soft spots**' which can significantly alter the stress distribution.
- It is **isotropic** ie the engineering properties are same in all directions with exceptional is the bedded sedimentary rocks.
- It has **linear elastic stress-strain properties** ie stress is proportional to strain.

Homogeneous vs Isotropic

Homogeneous refers to the uniformity of the structure of the matter.	Isotropic refers to the property of having uniform in every direction.
Structure of homogeneous material is uniform.	Structure of isotropic material can be either homogeneous or non-homogeneous.
Properties of homogeneous matter does not depend on the direction.	Properties of isotropic matter depends on the direction.
Air, clear solutions are example.	Water, glass are examples.

Designation of Stress & Strain

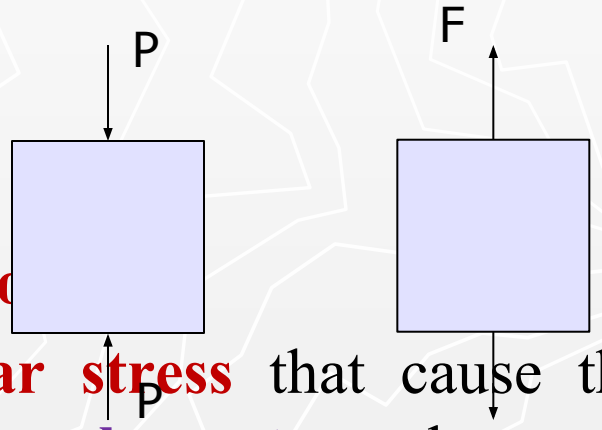
Stress:

- **Each face of the element of a material** is **subjected to a normal stress**, which could be either tension or compression.
- Geotechnical engineers use:

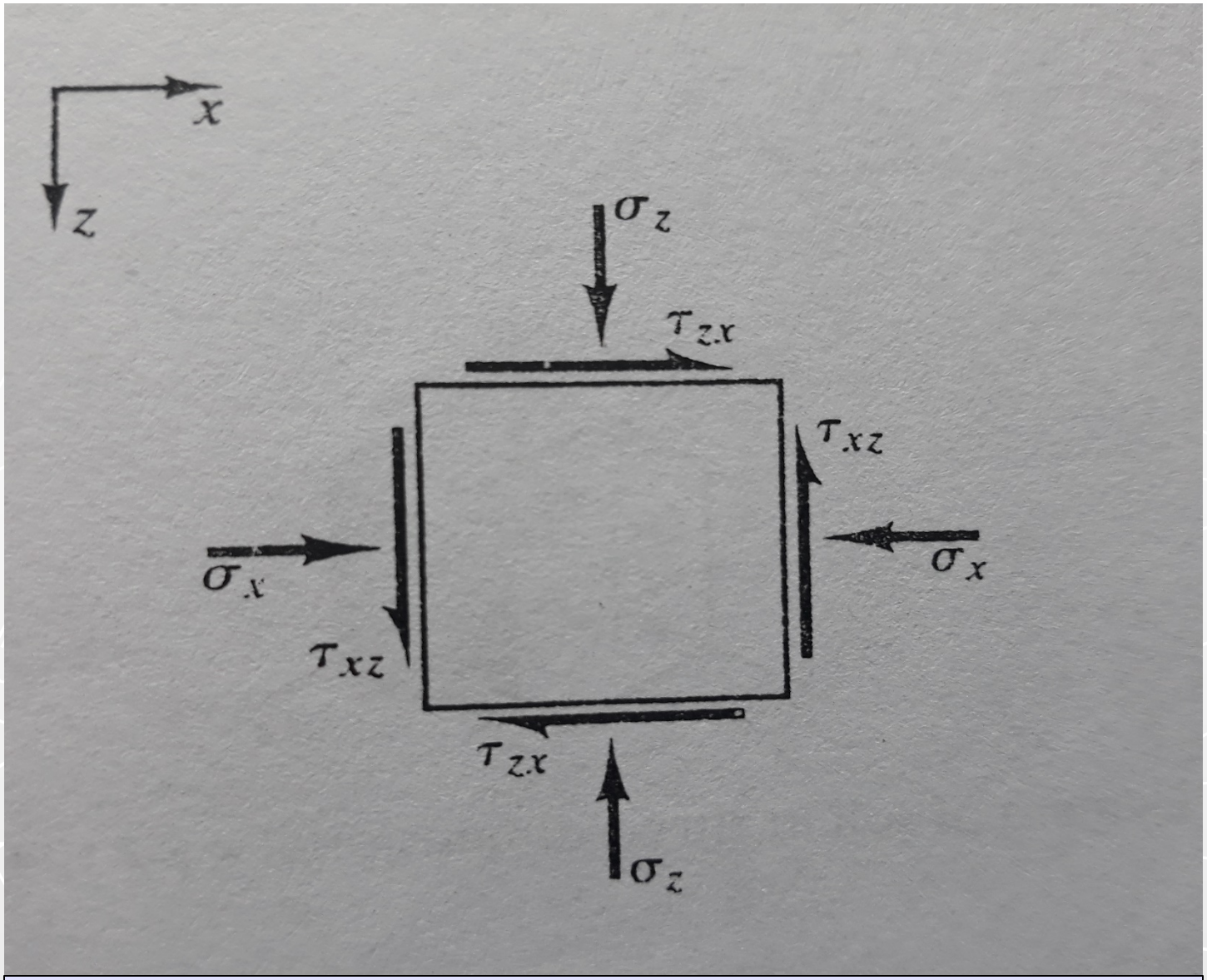
  + = compression

  - = tension

Because they **always deal with the compressio**



- For two dimensional problems, **+ve shear stress** that cause the element to **rotate counter clockwise and –ve shear stress** that cause it to rotate clock wise.



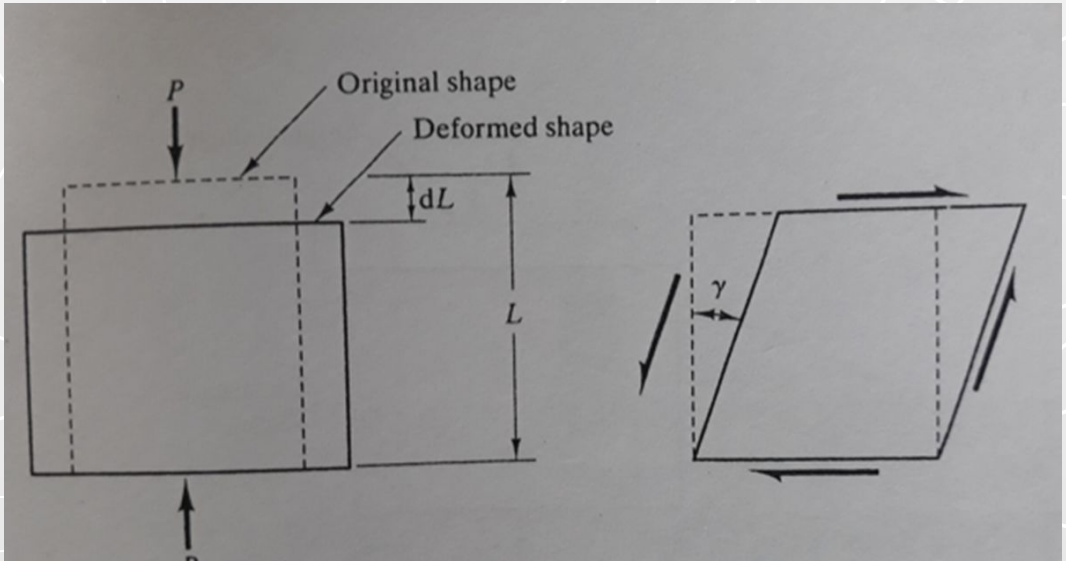
A two dimensional soil element with x-axis and z-axis

Strain:

- When material are **subjected to a stress**, **they respond to deformation** called strain.
- Normal stresses produce normal strain and shear stresses produce shear strain.
- **Normal Strain**, ϵ, is the change in length divided by the initial length.

$$\epsilon = \frac{dL}{L}$$

- **Shear Strain**, γ, is the angle of deformation, expressed in radians.



Modulus of Elasticity, Shear Modulus and Poisson's Ratio: In stress analysis E, G and ν to be constant

Modulus of Elasticity (E):

- Also known as Young's Modulus.

(the modulus takes its name from British scientist Thomas Young, who described its calculation in his *Course of Lectures on Natural Philosophy and the Mechanical Arts* in 1807).

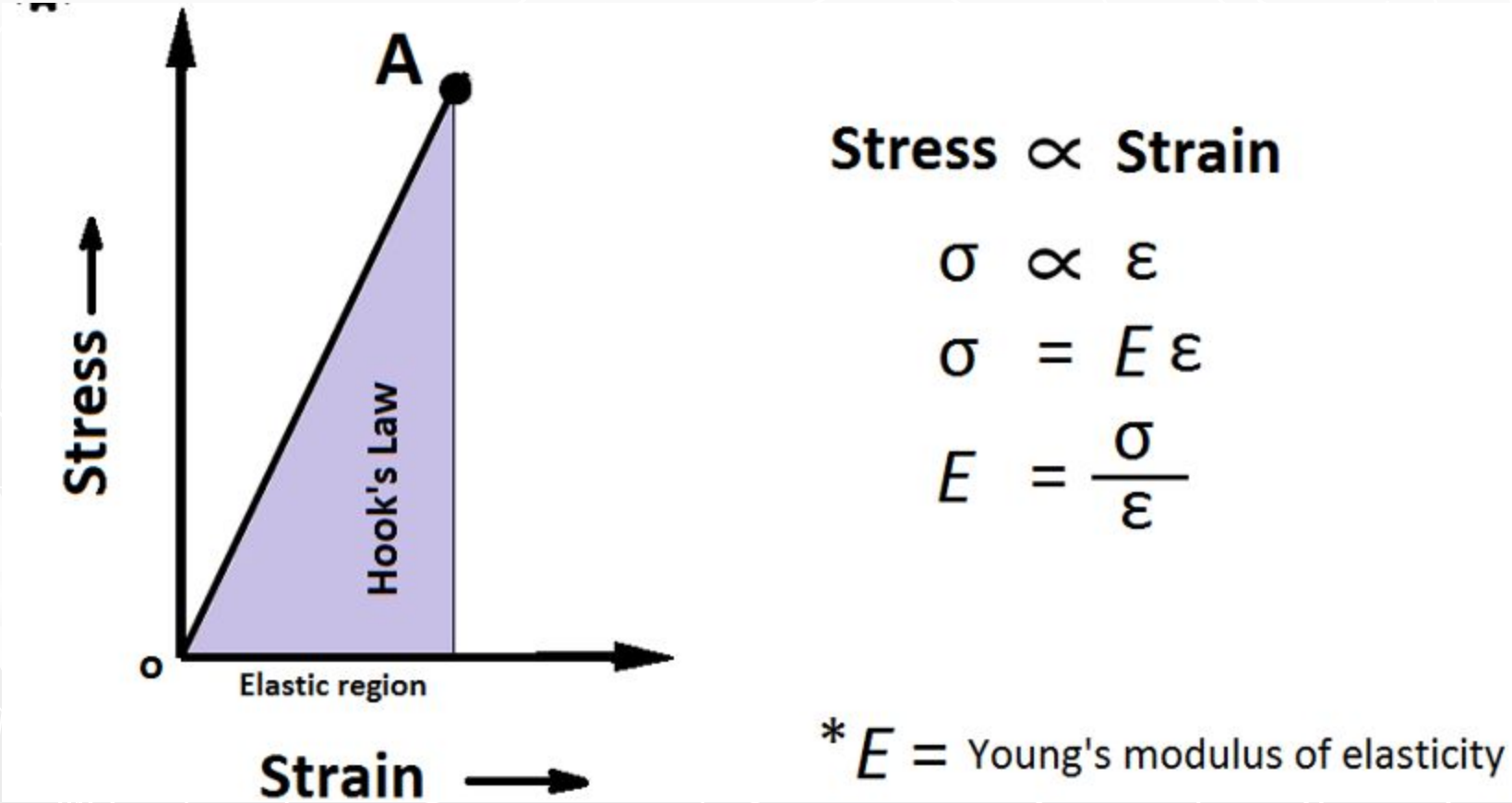
- $E = \frac{\sigma}{\epsilon} = \frac{Normal\ Stress}{Normal\ Strain}$

$\sigma = \frac{P}{A}$ $\epsilon = \frac{dL}{L}$

Large value of E indicates a material that is **very stiff** and does not experience much deformation under applied load.

Type of Soil	Strength of soil	Elastic modulus, E (MPa)
Fine sand	Loose	5–10
	Medium	10–25
	Dense	25–50
Silt	Soft	<10
	Stiff	10–20
	Hard	>20
Clay	Very soft	<3
	Soft	2–7
	Firm	5–12
	Stiff	10–25
	Very stiff	20–50
	Hard	40–80

Hook's Law: Law stating that the strain in a solid is proportional to the applied stress within the elastic limit of that solid.



$$\text{Stress} \propto \text{Strain}$$

$$\sigma \propto \varepsilon$$

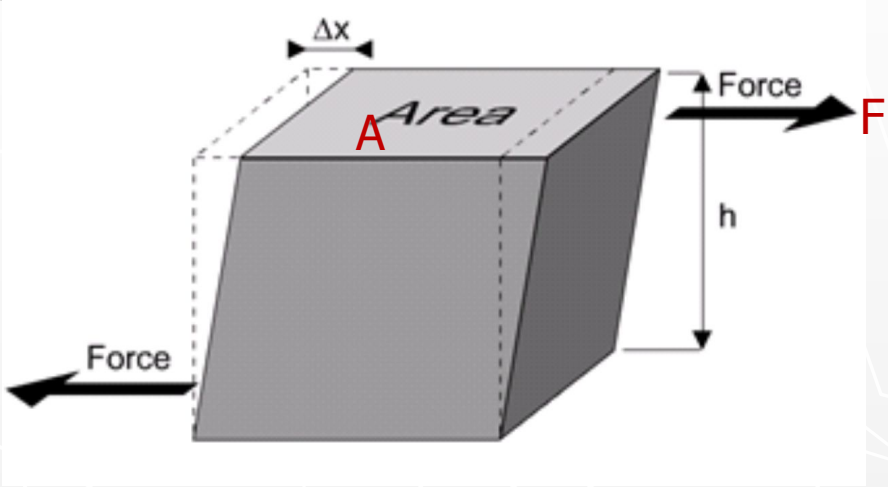
$$\sigma = E\,\varepsilon$$

$$E = \frac{\sigma}{\varepsilon}$$

* E = Young's modulus of elasticity

Shear Modulus (G or η):

- Also known as **modulus of rigidity**.
- Defined as **the ratio of shear stress to shear strain**.
- May be denoted by G .



$$G = \frac{\tau}{\gamma} = \frac{Shear\ Stress}{Shear\ Strain} = \frac{\frac{F}{A}}{\frac{\Delta x}{h}}$$

$$= \frac{\tau}{\gamma} = \frac{E}{2(1+\nu)}$$

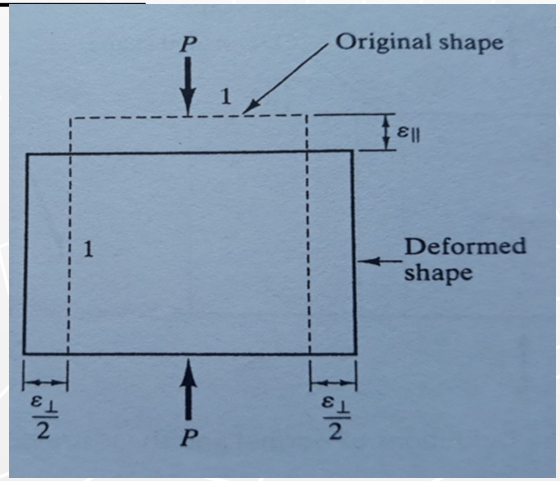
- **A large** value indicates **a soil is highly rigid**.
- A **small G value** indicates a **soil is soft or flexible**.
- One **definition of a fluid is a substance with a shear modulus of zero.**

Soil Type	Shear Modulus, (kPa)
Dense Sands & Gravels	69,000 - 345,000
Silty Sand	27,600 - 138,000
Medium Stiff Clay	6900 - 34,500
Soft Clays	2750 - 13,750

Poisson's Ratio :

- **Figure shows** the deformation of an unconfined element.
- Compressive strain parallel to stress is $\epsilon_{||}$
- Tensile strain perpendicular to stress $\epsilon_{\perp}$
- $\nu = \frac{\epsilon_{\perp}}{\epsilon_{||}} = \frac{Strain\ in\ the\ direction\ perpendicular\ to\ load}{Strain\ in\ the\ direction\ of\ load}$

$$= \frac{lateral\ strain}{Longitudinal\ strain}$$



$\nu = 0.0 - 0.5$

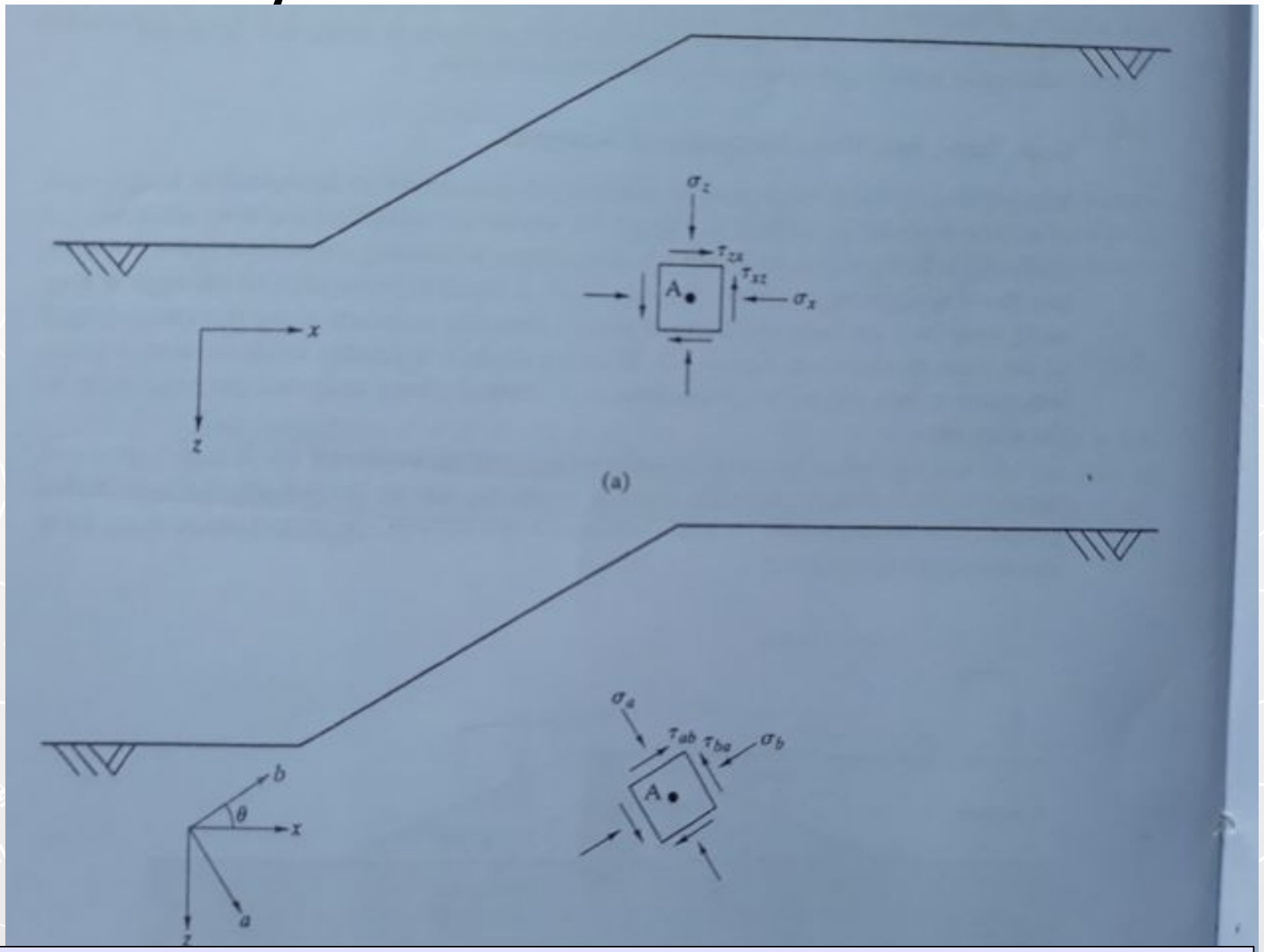
Concrete $\nu = 0.1 - 0.2$; Cork $\nu = 0.0$,

Rubber, $\nu = 0.49999$

For soils: 0.15 to 0.45 (soft clay to dense sand)

Material	*Poisson's ratio*
rubber	≈0.5
gold	0.42
saturated clay	0.40–0.50
magnesium	0.35
titanium	0.34
copper	0.33
aluminium-alloy	0.33
clay	0.30–0.45

Mohr's Circle Analysis



State of stress at a point under the ground represented in two different coordinate systems.

Mohr Circle

- At the point A, **under slope**, the **normal and shear stresses are acting on any two orthogonal planes**, define the stresses at this point.

- However, there are an **infinite number of pairs of orthogonal planes that can pass through point A**, each with their own set of normal and shear stresses.

- One of the **way to use graphical representation** **called Mohr Circle**, **to transform stresses from one pair of orthogonal planes** to any other pair of orthogonal planes.

- Mohr Circle was developed by **German Engineer Otto Mohr**.

- Mohr Circle **provides**, **how stresses are changed from one coordinate system to another.**

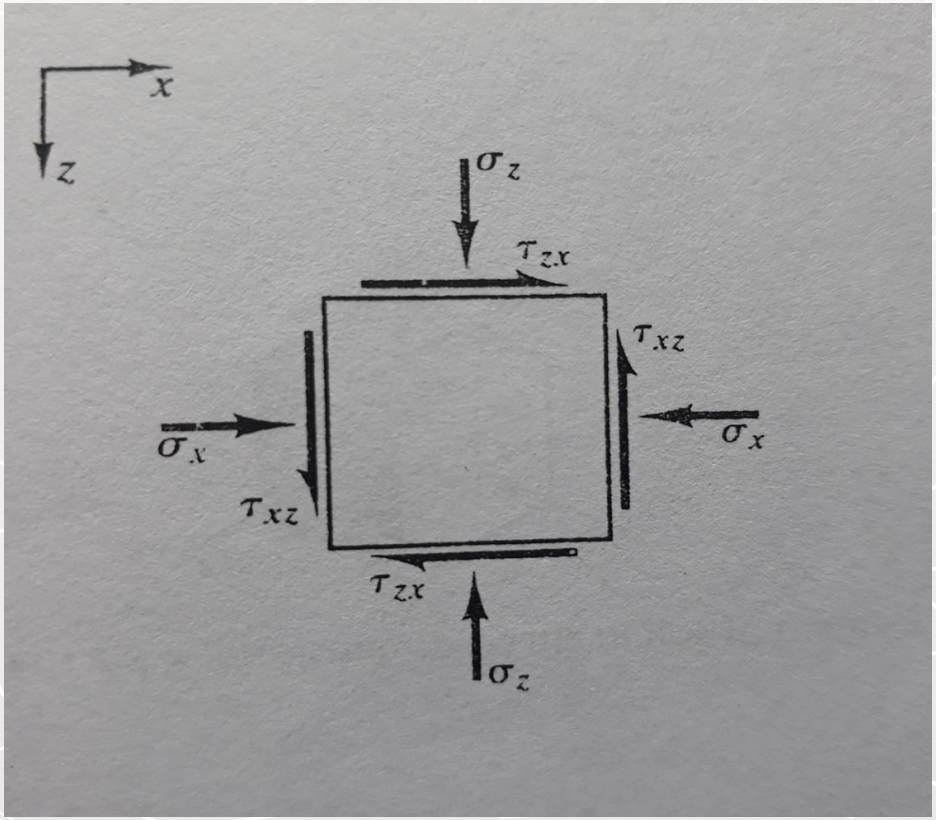
What is Mohr Circle? Write down the importance of Mohr Circle in the analysis of stresses in soils.

Mohr Circle?

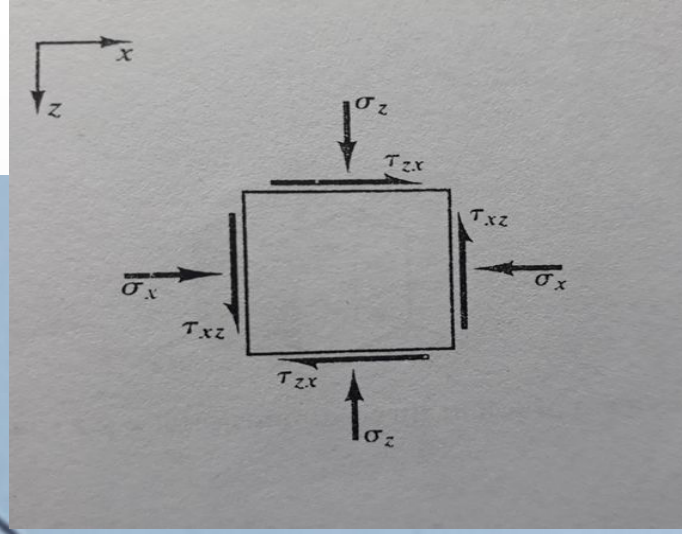
- A **graphical representation** of stresses developed by the German Engineer Otto Mohr.
- Mohr Circle **describes the two-dimensional state of stresses at a point on a circle** plotted on a normal vs shear stress diagram.
- Location of the **center and radius of the circle represent the state of stresses**, and
- **Points on the circle represent the shear and the normal stresses** for axes system rotated at different angle.

How to draw Mohr Circle-Pole Method?

- **Applied stresses in one coordinate system** ie the normal and shear stresses on two orthogonal planes in soil element shown in the figure.
- **Assume** the **value of stresses are known** and normal vertical stress is greater than normal horizontal stress ie $\sigma_z > \sigma_x$.



Mohr Circle of stress and the Pole Method



Points Z and X on the Circle represent the stresses for soil element shown above.

Creation of Mohr Circle

Following steps are followed to create Mohr Circle:

Step-1: Locate the center of circle on the $\sigma - \tau$ **plot**.

- Circle is **always centered on σ –axis** at the point $(\sigma_m, 0)$,

Step -2: Locate the points representing the known stresses and compete the circle:

- Stresses acting on the Z and X planes will be represented by two **points lying on the circle**.
- Here τ_{zx} **is** $-$ **ve** because of the element rotates clockwise and τ_{xz} **is +ve** for counterclockwise rotation.

- Assuming $\sigma_z > \sigma_x$, plot point Z, representing the stresses on the Z-plane (plane perpendicular to Z-axis) and point X representing stresses on the X-plane (plane parallel to X-axis).

- Knowing the **coordinates of points Z and X**, we can compute the radius of the circle.

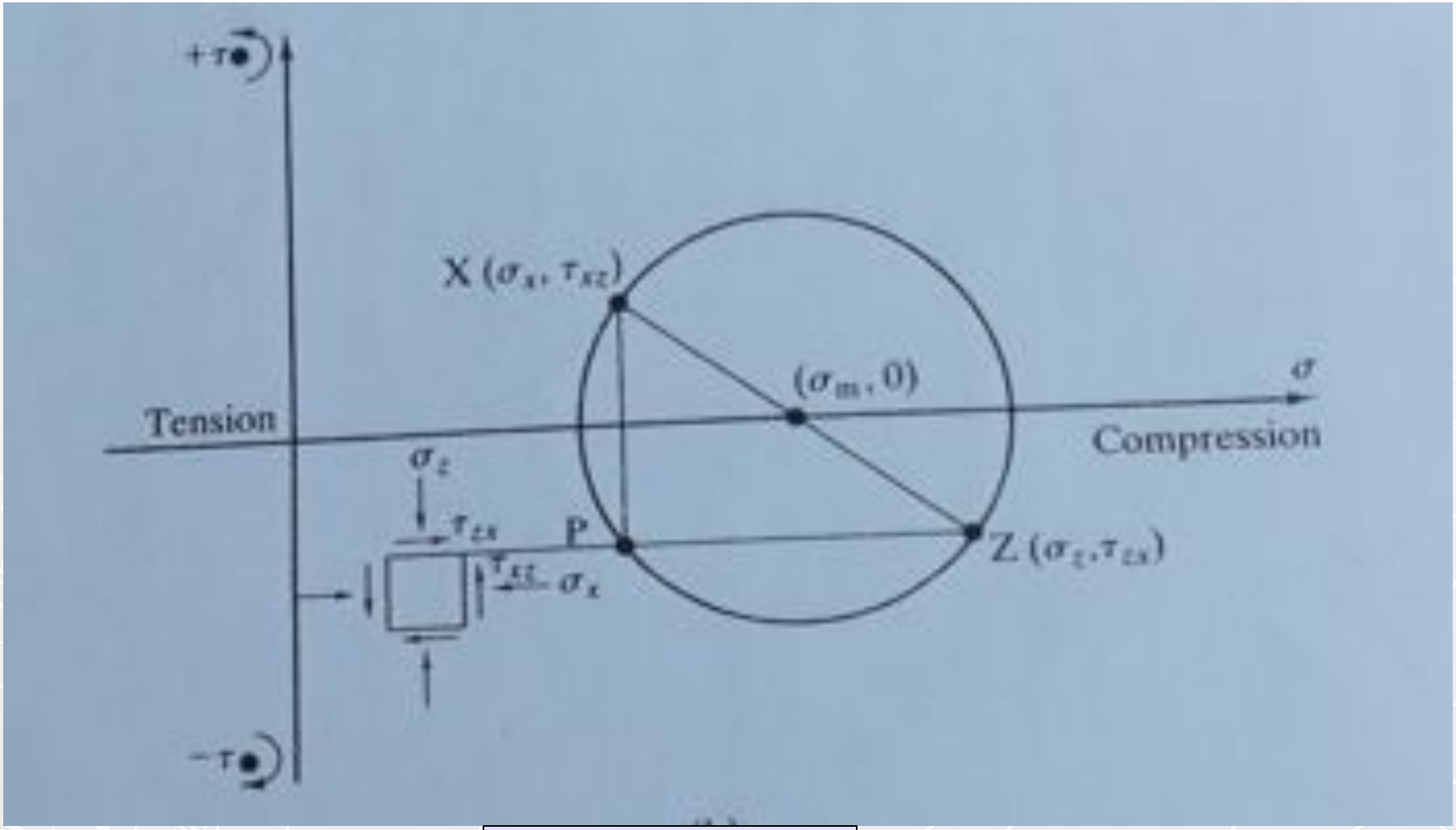
$$R = \frac{\sqrt{(\sigma_z - \sigma_x)2 + (2\tau_{zx})2}}{2} = \sqrt{(\sigma_z - \sigma_x)^2/2 + (\tau_{zx})^2}$$

- Now, **Mohr Circle is created**.

Step - 3: Locate the pole or **origin of plane** on the Mohr Circle.

- Pole is a **special point** on Mohr Circle.
- "A straight line drawn **through** the pole, parallel to a given plane will **intersect the Mohr Circle at the point** which represents the stresses acting on the plane".
- **We know point Z** (plane parallel to Z-axis and plane on which σ_z and τ_{zx} act), represents the stresses on the Z-plane, **locate the pole by drawing a horizontal line from point Z ,** to where it intersect the circle at point P**.**

Mohr Circle of stress and the Pole Method



Location of Pole 'P'

Location of Pole 'P'

- Similarly, if we **draw a vertical line from point X**, (plane perpendicular to X-axis and plane on which σ_x and τ_{xz} act), it will also **intersect the circle at point P**.

- **Point P is the pole**.

- **Location of Pole on given Mohr Circle is unique.**

- Pole method **only works** if the sign conventions are compression is +ve for normal stress and clockwise is –ve for shear stresses.

Advantages of Pole Method

- **Soil element can be drawn** on the figure with its proper orientation.

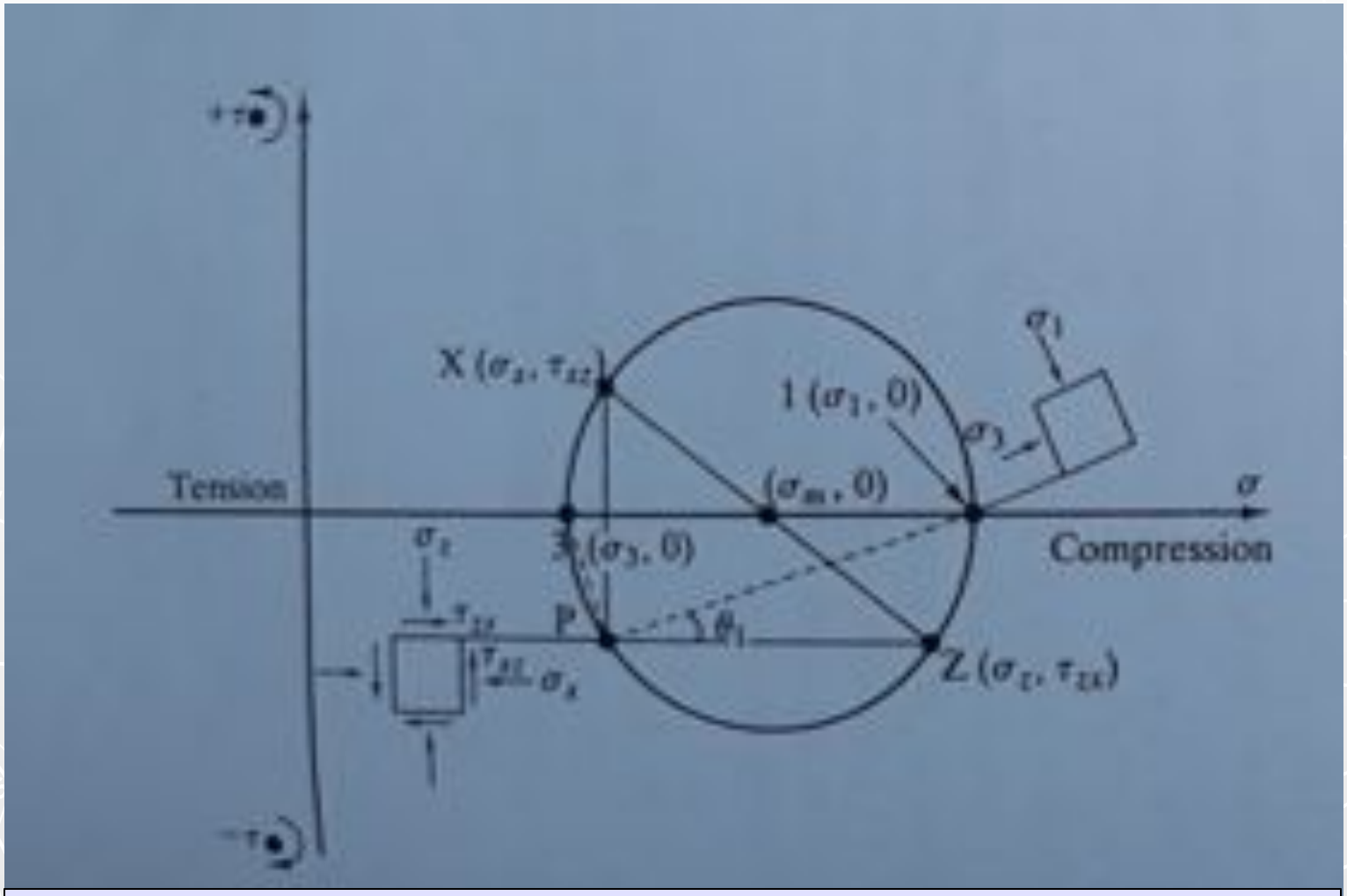
- Easy to ensure that **stresses are shown in proper direction**, using correct sign convention.

What is Pole Method of creating a Mohr Circle?

Describe the Pole method to draw a Mohr Circle with net diagram.

What do you mean by the Principal Stresses described in Mohr Circle? How you can determine the magnitude of principal stresses?

Principal Stresses



Determining the principal stresses by drawing lines from the pole to points 1 and 3.

Principal Planes and Principal Stresses:

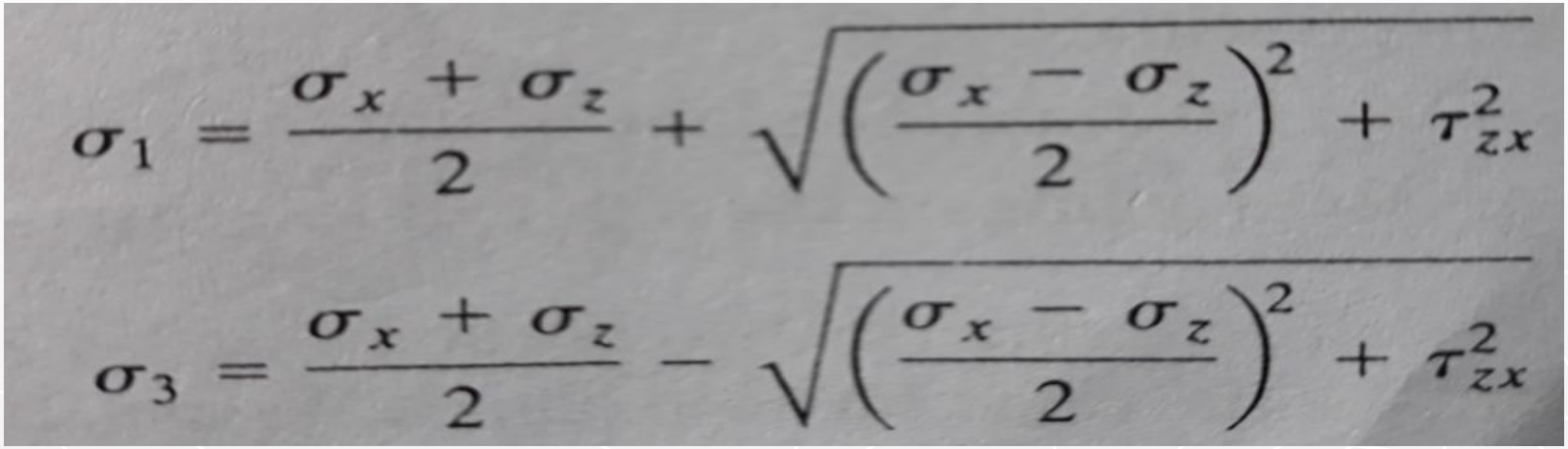
If the **soil element is rotated to certain angle**, the **shear stresses will be zero on all four sides.**

- Plane of this element are represented by **point 1** and **3**, known as **Principle Planes**.

Stresses acting on Principles Planes are known as Principle Stresses:

- **Major principal Stress, σ_1**: The greatest normal stress that acts on any plane.
- **Minor Principal Stress, σ_3**: The smallest normal stress that acts on any plane.
- These two stresses **σ_1 and σ_3 act at right angle** to each other.

- Principal Stresses can be determined by the following formula:



$$\sigma_1 = \frac{\sigma_x + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{zx}^2}$$

$$\sigma_3 = \frac{\sigma_x + \sigma_z}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{zx}^2}$$

Where, $\boldsymbol{\sigma_1}$= major principal stress

$\boldsymbol{\sigma_3}$= minor principal stress

$\boldsymbol{\sigma_x}$=horizontal normal stress

$\boldsymbol{\sigma_z}$ = vertical normal stress

$\boldsymbol{\tau_{zx}}$ =shear stress acting on horizontal plane.

The vertical and the horizontal stresses at a certain point in a soil are as follows:

σ_x=2100 lb/ft2

σ_z=3000 lb/ft2

$\boldsymbol{\tau_{zx}}$ = 300 lb/ft2

Determine the magnitudes and the direction of the major and minor principal stresses.

Solution:

- Construction of the Mohr Circle

Center is located at

$$\sigma_m = \frac{\sigma_1+\sigma_3}{2} = \frac{2100+3000}{2} = 2550 \text{ lb/ft2}$$

Radius of the circle is given by

$$r = \sqrt{\left(\frac{\sigma_z-\sigma_x}{2}\right)^2+(\tau_{zx})^2}$$

$$= \sqrt{\left(\frac{3000-2100}{2}\right)^2+(300)^2}$$

$$= 300 \text{ lb/ft2}$$

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Where, $\boldsymbol{\sigma_1}$= major principal stress

$\boldsymbol{\sigma_3}$= minor principal stress

$\boldsymbol{\sigma_x}$=horizontal normal stress

$\boldsymbol{\sigma_z}$ = vertical normal stress

$\boldsymbol{\tau_{zx}}$ =shear stress acting on horizontal plane.

- Compute the principal stresses:

  σ_1= 2550+541= 3091lb/ft2

  σ_3= 2550-541 = 2009 lb/ft2

- Direction of principal stresses are:

$$\theta_1 = \frac{1}{2}\cos^{-1}\sqrt{\frac{1}{1 + [2\tau_{zx}/(\sigma_z - \sigma_x)]^2}}$$

θ_1=16.8º

Thank you