CE341/EWCE341: Soil Mechanics I

Chapter 06: Shear Strength of Soil

Professor Md. Zoynul Abedin
Dept. of Civil Engineering, MIST
Dhaka, Bangladesh

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Shear Strength of Soil

6.1 Shear Failure Mechanism Within Soil Mass

If the load or stress in a foundation or earth slope is increased to cause an unacceptably large deformation, the soil in the foundation or slope is considered as failed against its strength.

As the soil is a granular material the excessive deformation generally occurs due to sliding, rolling or rearrangements of the particles with in a soil mass or simply due to shearing. This phenomenon is illustrated in Figs. 6.1 and 6.2.

Shear Strength of Soil

6.1 Shear Failure Mechanism Within Soil Mass (Contd.)

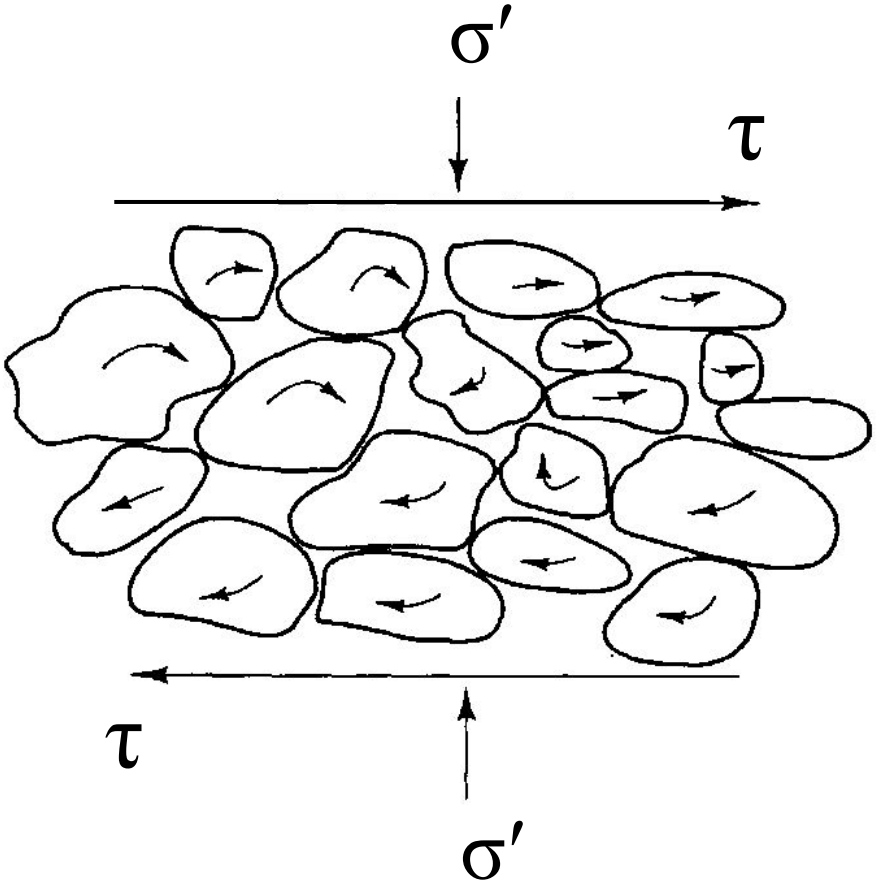


Fig. 6.1 Illustration of soil failing in shear due to sliding and rolling of particles.

Shear Strength of Soil

6.1 Shear Failure Mechanism within Soil Mass (Contd.)

In case of footing, the compressive stress at its base due to applied load from column causes the soil immediately below the footing a tendency to move in the outward and upward directions. Thus, the failure line will follow the inclined resultant path causing slippage or shear along this surface, Fig. 6.2(a). The failure mechanism is further illustrated in Fig. 6.2(b).

Thus, as soil is a granular material, whatever may be the external loading, the eventual failure within the soil mass would occur due to slippage on a particular surface. As such, the geotechnical

Shear Strength of Soil

6.1 Shear Failure Mechanism within Soil Mass (Contd.)

In case of footing, the compressive stress at its base due to applied load from column causes the soil immediately below the footing a tendency to move in the outward and upward directions.

Thus, the failure line will follow the inclined resultant path causing slippage or shear along this surface, Fig. 6.2(a). Failure mechanism is also illustrated in Fig. 6.2(b). The applied compressive load eventually causes the soil to fail in shear in a rupture surface.

Shear Strength of Soil

6.1 Shear Failure Mechanism within Soil Mass (Contd.)

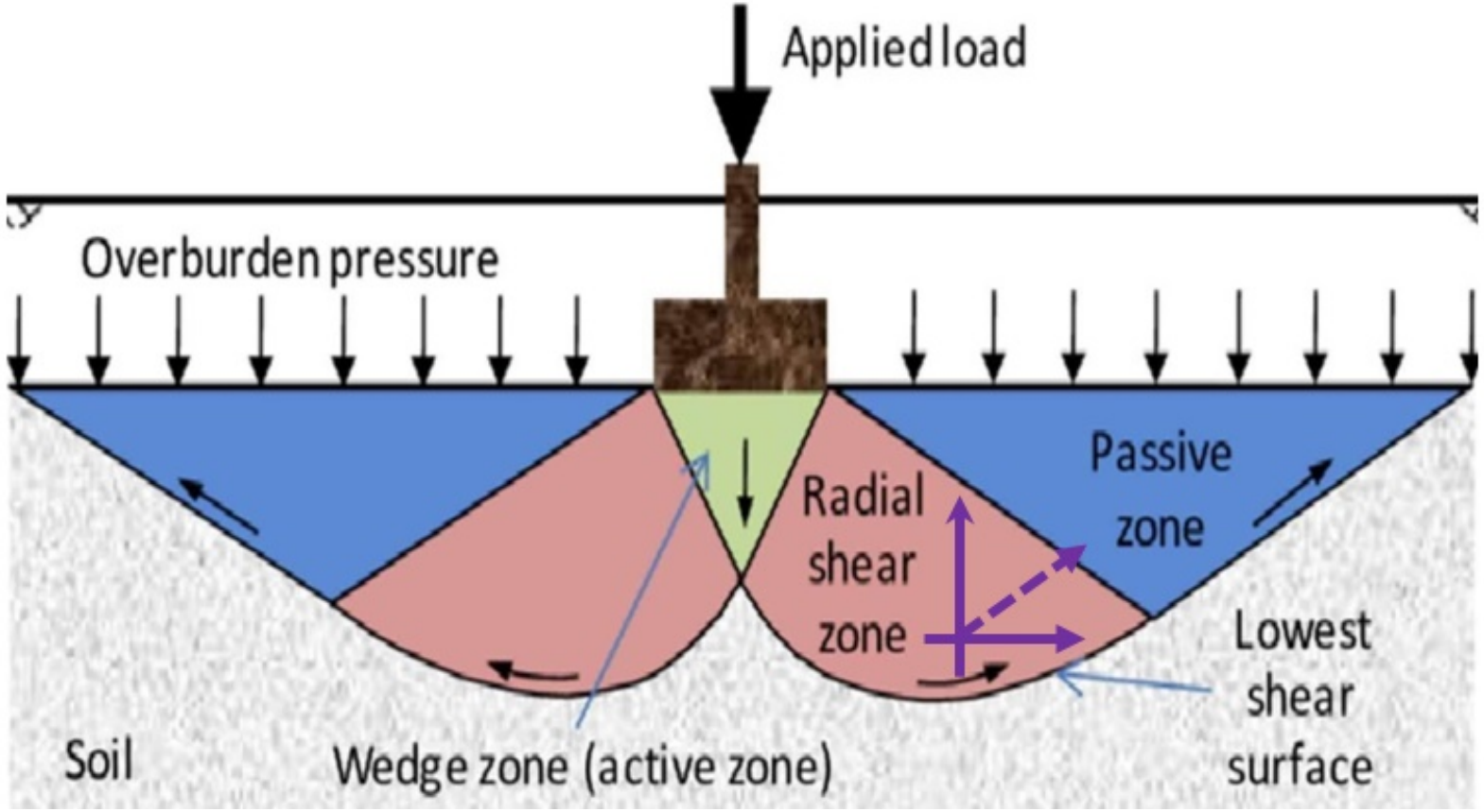


Fig. 6.2 (a)Failure mechanism in soil mass due to footing loading.

Shear Strength of Soil

6.1 Shear Failure Mechanism within Soil Mass (Contd.)

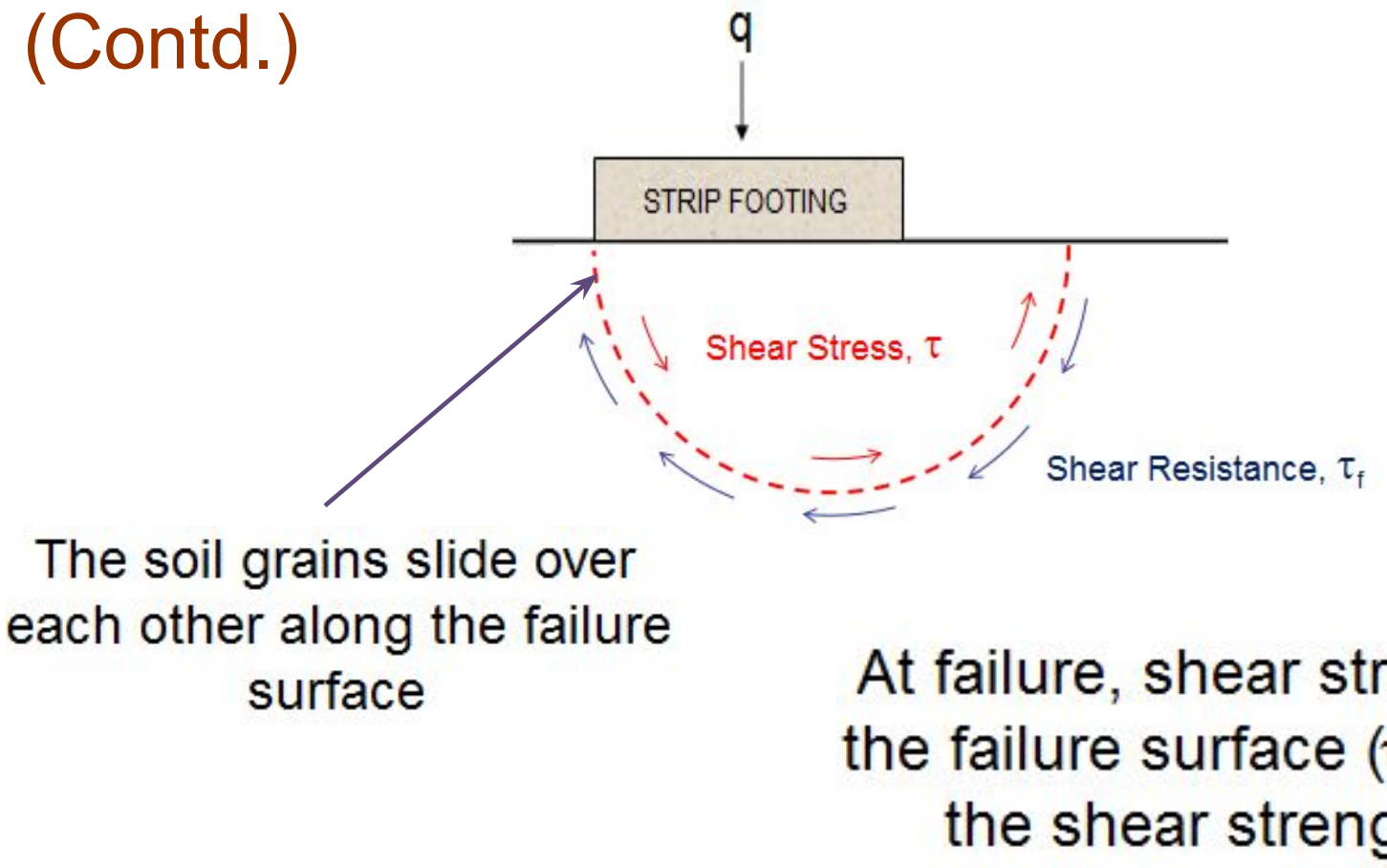


Fig. 6.2 (b) Failure mechanism illustration for strip footing.

Shear Strength of Soil

6.1 Shear Failure Mechanism Within Soil Mass (Contd.)

Some examples of shear failure are illustrated in Figs. 6.3. Application of shear strength are illustrated in Figs. 6.4 (a) to 6.4(e).

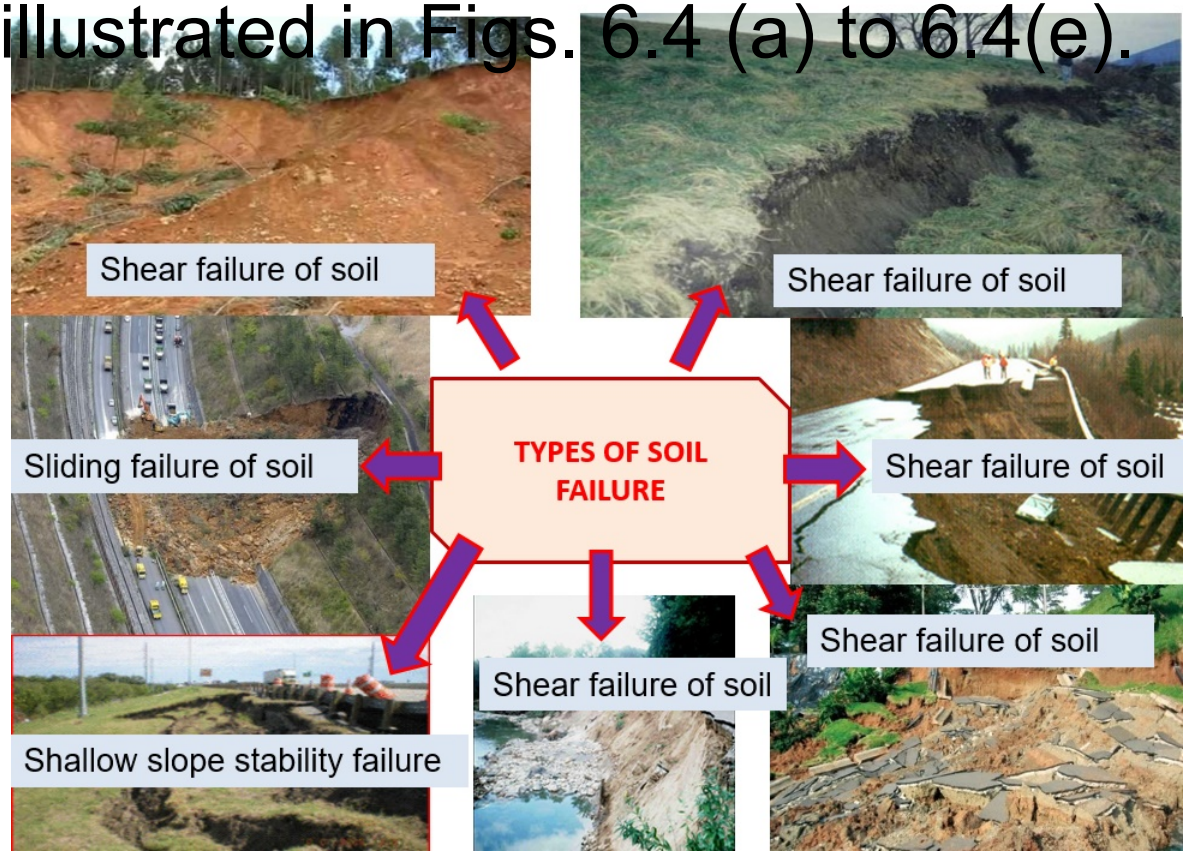
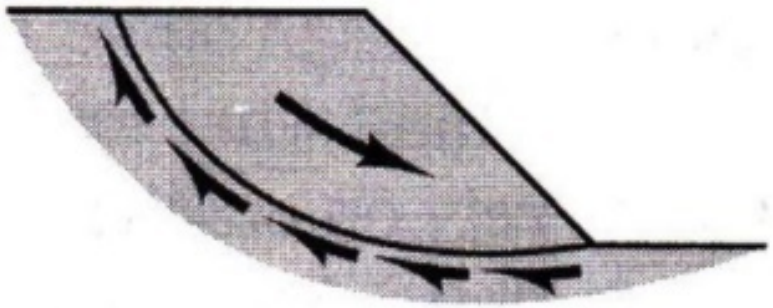


Fig. 6.3 Examples of some common shear failures.

Shear Strength of Soil

6.1 Shear Failure Mechanism Within Soil Mass (Contd.)



Earth slopes

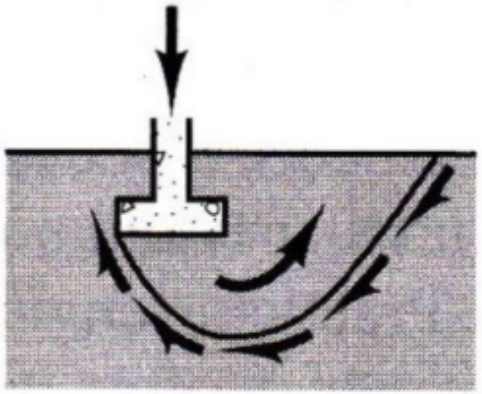
When the ground surface is inclined, gravity produces large geostatic stresses in the soil or rock.

If the stresses exceed the shear strength, a landslide occurs

Fig. 6.4 (a) Application of shear strength in stability of earth slope .

Shear Strength of Soil

6.1 Shear Failure Mechanism Within Soil Mass (Contd.)



Structural foundations

Load from a structure are transferred to the ground through structural foundations

This produces compressive and shear stress in the nearby soil.

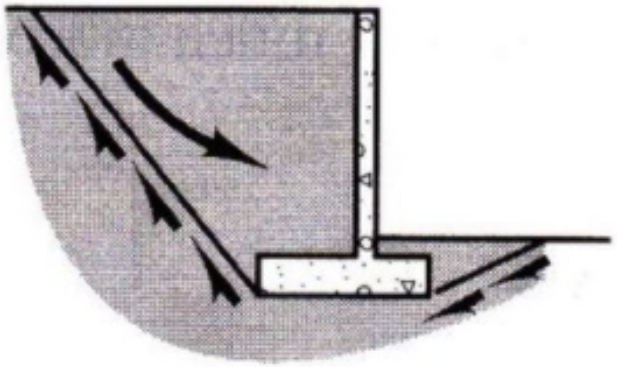
If the shear stress exceed the shear strength, thus producing a shear failure.

This is known as bearing capacity failure and can cause the structure to collapse

Fig. 6.4 (b) Application of shear strength in stability of structural foundation.

Shear Strength of Soil

6.1 Shear Failure Mechanism Within Soil Mass (Contd.)



Retaining walls

The weight of soil behind a retaining wall produces shear stresses

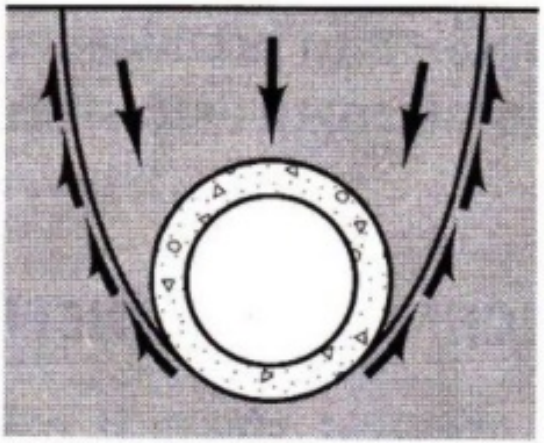
Its shear strength resists some of this stress and the wall resists the rest

Thus, the load carried by the wall depends on the shear strength of the retained soil.

Fig. 6.4 (c) Application of shear strength in stability of retaining wall.

Shear Strength of Soil

6.1 Shear Failure Mechanism Within Soil Mass (Contd.)



Tunnel linings

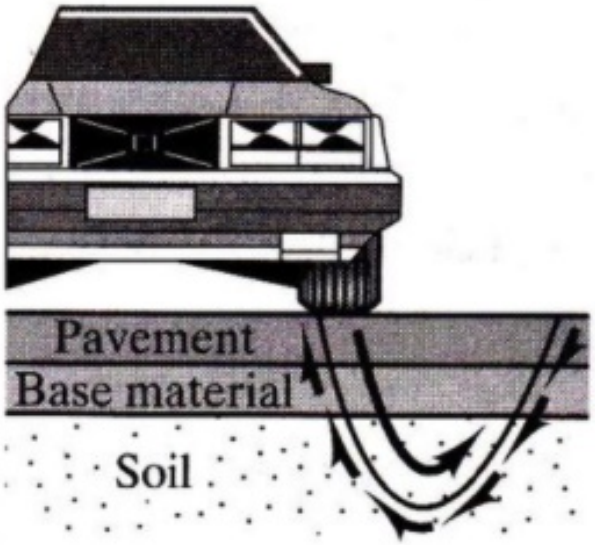
Tunnels in soil or weak rock normally require linings of steel or concrete for support

Such linings must resist pressures exerted by the surrounding ground, thus keeping the tunnel from collapsing.

Fig. 6.4 (d) Application of shear strength in stability of tunnel lining.

Shear Strength of Soil

6.1 Shear Failure Mechanism Within Soil Mass (Contd.)



Highway pavements

Wheel loads from vehicles spread through the pavement and into the ground below.

These loads produces shear stresses that could causes a shear failure.

Fig. 6.4 (d) Application of shear strength in stability of highway pavement.

Shear Strength of Soil

6.1 Shear Failure Mechanism Within Soil Mass (Contd.)

The shear strength of a soil can be described as its maximum resistance to shearing stresses.

When this strength is exceeded, failure occurs along planes in the soil mass called slip surfaces.

The sliding capability of a soil, while to support a loading from a structure or to support its own overburden or to sustain a slope in equilibrium, is called shear strength of soil.

Shear Strength of Soil

6.2 Importance of Shear Strength in Soil Mechanics

In many of the soil mechanics problems, the shear strength of the soil emerges as one of the most important characteristics, for design and analytical purposes. Examples are:

- Foundation design
- Earth and rockfill dam design
- Highway and airfield design
- Stability of slopes and cuts
- Lateral earth pressure

Shear Strength of Soil

6.3 Influencing Factors of Shear Strength of Soil

As soil mass is a complex material formed naturally in an uncontrolled environment, shear strength of a soil cannot be interpreted in a simple manner. Soil can exhibit significantly different shear strength at various field conditions. The important influencing factors are:

- Heterogeneous nature of soil
- The water table location and moisture contents
- The drainage facility involving pore water pressure
- The type and nature of construction
- Stress history
- Structural disturbance of soil
- Chemical action

Shear Strength of Soil

6.3 Influencing Factors of Shear Strength of Soil

It is understood that unlike any other solid mass where one or more straight forward strengths properties (like tension, compression, shear, flexural, torsional etc.) are considered design factors, soil being a particulate material shear strength is the sole strength consideration, Fig. 6.5. However, shear strength of soil is a complex phenomenon involving the interrelated influencing factors mentioned above and that needed to have clear in views of each of them. In particular, the presence water in the voids of soil mass makes shear strength a real complex phenomenon

Shear Strength of Soil

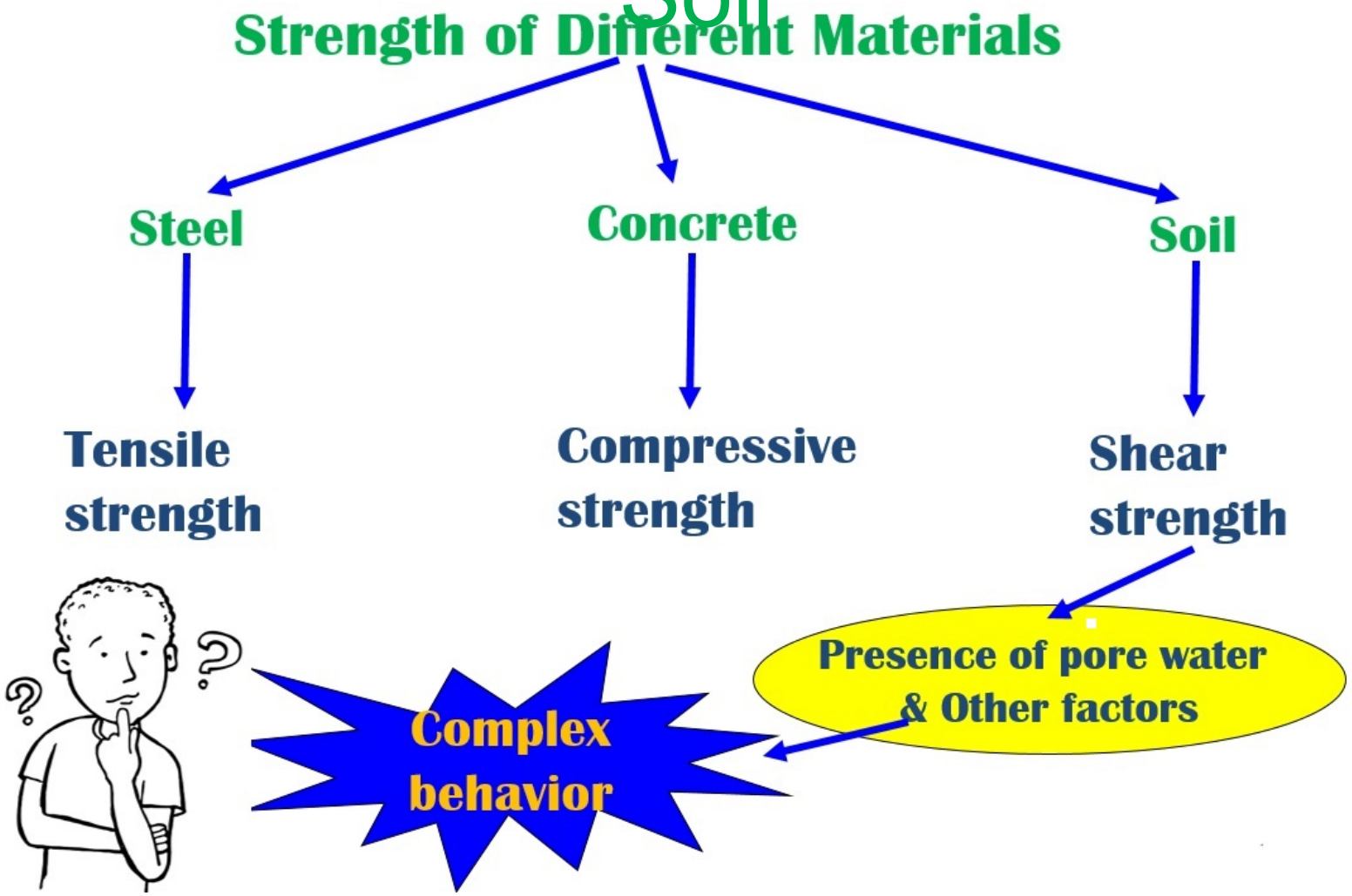


Fig. 6.5: Complexity of shear strength of soil.

Shear Strength of Soil

6.3 Influencing Factors of Shear Strength of Soil

While comparing shear strength of soil with that of other solid engineer materials, the following distinctions are to be made.

Solid Engineering Materials

- Shear strength is controlled by their molecular structure.
- Failure requires the breaking of molecular bonds that hold the material together. Thus, steel has a very strong molecular bond having high shear strength. Plastic on the other hand has a weaker molecular bond resulting in low shear strength.

Soil Mass

- Shear strength primarily depends on interaction between the particles, not their internal strength.

Shear Strength of Soil

6.4 Attributes of Shear Strength of Soil

The shear strength of soil may be attributed to three basic components, Fig. 6.6:

- Resistance due to interlocking of the particles.
- Frictional resistance between the individual soil grains, which may be either of the sliding or rolling frictions or the both.
- Adhesion between soil particles called cohesion.

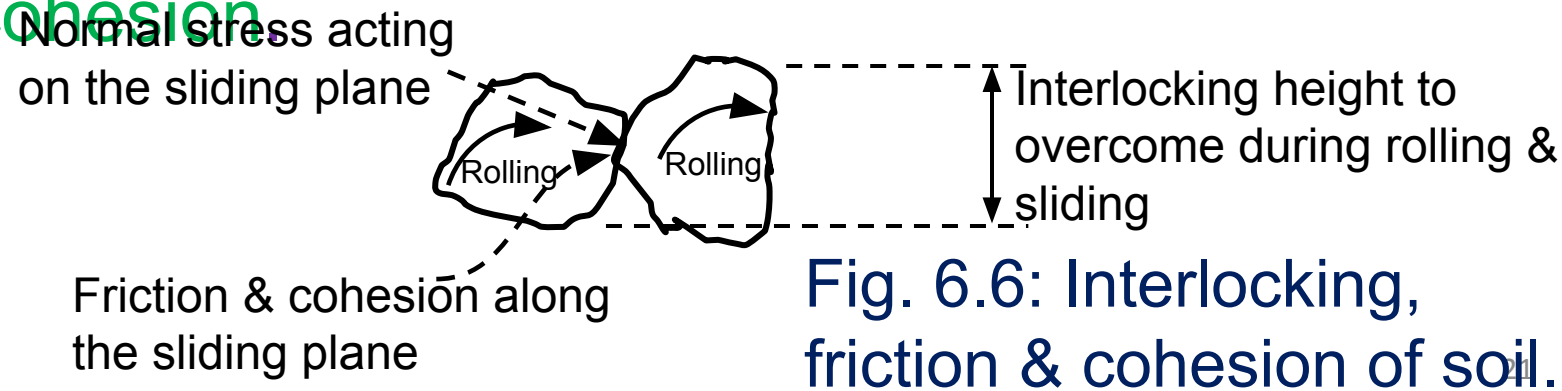


Fig. 6.6: Interlocking, friction & cohesion of soil.

Shear Strength of Soil

6.4 Attributes of Shear Strength of Soil

- First two sources are mainly responsible for the shear strength of granular soils.
- Second and third ones are for the cohesive soils.
- Whereas, third one is only responsible for shear strength of highly plastic clays.

It is neither easy nor practical to distinguish clearly the effects of these components on the shear strength of the soil, as these are in turn influenced by many agents.

Shear Strength of Soil

6.5 Basic Aspects in Understanding Shear Strength of Soil

- Friction between solid bodies.
- Cohesion between the soil particles.
- Stresses on the planes passing through a point represented by Mohr Circle diagram.

Shear Strength of Soil

6.5.1 Friction between Solid Bodies

This is similar to classic sliding friction problem from basic physics or mechanics.

- A block of weight, W is placed on a horizontal plane having frictional interface, Fig. 6.7.
- The forces acting on the block, Fig. 6.7a, are gravity and the reaction of the plane.
- Although a friction resistance is available, it does not come into play since there is no horizontal force applied to the block.

Shear Strength of Soil

6.5.1 Friction between Solid Bodies

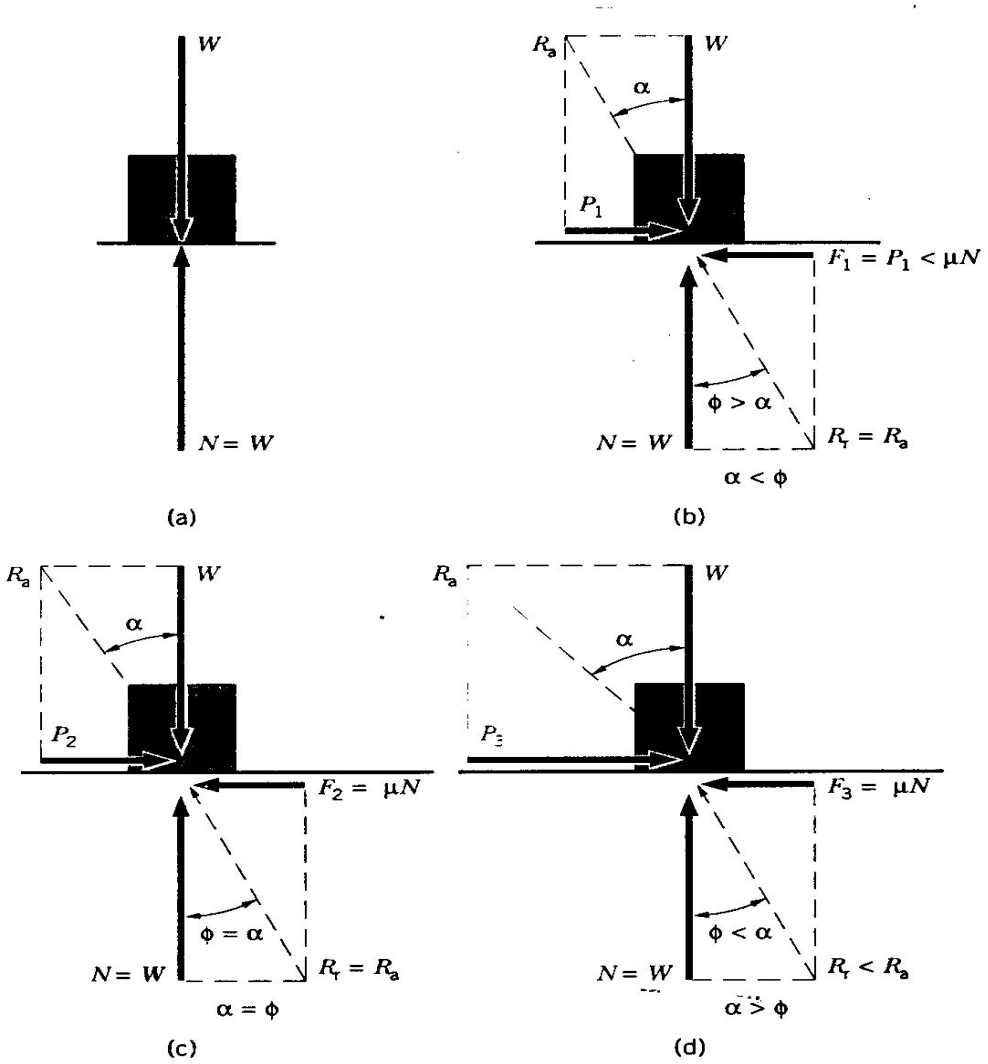


Fig. 6.7 Illustration of frictional forces. (a) Development of no friction; (b) Development of friction; (c) Friction impending motion; (d) Friction with motion.

Shear Strength of Soil

6.5.1 Friction between Solid Bodies

- Now, a horizontal force P_1 is applied to the block, Fig. 6.7b. If P_1 is small, the block will not move; the resultant R_a makes an angle α commonly known as angle of obliquity of force P_1.
- For equilibrium, a resisting horizontal force F_1 must exist to balance P_1. The angle α is less than the maximum angle available, for the maximum horizontal force, known as angle of friction, φ. In Figure, N is the component of the weight normal to the plane.

Shear Strength of Soil

6.5.1 Friction between Solid Bodies

- Now the horizontal force is increased to P_2, as shown in Fig 6.7c, the friction force also increases to a value of F_2.
- It reaches to a maximum value when the angle of obliquity, α is equal to the angle of friction, φ. At this point, a state of impending motion exists.
- For the range of value $0 < \alpha < \varphi$, no sliding occurs. At this point, the maximum available frictional force F can be related to the normal resisting force by a coefficient μ as given by Eq. (6.1).

$$F = \mu N \qquad (6.1)$$

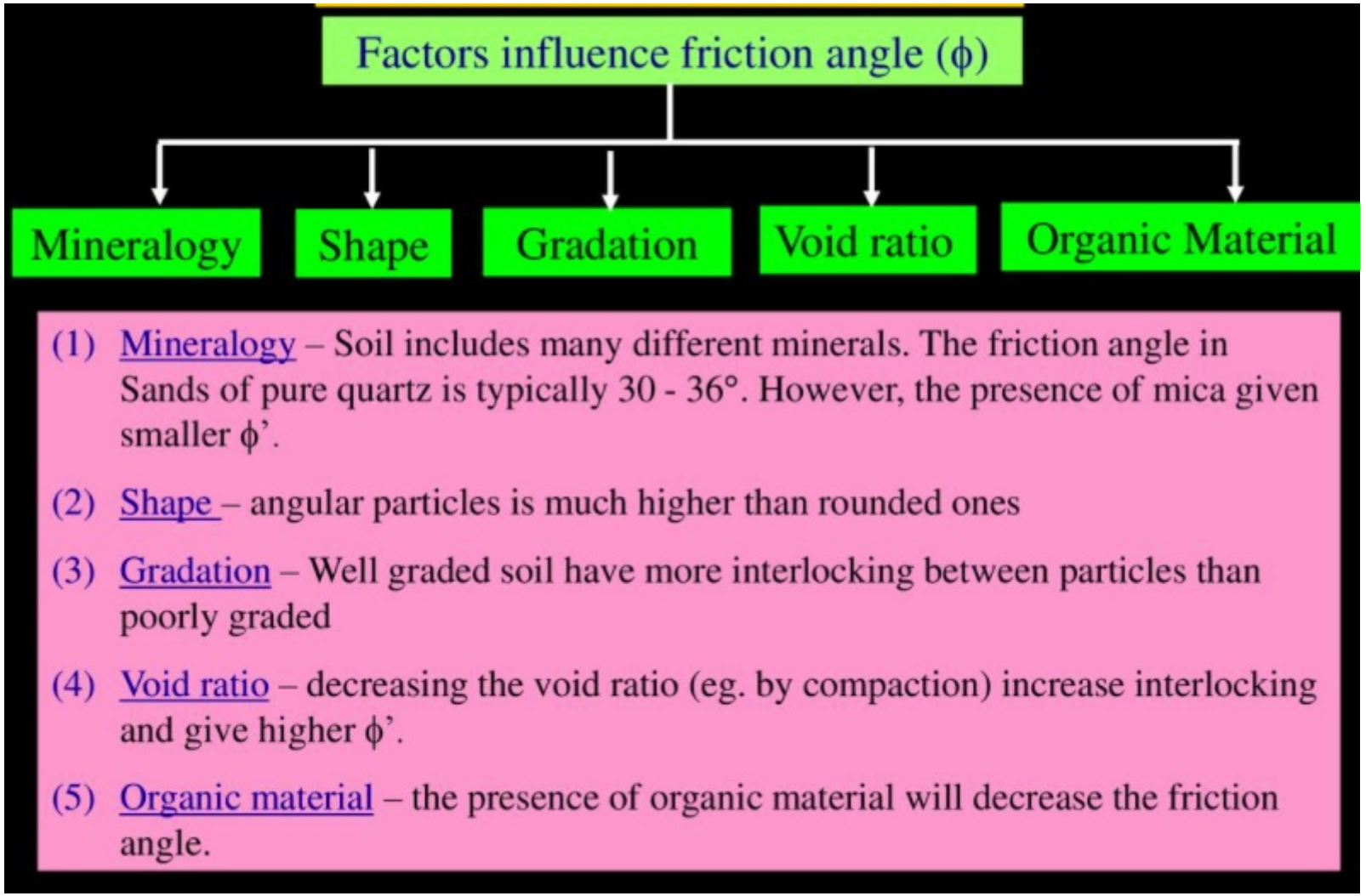
Shear Strength of Soil

6.5.1 Friction between Solid Bodies

- The coefficient of friction μ is independent of contact area. It is, however, strongly dependent on the nature of the surface in contact, the type of material, the condition (wet or dry) of the surface and so on.
- If the horizontal force is increased to a value of P_3 as shown in Fig. 6.7d, such that the angle α is greater than φ, the block will start sliding.
- The frictional force can not exceed the value given by $F = \mu N$, and therefore the block will accelerate in the direction of applied horizontal force. Fig. 6.7c shows that $\mu = F/N = \tan\varphi$, which also means that $\sigma/\tau = \tan\varphi$.

Shear Strength of Soil

6.5.1 Friction between Solid Bodies



(1) Mineralogy – Soil includes many different minerals. The friction angle in Sands of pure quartz is typically 30 - 36°. However, the presence of mica given smaller ϕ'.

(2) Shape – angular particles is much higher than rounded ones

(3) Gradation – Well graded soil have more interlocking between particles than poorly graded

(4) Void ratio – decreasing the void ratio (eg. by compaction) increase interlocking and give higher ϕ'.

(5) Organic material – the presence of organic material will decrease the friction angle.

Shear Strength of Soil

6.5.2 Concept of Cohesion

The soil mass consists of granular materials. There exists two forces namely, body force and Van der Wall force, between two bodies which usually govern the properties of granular mass.

The body force is usually referred to as gravitational force where the mass of the bodies is predominant and larger particles are influenced by this force.

Whereas, Van der Wall force is due to electrical surface charges and a bonding between the particles occurs in presence of water which helps to generate various chemical bonding among the

Shear Strength of Soil

6.5.2 Concept of Cohesion

This adhesion due to bonding is known as cohesion for soil.

In case of smaller soil particles the specific surface area (area per unit mass) is significantly higher as compared to the coarse grained soil.

As the electrical charges and hence the Van der Wall force are directly proportional to surface charge, the property of fine grained soil is governed by cohesion of the soil.

The body force here is insignificant because of their smaller mass weight.

Shear Strength of Soil

6.5.2 Concept of Cohesion

Similar thing happens to coarse grained soil; the Van der Wall force here has a very small influence.

It is possible to make a vertical cut in silts and clays and for this cut to remain standing, unsupported, for some time.

This cannot be done with a dry sand which, on removal of the cutting implement, will slump until its slope is equal to an angle known as the angle of repose.

In silts and clays, therefore, some other factor must contribute to shear strength.

This factor is called cohesion and results from the

Shear Strength of Soil

6.5.2 Concept of Cohesion

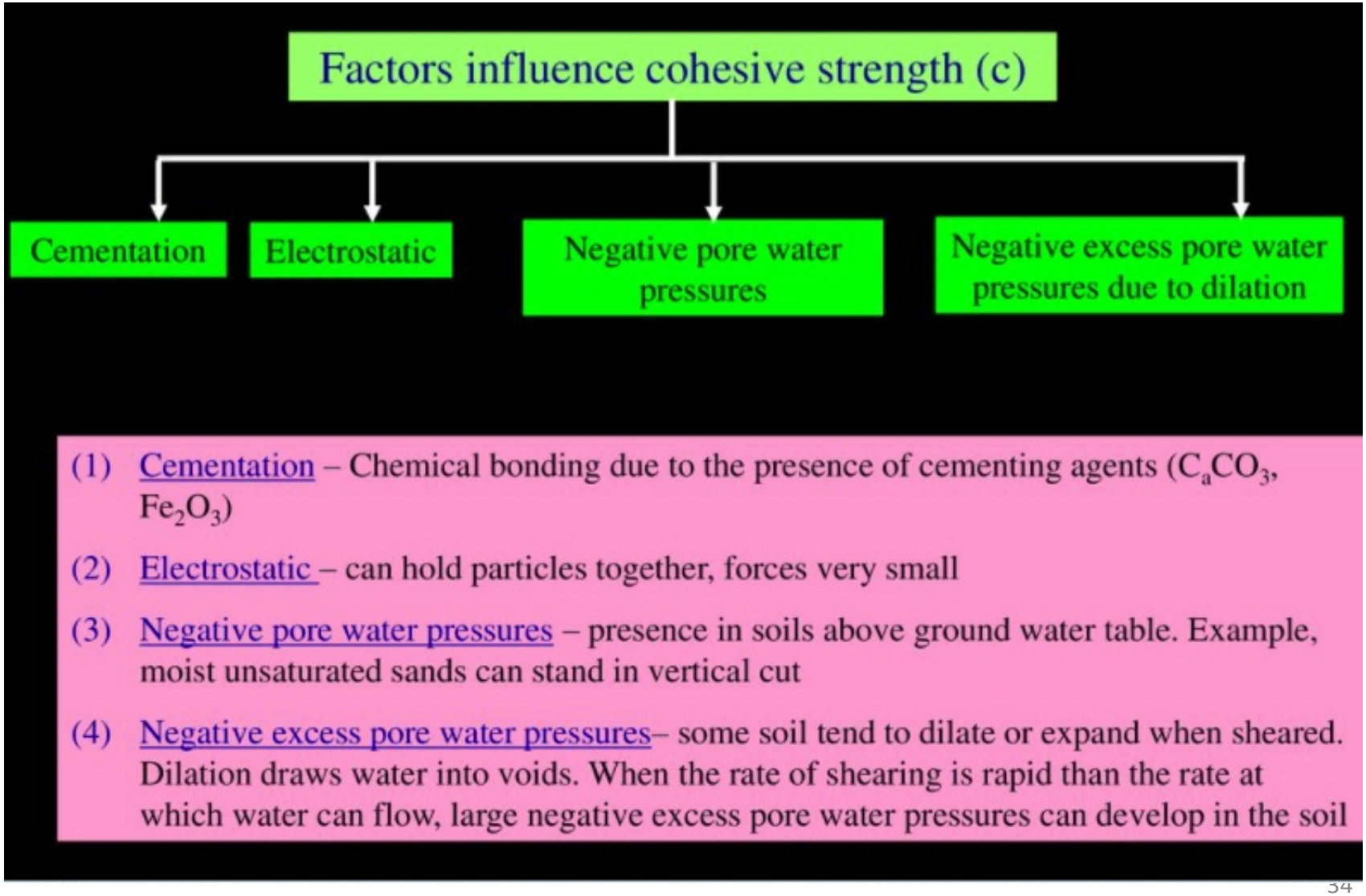
Water in a cohesive soil cannot drain quickly and so the soil is claimed to be in an undrained state, and the undrained cohesion is providing strength to the soil.

This cohesion parameter is also dependent on stress history that is mode of formation of soil deposit.

In soils like over-consolidated clays, partially saturated soils and cemented soils, the individual particles are bonded together giving rise to cohesion. As such, this parameter can be viewed

Shear Strength of Soil

6.5.2 Concept of Cohesion



(1) Cementation – Chemical bonding due to the presence of cementing agents (C_aCO_3, Fe_2O_3)

(2) Electrostatic – can hold particles together, forces very small

(3) Negative pore water pressures – presence in soils above ground water table. Example, moist unsaturated sands can stand in vertical cut

(4) Negative excess pore water pressures– some soil tend to dilate or expand when sheared. Dilation draws water into voids. When the rate of shearing is rapid than the rate at which water can flow, large negative excess pore water pressures can develop in the soil

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

According to principles of mechanics, due to external loading both normal and shear stresses act on any planes passing through a point. Once the stresses on two orthogonal planes passing through a point are known, they can be used to determine the stresses acting on any planes passing through that point at any other orientation.

These stresses may be determined both by (i) analytically using the principles of mechanics or (ii) by graphically using a diagram given by Mohr and commonly known as Mohr's circle of

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

The graphical device is based on identification of a unique point on a circle called the Pole of Mohr's circle or origin of planes.

This point has such a useful property that "any straight line drawn through this point (pole) will intersect the circle at a point which represents the state of stress on a plane inclined at the same orientation in the space of the line."

Thus, once pole is identified, a line parallel to the plane of interest can be drawn through pole that will intersect the Mohr circle at a point that gives the

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

- The Mohr's circle is drawn in a $\sigma - \tau$ $(normal\ stress - shear\ stress)$ plane using the normal stress (σ) and shear stress (τ) values acting on two orthogonal planes as two coordinates on the $\sigma - \tau$ plane. The line connecting these two points will act as the diameter of the Mohr's circle. The following sign conventions are used in drawing Mohr's circle diagram.

  - ✓ The compressive normal stress is considered as positive and tensile stress as negative.
  - ✓ The counter-clockwise shear stress is positive, while clockwise shear is negative.

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

Fig. 6.8 illustrates the construction of Mohr's circle diagram and the procedure will follow. The letter "P" is typically used to identify Pole on the Mohr

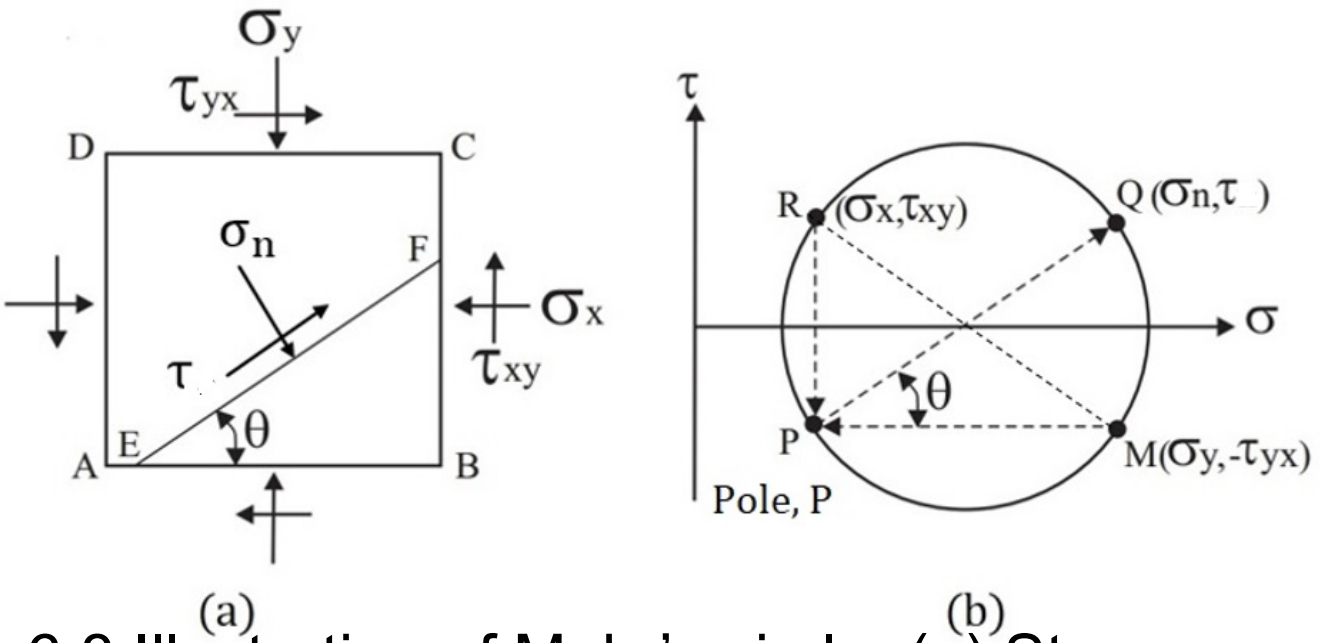


Fig. 6.8 Illustration of Mohr's circle: (a) Stresses on orthogonal planes; (b) Mohr's circle and its pole

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

- **Procedures of Construction of Mohr's Circle**

1. Points R (σ_x, τ_{xy}) and M $(\sigma_y, -\tau_{yx})$ are located on the "σ vs. τ" space (Fig. 6.8a).

2. Line RM is drawn which is the diameter of the Mohr's and accordingly Mohr circle is constructed.

3. To locate pole, a line MP through Point M $(\sigma_y, -\tau_{yx})$ which is parallel to the plane on which the stresses of Point M act. It will be a horizontal line from M to P (Fig. 6.8b) because σ_y and $-\tau_{yx}$ act on the horizontal plane DC.

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

- **Procedures of Construction of Mohr's Circle**

4. Similarly, a line RP through Point R (σ_x, τ_{xy}) is drawn parallel to the plane on which the stresses of point R act. It will be a vertical line from R to P (Fig. 6.8b) because σ_x and τ_{xy} act on the vertical plane BC.
5. As per the basic principles of geometry, the intersecting point P of lines MP and RP will lie on the circumference of the circle as $\angle RPM$ is a right angle and RM being the diameter of the circle.
6. Intersection point P is the Pole of the Mohr's circle, Fig. 6.8b.
7. A line PQ is drawn from P, that is parallel to the plane EF. The coordinates of Q, are the stresses (σ_n, τ) acting on plane EF.

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

- **Mohr's Circle Diagram used in Soil Mechanics**

Within the soil mass beneath the foundation generally there are basically two types of stresses like vertical (σ_v) and induced lateral (σ_h) are acting due to external loading and overburden pressure, Fig. 6.9a. They act on horizontal and vertical planes respectively, and termed as principal stresses.

The Mohr's circle for these conditions are drawn in Fig. 6.9b. The stresses on any plane at an orientation$(angle\ \theta)$ can be obtained from the Mohr's circle diagram as indicated by the coordinates of point Q (σ_n, τ), Fig. 6.9b.

It can be observed that the Mohr' circle thus constructed is symmetrical about σ —axis like a mirror image, Fig. 6.9b.

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

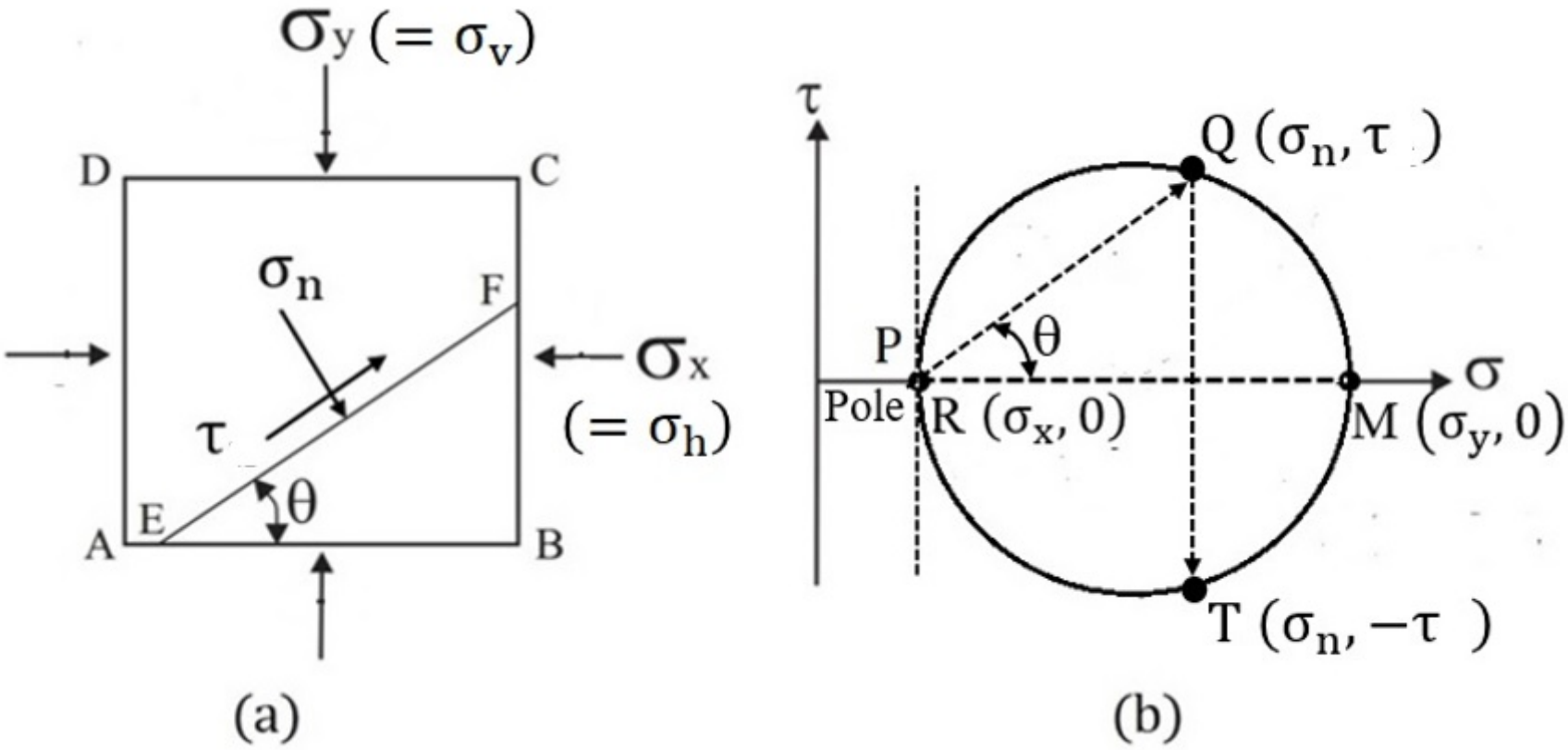


Fig. 6.9 Mohr' Circle diagram for principal stresses only.

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

Mohr's Circle Diagram used in Soil Mechanics

Corresponding to each points on the circumference of the circle above $\sigma -$ axis, there is a point opposite below the $\sigma -$ axis indicating exactly the same values of normal and shear stresses, but with the opposite sign of shear. This is illustrated by points $Q(\sigma_x, \tau)$ and $T(\sigma_n, -\tau)$ in Fig. 6.9b.

In soil failure, only the magnitude of shear is important and sign is not that important. The reason being, the failure will initiate on the weakest plane and it can be in any direction. That's why in soil mechanics the consideration of positive shear is enough that leads to construct half of the Mohr's circle as illustrated in Fig. 6.10.

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

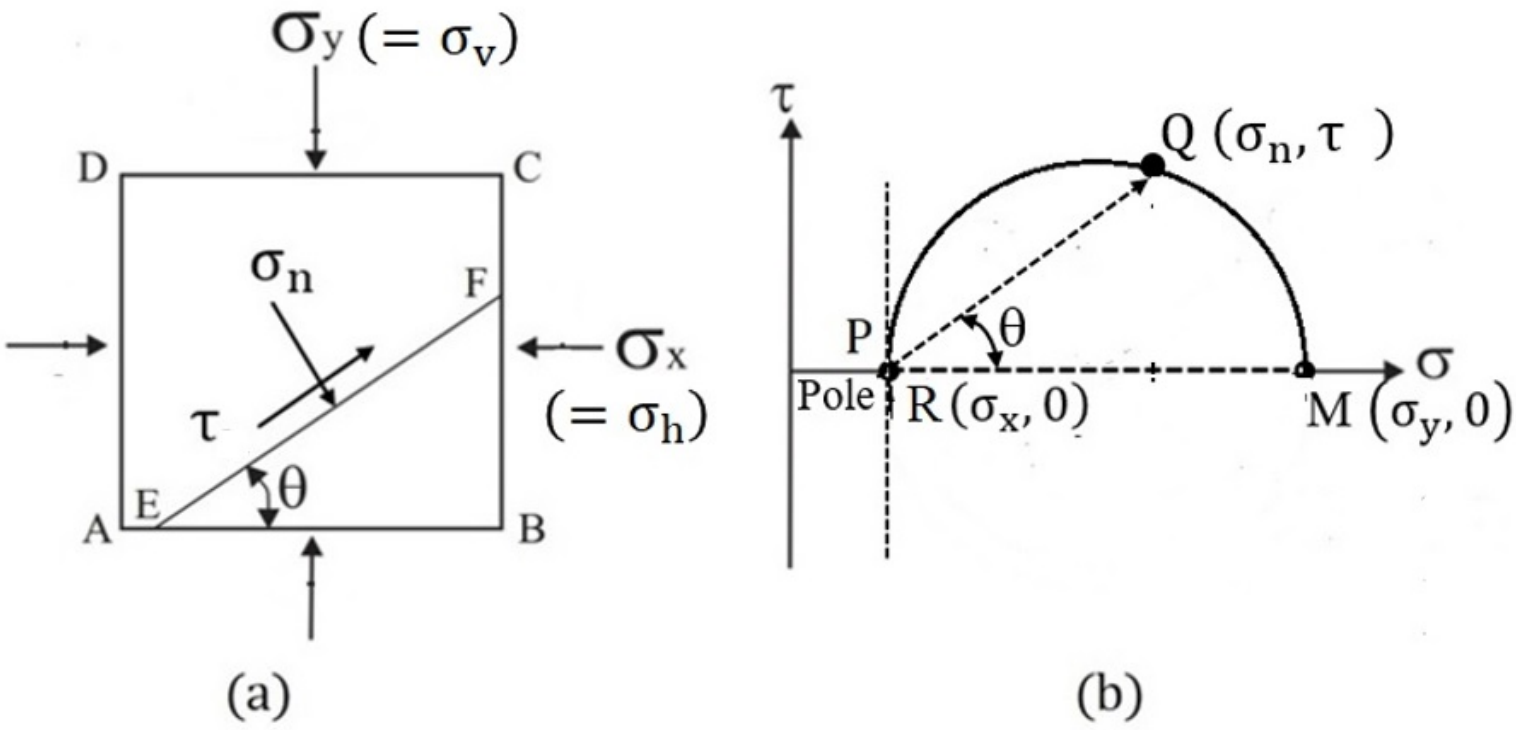


Fig. 6.10 Mohr' Circle diagram as used in Soil Mechanics.

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

- **Analytical Method**

To obtain the Mohr–Coulomb failure line (and thus the values of the shear strength parameters c' and φ'), it is normal practice to make use of a graphical construction known as the Mohr circle of stress. This can be explained by analytical method as well. Mohr's circles are widely used in stress analysis, and the explanation given here is limited to their use in soil mechanics. Let us consider the state of stress on an element within a body where the major principal stress and minor principal stress act in the vertical and horizontal directions, respectively.

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

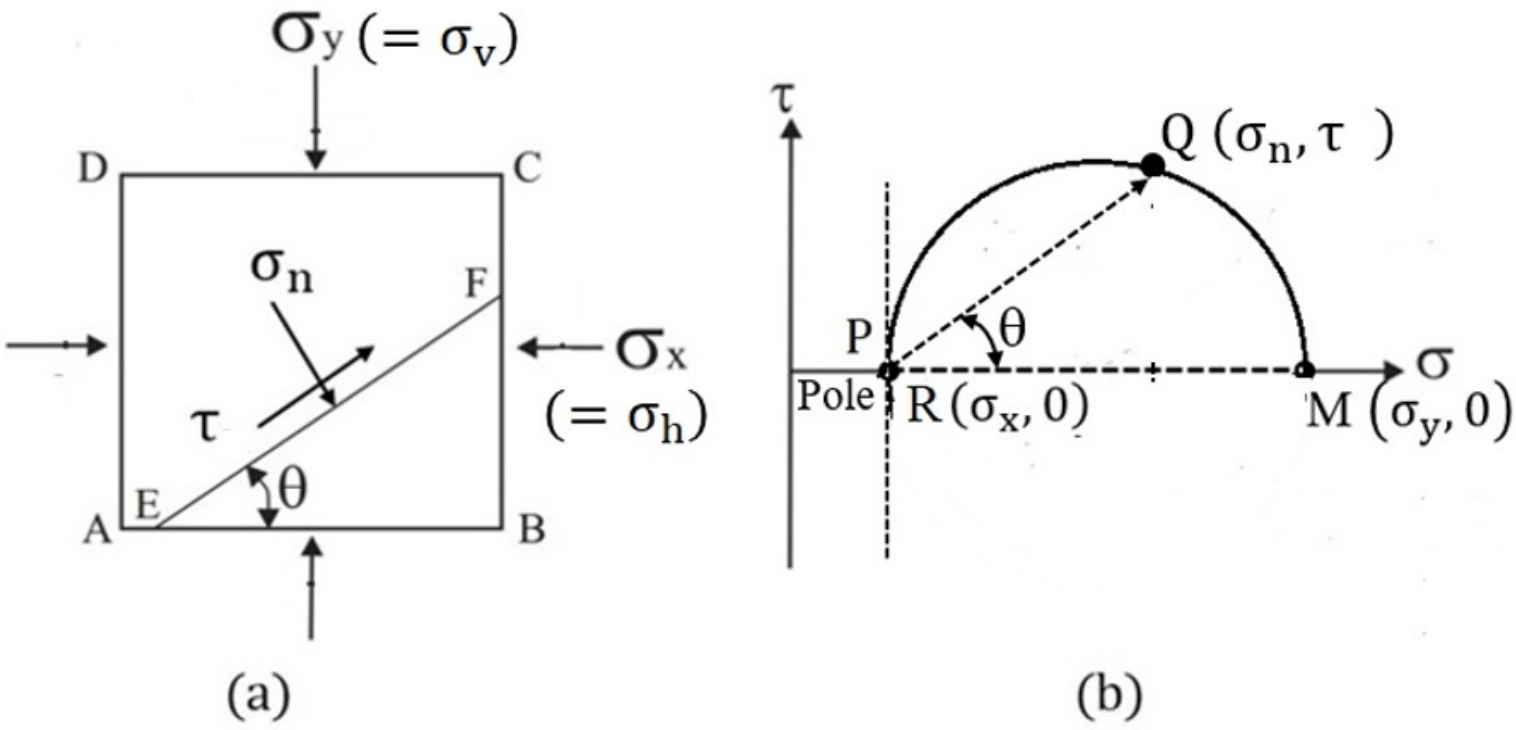


Fig. 6.11 Mohr' Circle diagram as used in Soil Mechanics.

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

- **Analytical Method**

This is illustrated in Fig. 6.11 and corresponds to the stress state in a triaxial test. It is intended to know the stress state on any other plane at an angle α to the horizontal since failure cannot occur on either the vertical or horizontal plane. There will be a normal stress σ_n and a shear stress τ on this plane, Fig. 6.11.

Considering the equilibrium of the element and Resolving parallel to the inclined plane yields

$$\tau a + \sigma_3 a \sin\theta \cos\theta = \sigma_1 a \cos\theta \sin\theta \qquad (6.1)$$

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

Analytical Method

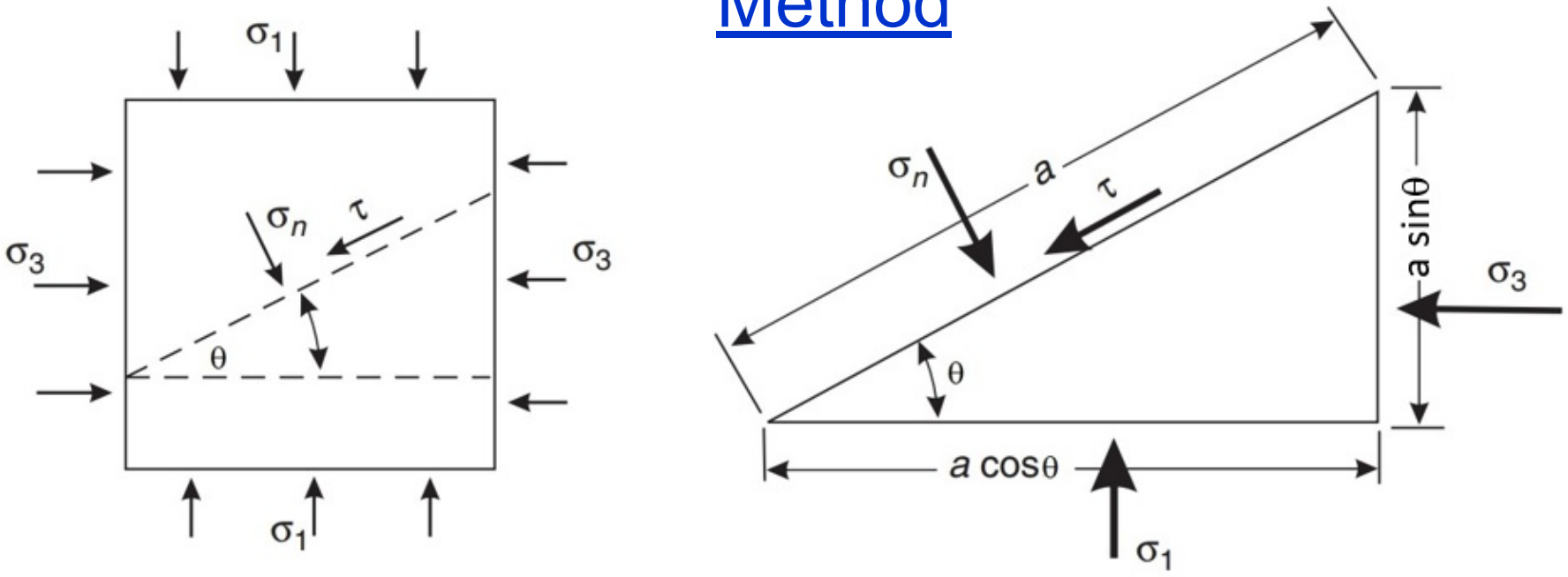


Fig. 6.12 Stresses on a element used to derive Mohr circle equations.

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

- **Analytical Method**

Thus,

$$\tau = (\sigma_1 - \sigma_3) \sin\theta \cos\theta$$

As $\sin 2\theta = 2 \sin\theta \sin\theta$

$$\tau = \frac{(\sigma_1 - \sigma_3)}{2} \sin 2\theta \qquad (6.2)$$

Resolving normal to the plane gives,

$$\sigma_n a = \sigma_1 a \cos\theta \cos\theta + \sigma_3 a \sin\theta \sin\theta$$

That is,

$$\sigma_n = \sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta$$

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

- **Analytical Method**

Now, $\cos 2\theta = 2\cos^2\theta - 1$ **and** $\cos 2\theta = 1 - 2\sin^2\theta$

Rearranging gives,

$$\sigma_n = \frac{(\sigma_1+\sigma_3)}{2} + \frac{(\sigma_1-\sigma_3)}{2}\cos 2\theta \qquad (6.3)$$

Now the Mohr's circle shown in Fig. 6.13 is taken into consideration. A line at angle θ has been drawn from point A to intersect the circle at point P. The magnitude of OD and DP is to be examined. The horizontal axis represents normal stress and the vertical axis represents shear stress.

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

- **Analytical Method**

By plotting the values of principal stress on the x axis, it is possible to determine the values of normal and shear stress on any other plane.

$$PD = CP \sin 2\theta = \frac{(OB - OA)}{2} \sin 2\theta$$

That is,

$$\tau = \frac{(\sigma_1 - \sigma_3)}{2} \sin 2\theta \qquad \textbf{(6.4)}$$

Now,

$$OD = OC + CD = \frac{(OA + OB)}{2} + CP \cos 2\theta$$

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

Analytical Method

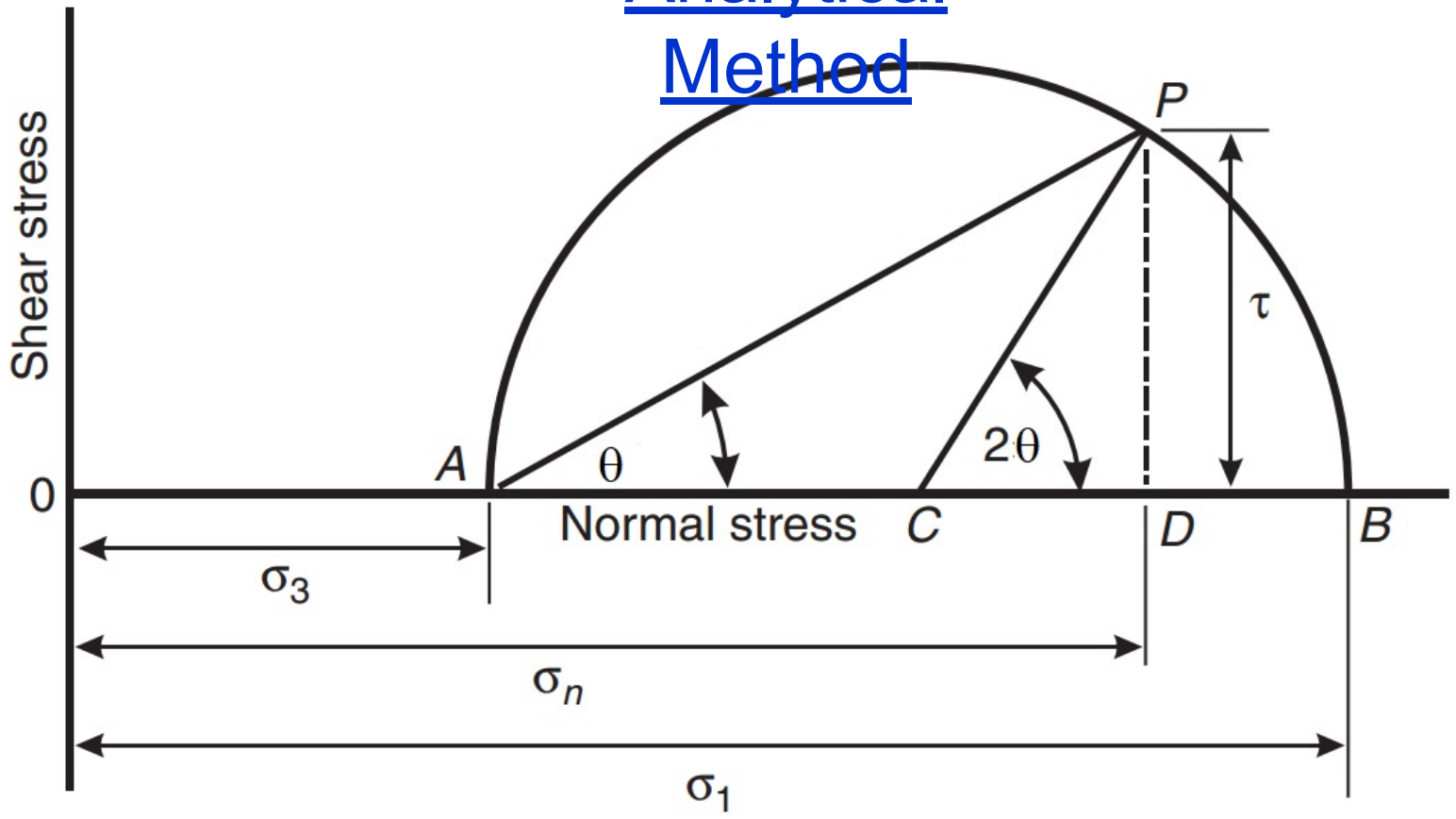


Fig. 6.13 Mohr's circle of stress (Analytical).

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

- **Analytical Method**

Hence,

$$\sigma_n = \frac{(\sigma_1+\sigma_3)}{2} + \frac{(\sigma_1-\sigma_3)}{2}\cos 2\theta \qquad (6.5)$$

These equations, Eqs. 6.4 and 6.5, are identical to those above (Eqs. 6.2 and 6.3) for the stresses σ_n and τ derived from the equilibrium analysis. Thus the distance OD gives the value of σ_n and DP gives the value of τ on a plane inclined at angle θ to the horizontal. This gives a simple graphical method for plotting the results of triaxial tests.

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

- **Limit Conditions of Mohr's Circle of Stresses**

It has been stated that the maximum shearing resistance is developed when the angle of obliquity equals its limiting value, φ. For this condition the line OD in Fig. 6.13 becomes a tangent to the stress circle, inclined at angle φ to axis OX.

An interesting point that arises from Fig. 6.13 is that the failure plane is not the plane subjected to the maximum value of shear stress. The criterion of failure is maximum obliquity, not maximum shear stress.

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

- **Limit Conditions of Mohr's Circle of Stresses**

The plane AE in Fig. 6.13 is subjected to a greater shear stress than the plane AD, it is also subjected to a larger normal stress and as such angle of obliquity is less than on AD, which is the plane of failure.

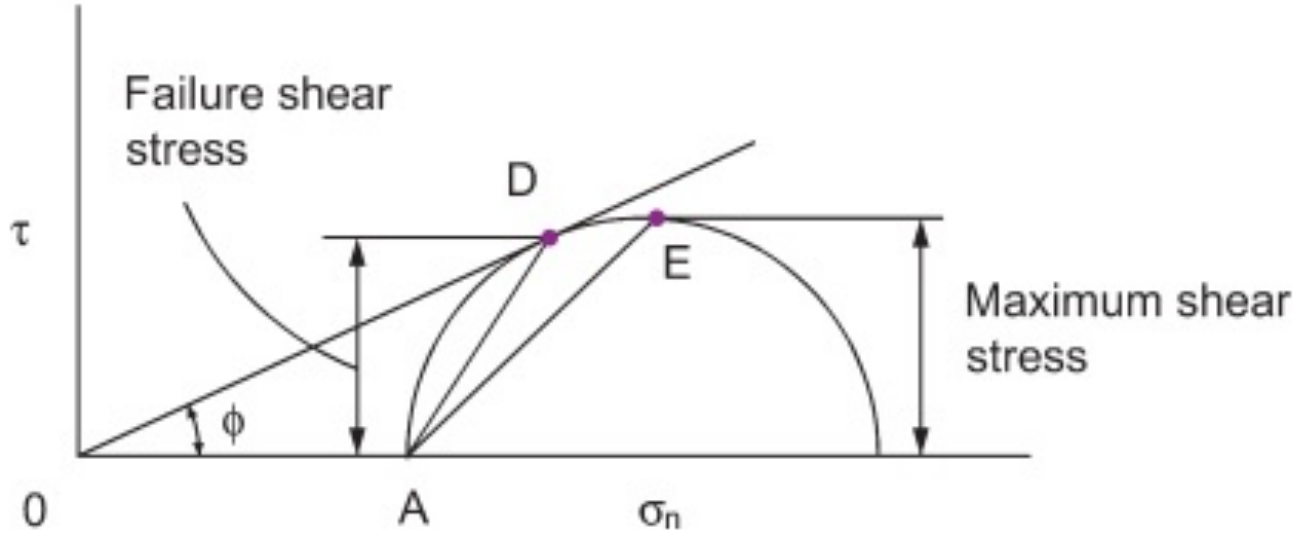


Fig. 6.13 Limit conditions of Mohr's circle of stresses

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

- **Strength Envelopes**

If φ is assumed constant for a certain material, then the shear strength of the material can be represented by a pair of lines passing through the origin, O, at angles $+\varphi$ and $-\varphi$ to the axis OX (Fig. 6.14). These lines comprise the Mohr strength envelope for the material.

In Fig. 6.14, a state of stress represented by circle A is quite stable as the circle lies completely within the strength envelope. Circle B is tangential to the strength envelope .

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

- **Strength Envelopes**

Circle B represents the condition of incipient failure, since a slight increase in stress values will push the circle over the strength envelope and failure will occur. Circle C cannot exist as it is beyond the strength envelope.

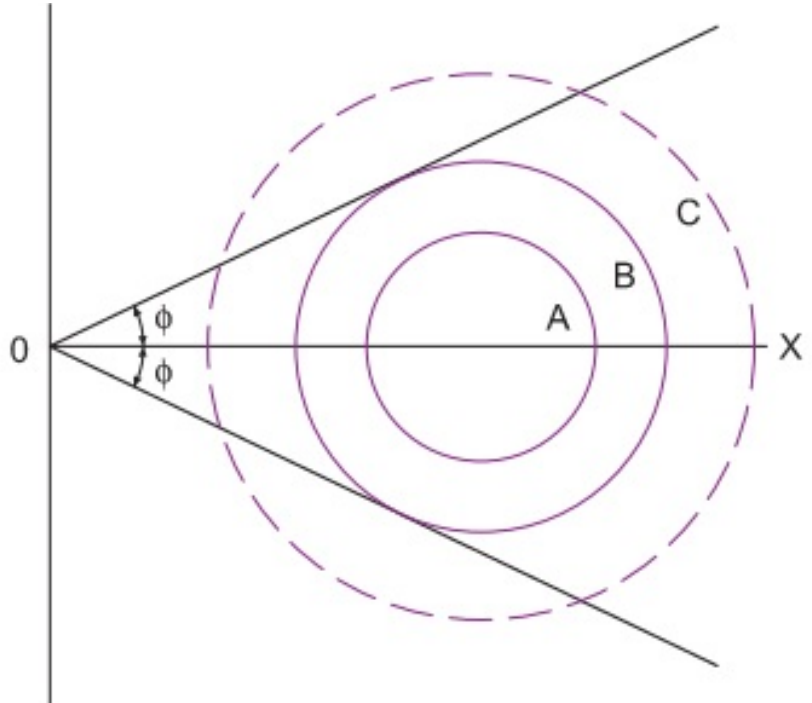


Fig. 6.14 Mohr strength envelope.

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

- **Relationship between $\emptyset$ and θ**

In Fig 6.15, $\angle DCO = 180^o - 2\theta$

In triangle ODC,

Now, $\angle DOC = \varphi$, $\angle ODC = 90^o$, $\angle OCD = 180^o - 2\theta$

These angles summate to $180^o - 2\theta$

That is, $\varphi + 90^o + 180^o - 2\theta = 180^o$,

Hence, $\theta = \frac{\varphi}{2} + 45^o$ **(6.6)**

Shear Strength of Soil

6.5.3 Mohr's Circle of Stresses

Relationship between φ and θ

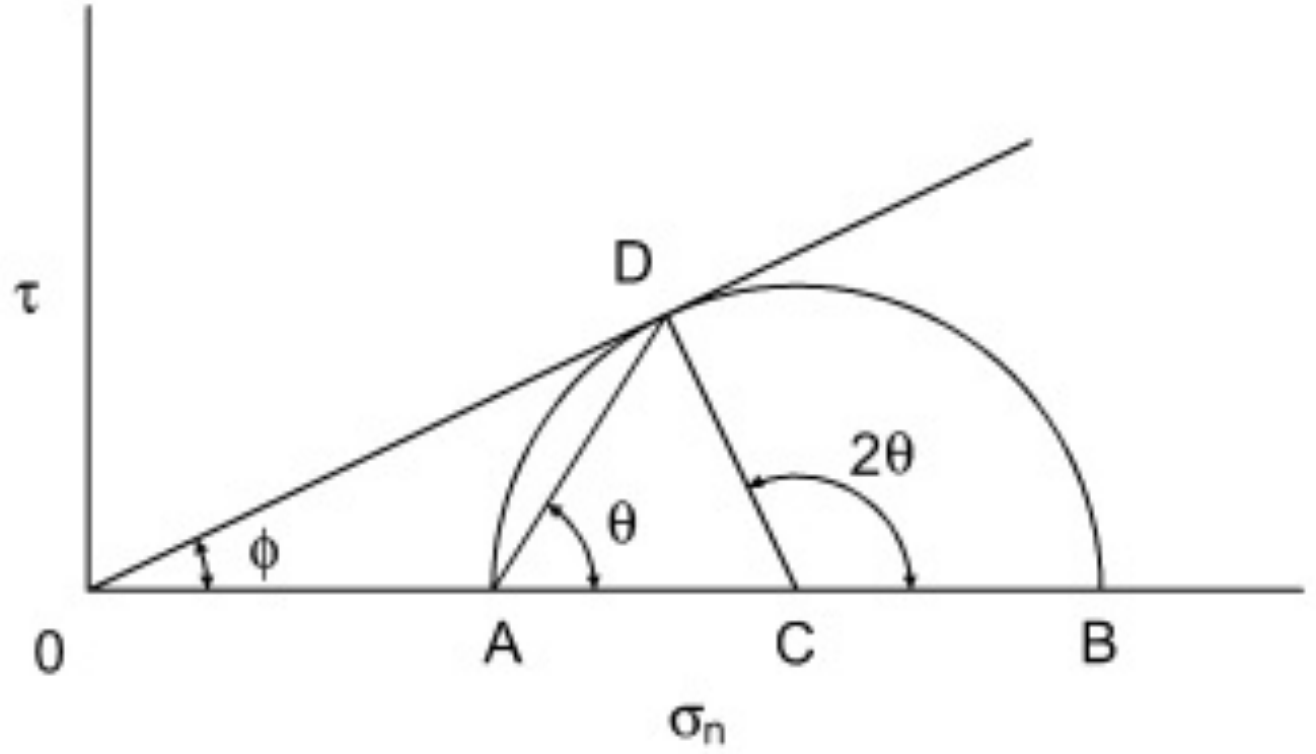


Fig. 6.15 Relationship between φ and θ.

Shear Strength of Soil

6.6 Failure of Soil Mass: Stress Strain Relations

In soils, failure occurs as a result of mobilizing the optimum (not maximum) shear stress the soil can sustain, so an understanding of shear strength is fundamental to the behaviour of a soil mass.

The shear strength of the soil, allied with a particular method of analysis, will determine the ultimate (failure) load which can be applied on a foundation resting on soil, or the ultimate force required to cause failure of a soil mass forming a slope. Shear stresses in a soil mass are only produced when shear strains can occur, so placing a foundation load on a soil mobilizes the available shear strength (or a part of it).

Shear Strength of Soil

6.6 Failure of Soil Mass: Stress Strain Relations

Strictly speaking, it is the shear strain, γ, which produces shear stress, τ (or vice versa), but this is not easy to determine, in situ or in laboratory tests, so direct strains, ε or just displacement δL, have to be used.

A typical stress-strain curve is shown in Fig. 6.16. As shear stress is applied the soil structure distorts. Initially, this distortion is proportional to the stress applied and if the stress is removed the distortions are recovered. These distortions are probably associated with small rotations at the numerous particle contacts and some elastic compression of the particles themselves. The soil in this region is said to behave in an elastic manner.

Shear Strength of Soil

6.6 Failure of Soil Mass: Stress Strain Relations

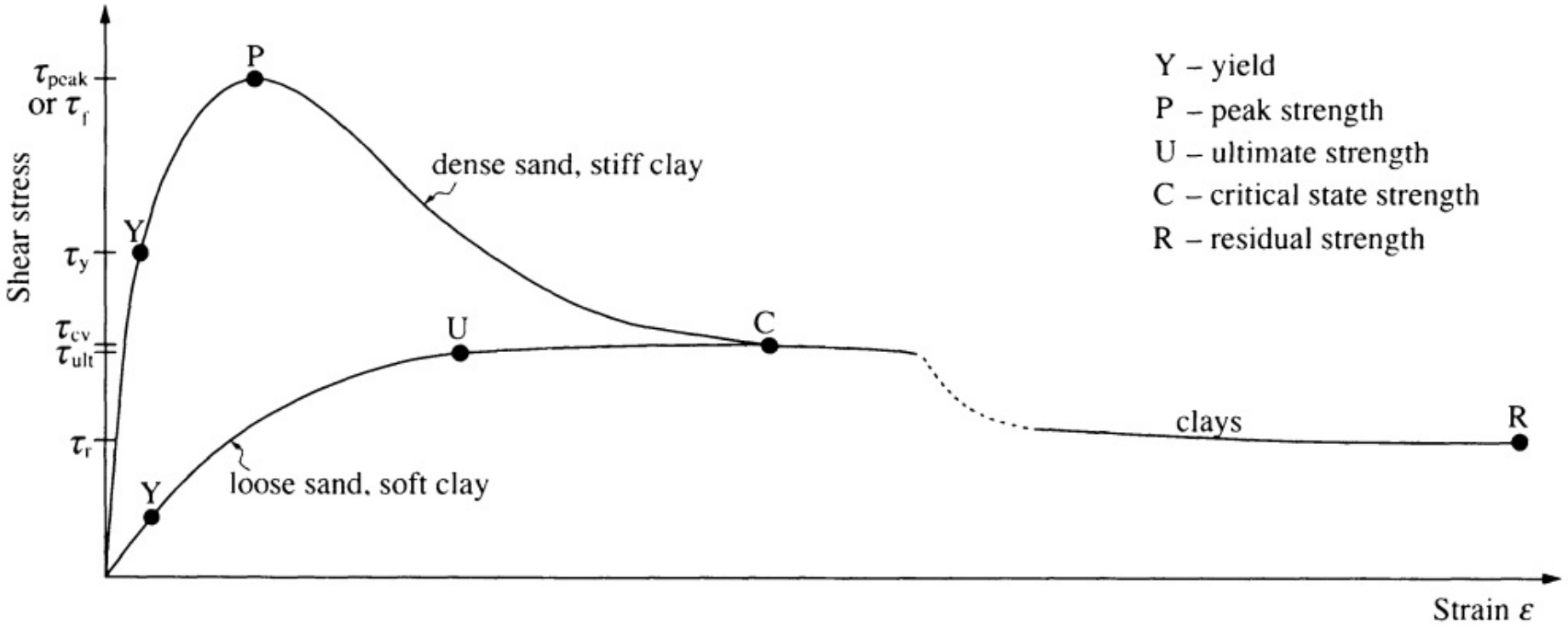


Fig. 6.16 Stress strain diagram of soil.

Shear Strength of Soil

6.6 Failure of Soil Mass: Stress Strain Relations

At a certain stress level, depending on the soil type, the soil structure will deform in a plastic manner by rearrangement of the particle locations. Strains from this point (yield point; point Y in Fig. 6.16) will comprise both elastic and plastic components but the plastic strains will not be recovered on removal of the stress since the soil particles have moved into a new arrangement.

As the soil is strained or 'worked' further additional shear stress can be sustained due to a process described as work-hardening.

Shear Strength of Soil

6.6 Failure of Soil Mass: Stress Strain Relations

In dense sands and stiff overconsolidated clays, this would be due to expansion of the mineral grain structure (dilatency) as more stress is required to achieve further strain.

After reaching a peak shear stress these soils typically display work-softening since strains beyond this value are being applied to a soil structure which has been weakened. This phenomenon would also apply to a soil which has developed cementation or chemical bonding since deposition.

Shear Strength of Soil

6.6 Failure of Soil Mass: Stress Strain Relations

A reduction of strength beyond a peak value is the brittleness; some materials such as sensitive clays and 'collapsing' soils will suddenly crush or collapse due to breakdown of interparticle bonds.

For loose sands and soft normally consolidated clays work-hardening would commence at much lower stress and strain levels as shearing occurs making the structure progressively more able to support more stress. These materials do not tend to display a peak value followed by work-softening or brittleness.

Shear Strength of Soil

6.6 Failure of Soil Mass: Stress Strain Relations

- Figure 6.16 shows that choosing the point at which soil has 'failed' requires a definition. This could be:

[1] Yield - If the soil is stressed any further beyond the point Y on Figure 6.16 the strains and movements of the earth structure (foundation, slope etc.) will be so large that they could be considered to have failed. The quantity T represents a yield stress.

[2] Peak Shear Strength - this is the maximum shear stress which can be sustained. It may be dangerous to rely on this value for some brittle soils, due to the rapid loss of strength if the soil is strained beyond this point.

Shear Strength of Soil

6.6 Failure of Soil Mass: Stress Strain Relations

- **[3] Ultimate Strength - for loose sands and soft clays work-hardening may continue to increase the shear stress that can be sustained, even at very large strains so a maximum strain limit must be imposed, usually related to the performance of the earth structure, say 10-20% strain, point U on 6.16.**

Shear Strength of Soil

6.6 Failure of Soil Mass: Stress Strain Relations

- **[4] Critical State Strength – this is sometimes referred to as the ultimate strength. After a considerable amount of shear strain, a soil may achieve a constant volume state (by the soil structure expanding or contracting) and it would continue to shear at this constant volume without change in volume or void ratio.**

These shear strains must be uniform throughout the soil and not localized. It is sometimes referred to as the constant volume strength (φ_{cv}).

Shear Strength of Soil

6.6 Failure of Soil Mass: Stress Strain Relations

- **[5] Residual Strength-** After a considerable amount of strain on a single slip zone or surface (point R on Fig. 6.16) the particles on either side of this surface may rearrange to produce a more parallel orientation and that may produce the lowest possible strength or residual strength. This strength is very important in the reactivation of old landslides and is more significant for platy minerals (e.g. clays). Friction between the surfaces of particles where they are in contact with each other is related to the effective stress and is represented by the angle of interparticle friction, φ_μ. This friction forms the basis for the above observed shear strengths, the differences being produced by rolling friction and dilatancy.

Shear Strength of Soil

6.7 Failure Theories: Types of Failure

A material is considered failed when a permanent or non-recoverable deformation occurs. There is either direct separation of particles as in case of brittle materials or slipping of particles as for ductile materials where plastic deformation also takes place.

Design of any structural element is based on the hypothesis that the material will not yield during the expected loading conditions. For uniaxial loading a structural element will be safe as long as the stress produced by the load is less than the yield stress of the material. In case of biaxial or triaxial loading system, it is not possible to predict directly by the

Shear Strength of Soil

6.7 Failure Theories: Types of Failure

A tension test of an axially loaded member is easy to conduct, and the results, for many types of materials, are well known. When such a member fails, the failure occurs at a specific principal stress (i.e., the axial stress), a definite axial strain, a maximum shear stress equal to one-half of the axial stress, and a specific amount of strain energy per unit volume of stressed material.

Since all of these limits are reached simultaneously for an axial load, it makes no difference which criterion (stress, strain, or energy) is used for predicting failure in another axially

Shear Strength of Soil

6.7 Failure Theories: Types of Failure

For an element subjected to biaxial or triaxial loading, however, the situation is more complicated because the limits of normal stress, normal strain, shear stress, and strain energy existing at failure are not reached simultaneously.

In other words, the precise cause of failure, in general, is unknown. In such cases, it becomes important to determine the best criterion for predicting failure, because test results are difficult to obtain and the possible combinations of loads are endless.

Shear Strength of Soil

6.7 Failure Theories: Types of Failure

Several theories have been proposed for predicting failure of various types of material subjected to many combinations of loads.

Unfortunately no single theory agrees with test data for all types of materials and all combinations of loads. Several of the more common theories of failure for both ductile and brittle material are listed below.

Shear Strength of Soil

6.7 Failure Theories: Types of Failure

Ductile Failure: The material continues to stretch linearly until the yield stress of the material is reached. At this point the material begins to behave differently. Planes of maximum shear exist in the material at 45°, and the material begins to slide along these planes.

Brittle Failure: Brittle materials tend to fail suddenly by fracture, with little evidence of yielding; therefore, the limiting stress appropriate for brittle materials is the fracture stress (or the ultimate strength) rather than the yield strength. Furthermore, the tensile strength of a brittle material is often different from its

Shear Strength of Soil

6.7 Failure Theories: Types of Failure

For Ductile Materials

- Maximum Shear Stress Theory (Tresca Theory)
- Maximum Strain Energy Theory (Haigh's Theory)
- Maximum Shear Strain (Distortion) Energy Theory (Von-Mises or Von-Mises-Hencky Theory)

Brittle Materials

- Maximum Normal Stress Theory (Rankine Theory)
- Mohr Failure Theory

Shear Strength of Soil

6.7.1 Mohr Coulomb Failure Theory

It is understood that soil is neither a purely ductile or nor a purely brittle material. However, its significantly different tensile and compressive strengths resembles it closer to brittle material and the theory of Mohr along with the hypothesis of Coulomb may be considered sufficiently accurate for soil engineering purposes.

More so, their theories were modified to take into account the effect of pore pressure on the shear strength. Over the years various yield theories have been proposed for soils.

Shear Strength of Soil

6.7.1 Mohr Coulomb Failure Theory

The best known failure are: the Tresca theory, the von Mises theory, the Mohr–Coulomb theory and the critical state theory.

The Mohr–Coulomb theory does not consider the effect of strains or volume changes a soil experiences on its way to failure; nor does it consider the effect of intermediate principal stress, σ_2 . Nevertheless satisfactory predictions of soil strength are obtained. As it is simple to apply, the Mohr–Coulomb theory is widely used in the analysis of most practical problems which involve soil strength.

Shear Strength of Soil

6.7.1 Mohr Coulomb Failure Theory

The Mohr strength theory is really an extension of the Tresca theory, which in turn was probably based on Coulomb's work – hence the title. The theory assumes that the difference between the major and minor principal stresses is a function of their sum, i.e. $(\sigma_1 - \sigma_3) = \int(\sigma_1 + \sigma_3)$. Any effect due to σ_2 is ignored.

A typical example of a Mohr circle diagram is shown in Fig. 6.17. The intercept on the shear stress axis of the strength envelope is the intrinsic pressure, i.e. the strength of the material when under zero normal stress. This intercept is called cohesion in soil mechanics.

Shear Strength of Soil

6.7.1 Mohr Coulomb Failure Theory

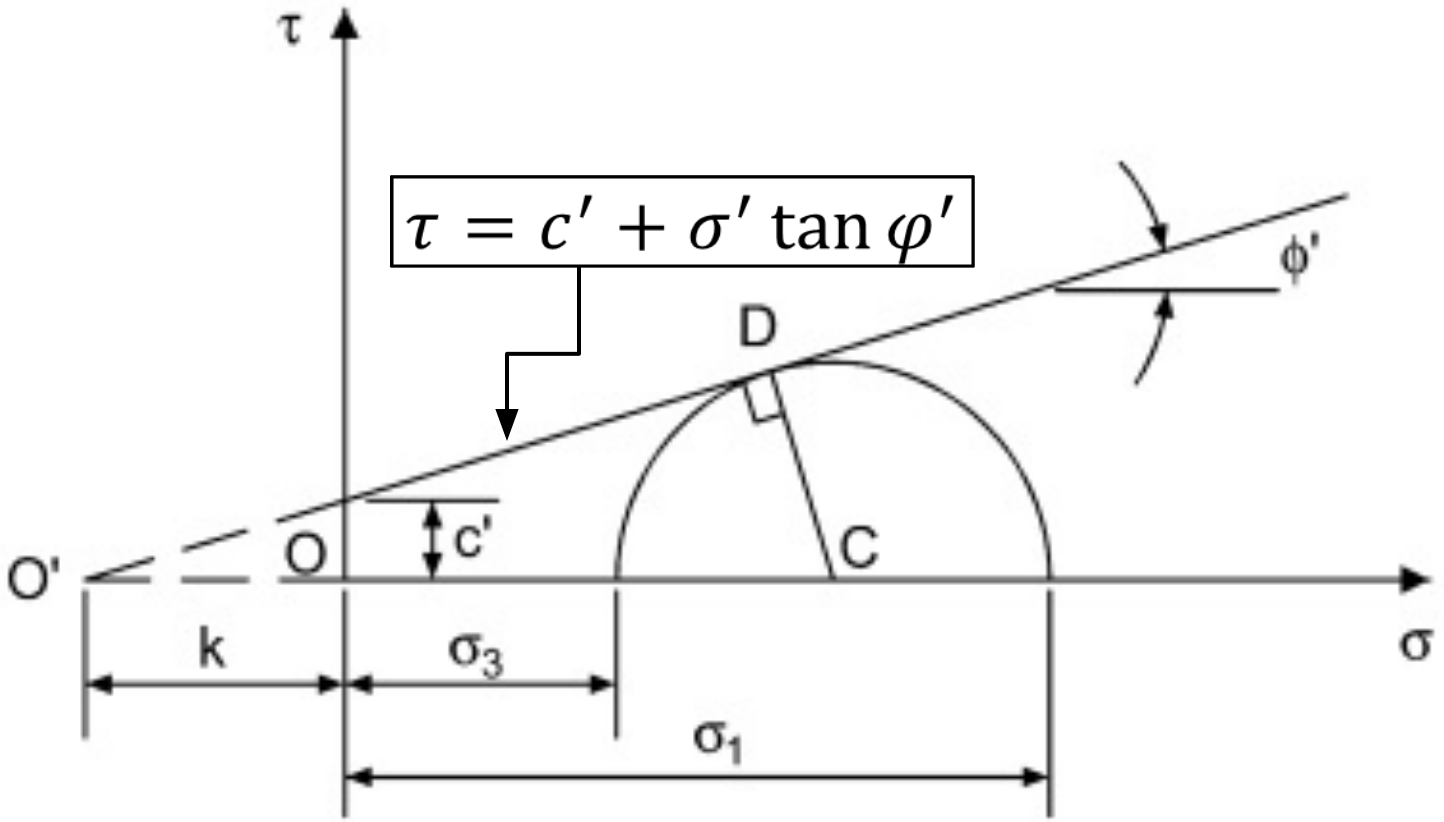


Fig. 6.17 Mohr circle diagram.

Shear Strength of Soil

6.7.1 Mohr Coulomb Failure Theory

- In Fig. 6.17, $\sin\varphi = \frac{DC}{O'C} = \frac{\frac{(\sigma_1-\sigma_3)}{2}}{K+(\sigma_1+\sigma_3)} = \frac{\sigma_1-\sigma_3}{2K+\sigma_1+\sigma_3}$

Hence, $\sigma_1 - \sigma_3 = 2K\sin\varphi + 2K(\sigma_1 + \sigma_3)$

Now, $\mathrm{K} = \mathrm{c}\cot\varphi$

$$\Rightarrow (\sigma_1 - \sigma_3) = 2c\cos\varphi + (\sigma_1 + \sigma_3)\sin\varphi$$

The equation can be expressed in terms of either total stress (as shown) or effective stress:

$$(\sigma'_1 - \sigma'_3) = 2c\cos\varphi' + (\sigma'_1 + \sigma'_3)\sin\varphi'$$

Shear Strength of Soil

6.7.1 Mohr Coulomb Failure Theory

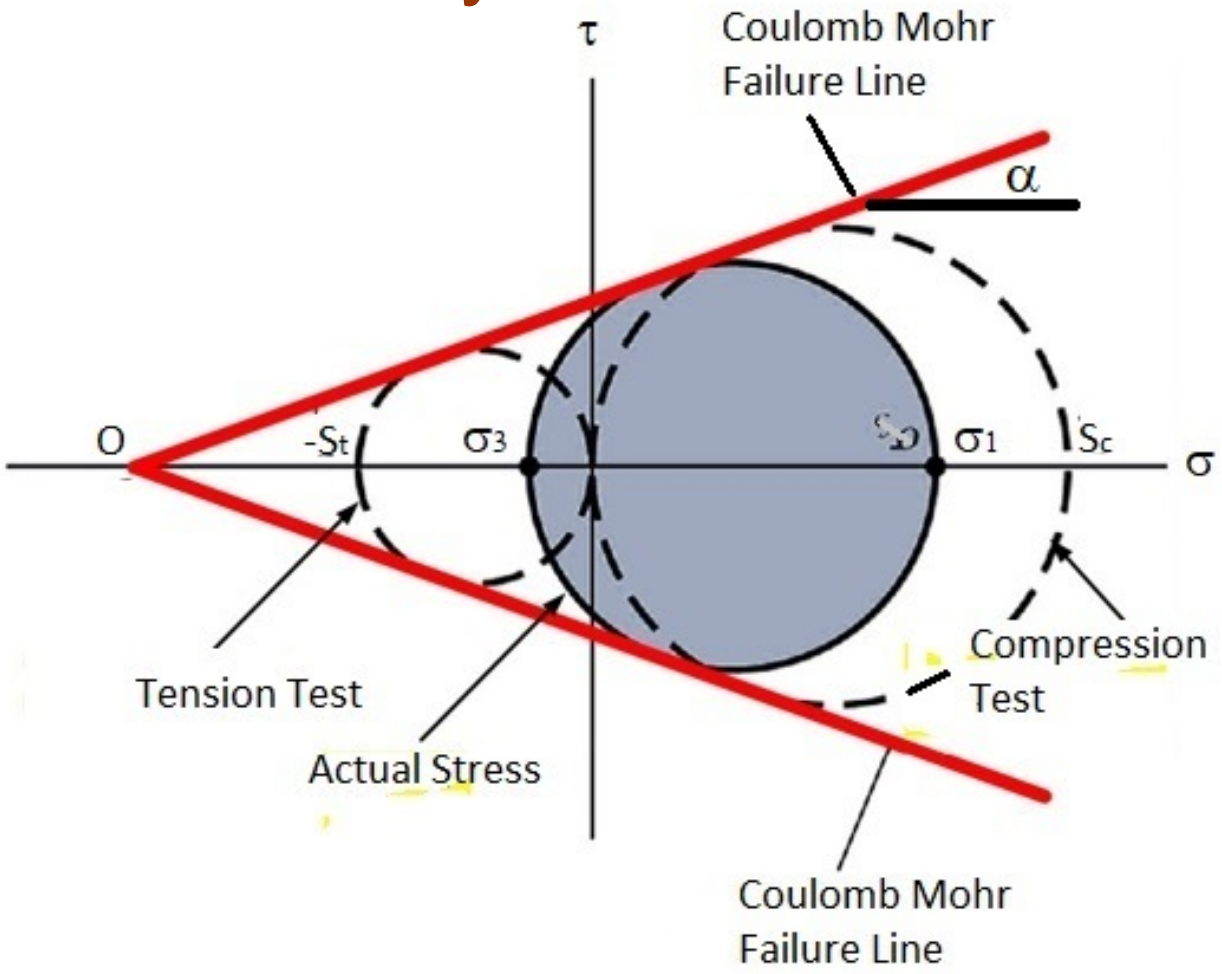


Fig. 6.18 Illustration of Coulomb-Mohr's theory

Shear Strength of Soil

6.7.1 Mohr Coulomb Failure Theory

Mohr's theory suggests that failure occurs in a material whenever Mohr's circle for the combination of stresses at a point in a body exceeds the "envelope" defined by the Mohr's circles for the tensile and compressive tests, as shown in Figs. 6.18.

Coulomb (1776) suggests that soil fails because the strength provided by interparticle friction and cohesion is exceeded by the applied shearing stress on the slip surface. The stresses acting on the surface, inclined at an angle α to the horizontal, are shown on Figs. 6.18.

Shear Strength of Soil

6.7.1 Mohr Coulomb Failure Theory

It can be seen that the shear resistance offered by a particular soil is made up of the two components of friction and cohesion.

Frictional resistance does not have a constant value but varies with the value of normal stress acting on the shear plane, whereas cohesive resistance has a constant value which is independent of the value of σ_n.

Shear Strength of Soil

6.7.1 Mohr Coulomb Failure Theory

In 1776, Coulomb suggested that the equation of the strength envelope of a soil could be expressed by the straight line equation relating the parameters cohesion and friction as.

$$\tau = c + \sigma_n \tan \varphi \qquad (6.7)$$

Where, τ is the shear strength, c and φ are shear parameters known as apparent cohesion and angle of internal friction.

Shear Strength of Soil

6.7.1 Mohr Coulomb Failure Theory

General Expression for Shear Strength

The general expression for soil shear strength, used almost universally in the world today, is

$$\tau \text{ or } s = c' + (\sigma - u)\tan\varphi' \qquad \text{(6.8a)}$$

or, $$\tau \text{ or } s = c' + \sigma' \tan\varphi' \qquad \text{(6.8b)}$$

Where,

$\tau \text{ or } s$ = shear strength or shearing resistance.

σ = total normal stress on shear plane

u = pore pressure on shear plane

σ' = effective normal stress on shear plane

Shear Strength of Soil

6.7.1 Mohr Coulomb Failure Theory

Equation (6.7) is represented graphically in Fig. 6.17. This equation often termed as Mohr-Coulomb failure envelope.

Coulomb's equation defines the shear stress in terms of friction angle φ and the cohesion intercept c.

These are empirical constants only and depends on the natural state of the soil as well as on the method of testing.

Shear Strength of Soil

6.7.1 Mohr Coulomb Failure Theory

Apparent Cohesion

This is caused by surface tension, acting at the ends of the thin film of water between the contact surfaces of fine-grained, moist soils.

The apparent cohesion disappears if the soil is flooded or completely dried. This is why sand particles stick together, whilst partially saturated. Soils are often referred to as:

- φ – soil or cohesionless soil, when $c = 0$
- c – soil or cohesive soil, when $\varphi = 0$
- $c - \varphi$ soil when neither φ nor c is zero.

Shear Strength of Soil

6.7.2 Factors Influencing Shear Strength and Parameters c and $\emptyset$

- **Drainage Condition** – Undrained and drained condition
- **Soil Composition** - mineralogy, grain size and grain size distribution, shape of particles, pore fluid type and content, ions on grain and in pore fluid.
- **Initial State** - State can be described using terms such as: loose, dense, over-consolidated, normally consolidated, stiff, soft, etc.
- **Structure** - Refers to the arrangement of particles within the soil mass; the manner in which the particles are packed or distributed. Features such as layers, voids, pockets, cementation, etc, are part of

Shear Strength of Soil

6.7.3 Drained and Undrained Condition

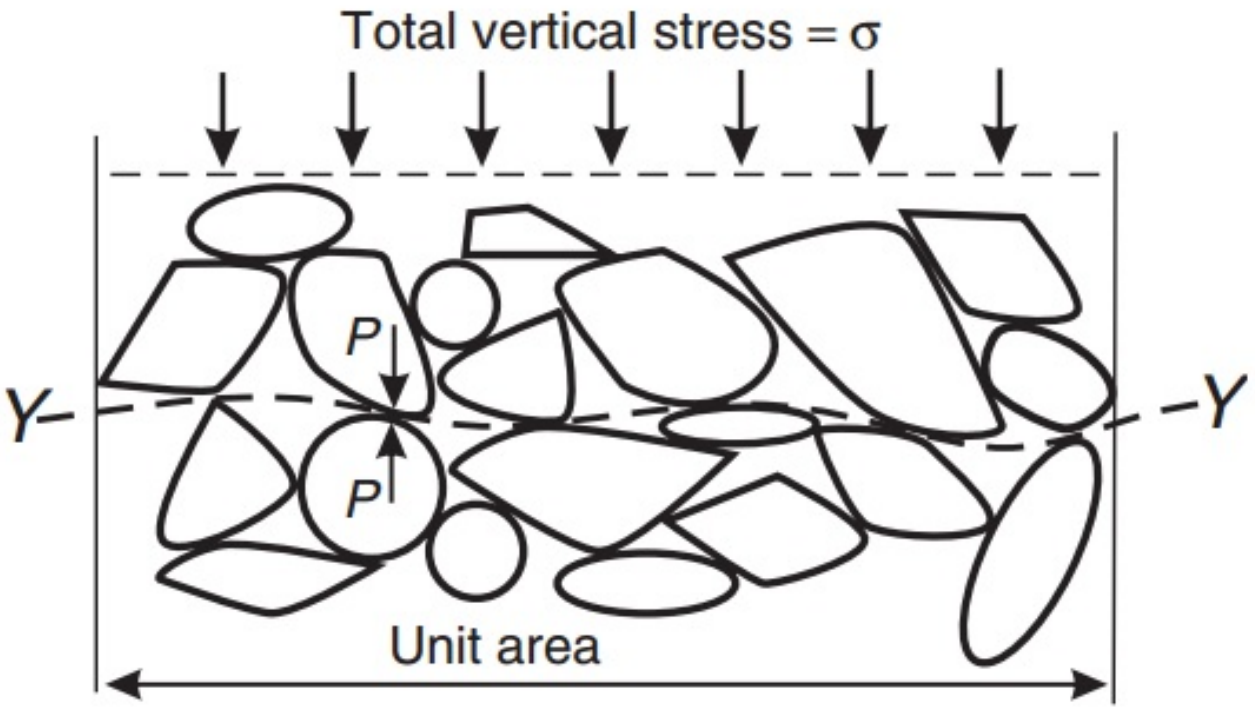
The terms drained and undrained are widely used in soil mechanics and should be clearly understood. The terms refer to whether or not any movement of water occurs in the soil. Whether the soil behaves in a drained or undrained manner depends primarily on two factors:

(a) The rate at which the load is applied relative to the permeability and compressibility of the soil.

(b) The opportunities that exist for water to drain from the soil as the load is applied, in

Shear Strength of Soil

6.7.3 Drained and Undrained Condition

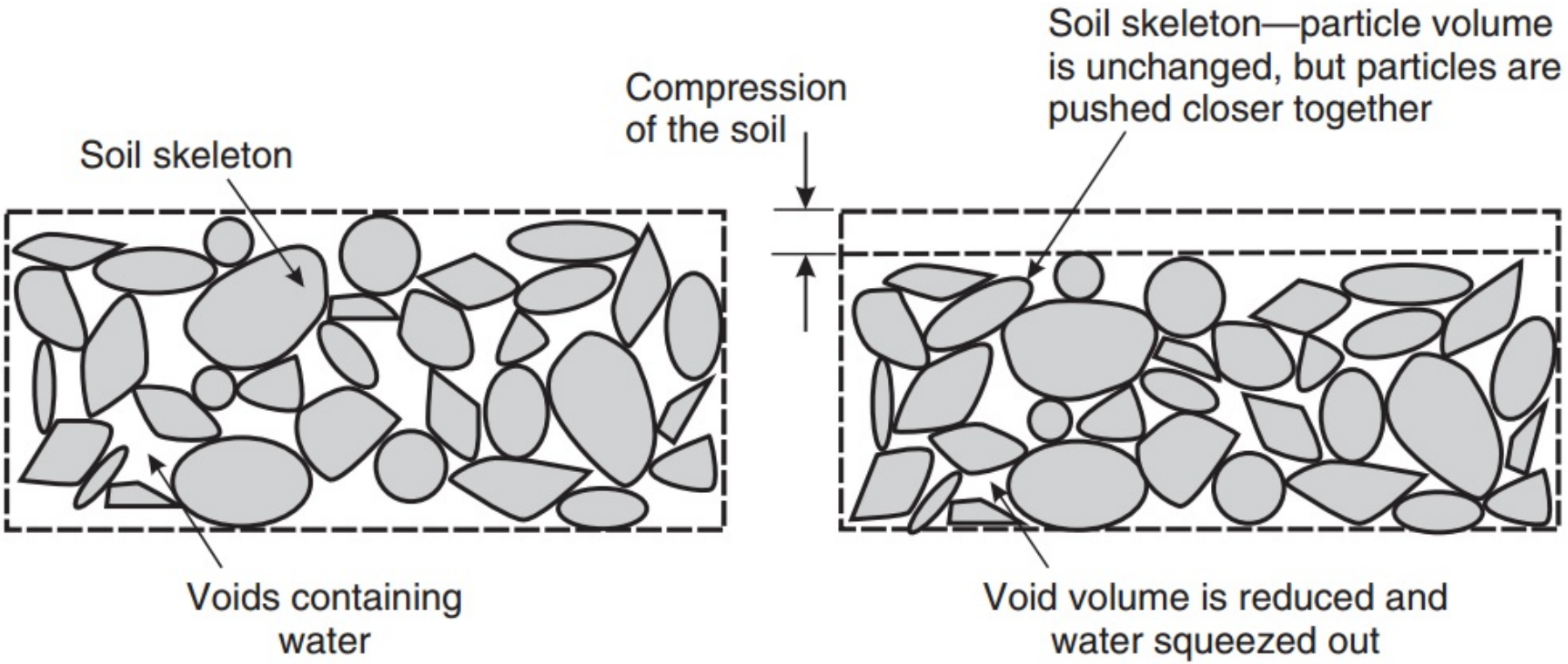


$$\sigma' = \sigma - u$$

Fig. 6.19 Total and effective stress in a soil.

Shear Strength of Soil

6.7.3 Drained and Undrained Condition



(a) Undrained condition (b) Drained condition

Fig. 6.20 Illustration of undrained and drained Conditions.

Shear Strength of Soil

6.7.3 Drained and Undrained Condition

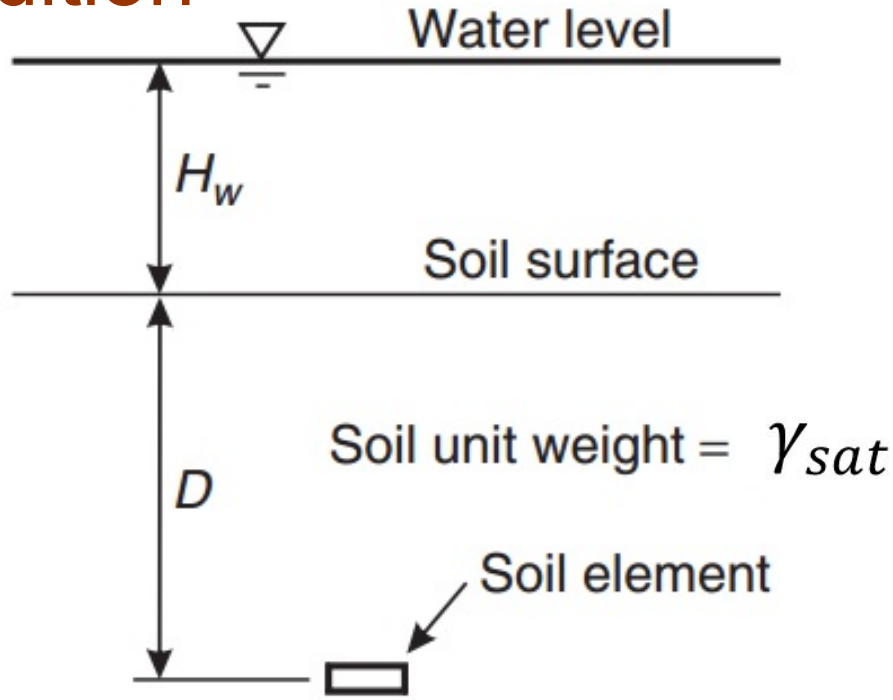


Fig. 6.21 Total and effective stresses.

$$Effective\ stress, \sigma' = \sigma - u$$

$$= (\gamma_w H_w + \gamma_{sat} D) - \gamma_w (H_w + D) = (\gamma_{sat} - \gamma_w) D = \gamma' D$$

Shear Strength of Soil

6.7.3 Drained and Undrained Condition

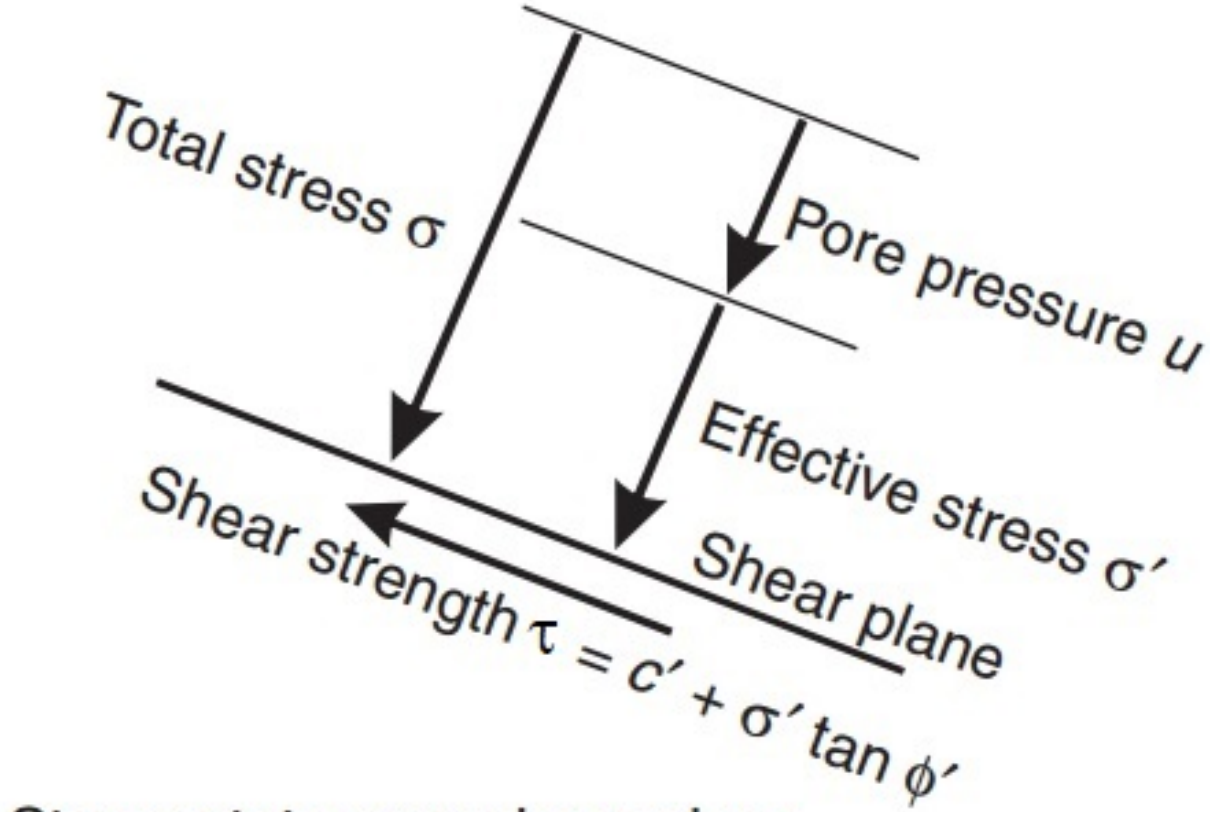


Fig. 6.22 Stress state on a shear plane.

Shear Strength of Soil

6.7.3 Drained and Undrained Condition

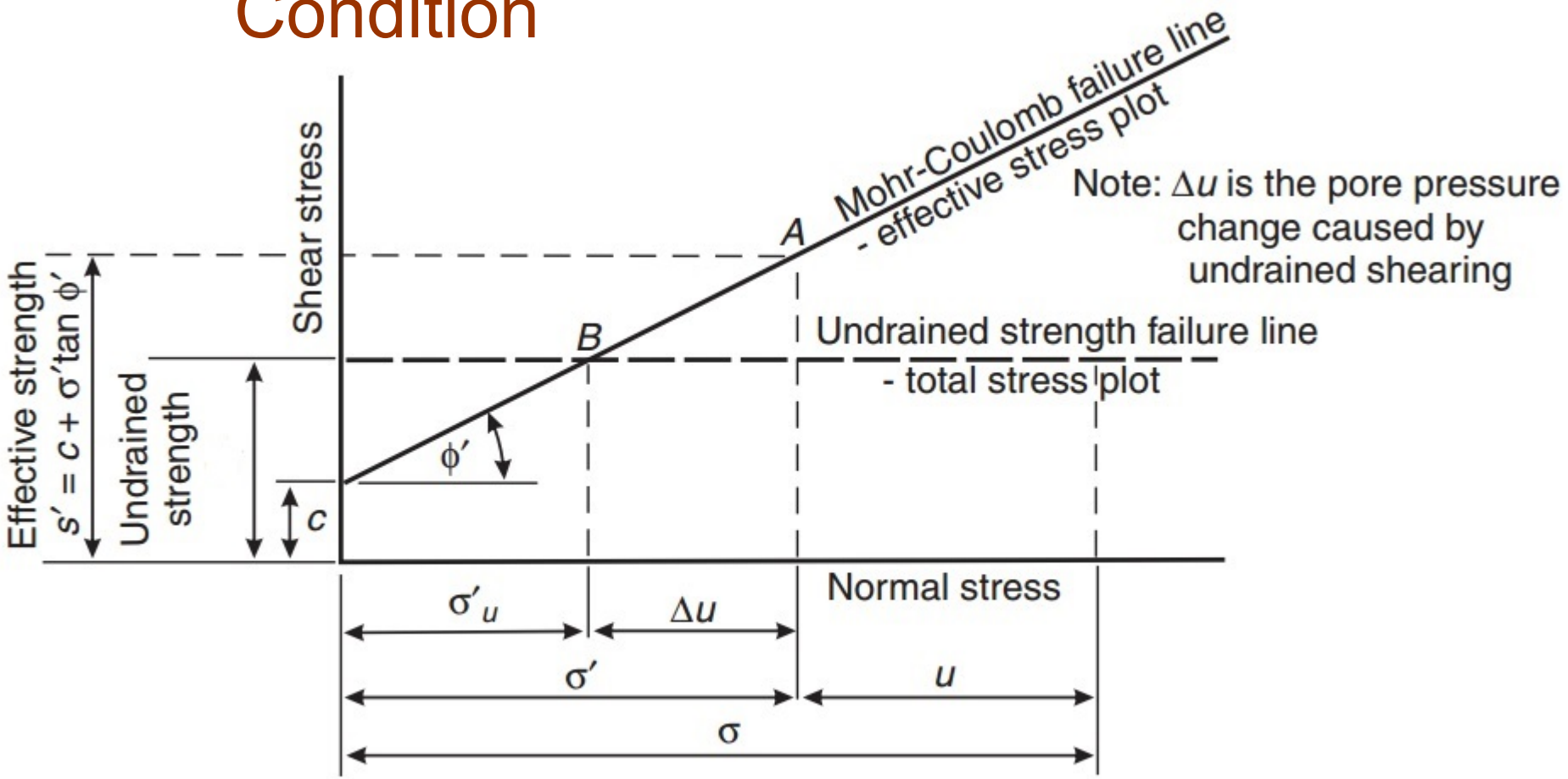


Fig. 6.23 Mohr-Coulomb representation of stress state.

Shear Strength of Soil

6.7.4 Undrained Condition in Clay

When a building or an embankment load is applied on a clayey ground, it is often assumed that the entire load is applied instantaneously.

Clays have low hydraulic conductivity, hence soon after the loading with very little time for any drainage (i.e., short-term), it is fair to assume that the clay is loaded under undrained conditions (Fig. 6.20).

Under undrained conditions, the clay is often analyzed in terms of total stresses, without worrying about the effective stresses and pore water pressures induced within the clay which are

Shear Strength of Soil

6.7.4 Undrained Condition in Clay

Here, no attempt is made to separate the stresses carried by the soil skeleton formed by the clay particles and the pore water.

The entire clay is treated as one homogeneous material and the shear strength must be analyzed accordingly.

Shear Strength of Soil

6.7.5 Drained Condition in Clay

Drained condition occurs when there is change in pore water pressure due to external loading. (Fig. 6.14)

After a long time of loading (i.e., long-term), the story is different than a short term loading. The clay would have fully drained, with no excess pore water pressures, and the loading can be treated as drained loading, where the analysis can be carried out in terms of effective stresses.

There can still be static pore water pressures due to the phreatic surface (i.e., water table). Similarly, if it is known that the clay is loaded very slowly without any buildup of excess pore water pressures, such situations can also be analyzed as drained loading, in

Shear Strength of Soil

6.7.6 Drained and Undrained Condition in Sandy Soil

In granular soils, where the drainage is always good with little or no build-up of excess pore water pressures, all loadings are under drained conditions.

Shear Strength of Soil

6.7.7 Shear Strength of Coarse and Fine Grained Soils

In coarse-grained soils, the shear strength depends largely on:

- Surface roughness of the grains.
- Interlocking of grains and state of compaction.
- Magnitude of contact pressures.
- Adhesion in finer-grained wet soils due to thin water layer between the contact points

Shear Strength of Soil

6.7.7 Shear Strength of Coarse and Fine Grained Soils

In fine-grained soil the shear strength is assumed to be dependent largely on:

- Water content of the soil
- Shape, size and packing of the particles
- Adhesion due to the thin film of water between the contact surfaces

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

- **Shear strength (its parameters c and φ) of soils can be determined both in the laboratory or in the field.**
- **The tests for studying the shear strength and related deformation characteristics of soils include the following list.**

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

- Direct shear test
- Triaxial test
  - ✔ Triaxial Compression
  - ✔ Triaxial Extension
- Unconfined compression test
- Vane shear test
- Torsion shear test
- Ring shear test (double shear)
- Hollow cylinder test
- Plain strain test
- Cuboidal or true triaxial test
- Borehole shear test

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

- The first three, direct shear, triaxial and unconfined compression are the most common laboratory tests. The fourth one, vane shear test, is in principle similar both in the laboratory and in the field except that the apparatus used in the field is larger in size for practical reasons. The vane shear test is usually done to determine the undrained shear strength parameter (c) of saturated cohesive soil.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test

The direct shear test is a **simple and widely used test.** The direct shear apparatus is essentially a rectangular or circular box having separated lower and upper halves (Fig. 6.24a, b, c & d). The lower half of the box is fixed to a frame whereas, the upper section is capable of moving horizontally relative to the lower one.

The soil sample is placed in the box, with approximately half of the sample within either section. The sample size is usually 60 mm square or 75 mm circular having a thickness in

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test

In case of cohesive soil, prism of soil is either prepared or taken from the undisturbed sample using a cutter of similar dimension of shear box.

For cohesionless soil, the specimen has to be prepared in the box at the required void ratio. Porous disc may be placed on top and bottom of the specimen to facilitate the drainage conditions. A normal load is applied to the plane of shear via a loading plate placed over the sample (Fig. 6.24a).

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test



Fig. 6.24 (a) Direct shear test apparatus.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test

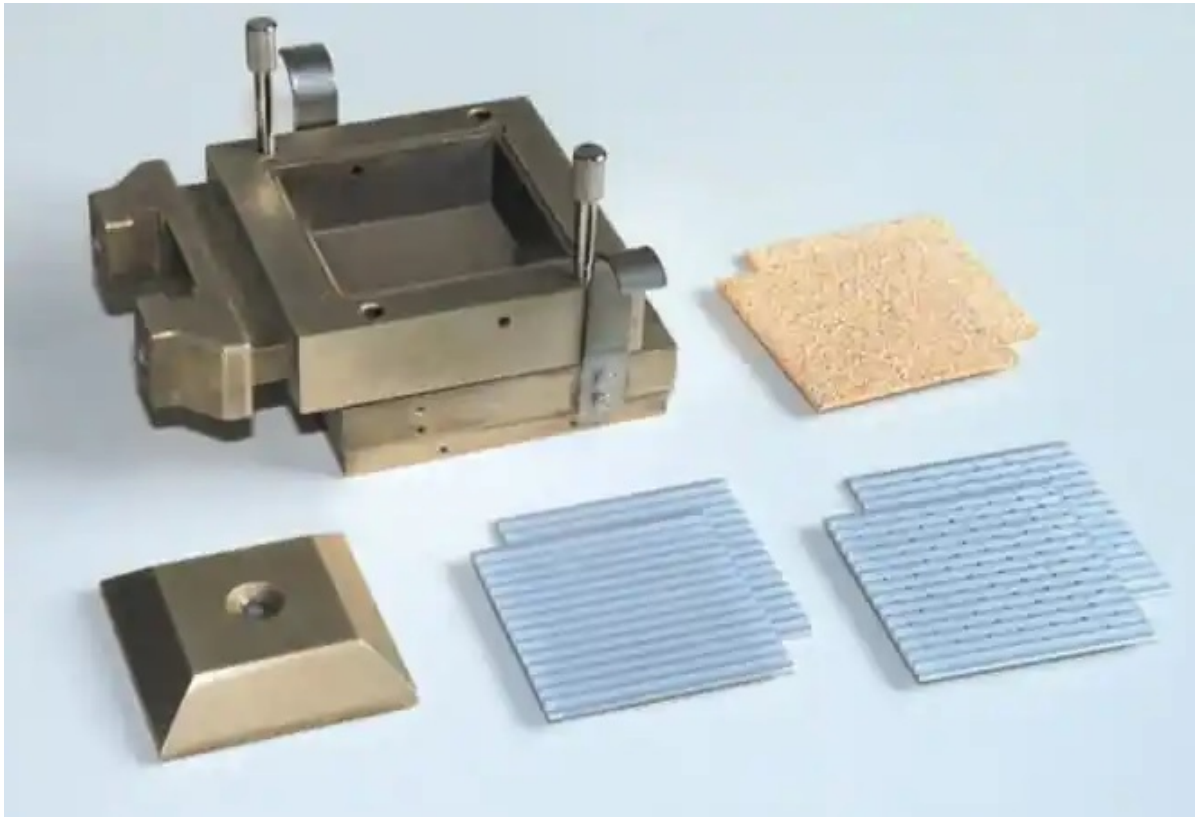


Fig. 6.24(b) Direct shear apparatus: shear box and accessories.

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test

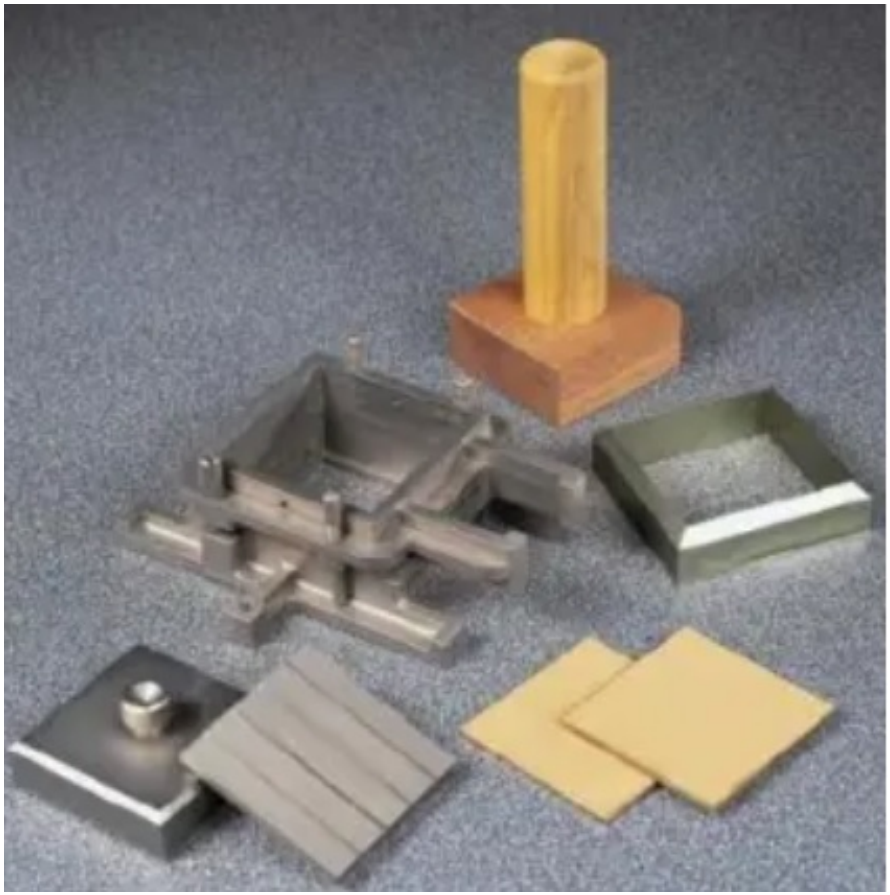


Fig. 6.24(b) Direct shear apparatus: shear box and accessories.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test

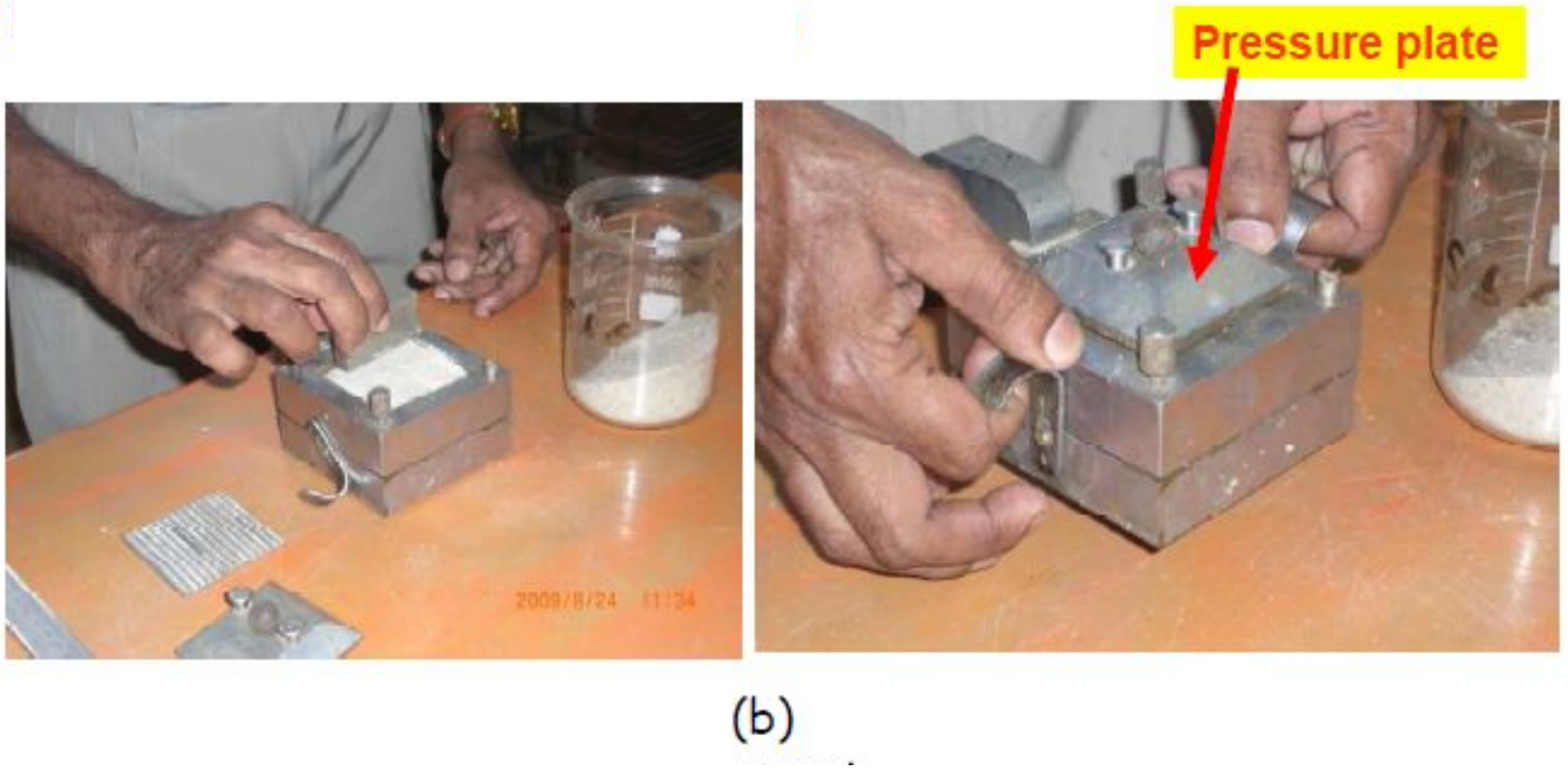


Fig. 6.24(b) Direct shear apparatus: sand sample preparation.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test

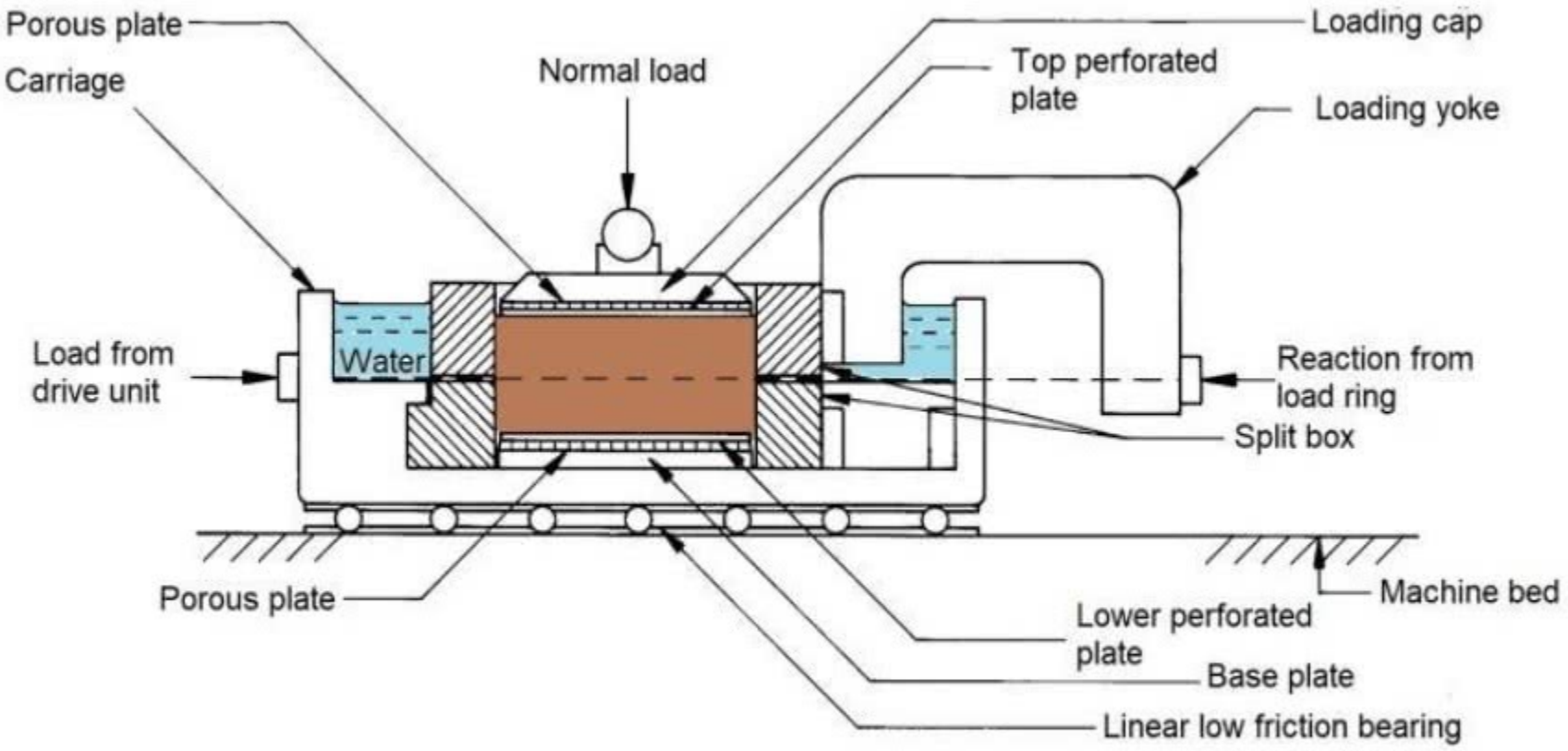


Fig. 6.24 (c) Direct shear apparatus: Schematic diagram.

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test

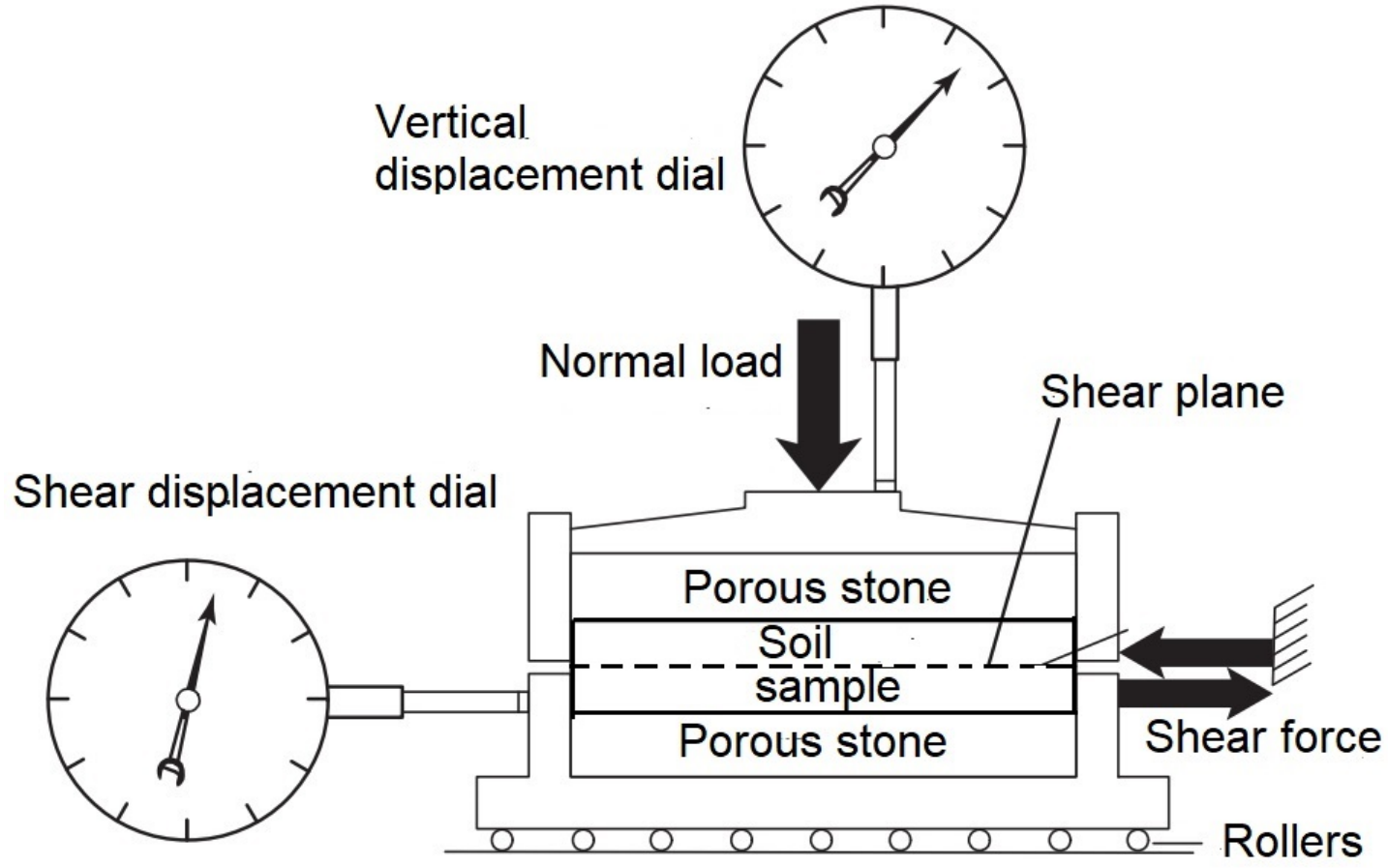


Fig. 6.24 (c) Direct shear apparatus: Schematic diagram

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test

The upper half of the shear box is then moved laterally forcing the sample to shear across the plane between the two halves of the box either by controlling rate of strain or stress.

In strain controlled apparatus, the shearing deformation is continuously applied at a constant speed and shear force is measured by means of a proving ring or a load transducer. Such a test can be continued even after the failure of the specimen.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test

In stress controlled apparatus, the magnitude of the shear stress is increased uniformly or in increments. Each increment is applied and held constant until the shearing deformation ceases. A stress controlled test can not be continued as soon as the specimen fails.

Test procedure is repeated using at least three identical specimen with variable normal stresses. It is sometimes convenient to interpret the results, if the normal stresses are so chosen that one is closer to the existing effective overburden and the

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test

The magnitude of the shearing force is recorded and the shear displacement is maintained to obtain the peak shear stress (failure stress) from the shear stress-strain plot for a particular normal load. (Figs. 6.25a)

Changes in sample thickness that occurs during the shearing process are also recorded so that the volume change versus shearing stress or shearing strain can be studied. (Figs. 6.25b)

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

- **Direct Shear Test**

The peak value of shear stress or the maximum value at a relatively higher shear deformation (normally 15 per cent) gives shear strength of the soil at a particular normal load.

A best possible straight line is fitted to the observed point to represent the failure envelope. The shear strength parameters c' and φ' is thus obtained from the plot. (Fig. 6.25c).

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test

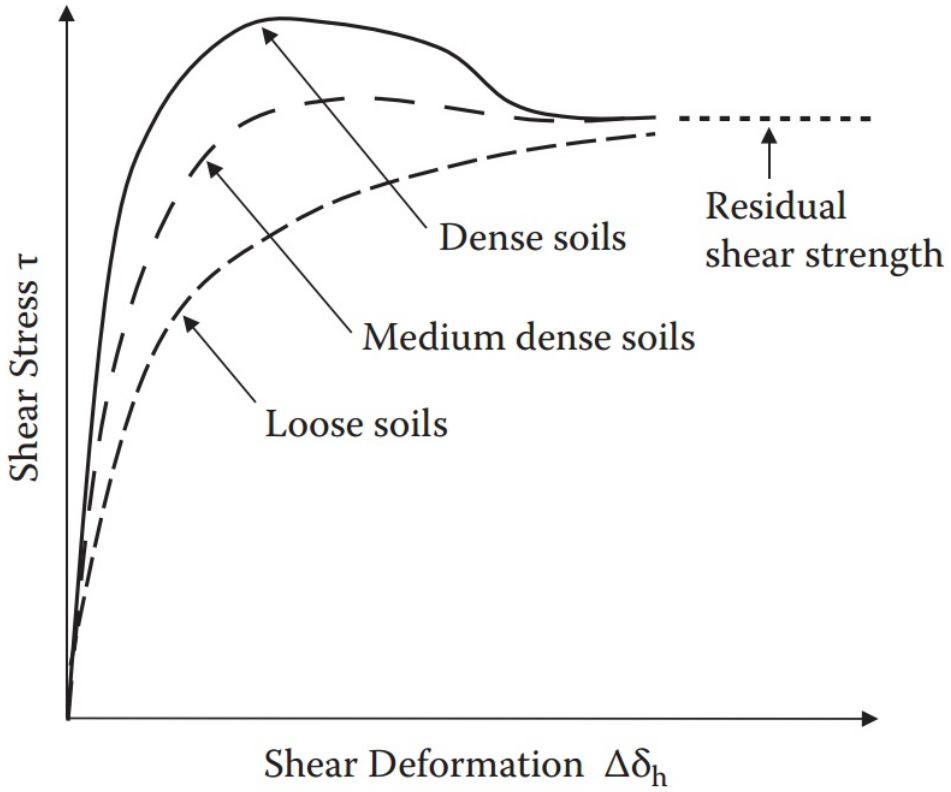


Fig. 6.25a Typical direct shear test results.

6.7.8 Determination of Shear Strength Parameters

Direct Shear

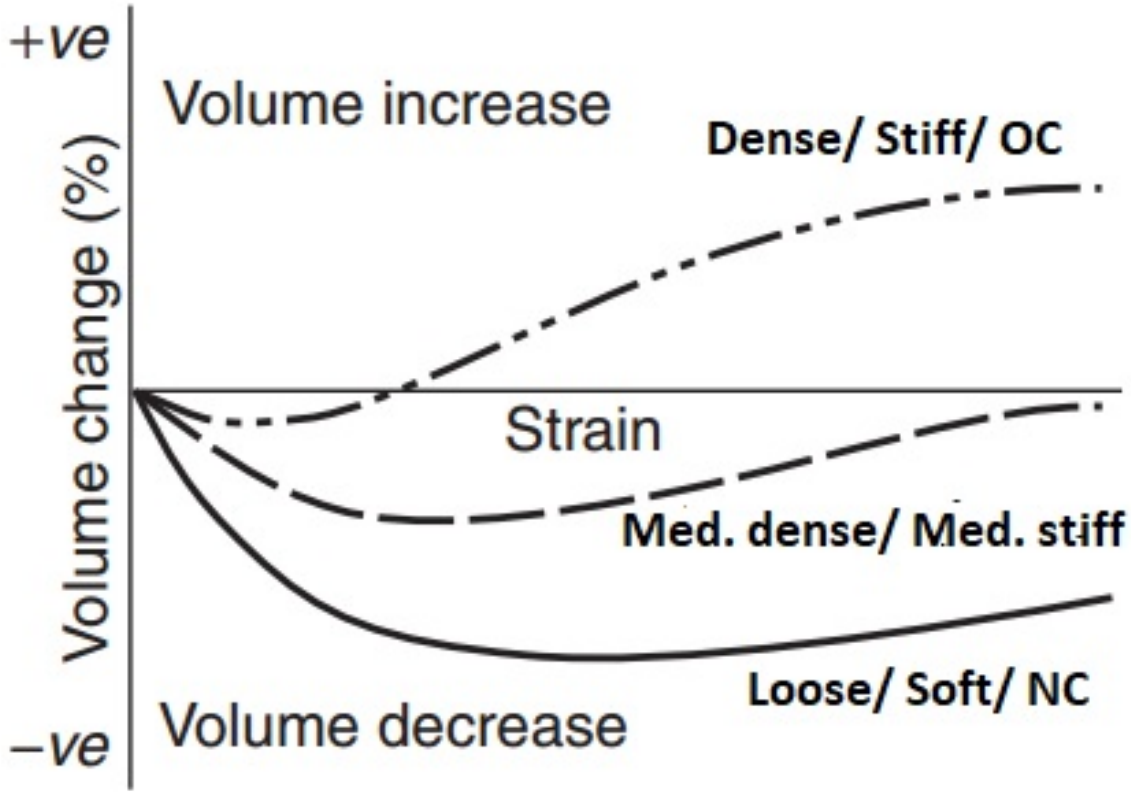


Fig. 6.25b Typical direct shear test results.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test

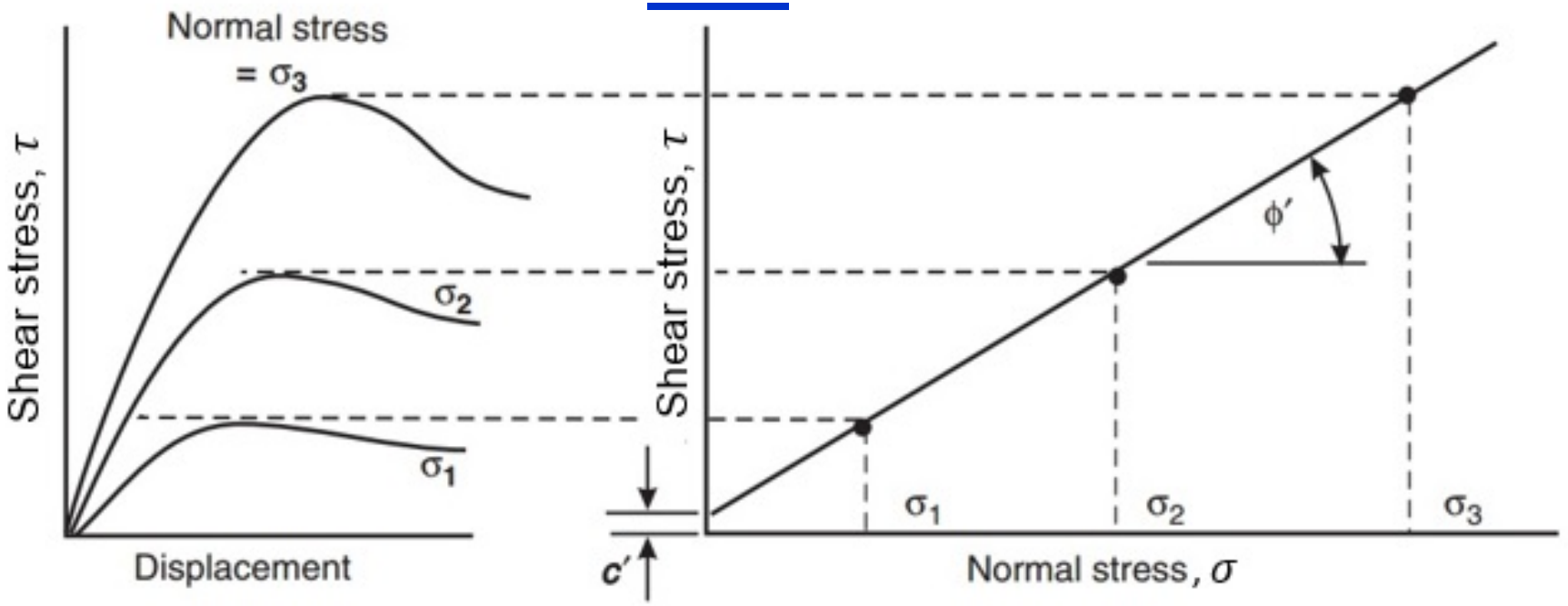


Fig. 6.25c Typical direct shear test results.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

- **Direct Shear Test**

Usually, the drained strength parameters are obtained from direct shear test. For granular soil the cohesion is zero and the available strength parameter would be only φ'. For cohesive soil the parameter would depend on the stress history (normally consolidated or overconsolidated) of the soil. Similar to sandy soil normally consolidated clay soil will have $c' = 0$. Overconsolidated soil will have a value of c as indicated in Fig. 6.27. Typical values of φ' for cohesionless soils are shown in Table 6.1 and Fig. 6.28.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test

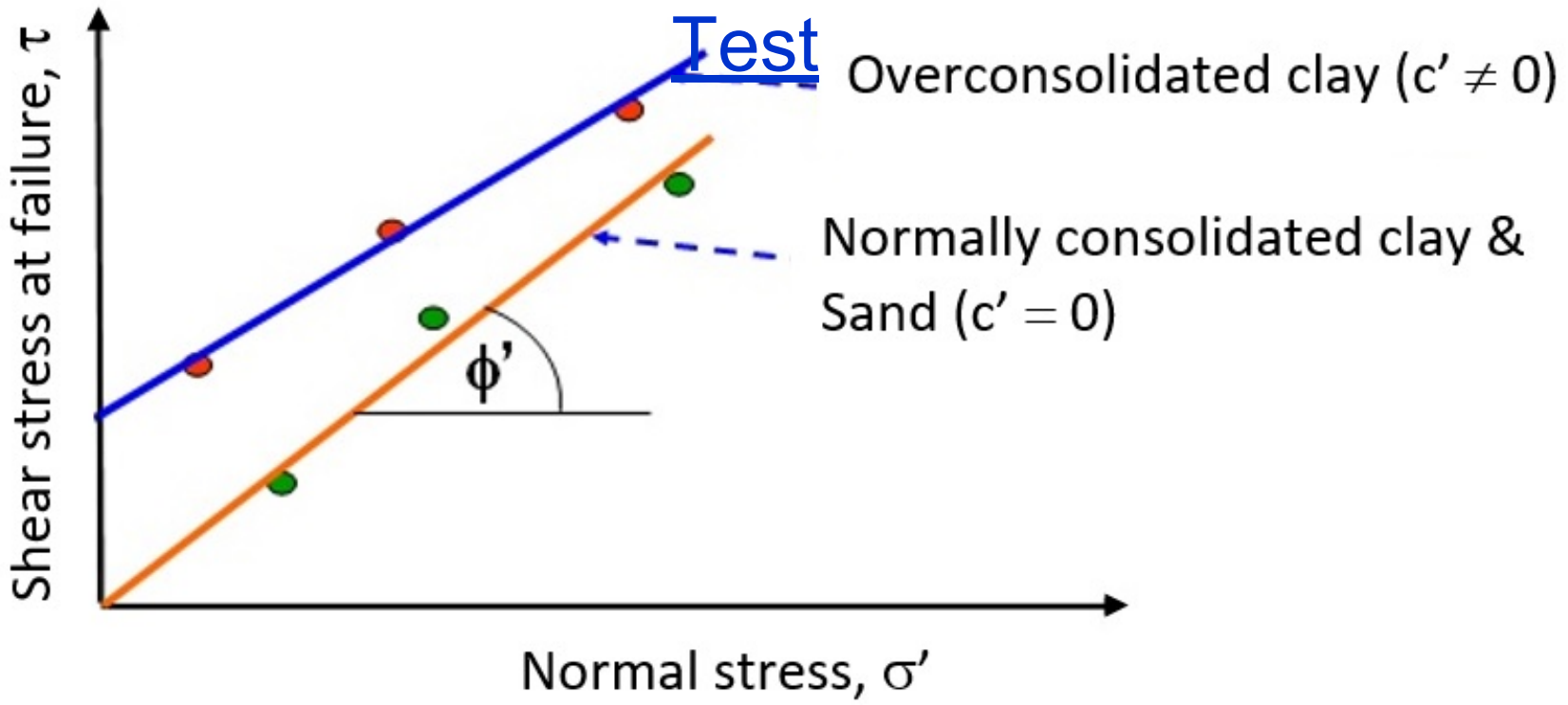


Fig. 6.26 Direct shear test results for sandy and clayey soil.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Table 6.1 Typical values of φ' for granular soil (after Terzaghi and Peck, 1967)

Soil type	Angle of internal friction, ϕ′(degrees)	
	Loose	Dense
Nonplastic silt	27-30	30-34
Silty sand	27-33	30-35
Uniform sand	28	34
Well graded sand	33	45
Sandy gravel	35	50

6.7.8 Determination of Shear Strength Parameters

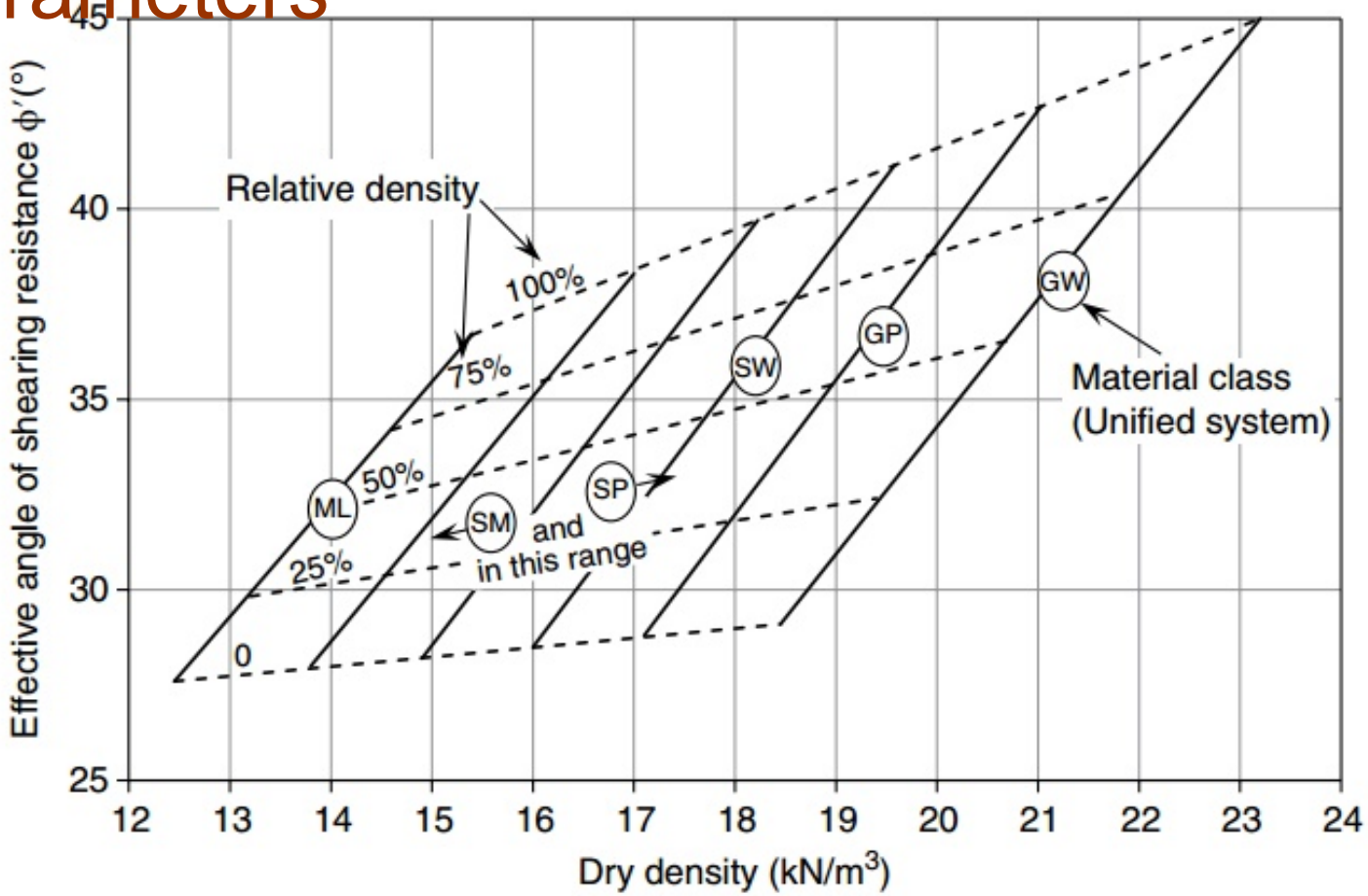


Fig. 6.28 Typical values of density and angle of internal friction of cohesionless soils.

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test: Advantages and Limitations

Advantages and limitations of the direct shear test include the following:

- It is simple and quick test, and easy to perform.
- Failure occurs on a predetermined plane and not on the weakest plane.
- Undisturbed samples are difficult to prepare.
- Procedure of specimen saturation is arbitrary. Cannot be ascertained about the achieved degree of saturation of the soil specimen.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Direct Shear Test: Advantages and Limitations

- For highly plastic soil complete shear may not occur on the shear surface..
- Drainage cannot be controlled, so that undrained tests are not possible.
- Drainage cannot be controlled, so that undrained tests are not possible.
- Principal stresses are not known, especially the lateral stress.
- Area of sample changes as the test proceeds, and area corrections are indeterminate.

6.7.8 Determination of Shear Strength Parameters

Triaxial Test

The triaxial test is perhaps the most versatile and sophisticated test for determining the shear strength parameters of any type of soils.

Considering the mode of application of deviatoric stress, laboratory triaxial tests are basically of two types.

- ✔ Triaxial compression test, and
- ✔ Triaxial extension test

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

- **Triaxial Test**

In fact, in the field there are also three ways in which the stresses are changed: by compression, extension and direct shear. Thus, the question of triaxial compression and triaxial extension comes in to picture, though triaxial compression is the most common type of test.

With the first two that is compression and extension, the shear stress is applied indirectly by changes in the principal stresses (σ_v and σ_H or σ_1 and σ_3) while the latter occurs where the shear stress is applied directly.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Test

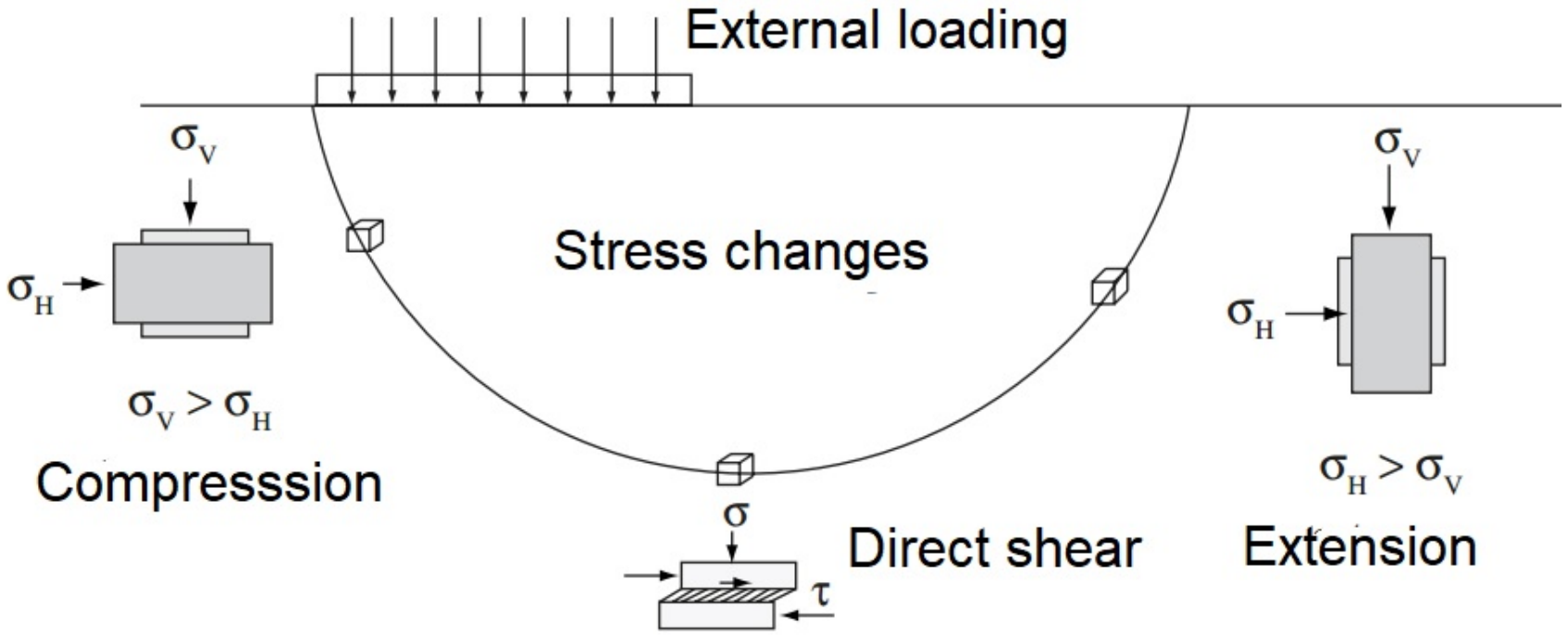


Fig. 6.29 Stress changes within soil mass.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test

Triaxial tests constituents of three steps.

Step 1: Preparation of soil sample

Step 2: Saturation of soil sample (using cell pressure and back pressure)

Step 3: Application of cell pressure (Loading stage 1 representing construction stage in the field)

Step 4: Application of additional load or deviatoric load and shearing (Loading stage 2 representing live load).

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test

The test procedure may be stated as follows.

- A cylindrical soil specimen usually 38 mm in diameter and 76 mm in length is prepared and encased in a thin rubber membrane which extends over a top cap and bottom pedestal.
- The specimen of other dimensions can also be used provided that the length to diameter ratio lies in between 2:1 to 3:1. Usually, three specimens are used for three loading conditions (cell pressure).

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test

- To facilitate proper drainage condition of the sample, porous stones/discs and filter papers are used at the edges of the sample.
- For enhancing further drainage, a slitted filter paper strip is also wrapped around the cylindrical specimen prior to encasing it within the rubber membrane.
- The specimen assembly is then placed at the base of triaxial setup within a cylindrical perspex cell. The porous discs at the top cap and bottom of the specimen are connected via nylon tubes

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test

- Another connection from cell to outside is provided to facilitate cell pressure. A photograph of triaxial test set up with soil test specimen and accessories are shown in Figs. 6.30(a) and 6.30(b). A schematic diagram of triaxial setup is shown in Fig. 6.30(c).
- The prepared soil sample is properly placed in the cylinder cell, the cell is then filled with water via cell pressure control tube.
- The specimen is made saturated maneuvering cell pressure and back pressure (using top

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

- **Triaxial Compression Test**

- On ascertaining saturation of the specimen an all around cell/confining pressure is applied. The cell pressure is designated as σ_3.
- Then a vertical load commonly referred to as deviator stress, $\Delta\sigma_1$ is applied and steadily increased until the failure of the specimen. Thus the normal stress in the vertical direction at failure becomes, $\sigma_1 = \sigma_3 + \Delta\sigma_1$.
- The pore water pressure in the sample can be measured through a porous disc at the base pedestal and connected through a water filled duct to a pressure transducer

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test



Fig. 6.30(a) Photograph of triaxial cell.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test



Fig. 6.30 (b) Triaxial compression test specimen and accessories.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test

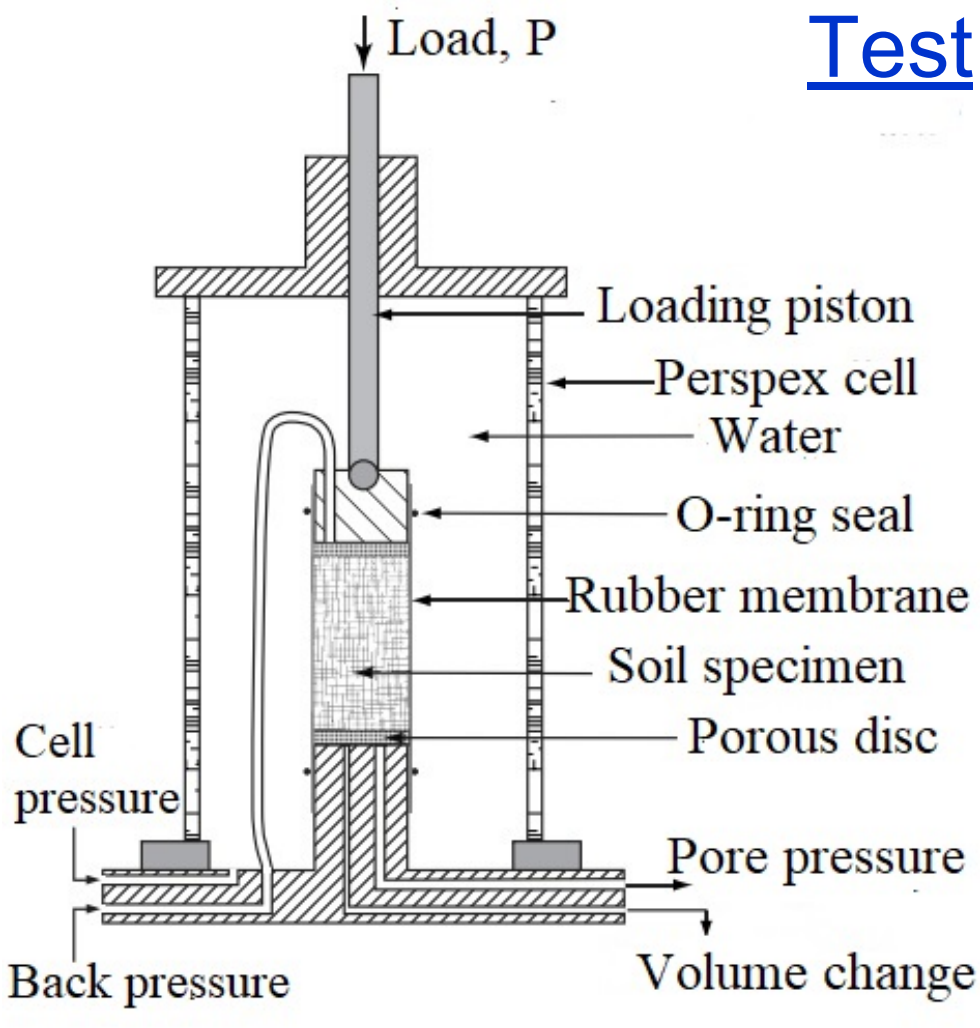


Fig. 6.30(c) A schematic diagram of triaxial compression test set up.

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test

- To minimize friction at the top and bottom of sample and to allow an unrestricted lateral deformation during shear, greased rubber discs are placed between sample and end caps.
- The procedure is repeated for two other identical test specimens using different confining pressures and corresponding deviator loads are recorded. A Mohr's circle for each combination of confining pressure (σ_3), and total axial stress ($\sigma_3 + \Delta\sigma = \sigma_1$) is constructed. The tangent to the resulting circles becomes the Mohr envelope. They are illustrated in Figs. 6.30(a) 6.30(b).

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test

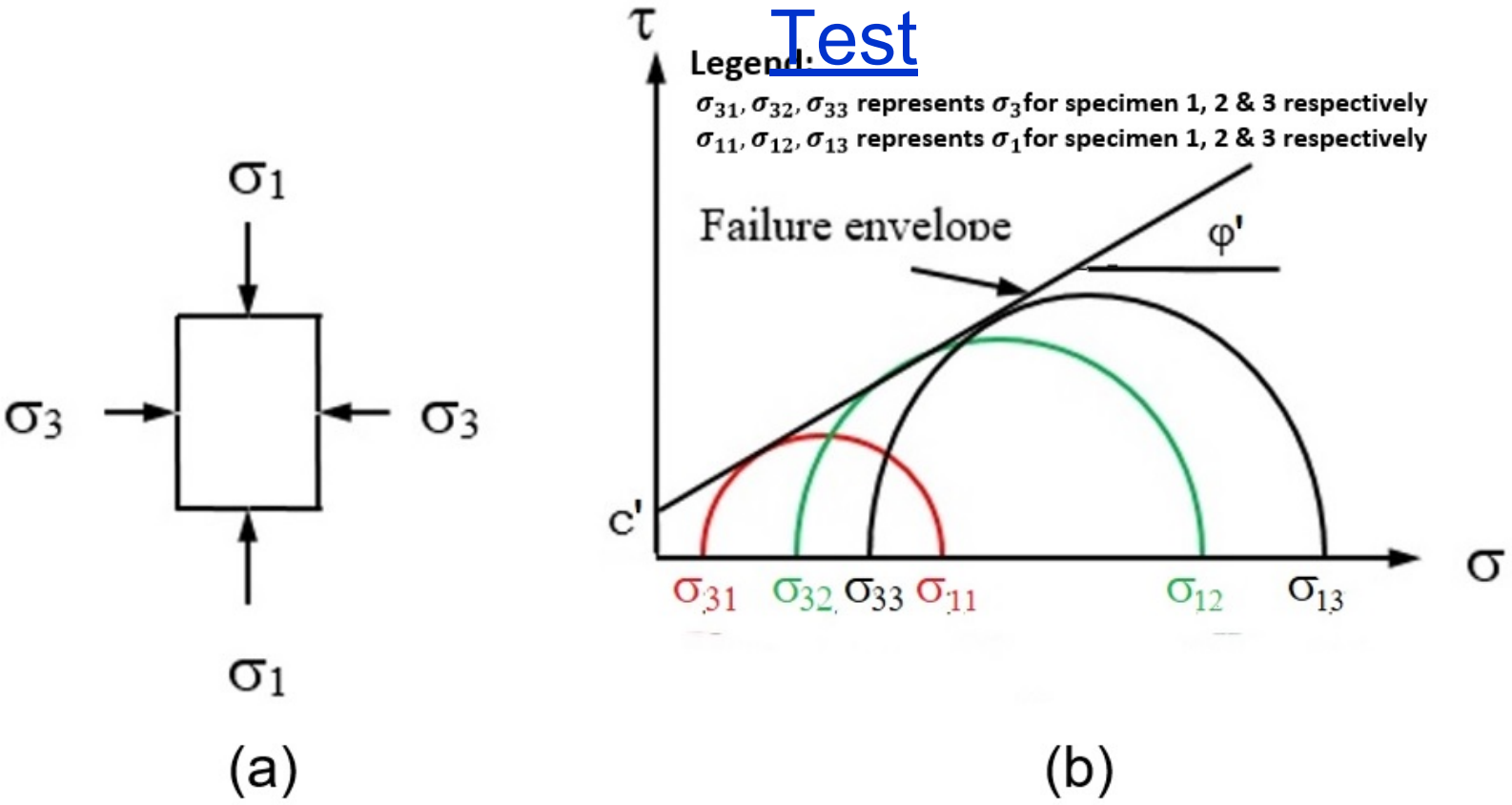


Fig. 6.31 Triaxial test; (a) Representation of stressed specimen; (b) Mohr's circle and failure envelope.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test

The triaxial test can be manipulated to represent reasonable insitu conditions. The apparatus is designed to allow;

- Control of confining pressure and deviator stress
- Control of pore water pressure and drainage of water to simulate undrained and drained conditions
- Saturation of soil sample

As such, a wide variety of tests are possible using triaxial testing apparatus especially to depict insitu loading and drainage conditions, as illustrated in Fig.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test:

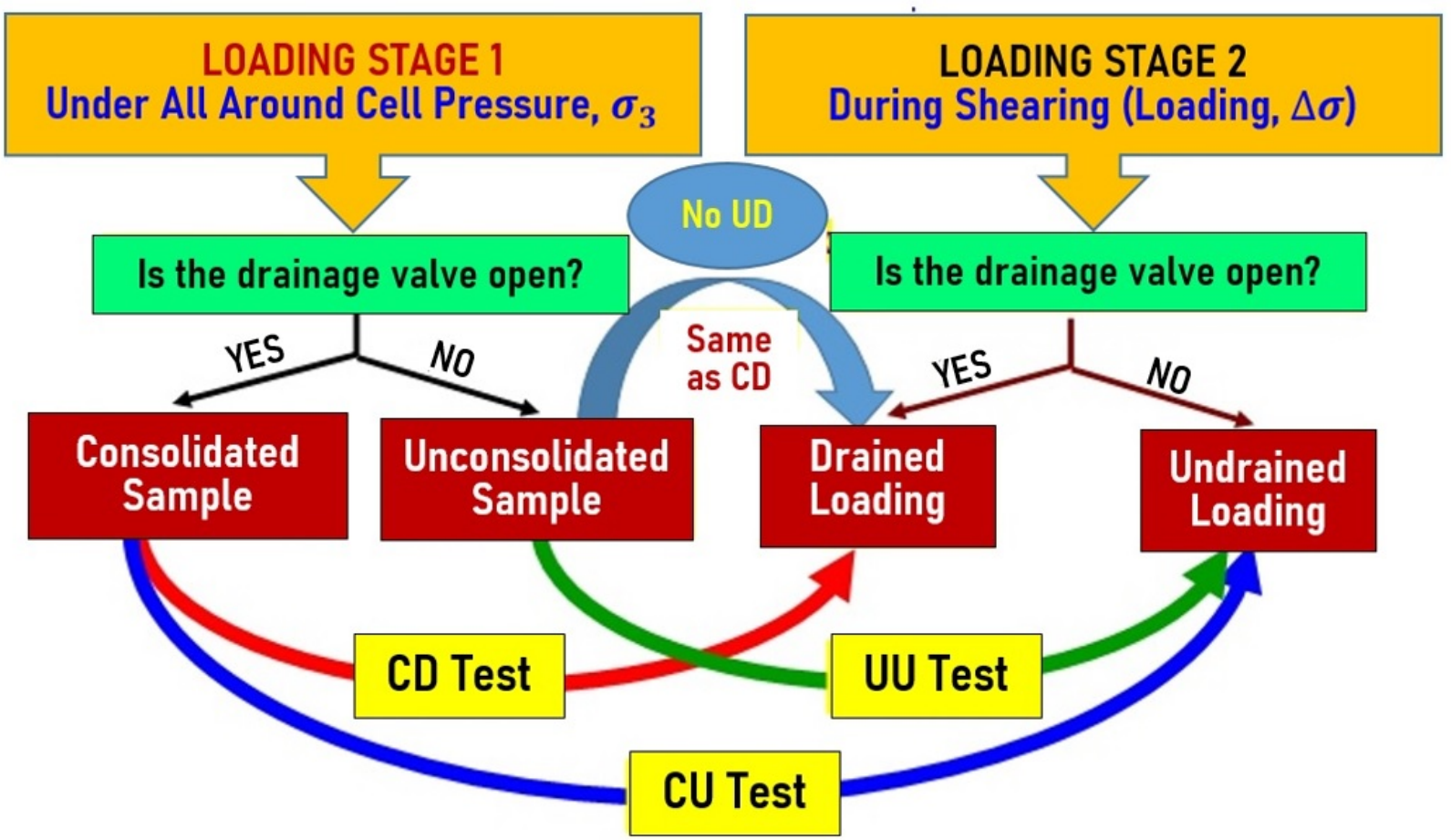
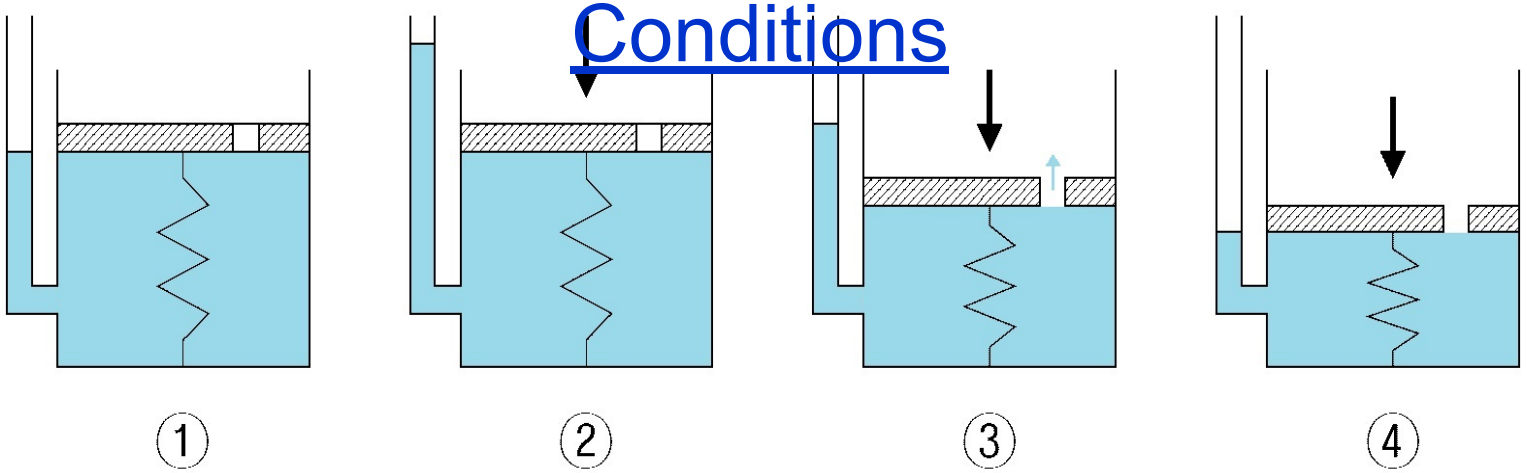


Fig. 6.32 Types of triaxial test depicting drainage and loading conditions.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test: Spring Analogy of Drained and Undrained Conditions



1. Initial Condition before the application of load; $u = 0$
2. Immediately after load application; t ≅ 0; $\sigma = u$
3. When t > 0; $\sigma = \sigma' + u$
4. When t = ∝; $\sigma = \sigma'$

Fig. 6.33 Triaxial test: spring analogy depicting porewater pressure

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test: Types

Types of triaxial tests are further illustrated by an analogous example of entering 50 and 60 people groups, for example, respectively in a room at two different times through the entrance, and leaving the room depending on whether the exit of the room is open closed at the time of entry. Thus, final number of people staying at the room can be estimated for different exit provisions from the room. This can be considered analogous to the porewater pressure generated within triaxial soil specimen at two different loading stages. The analogy is demonstrated through Figs. 6.34(a) to 6.34(d) that suggests that triaxial shear tests could be of 3 types like UU, CU and CD

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test: Types

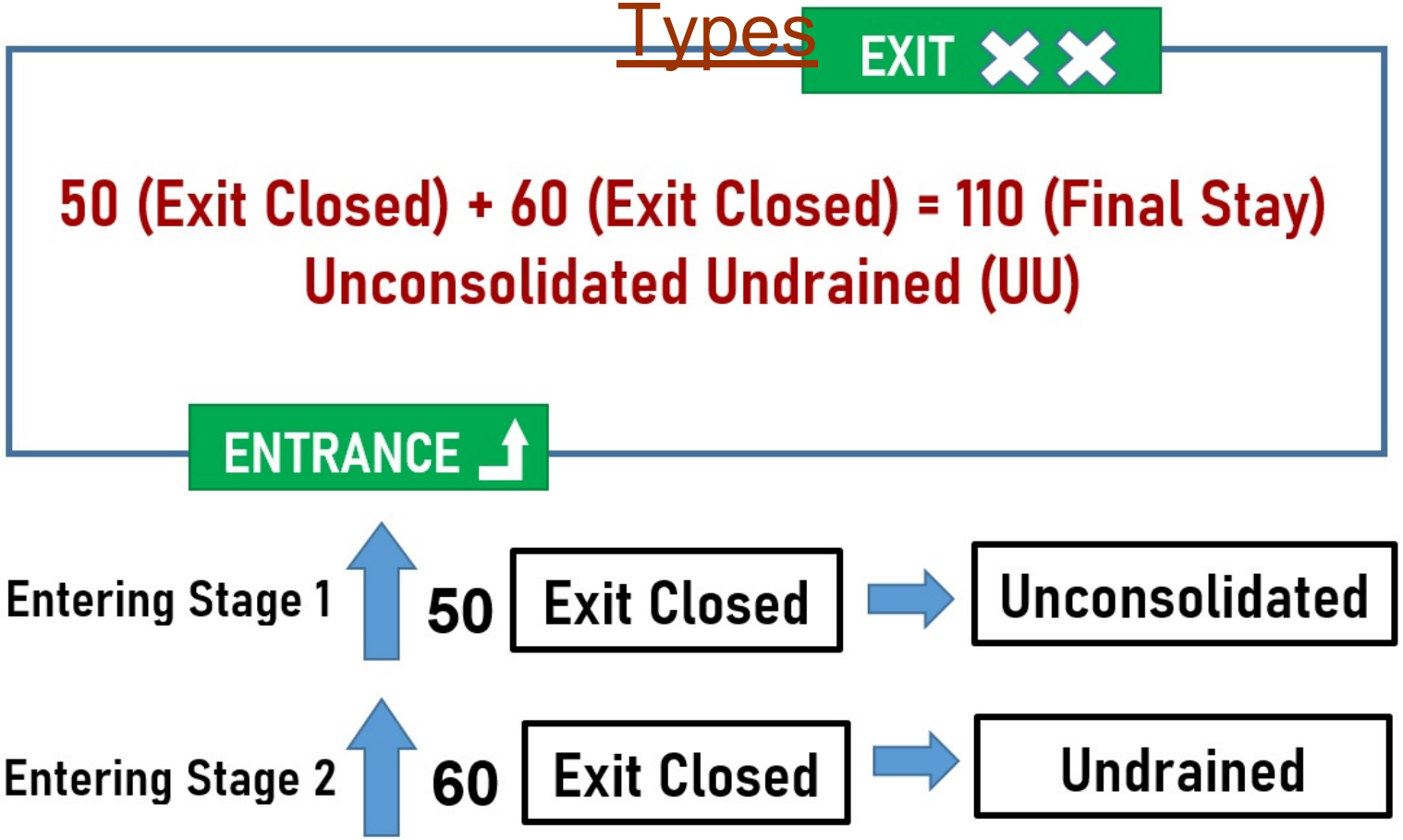


Fig. 6.33 Analogous example of UU triaxial test

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test: Types

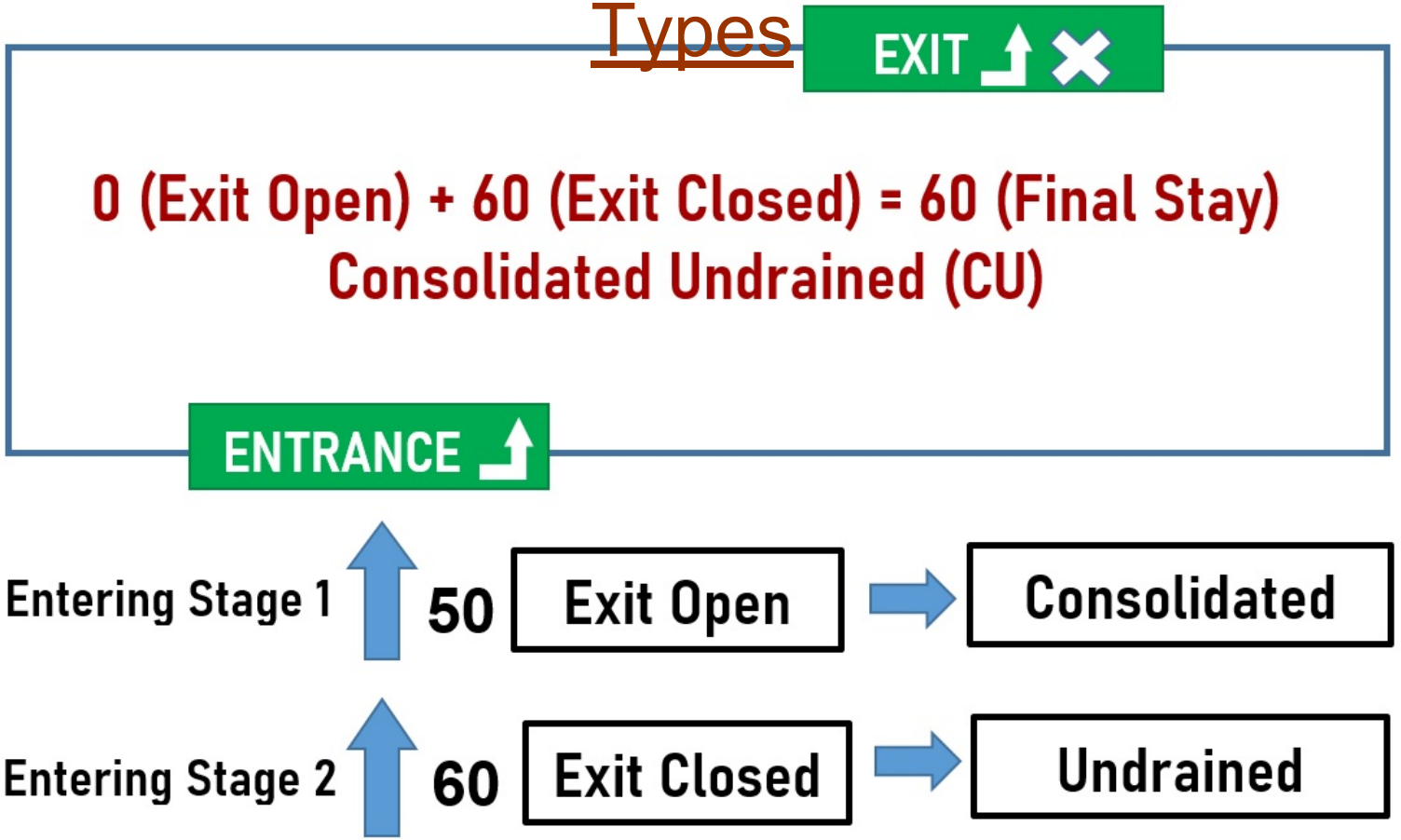


Fig. 6.33(b) Analogous example of CU triaxial test

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test: Types

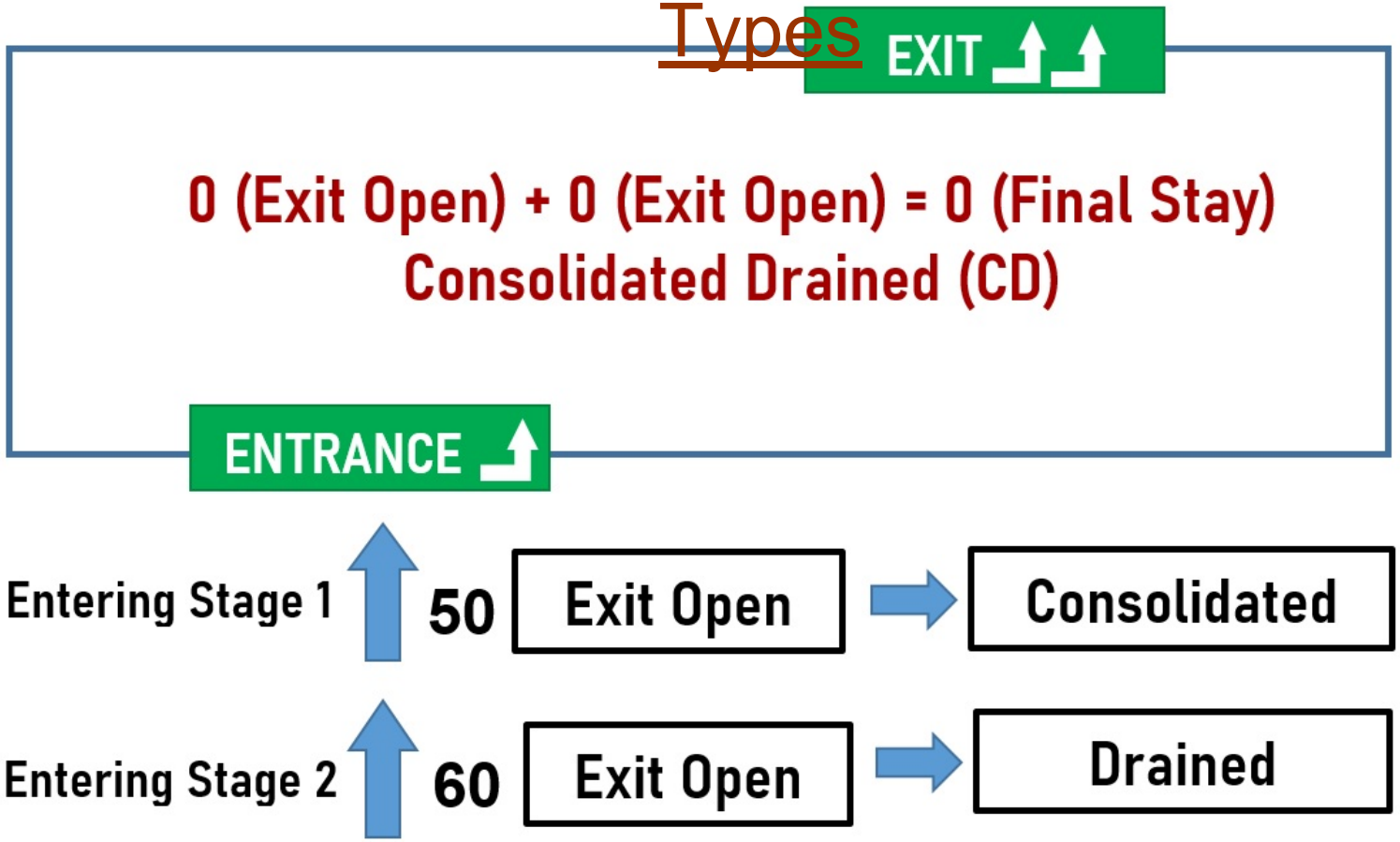


Fig. 6.33(c) Analogous example of CD triaxial test

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test:

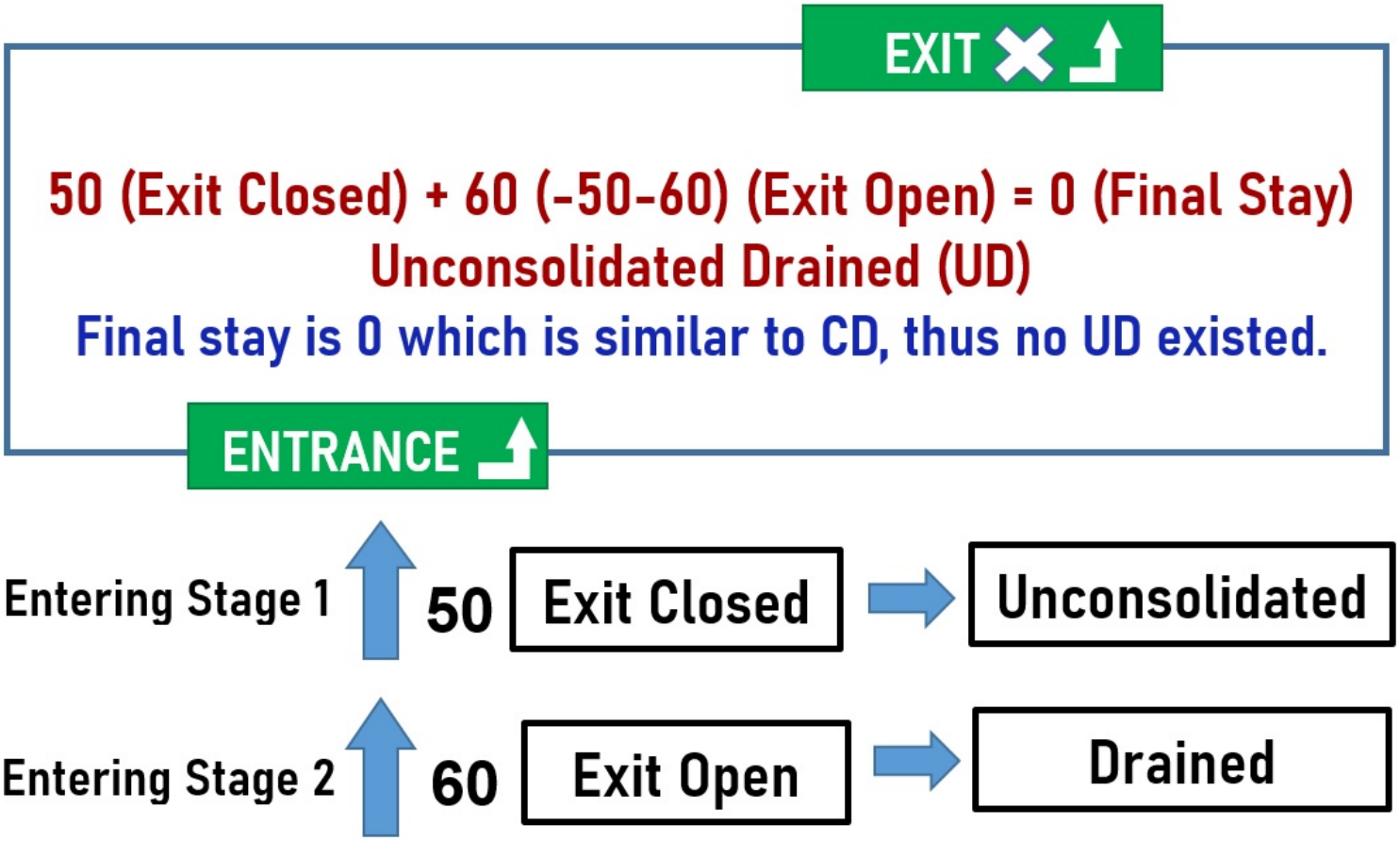


Fig. 6.33(d) Analogous example of UD triaxial test that does not exist

6.7.8 Determination of Shear Strength Parameters

Triaxial Compression Test: Types

Thus, the three test conditions are adequate for most of the engineering purposes. They are sometimes also designated as Quick or Q, Qc or R and slow or S test respectively.

- Unconsolidated Undrained test (UU/Quick/Q test)

  [A special UU-test done on saturated cohesive soil that is also known as Quick Undrained or QU-test]
- Consolidated Undrained test (CU/ Q_c/ R test*)
- Consolidated Drained Test (CD/ Slow/ S test)

(*it is simply because, R lies between Q and S in

6.7.8 Determination of Shear Strength Parameters

Applications of the UU – Triaxial Test

UU-test can be used to simulate the soil conditions in practical problems, where there is imperceptible pore water pressure dissipation during the imposition of external loading.

In other words, the loads are placed over such a short period that there is only negligible outflow of water from soils of low permeability, hence there is not time for consolidation and strength increase.

The soil is therefore weakest at this juncture, but gets stronger with the passage of time.

6.7.8 Determination of Shear Strength Parameters

Applications of the UU – Triaxial Test

Some of the practical problems are:

- Excavation and end of construction of foundations in partially saturated soil.
- Stability of compacted fill of low permeability.
- During the construction of earth dam, where large pore pressures can develop, owing to the speed of construction.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Application of Quick-Undrained QU-Test

Similar to the UU-test, the QU-test is applied to the determination of short-term stability of structures, constructed in soils of low permeability.

The test is not applicable, when either the period of construction is exceedingly long or the drainage path from the soil is too short as these allow faster outflow of water than assumed and simulated. A few examples are as follows.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Application of Quick-Undrained QU-Test

(a) Retaining Walls: Total stress analysis applies when −

- The soil is saturated and little or no drainage occurs during construction.
- Temporary excavation is supported by a structure and insignificant pore water outflow is expected during its short life-time.

(b) Excavation for foundation:

- The clay is weakest at the end of excavation, when the removal of the

6.7.8 Determination of Shear Strength Parameters

Application of Quick Undrained QU-Test

(b) Excavation for foundation (Contd.):

- After the construction of the footing, the applied load, which is usually larger than the removed overburden reduces the uplift and then strengthens the soil by consolidating it.
- Short-term stability of natural ground under embankment, yet to be constructed, may be analysed in terms of total stresses, proving the period of construction is too short for drainage to occur.

6.7.8 Determination of Shear Strength Parameters

Some Applications of CU-Test

- **(a) Foundations for structures,** where the weight of the structure consolidates the supporting soil and then, there is a sudden increase in the loading. This occurs repeatedly in water tanks and grain silos.

- **(b) Foundation of earth dams and embankments,** where some consolidation could occur over extended construction period.

- (c) To determine the effective stress parameters c' and φ' of saturated soil.

6.7.8 Determination of Shear Strength Parameters

Some Applications of CD Test

In general, the test is used in problems, where water can drain under loading.

- Foundations and piles in sand or gravel.
- Retaining walls in sand or gravel.
- Sudden variation of water level in sand or gravel slopes of rivers or reservoirs e.g. sudden drawdown.
- Earth retaining structures and fills.
- Foundation of earth or other structures, where some consolidation could occur due to slow progress of construction.
- Stability during construction in fissured clay

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Preparation, Saturation and Testing of Triaxial Specimen

As mentioned earlier, there are four successive steps in a triaxial test namely preparation of soil specimen, saturation of soil specimen, application of cell pressure and application of excess axial stress or deviator stress.

The preparation of soil specimen, for all the tests is essentially similar with only a minor exception that in case of UU test the drainage accessories like wrapping of filter paper around the soil specimen can be omitted.

By increasing the cell pressure and the back pressure in a triaxial specimen of clay the voids are eventually filled with water by water entering the specimen from

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Preparation, Saturation and Testing of Triaxial Specimen

- The procedure is first to apply a cell pressure and to record the pore water pressure. Then the back pressure (in the pore water) is increased and the pore pressure recorded when it virtually equals the back pressure applied. The back pressure must be about 10 kN/m^2 less than the cell pressure to ensure positive effective stress in the soil. The pore pressure coefficient B is determined from the relation $\Delta u_c = B\Delta\sigma_c$ and when the value of B reaches about 0.97 the specimen may be assumed to be sufficiently fully saturated.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Preparation, Saturation and Testing of Triaxial Specimen

- The dissipation of pore water pressure or drainage during third and fourth stages of testing depends on the test type.

Often pore water pressure is measured in consolidated undrained test to estimate the effective stress parameters. Such a test is designated as $\overline{CU}$ test. Table. 6.2 presents the drainage and loading conditions during third and fourth stages of all types tests.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Preparation, Saturation and Testing of Triaxial Specimen

Table 6.2 Drainage and loading conditions in triaxial tests

Type of test	Application of cell pressure	Application of additional axial stress
Unconsolidated Undrained (UU)	Drainage is not allowed. Sample is unconsolidated.	Drainage is not allowed. That is, failure occurs in undrained condition.
Consolidated Undrained (CU)	Drainage is allowed. That is, sample is consolidated.	Drainage is not allowed. That is, failure occures in undrained condition.
Consolidated Drained (CD)	Drainage is allowed. That is, sample is consolidated.	Drainage is allowed. That is, failure occurs in drained condition.

6.7.8 Determination of Shear Strength Parameters

Preparation, Saturation and Testing of Triaxial Specimen

The triaxial test data obtained from a test on each of the specimen are separately plotted as deviator stress versus strain diagram to obtain the peak or maximum deviator stress and the corresponding axial strain.

The excess pore water pressure reading is used to calculate the effective principal stresses where appropriate. Thus the principal stresses on the soil sample are estimated and Mohr circle are drawn.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Test: Parameters for Cohesive Soil

The shear strength of cohesive soils depends on factors like degree of saturation, stress history, loading rate and drainge conditions.

Cohesive soils are often treated in a saturated state to represent the worst condition that can prevail in the field.

6.7.8 Determination of Shear Strength Parameters

UU Triaxial Test on Unsaturated Cohesive Soil

During this test, the soil specimen is sheared without allowing the outflow of pore water. This is to simulate ground conditions in actual engineering problems, where there is no time for pore pressure dissipation, during the construction of structures in cohesive soils of low permeability. There are two standard tests of this type:

1. Undrained test without any saturation or on partially saturated soil.

2. Undrained test on saturated cohesive soils, usually referred to as ‘Quick’ or QU-test, applicable to both normally and overconsolidated clays

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

UU Triaxial Test on Unsaturated Cohesive Soil

- In UU-test without saturation, both the total stress as well as the pore pressure are measured. The presentation of the results are in terms of effective stresses, hence the Mohr's circle are drawn.

The failure envelope is normally curved when expressed in terms of total stresses, and approximates a straight line, when drawn in terms of effective ones. For this reason the use of the effective stress parameters are preferred. The shear strength parameters of unsaturated soil obtained from UU test are c'_u and φ'_u known as effective undrained cohesion and angle of friction respectively, Fig. 6.37.

6.7.8 Determination of Shear Strength Parameters

UU Triaxial Test on Unsaturated Cohesive Soil

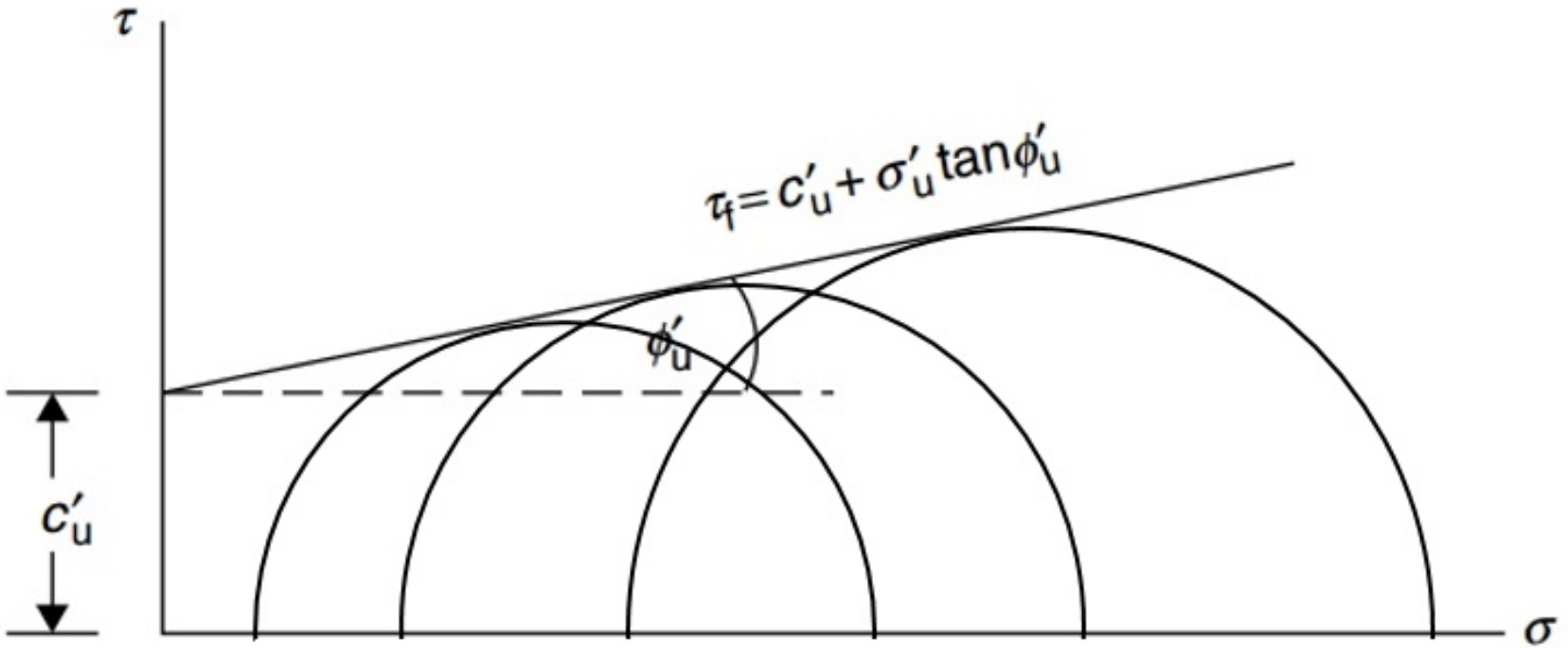


Fig. 6.37. UU Triaxial test on unsaturated cohesive Soil.

6.7.8 Determination of Shear Strength Parameters

UU Triaxial Test on Unsaturated Cohesive Soil

This test can be used to simulate the soil conditions in practical problems, where there is imperceptible pore water pressure dissipation during the imposition of external loading. In other words, the loads are placed over such a short period that there is only negligible outflow of water from soils of low permeability, hence there is no time for consolidation and strength increase.

The soil is therefore weakest at this juncture, but gets stronger with the passage of time. Some of the practical problems are stated as follows.

6.7.8 Determination of Shear Strength Parameters

UU Triaxial Test on Unsaturated Cohesive Soil

- Excavation and end of construction of foundations in partially saturated soil.
- Stability of compacted fill of low permeability.
- During the construction of earth dam, where large pore pressures can develop, owing to the speed of construction.

6.7.8 Determination of Shear Strength Parameters

UU Triaxial Test on Saturated Clay or Quick Undrained (QU) Test

- The QU-test is so called because the specimen is sheared, usually over a period of 15 to 20 minutes. Pore pressure is not measured and no drainage is allowed from the sample.

The test is carried out usually on saturated, cohesive soils and the results are presented in terms of total stresses and shear strength parameters c_u and φ_u. The failure envelope depends not only on the degree of saturation, but on the stress history of the soil.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

UU Triaxial Test on Saturated Clay or Quick Undrained (QU) Test

- In this test a minimum of three soil specimens are subjected to different confining pressure σ_3 and then loaded to failure.

A cell confining pressure and slightly small amount of back water pressure in several steps if required may be applied to the soil and allowed to stay for some time to saturate the sample.

The degree of saturation of the soil specimen can be checked from the pore pressure parameter, B.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

UU Triaxial Test on Saturated Clay or Quick Undrained (QU) Test

When a saturated cohesive soil is subjected to a stress increase, it will respond simply by increasing the pore water pressure by that amount.

So, any change in all around pressure or axial stress will not bring any change in the structure or arrangement of soil particles within the soil mass.

Hence the undrained strength remains constant and independent of cell pressure. Thus, the soil will fail at an equal deviator stress for all the 3 cell

6.7.8 Determination of Shear Strength Parameters

UU Triaxial Test on Saturated Clay or Quick Undrained (QU) Test

Deviator stress and hence the diameter of the Mohr's circle being equal for all cell pressures, the failure envelope will be a horizontal line thus depicting that only cohesion parameter will contribute to generate shear strength of soil.

The cohesion parameter at this condition is termed as undrained cohesion and designated as c_u. The other parameter φ would be zero. A typical Mohr's circle and

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

UU Triaxial Test on Saturated Clay or Quick Undrained (QU) Test

The failure envelope in this case can be defined as:

$$\tau_d = s_u = c_u \qquad (6.9)$$

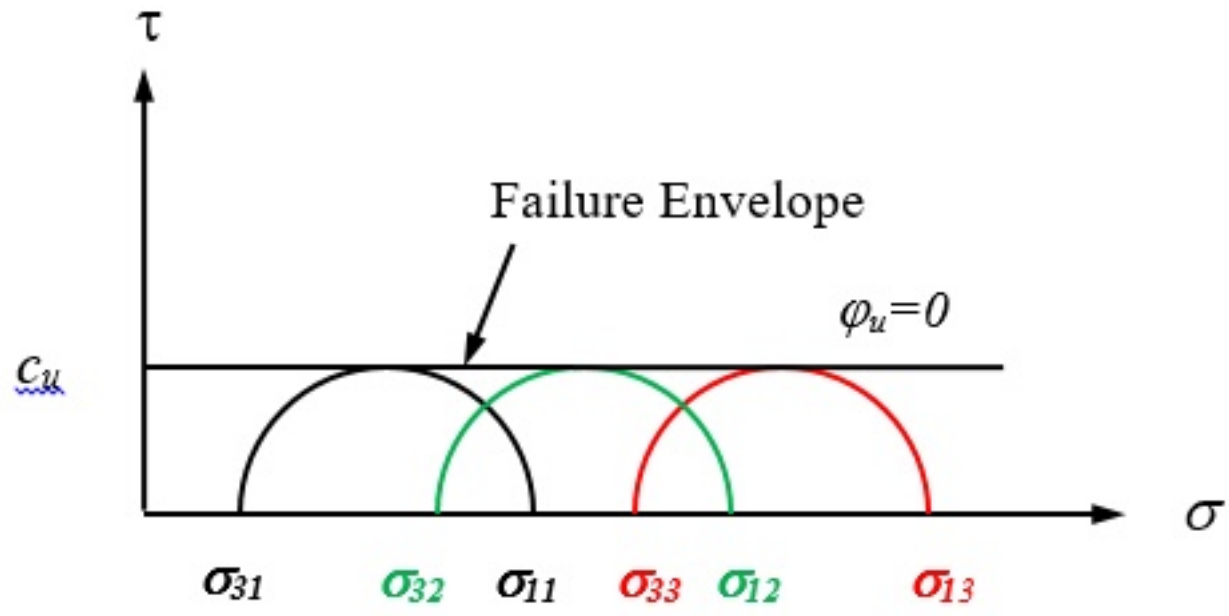


Fig. 6.38 Unconsolidated undrained (UU) test of saturated clay

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

CU Triaxial Test of Clayey Soil

In this test soil sample is initially consolidated under confining pressure. The volume change reading can also be taken at this stage to estimate the initial void ratio of the specimen.

After consolidation, the confining pressure and back pressure may be increased by equal amount thus keeping the effective cell (confining) pressure constant.

The back pressure is applied to saturate the soil sample and to facilitate the measurement of negative pore water pressure during application of

6.7.8 Determination of Shear Strength Parameters

CU Triaxial Test of Clayey Soil

- The soil sample is sheared at this elevated cell pressure σ_3 with a deviator stress of $\Delta\sigma$, that is axial stress of $\sigma_1 = \sigma_3 + \Delta\sigma$.

Morh's circles are drawn for the three specimens to determine the failure envelope and strength parameters in terms of total stress, Fig. 6.39(a).

The strength parameters are designated as c_{cu} and φ_{cu}. The failure envelope will be represented by Eq. (6.10).

$$\tau_f = c_{cu} + \sigma \tan \varphi_{cu} \qquad (6.10)$$

6.7.8 Determination of Shear Strength Parameters

CU Triaxial Test of Clayey Soil

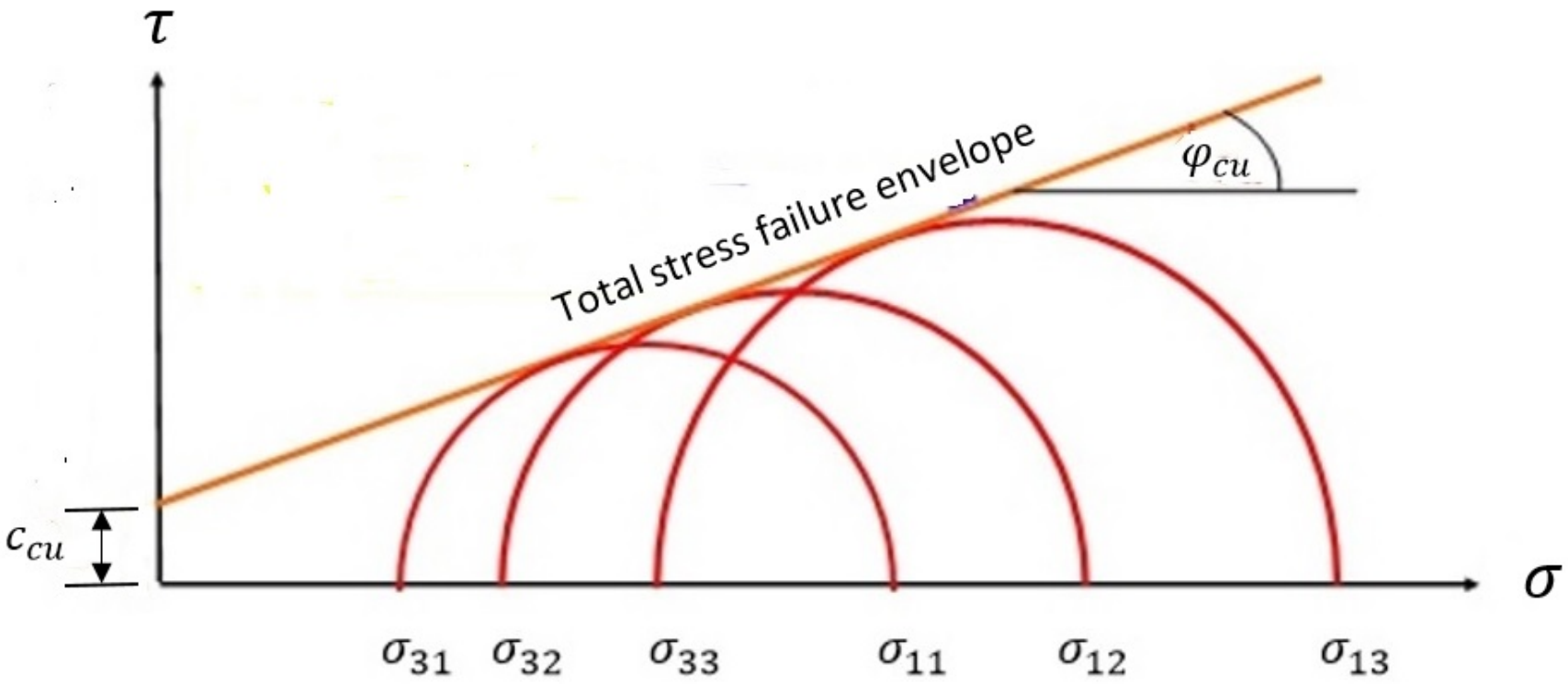


Fig. 6.39 Consolidated undrained (CU) strength of saturated clay

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

CU Triaxial Test of Clayey Soil

- Consolidated undrained test can also be carried out with pore water pressure measurement while shearing the soil sample.

The effective stress parameters in such a case are determined drawing Mohr circle and hence the failure envelope taking in to consideration the effective cell pressure σ_3' as $(\sigma_3 - u_f)$ and effective axial stress σ_1' as $(\sigma_1 - u_f)$. Here u_f is the pore water pressure at failure.

Effective minor and major principal stresses σ_3' and σ_1' respectively are to be computed for all the 3 specimens.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

CU Triaxial Test of Clayey Soil

- **The effective strength parameters are c' and φ' respectively for cohesion and friction and the equation for failure envelope (Figs. 6.40a & 6.40b) is given by:**

$$\tau_f = c' + \sigma' \tan \varphi' \qquad (6.11)$$

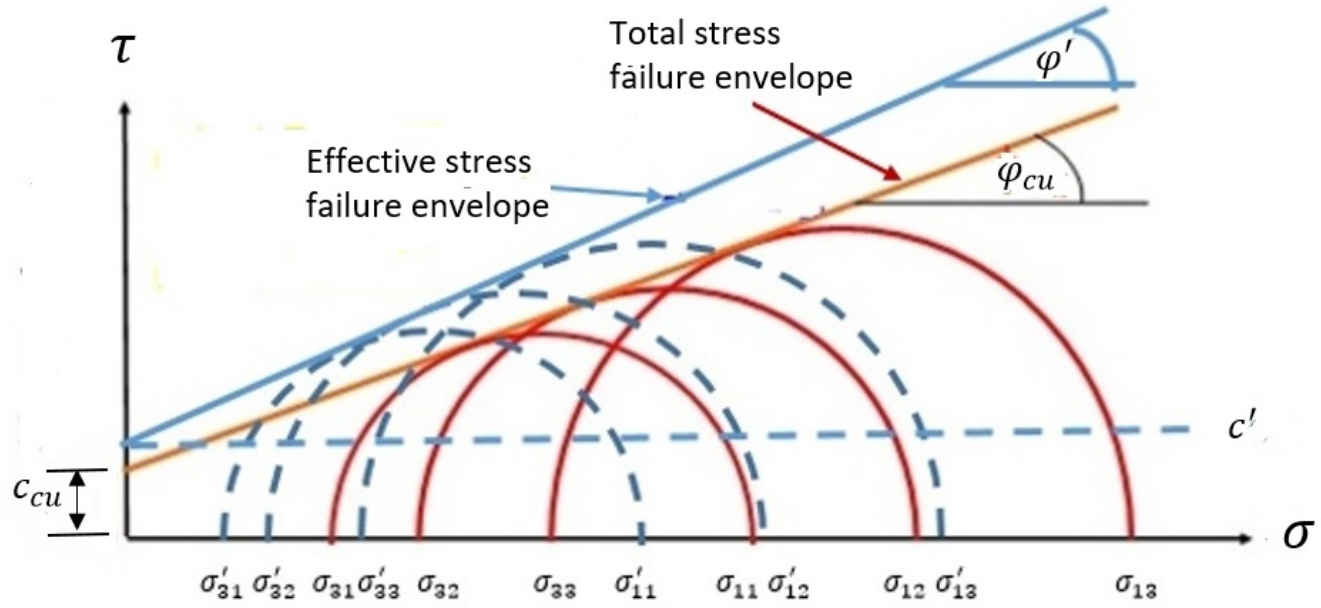


Fig. 6.40(a) Total and effective stress parameters from a

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

CU Triaxial Test of Clayey Soil

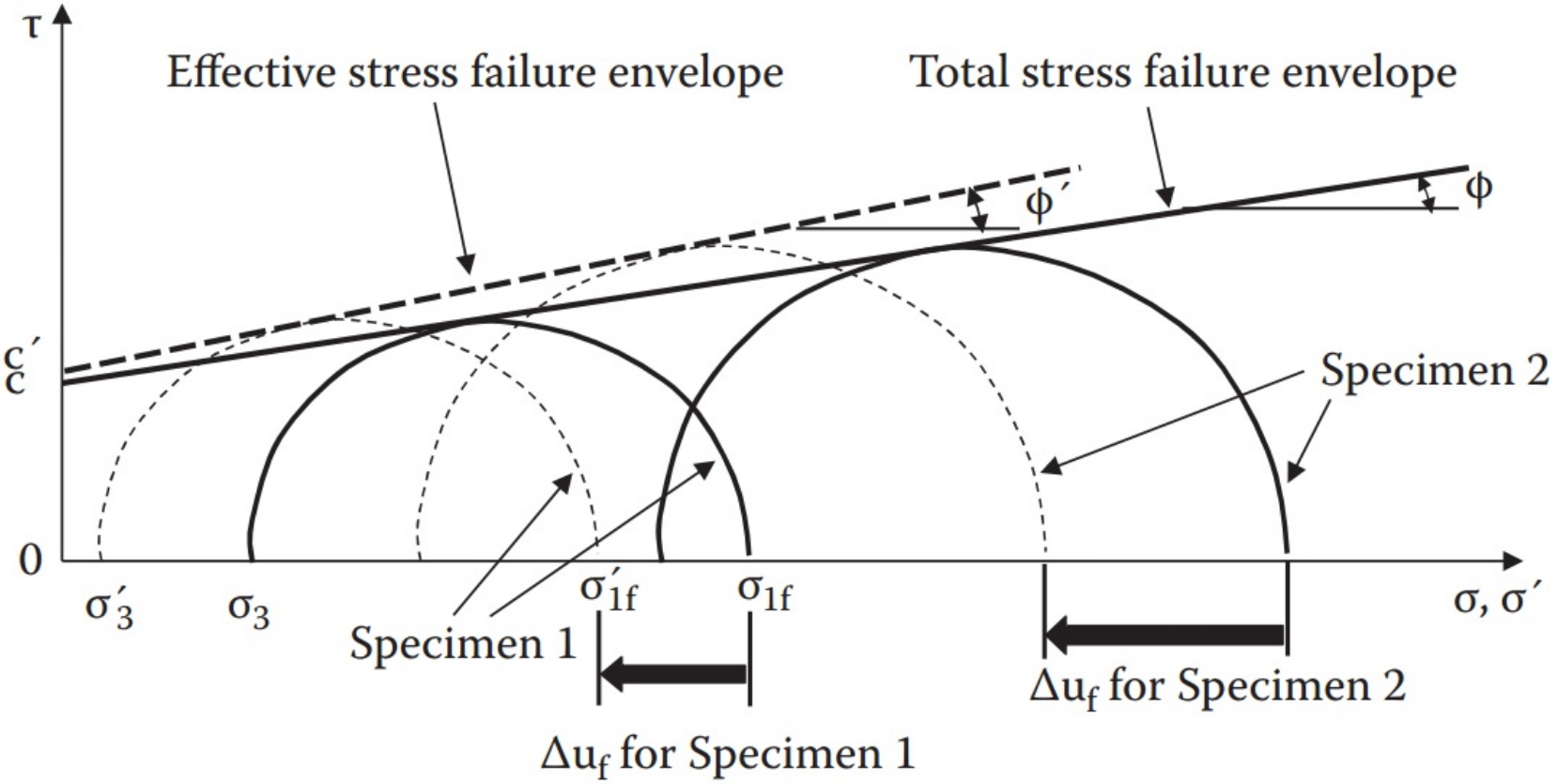


Fig. 6.40(b) Total and effective stress parameters from a CU test.

6.7.8 Determination of Shear Strength Parameters

CD Triaxial Test of Clayey Soil

- In this type of test, the sample is first consolidated under confining or cell pressure. Back pressure may be applied in a similar manner to that of CU test to saturate the sample, if necessary.

Thus the confining pressure σ_3 here is the effective confining pressure c_3'. The specimen is then sheared to failure with a very small strain rate so that no excess pore water pressure can develop on the failure surface.

The pore water pressures, thus, remains zero at all the stages of the test thus prevailing effective stress conditions all through

6.7.8 Determination of Shear Strength Parameters

CD Triaxial Test of Clayey Soil

- The volume change reading can also be taken to examine the void ratio of the soil at failure condition. Mohr's circle in terms of effective minor and major principal stresses at failure for all the three specimen are drawn to obtain the effective shear strength parameters c' and φ'.

The failure envelope is of typical in nature as shown in Fig. 6.40, and c and φ would be replaced by the parameters c' and φ'. The failure envelope can be given by:

$$\tau_f = c' + \sigma' \tan \varphi' \qquad (6.12)$$

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

CD Triaxial Test of Clayey Soil

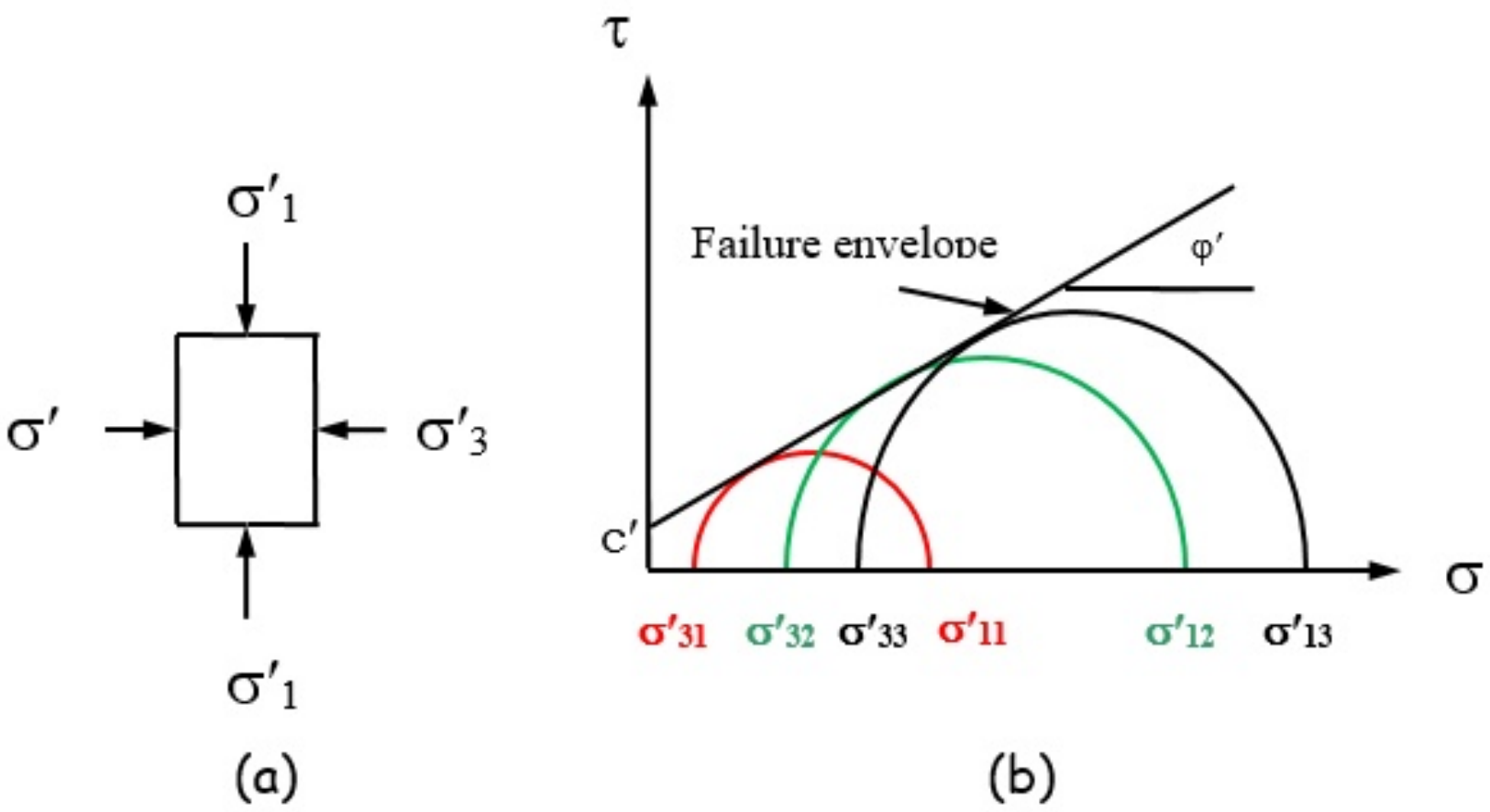


Fig. 6.41 Effective stress parameters from a CD test of clay soil.

6.7.8 Determination of Shear Strength Parameters

CD Triaxial Test of Clayey Soil

- The parameters c' and φ' are often replaced by c_{cd} and φ_{cd} to remind that these parameters are for drained condition.

The parameters obtained from a drained test always refers to effective stress conditions though these can also be obtained from a CU test and that should be specifically spelled out.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Test of Sandy Soil

When load is applied to a soil, the soil is stressed and causes the soil-water matrix to compress and some of water is squeezed out.

Because sandy soils have a high hydraulic conductivity (coefficient of permeability), this water is able to move quickly and easily.

The potential drainage rate is at least as high as the loading rate. This condition is known as drained condition and the stresses acting on a sandy soil are always the effective stresses.

6.7.8 Determination of Shear Strength Parameters

Triaxial Test of Sandy Soil

Hence, in case of sandy soils the measured strength parameters from shear tests are always the effective one and it is almost independent of type of triaxial tests.

As sandy soil has smaller surface forces, the cohesion parameter can be considered as zero (thus named as cohesionless soil) the strength envelope can be explained by the following equation.

$$\tau_f = \sigma' \tan \varphi' \qquad (6.13)$$

As such, only one Mohr's circle is required to define the failure envelope as another point is the origin of $\sigma - \tau$ coordinate system.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Test of Sandy Soil

- However, confining pressure has a little effect on strength the parameter and as such, conventionally three specimen are tested at different confining pressures.

Mohr's circles are drawn to obtain the failure envelope of the form represented by Eq. (6.13) and thus the angle of internal friction φ' is determined.

A typical Mohr circle diagram for triaxial test on dry cohesionless soil is shown in Fig. 6.42, where the strength parameter c is zero.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Triaxial Test of Sandy Soil

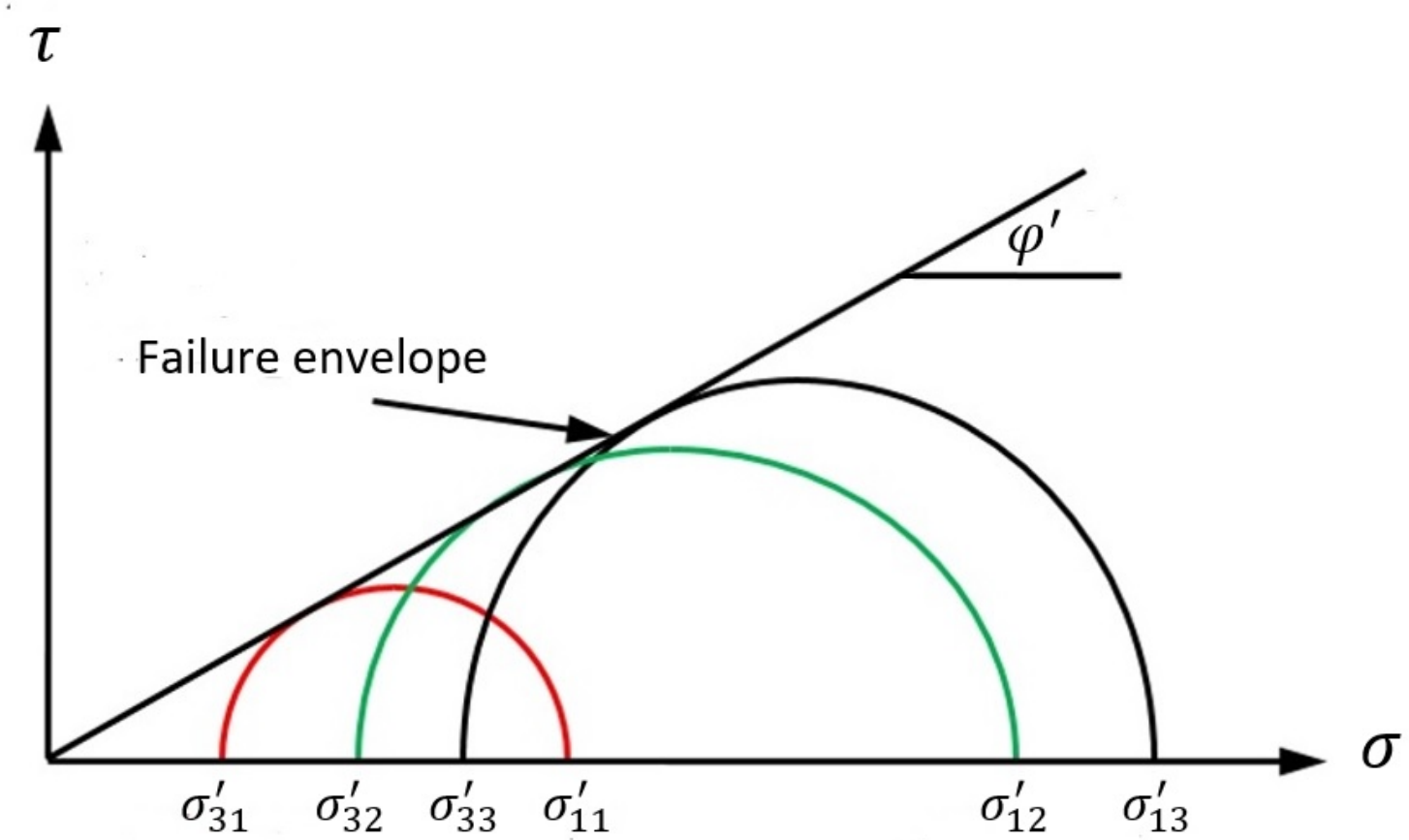


Fig. 6.42. Shear strength parameters of sandy soil from a triaxial test.

6.7.8 Determination of Shear Strength Parameters

Triaxial Test of Sandy Soil

A moist sandy soil might show a very small value of apparent cohesion which is due to surface tension.

In all the shear strength test of sand, volume change is a very important factor. Hence, it is preferable to record the volume change readings for better explanation of the test results.

Due to the mode of formation of failure planes, the shear strength parameter φ' obtained from direct shear test overestimates the triaxial φ' value approximately by 10%.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Shear Strength and Deformation of Sandy Soil

Figures 6.43(a) and 6.43(b) show the results of triaxial tests on samples of a sand prepared in a loose and a dense state tested at the same confining pressure.

With the dense sample, the deviator stress increases to reach a clear peak value and then decreases to eventually level off at a constant value. The volume shows an initial slight decrease followed by a steady increase and then levels off to a constant value. The loose sample, however, does not show a clear peak value of deviator stress.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Shear Strength and Deformation of Sandy Soil

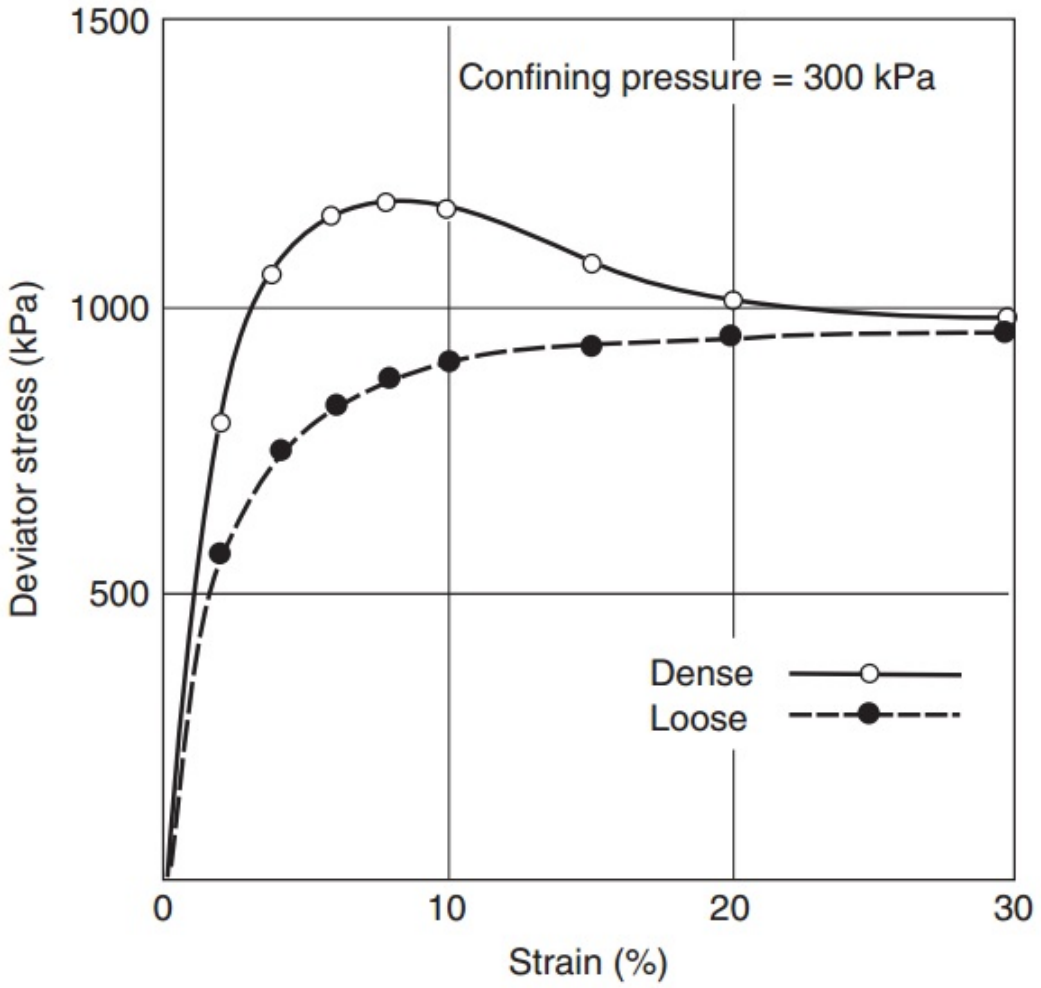


Fig. 6.43(a) Drained triaxial test on sand in loose and dense state; (a) Deviator stress strain diagram.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Shear Strength and Deformation of Sandy Soil

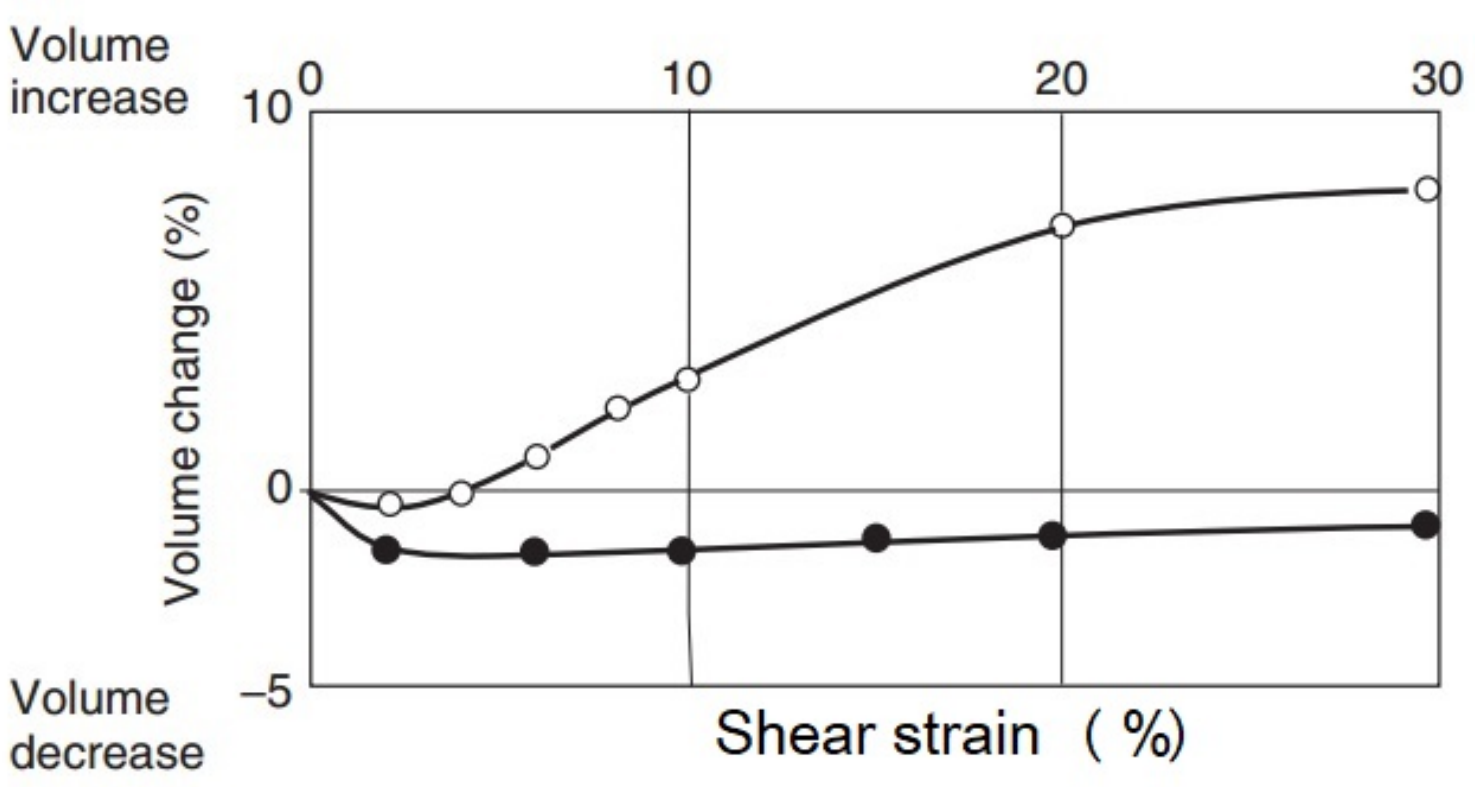


Fig. 6.43(b) Drained triaxial test on sand in loose and dense state; (b) Deviator stress strain diagram.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Shear Strength and Deformation of Sandy Soil

The strength simply rises and levels off to a constant value, which is very close to the value obtained from the dense sample.

The volume of the loose sample shows a slight decrease before leveling off to a constant value. In each test, both the deviator stress and volume become constant at large strains. If measurements are made of the density (or void ratio) of the samples once these “constant” or ultimate states are reached, it is found that the values are essentially the same. This ultimate state is known as the critical state. In this state, deformation will continue at constant deviator stress and constant volume.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Shear Strength and Deformation of Sandy Soil

- Figure 6.44 shows results of a set of drained shear box tests on identical sand samples plotted as stress versus displacement curves and failure linesin terms of both the peak strength and critical state strength. Sands are by nature cohesionless, so the failure lines normally go through the origin. Two values of the friction angle φ' are thus determined, the peak value (simply designated φ' and the critical state value, which is commonly designated φ'_{crit} or φ'_{cv}. The cv suffix refers to constant volume. Typical values of φ' for sand are between about 35° and 45°, the lower values normally associated with the loose state and the high values with the dense state.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Shear Strength and Deformation of Sandy Soil

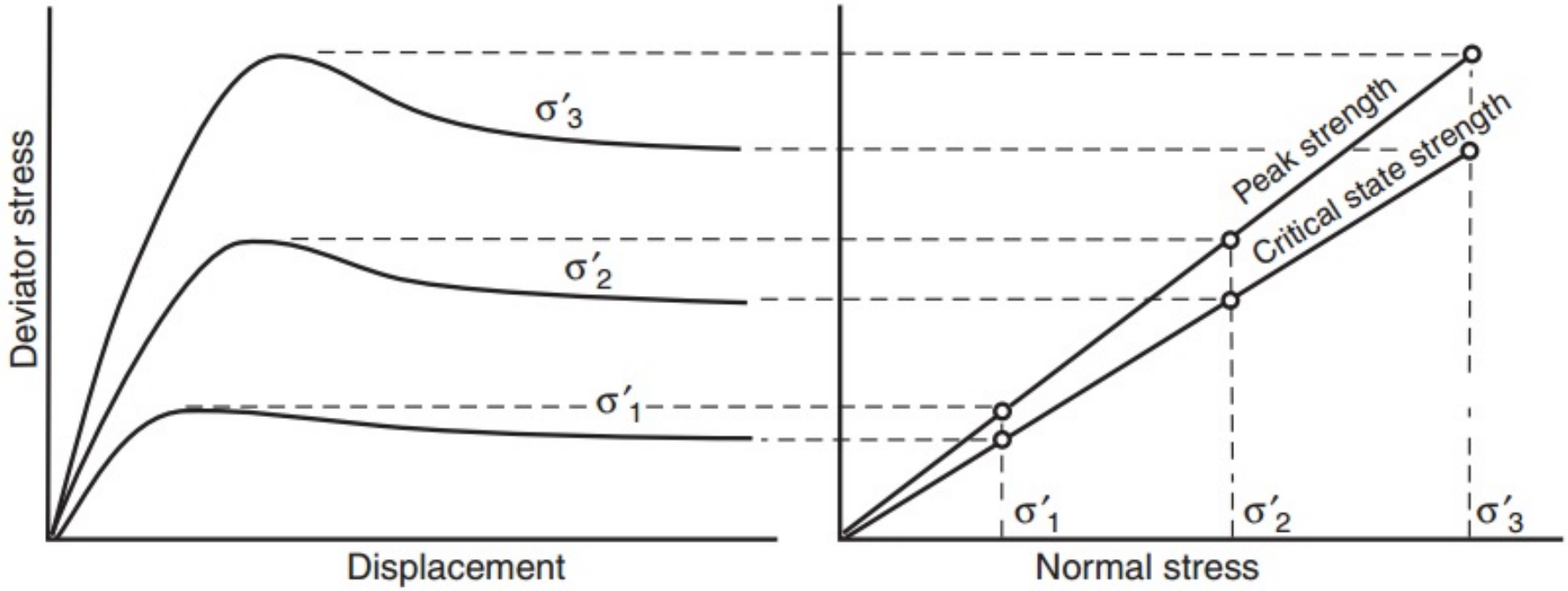


Fig. 6.44 Peak strength and critical state strength of sand.

6.7.8 Determination of Shear Strength Parameters

Normally Consolidated and Over Consolidated Clay

Clay may exist in the field under two conditions:

- Existing effective overburden pressure (σ'_o) is greater than or equal to the maximum of effective pressures subjected in the past called overconsolidation pressure (σ'_{oc}). This is normally consolidated soil.
- Existing effective overburden pressure (σ'_o) is less than the maximum past pressure (σ'_{oc}). This type of soil is known as overconsolidated soil.

The ratio σ'_{oc}/σ'_o is known as over consolidation ratio (OCR).

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Normally Consolidated and Over Consolidated Clay

Strength characteristics of normally consolidated and over consolidated clays (saturated) differ because of the fact that their depositional modes and drainage behaviour are responsible to impart the cohesion and frictional properties between the particles. As such the strength characteristics of clay soils are dependent on OCR that can be described as under.

- The UU test on both normally consolidated and overconsolidated clays gives c_u parameter and shear strength in this case equals to c_u only.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Normally Consolidated and Over Consolidated Clay

- For a CU test with pore pressure measurement or CD test, normally consolidated soil gives φ' (drained φ) parameter resulting in a zero cohesion intercept.

For over consolidated soils we get both c' and φ'. The failure envelopes for overconsolidated clays are nonlinear. For a over consolidated soil, if the failure envelope of the normally consolidated range is extended backwards it will pass through the origin giving $c' = 0$. Figure 6.45 illustrates the shear parameters as obtained from NC and OC range of loading.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Normally Consolidated and Over Consolidated Clay

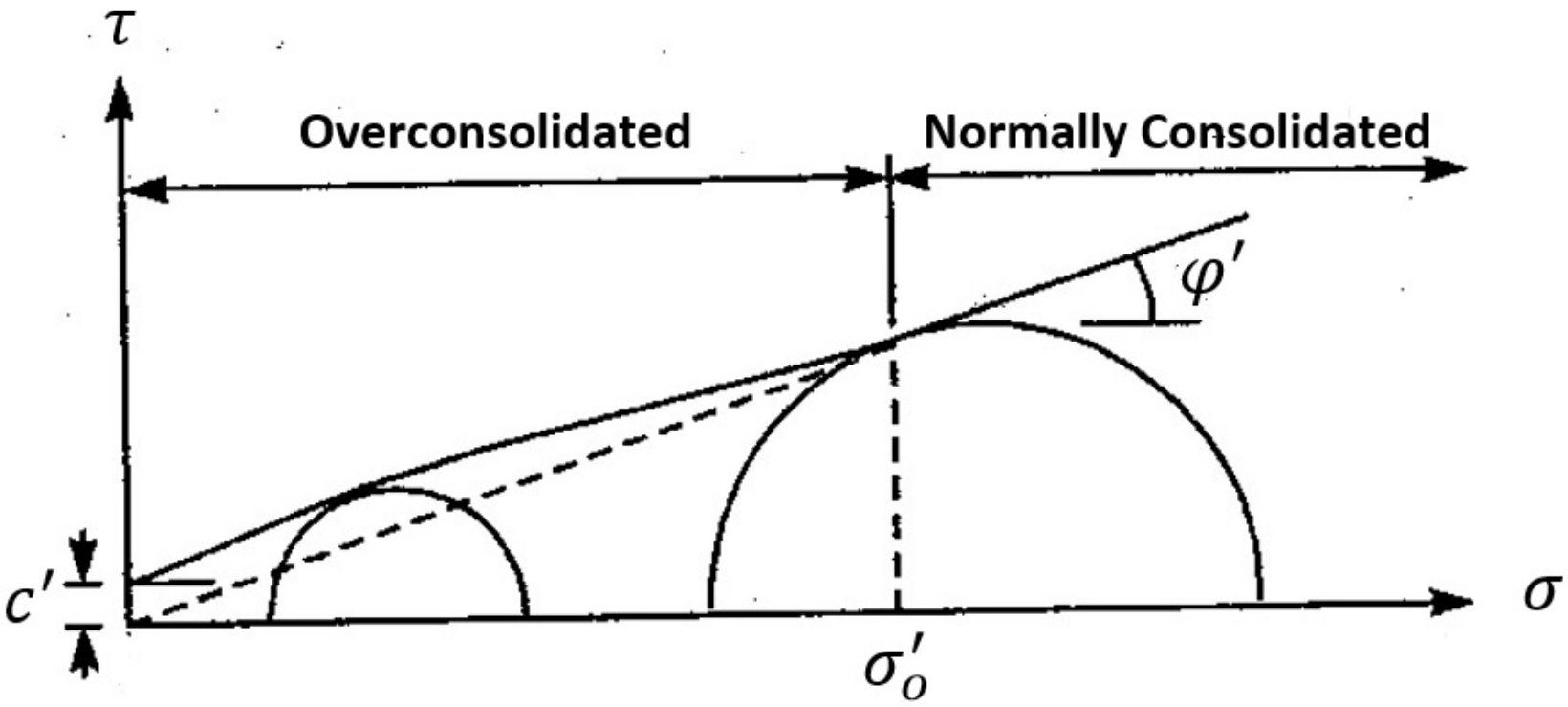


Fig. 6.45 Soil strength parameters as obtained for normally consolidated and overconsolidated range of loading.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Normally Consolidated and Over Consolidated Clay

- To illustrate the soil parameters for normally consolidated (the present overburden stress is the maximum in its life) and over-consolidated (the present overburden is not the maximum in its life) soil deposits, illustration examples are taken for alluvial river deposits as shown in Fig. 6.46. Layer 1 is deposited at a time when the soil layer was slurry having both normal stress $(\sigma)$and and shear stress (τ) of 0. Subsequently, several layers were deposited (up to Layer 5) and that acted as overburden on layer 1. They helped gaining the stiffness and thus its shear strength with increasing overburden normal load.

6.7.8 Determination of Shear Strength Parameters

Normally Consolidated and Over Consolidated Clay

- This is typically a case of normally consolidated deposit. These are illustrated in Figs. 6.46(a) to 6.46(e).

It is assumed that soil layers 2 to 5 suddenly eroded leaving behind layer 1 with some stiffness $(\tau \neq 0)$. Again soil started depositing that increases stiffness and both normal stress and shear stress of layer 1. This is typical of overconsolidation and illustrated in Figs. 6.46(f) to 6.46(j). The stress history and shear strength parameters are presented in Figs. 6.47.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Normally Consolidated and Over Consolidated Cla

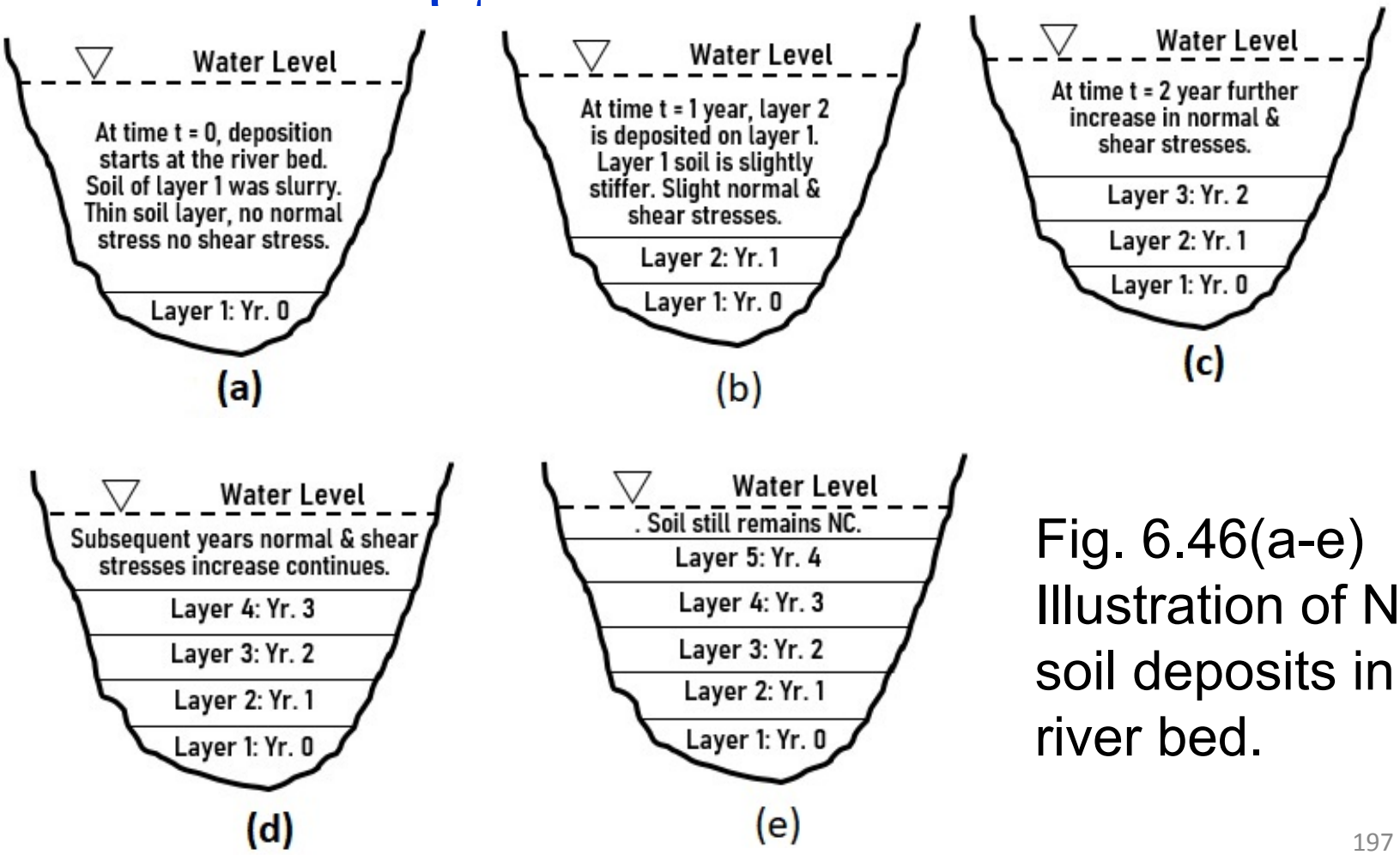


Fig. 6.46(a-e) Illustration of NC soil deposits in river bed.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Normally Consolidated and Over consolidated Clay

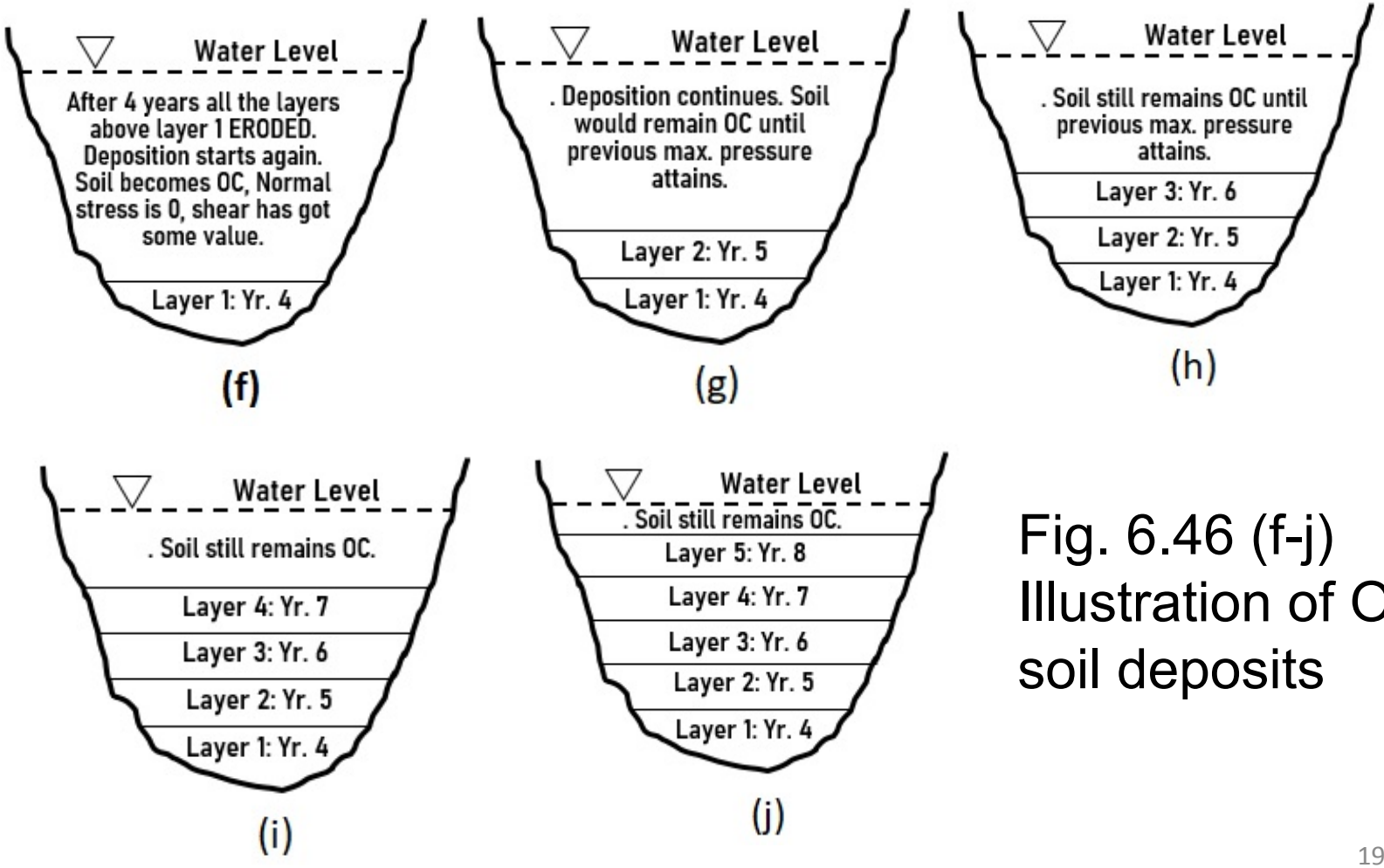


Fig. 6.46 (f-j) Illustration of OC soil deposits

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Normally Consolidated and Over Consolidated Clay

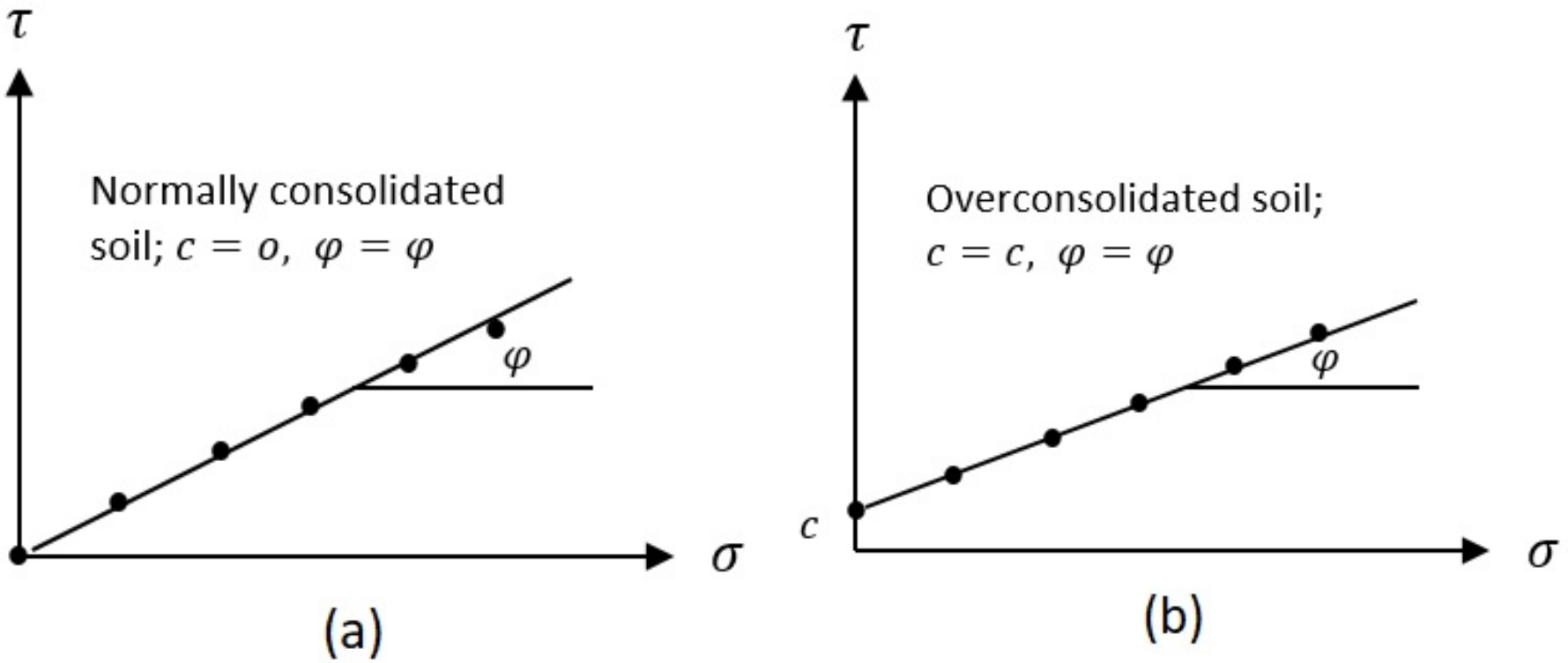


Fig. 6.47. Stress history and shear strength parameters; (a) Normally consolidated soil, (b) overconsolidated soil.

6.7.8 Determination of Shear Strength Parameters

Porewater Pressure Parameters

- The gradual increase in the total pressures σ_c and σ_v, during a triaxial test, induces corresponding increases (Δu) in the pore pressure within the soil mass. The correspondence between these quantities was derived by Skempton (1954), in terms of two coefficients A and B. The effect of σ_c and σ_v on the variation of u is considered separately. Figure 6.48 depicts the application of hydrostatic cell pressure .

6.7.8 Determination of Shear Strength Parameters

Porewater Pressure Parameters

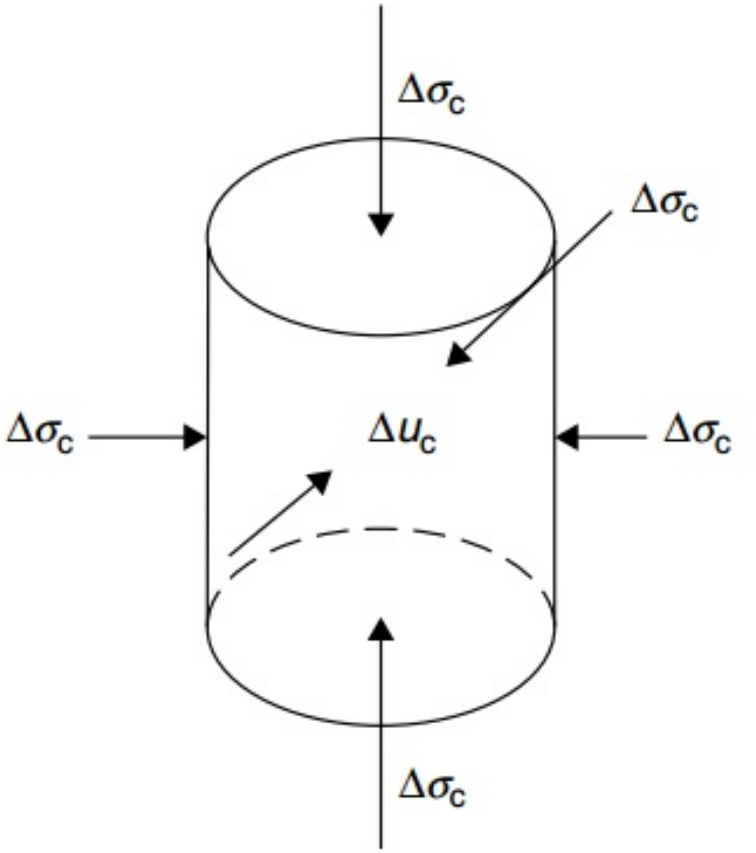


Fig. 6.48 Application of hydrostatic pressure (cell pressure) and generation of porewater pressure.

6.7.8 Determination of Shear Strength Parameters

Porewater Pressure Parameters

- **Increase in Cell Pressure**

In Fig. 6.48, $\Delta\sigma_c$ is the small increment in the cell-water pressure σ_c. It induces a small increase Δu_c in the porewater pressure. The magnitude of Δu_c depends also on the degree of saturation. The increase is given by:

$$\Delta u_c = B\Delta\sigma_c \qquad (6.14)$$

Where B is called the pore pressure parameter. It is found by increasing the cell pressure to any desired value $\Delta\sigma_c$ and measuring the corresponding pore pressure Δu_c

6.7.8 Determination of Shear Strength Parameters

Porewater Pressure Parameters

- **The magnitude of B is an indication of the degree of saturation as follows.**

$$B = 1 \qquad Saturated\ soil;$$

$$0 < B < 1 \qquad Partially\ saturated\ soil;$$

$$B = 0 \qquad Dry\ soil$$

Therefore, this first stage of a triaxial test may be used to determine the state of saturation.

6.7.8 Determination of Shear Strength Parameters

Porewater Pressure Parameters

- **Increase in Deviator Stress**

If now, the vertical pressure is increased without allowing water outflow from the specimen, then it induces further pore pressure increase. Figure 6.49 shows the addition of the deviator stress to the cell pressure. A small increment in the deviator stress, $\Delta\sigma_d$ is applied by the loading ram in the triaxial test. The induced pore pressure is given by:

$$\Delta u_d = \bar{A}\Delta\sigma_d \qquad (6.15)$$

6.7.8 Determination of Shear Strength Parameters

Porewater Pressure Parameters

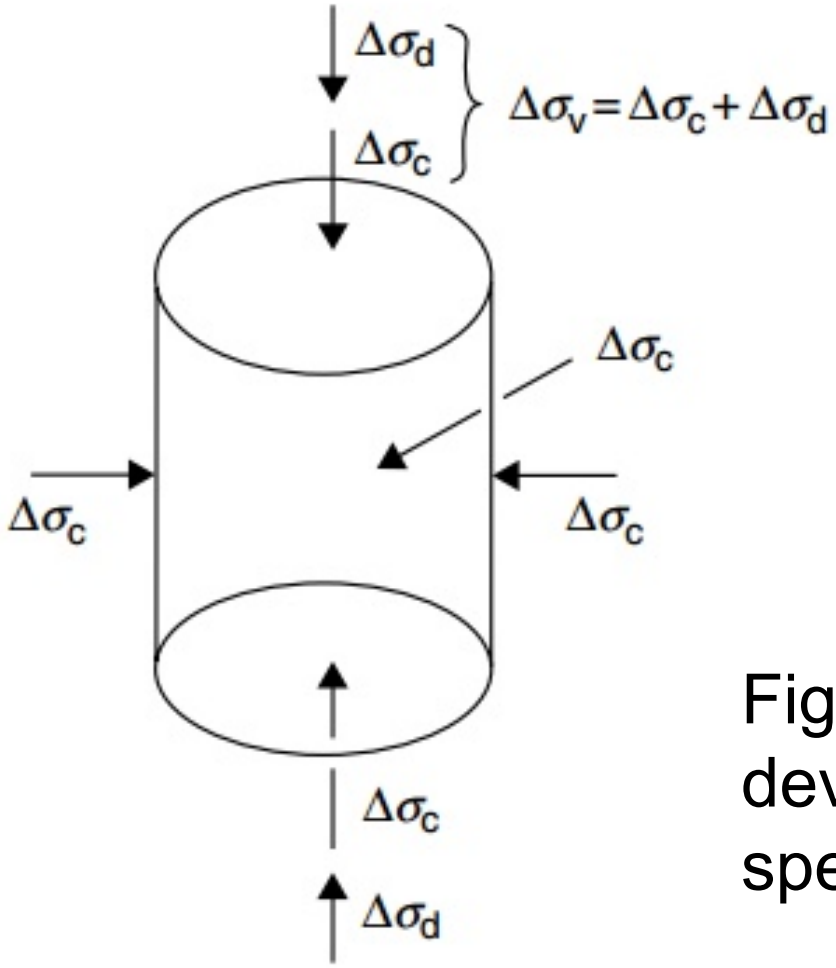


Fig. 6.49 Application of deviator stress in triaxial specimen.

6.7.8 Determination of Shear Strength Parameters

Porewater Pressure Parameters

Increase in Deviator Stress (Contd.)

Where, $\overline{A}$ is another pore pressure parameter. It is calculated from the pore pressure measured for the range of $\Delta\sigma_d$ and it depends on overconsolidation ratio.

Total Excess Porewater Pressure

Combining the above two increments (Δu_c and Δu_d), the overall change in the pore pressure, during a triaxial test, can be determined as follows.

6.7.8 Determination of Shear Strength Parameters

Porewater Pressure Parameters

- **Total Excess Porewater Pressure**

$$\Delta u = \Delta u_c + \Delta u_d = B\Delta\sigma_c + \bar{A}\Delta\sigma_d$$

$$= B\left(\Delta\sigma_c + \frac{\bar{A}}{B}\Delta\sigma_d\right) = B(\Delta\sigma_c + A\Delta\sigma_d) \qquad (6.16)$$

Thus, Δu can also be expressed as

$$\Delta u = B\left[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)\right] \qquad (6.17)$$

Where, $A = \frac{\bar{A}}{B}$ or $\bar{A} = AB$ and A is also a pore pressure parameter. The value of A depends largely on the stress history of the soil. Figures 6.50(a) and 6.50(b) shows the parameters A and B that varies with soil properties. Values are also provided in Table 6.7.

6.7.8 Determination of Shear Strength Parameters

Porewater Pressure Parameters

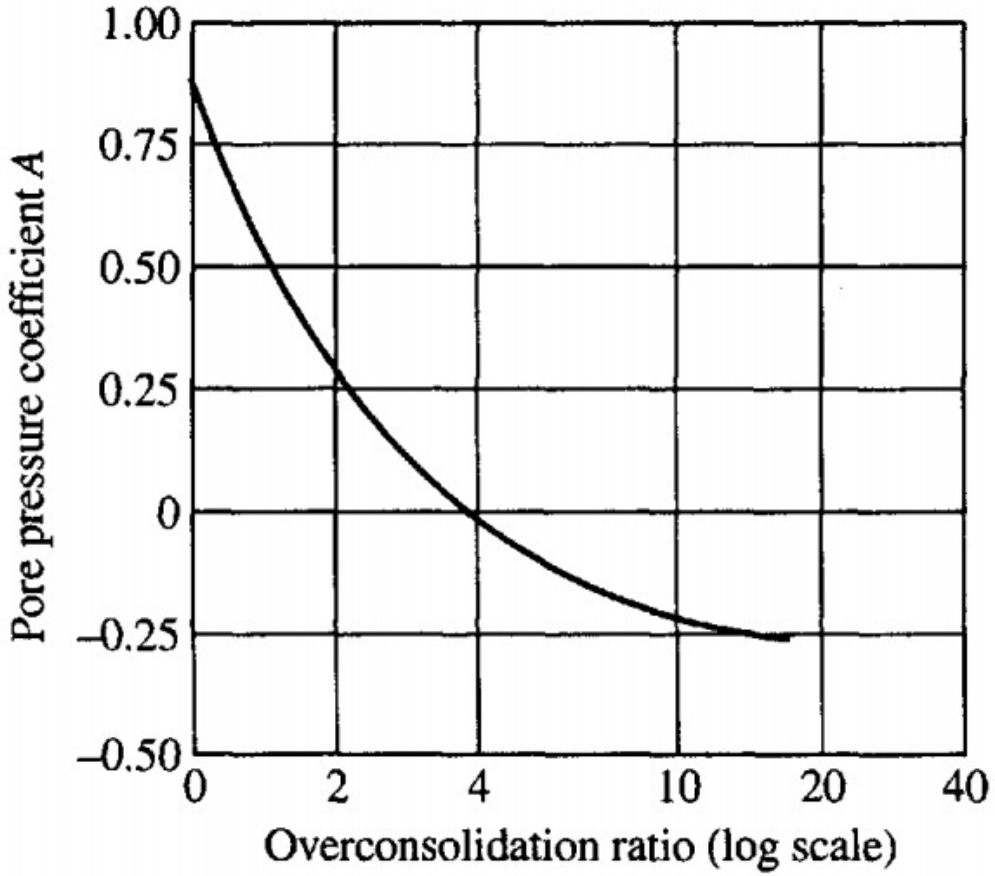


Fig. 6.50(a) Porewater pressure parameter A with overconsolidation ratio.

6.7.8 Determination of Shear Strength Parameters

Porewater Pressure Parameters

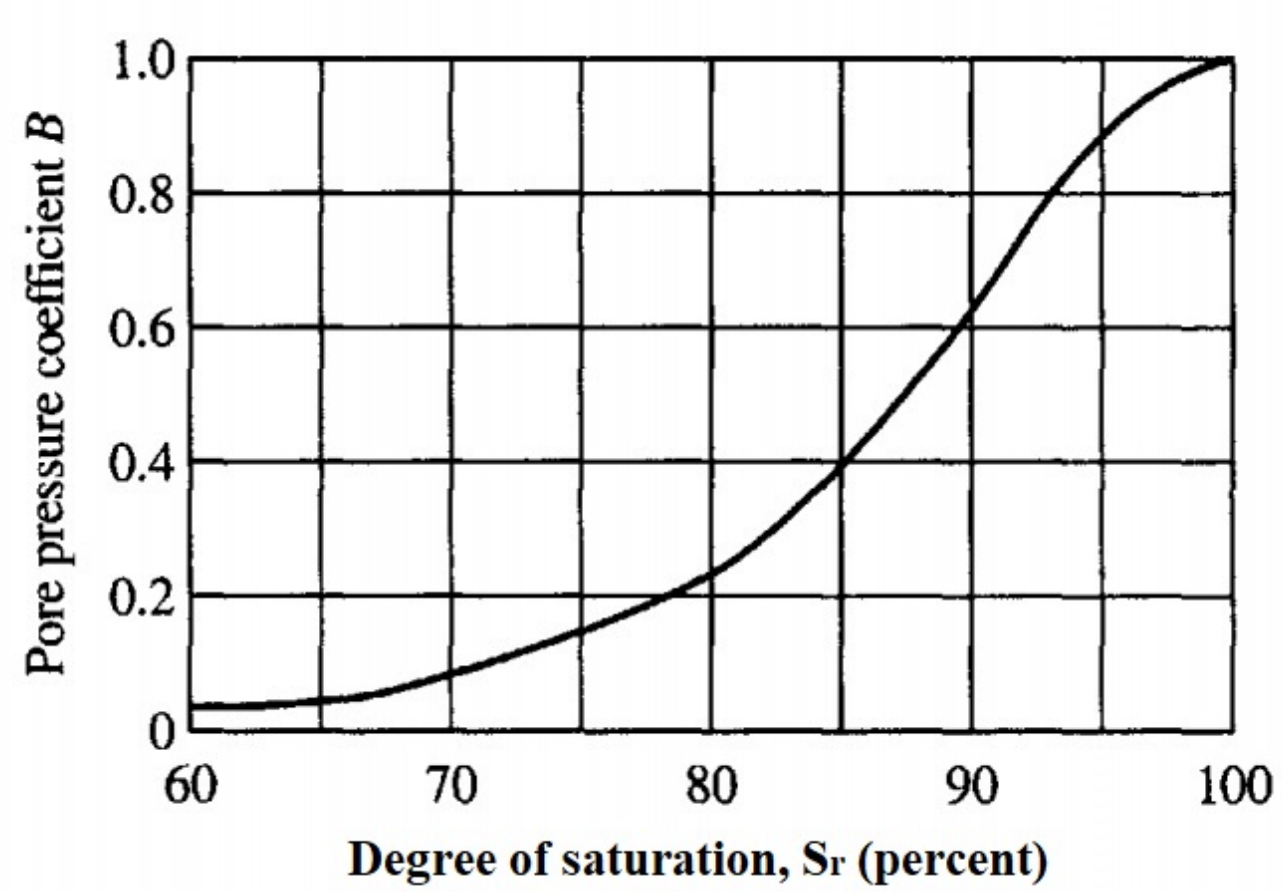


Fig. 6.50(b) Porewater pressure parameter B with degree of saturation.

6.7.8 Determination of Shear Strength Parameters

Porewater Pressure Parameters

Table 6.3 Typical values of porewater pressure parameters		
Type of Soil	Volume change	Value of A
Highly sensitive clay	Large contraction	+ 0.75 to + 1.50
Normally consolidated clay	Contraction	+ 0.50 to + 1.00
Compacted sandy clay	Slight contraction	+ 0.25 to + 0.75
Lightly overconsolidated clay	None	+ 0.00 to + 0.50
Compacted clay gravel	Expansion	

6.7.8 Determination of Shear Strength Parameters

Porewater Pressure Parameters

Parameter B in Saturating Triaxial Soil Specimen

The parameter B is applied to the triaxial test to ensure the state of full saturation. By increasing the cell pressure and the back pressure in a triaxial specimen of clay the voids are eventually filled with water by water entering the specimen from the back pressure system and air dissolving in the water. The procedure is first to apply a cell pressure and to record the pore water pressure.

6.7.8 Determination of Shear Strength Parameters

Porewater Pressure Parameters

Parameter B in Saturating Triaxial Soil Specimen

Then the back pressure (in the pore water) is increased and the pore pressure recorded when it virtually equals the back pressure applied. The back pressure must be about 10 kN/m^2 less than the cell pressure to ensure positive effective stress in the soil.

The B parameter is determined from Equation (6.14) and when it reaches about 0.97 the specimen may be assumed to be sufficiently fully

6.7.8 Determination of Shear Strength Parameters

Porewater Pressure Parameters

Table 6.4 Example of estimating pore pressure parameter B

Cell pressure (kN/m^2)	Back pressure (kN/m^2)	Back pressure valve	Pore pressure (kN/m^2)	Δu (kN/m^2)	$\Delta\sigma_3$ (kN/m^2)	Parameter, B
0	0	Closed	- 4			
50	--	Closed	7	11	50	0.22
50	40	Open	39	--	--	--
100	--	Closed	62	23	50	0.46
100	90	Open	89	--	--	--
150	--	Closed	126	37	50	0.74
150	140	Open	139	--	--	--
200	--	Closed	182	43	50	0.86
200	190	Open	190	--	--	--
300	--	Closed	285	95	100	0.95

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Unconfined Compression Test

It is apparent from Fig. 6.38 that the test results of UU triaxial test are the same regardless of the cell pressure. As such, it is not necessary to use a triaxial cell for measuring the undrained shear strength and a simple compression test on an unconfined cylindrical sample can be used. The compressive strength measured in this way is known as the unconfined compressive (UC) strength. Unconfined compression test is a special case of UU triaxial test where the minor principal stress or cell pressure is kept as zero. No rubber membrane is required to encase the

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Unconfined Compression Test

As such, the apparatus becomes much simpler. This is the easiest, quickest and simplest test for determining the undrained strength of saturated clay soil. This test can also be done on unsaturated sample, keeping in mind that undrained shear strength of clay soil varies with the moisture content and density.

Similar to triaxial tests, three identical cylindrical specimens are tested for axial compression to failure in an unconfined compression machine and their justified average values of peak axial strength are taken as the unconfined compressive strength q_u

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Unconfined Compression Test

The unconfined compression testing machine consists of a loading frame where the cylindrical soil sample can be placed in between base platform and a top cap. The top cap is attached to a proving ring or load transducer for measuring the load. The bottom platform can be raised by gear system or by a motor for the application of load and deformation. Axial deformation is measured by using a deformation dial gauge or linear variable differential transformer (LVDT). A view of the unconfined compression test machine

6.7.8 Determination of Shear Strength Parameters

Unconfined Compression Test



Fig. 6.51 View of unconfined compression testing machine.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Unconfined Compression Test

- The specimens must be sheared rapidly to failure. If the time is too long, evaporation and surface drying may result in high strength. Typical time to failure is 5 to 15 minutes. If the sample does not show any sign of failure, the unconfined compressive value is taken corresponding to 15 percent strain. The failure envelope for unconfined compressive strength is shown in Figs. 6.52(a) and 6.52(b). It is be apparent from Fig. 6.50(b) that the undrained shear strength (c_u) is half the unconfined compressive strength (q_u) provided the soil is fully saturated.

6.7.8 Determination of Shear Strength Parameters

Unconfined Compression Test

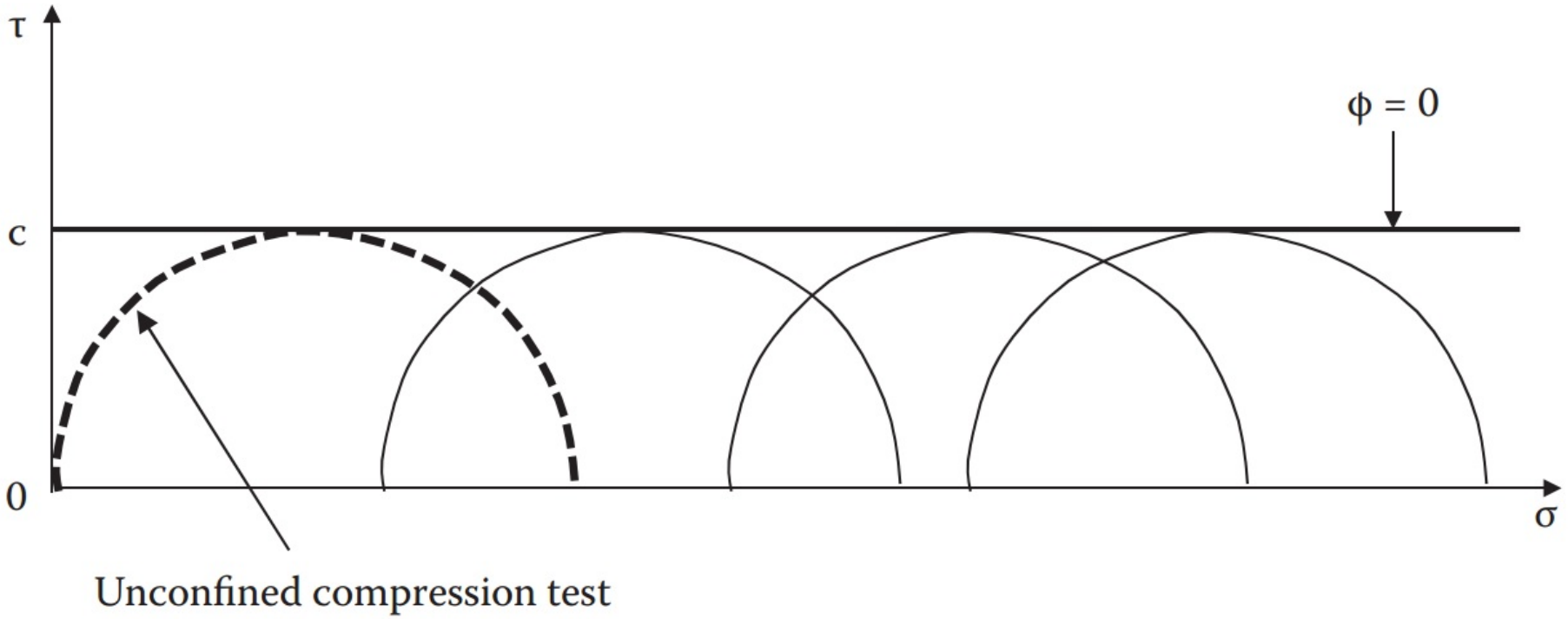


Fig. 6.52(a) Mohr circle for UU triaxial & unconfined compression test.

6.7.8 Determination of Shear Strength Parameters

Unconfined Compression Test

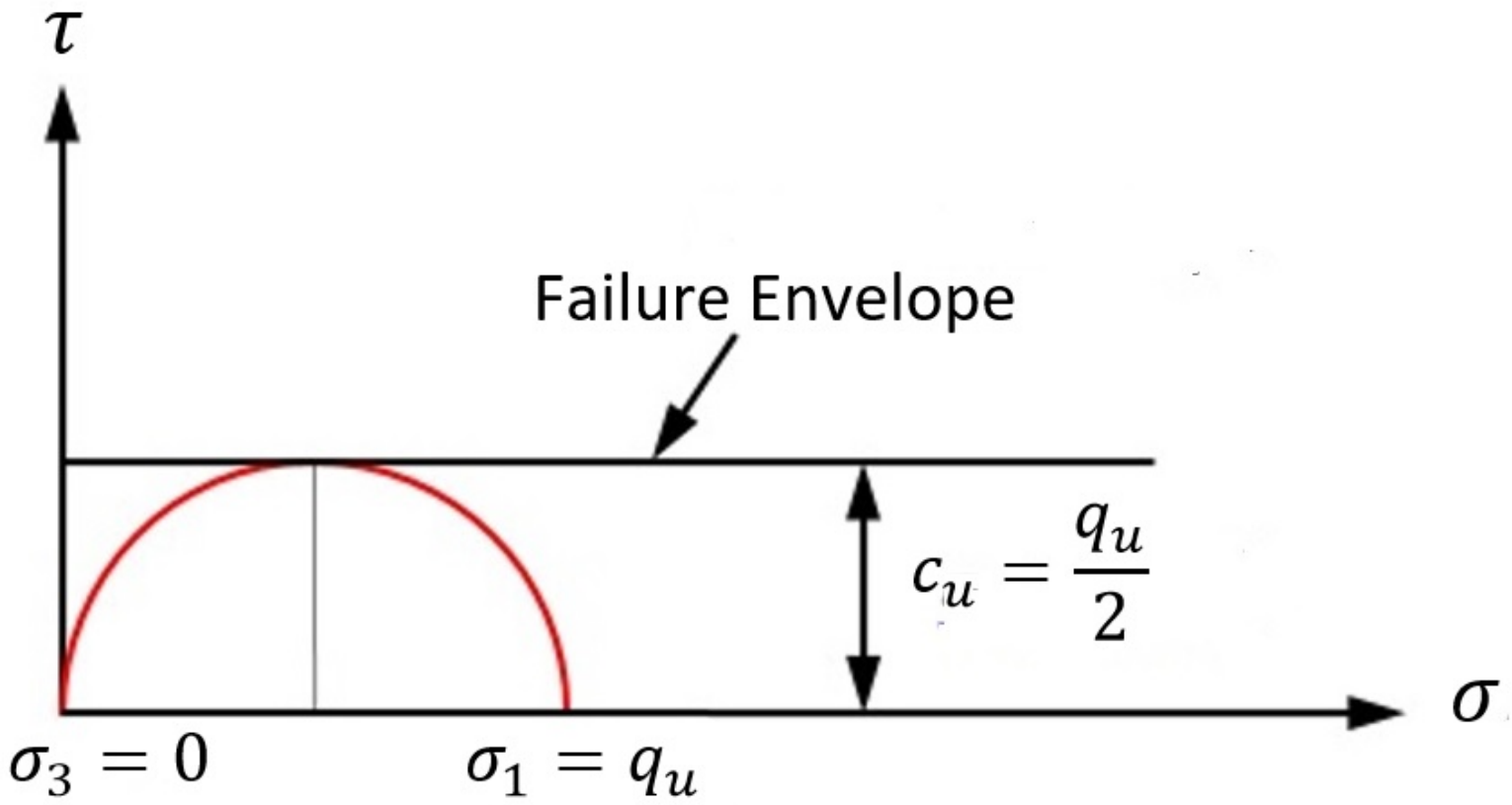


Fig. 6.52(b) Mohr circle for unconfined compression test.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Unconfined Compression Test

- **Unconfined compressive strength gives also a measure of consistency of clay soil as expressed in Table 6.5.**

This test can be done both on undisturbed and remoulded clay soil samples. Their values usually gives some useful estimation about the sensitivity of clay.

Sensitivity describes the effect of stress and environment with time on the undrained strength of soil. Sensitivity (S_t) is defined as the ratio of undrained shear strength of clay soil in undisturbed condition to that at remoulded state at the same water content.

6.7.8 Determination of Shear Strength Parameters

Unconfined Compression Test

Table 6.5 Consistency and unconfined compressive strength of clay soil	
Consistency	Unconfined compression strength, q_u (kN/m^2)
Very soft	
Soft	
Medium	
Stiff	
Very stiff	
Hard	

6.7.8 Determination of Shear Strength Parameters

Unconfined Compression Test

Sensitivity (S_t) describes the effect of stress and environment with time on the undrained strength of soil, and it is defined as the ratio of undrained shear strength of clay soil in undisturbed condition to that at remoulded state at the same water content. Though, it is referred to undrained strength, it is customary to use unconfined compressive strength (q_u) to define its sensitivity. Hence,

$$S_t = \frac{q_{u\,(undisturbed)}}{q_{u\,(remoulded)}} \qquad (6.18)$$

The sensitivity of a soil measures the reduction of shear strength that might occur due to disturbance of sample.

6.7.8 Determination of Shear Strength Parameters

Unconfined Compression Test

Bjerrum (1954) gives a sensitivity classification of soil as presented in Table 6.6.

Table 6.6 Sensitivity classification of clay soil

Sensitivity Value	Soil Description
Less than 2	Insensitive
2 -4	Medium sensitive
4 - 8	Sensitive
8 - 16	Very sensitive or Extra Sensitive
16 - 32	Slightly quick
32 - 64	Medium quick
Greater than 64	Quick

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Unconfined Compression Test

Highly overconsolidated soils are insensitive. This is partly due to low natural water content in the soil deposit. However, sensitivity most of the clay deposits ranges from 2 to 4. The decrease in strength of clay soil is attributed due to disturbance of adsorbed water in clay layers.

Thixotropy of Clay

Clay loses strength when remoulded, and its magnitude depends on its sensitivity. If the remoulded soil is left undisturbed at same water content, partly of its strength is regained due to reorientation of clay particles. This is known as

6.7.8 Determination of Shear Strength Parameters

Vane Shear Test

The vane shear test (VST) can be traced back to 1919 when it was first used in Sweden. The VST is used to determine the undrained shear strength of soft fine-grained plastic soils (clays and silts) having shear strength less than 100 kN/m^2 .

It can be performed either in the field with a field vane or on the sample taken to the laboratory with a mini vane or a hand vane. The test is most frequently carried out in a borehole in the field.

6.7.8 Determination of Shear Strength Parameters

Vane Shear Test

The apparatus consist of a four-bladed vane attached to a torque-measuring apparatus, by means of extension rods, as shown in Figs. 6.53(a), (b) and (c).

It happens sometimes, that the clay to be tested is so plastic, that it cannot be extruded from the ground, without causing extensive disturbance to its structure.

Because of its softness, the soil cannot be tested in the triaxial apparatus either. This applies especially to sensitive clays

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Vane Shear Test

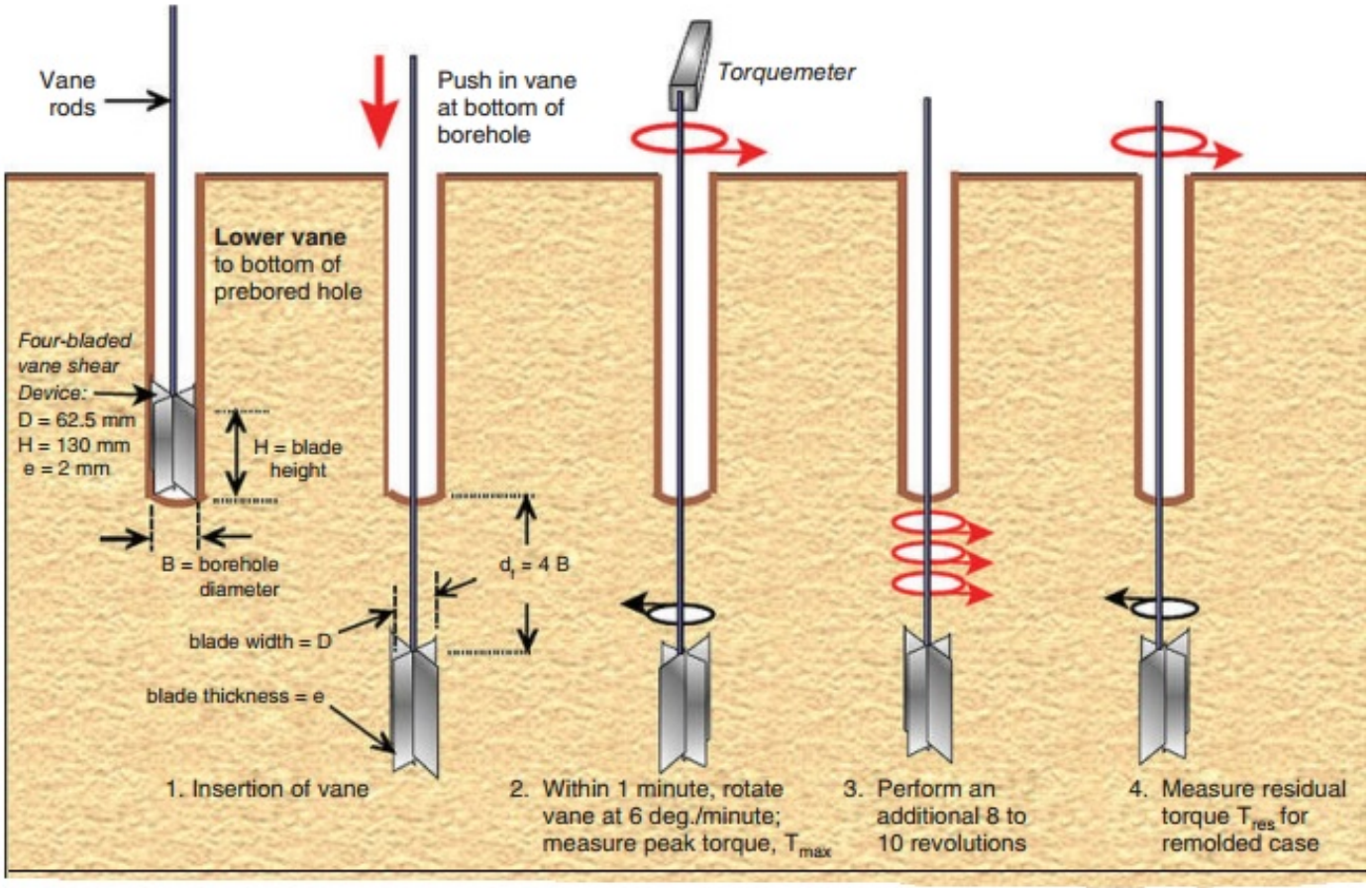


Fig. 6.53(a) Vane shear test.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Vane Shear Test

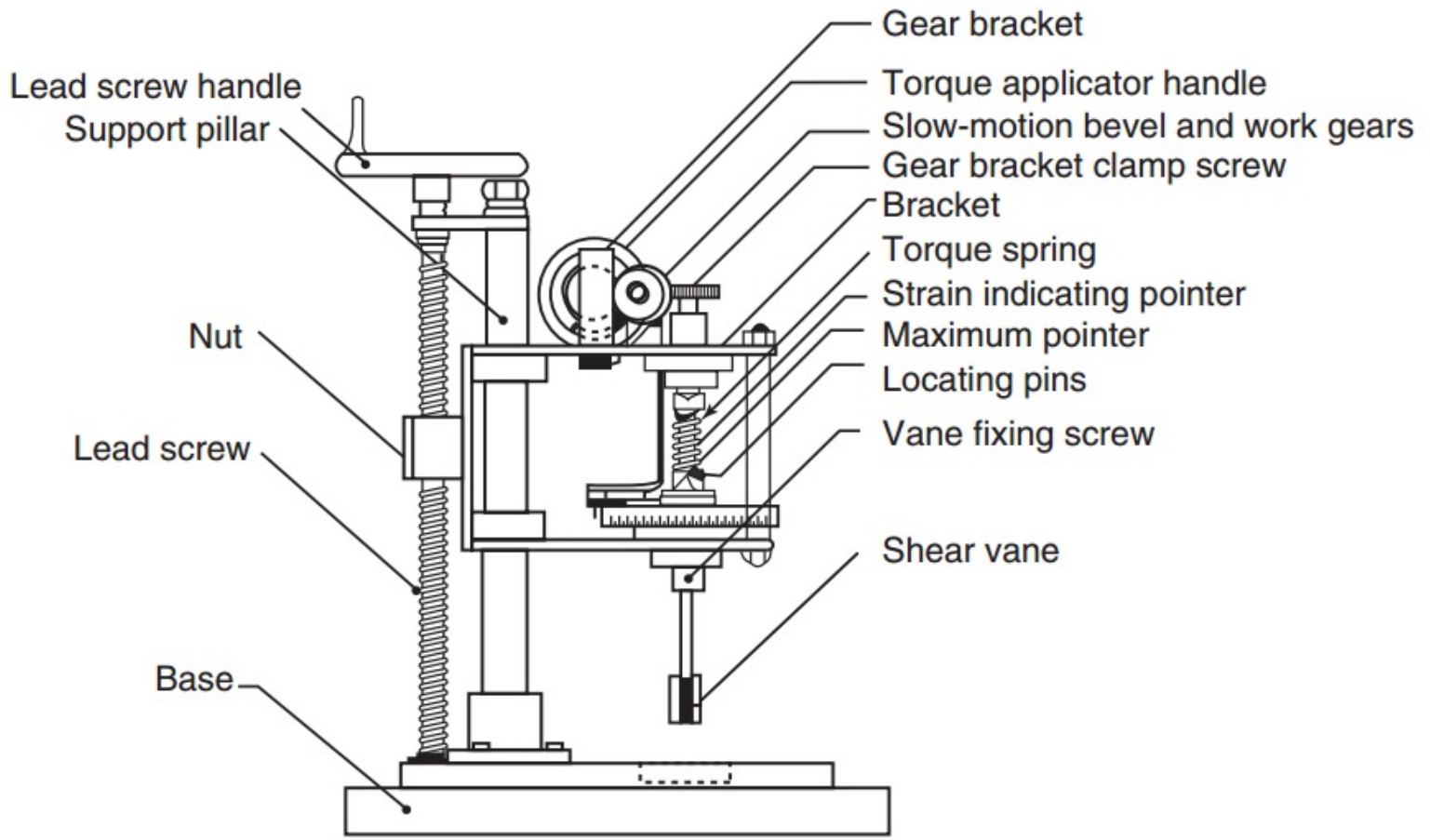


Fig. 6.53(a) Schematic of laboratory vane shear apparatus.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Vane Shear Test

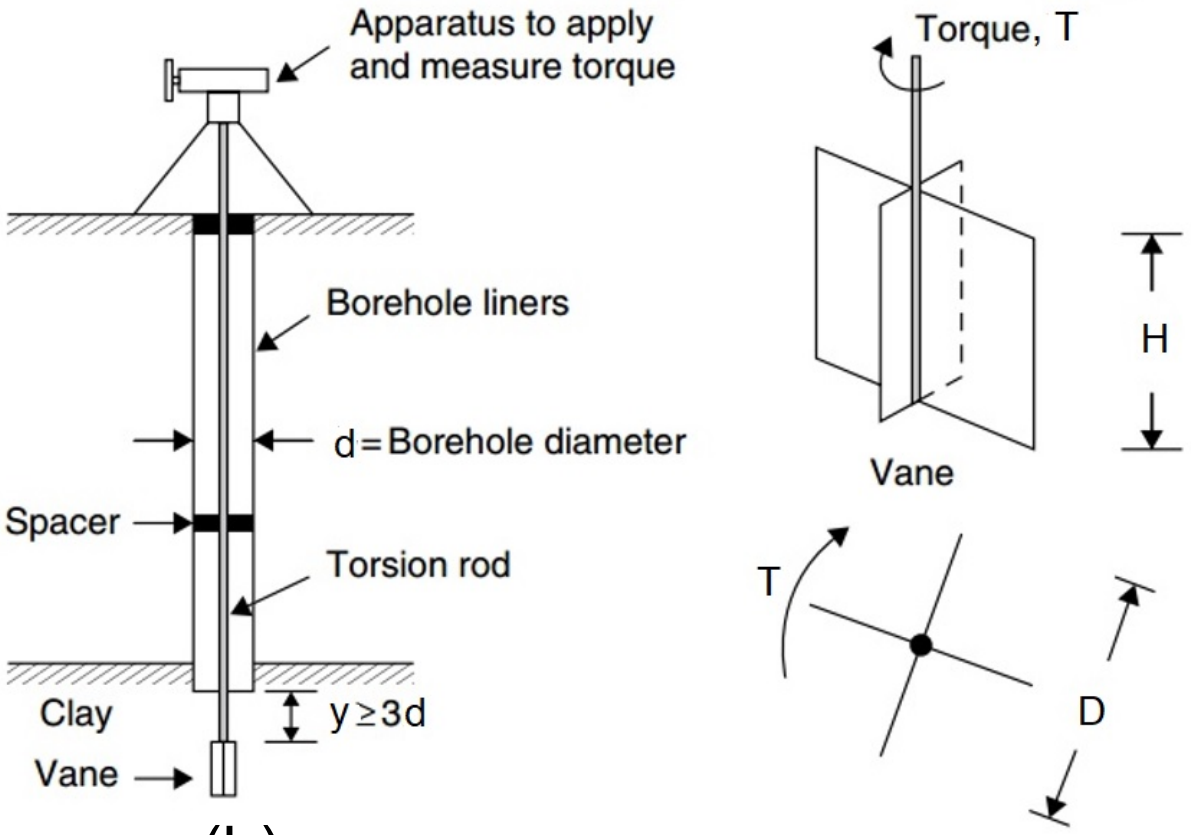


Fig. 6.53 Vane shear test; (a) General arrangements; (b) Vane geometry.

6.7.8 Determination of Shear Strength Parameters

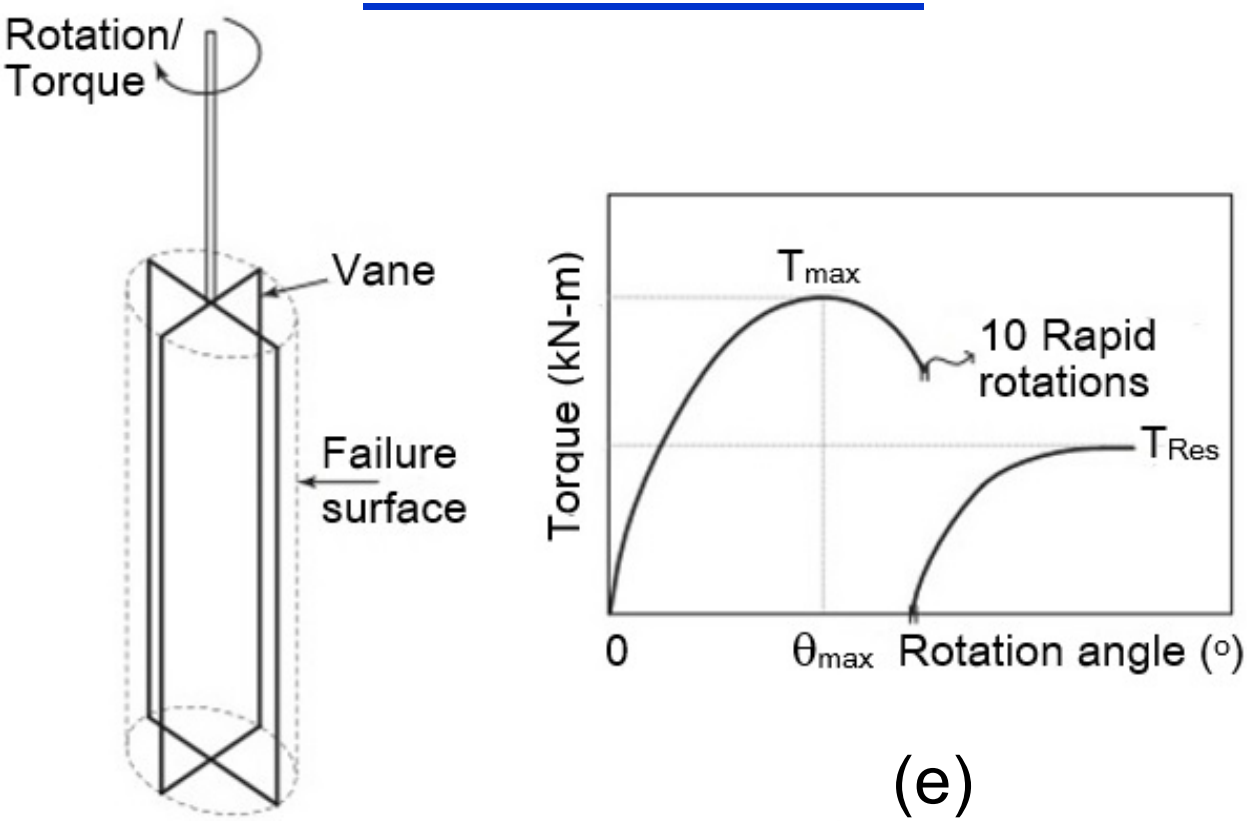
Vane Shear Test

- The vane is made of two perpendicular blades, each having a 2 to 1 height to width ratio. The width of the field vanes varies from 38 to 92 mm; the larger vanes are used in softer soils. The width of the lab vanes varies from 10 to 20 mm.

The VST consists of pushing a vane at the end of a rod into the soil until the desired depth is reached. Once the testing depth is reached, the vane is rotated at a slow rate (<1 degree/min) while measuring the torque developed and the rotation angle (Fig. 6.53d, Fig. 6.53e). Peak value of the torque is recorded as T_{max}. The blade is then rotated at least 10 times rapidly and a new maximum torque value, T_{Res}, is measured (Fig 6.53e).

6.7.8 Determination of Shear Strength Parameters

Vane Shear Test



(d) (e)

Fig. 6.53 Vane shear test; (d) Vane rotation; (e) Rotation torque relation.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Vane Shear Test

A formula between shear strength and torque can be derived considering the distribution of stresses as shown in Fig. 6.54 and on the following assumptions.

- The soil is purely cohesive, homogeneous and isotropic.
- The insertion of blade causes no disturbance to the soil.
- No drainage takes place during shearing.
- Failure takes place by shearing over the surface of the cylinder generated by the rotating vane.
- No progressive failure takes place in the soil and

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Vane Shear Test

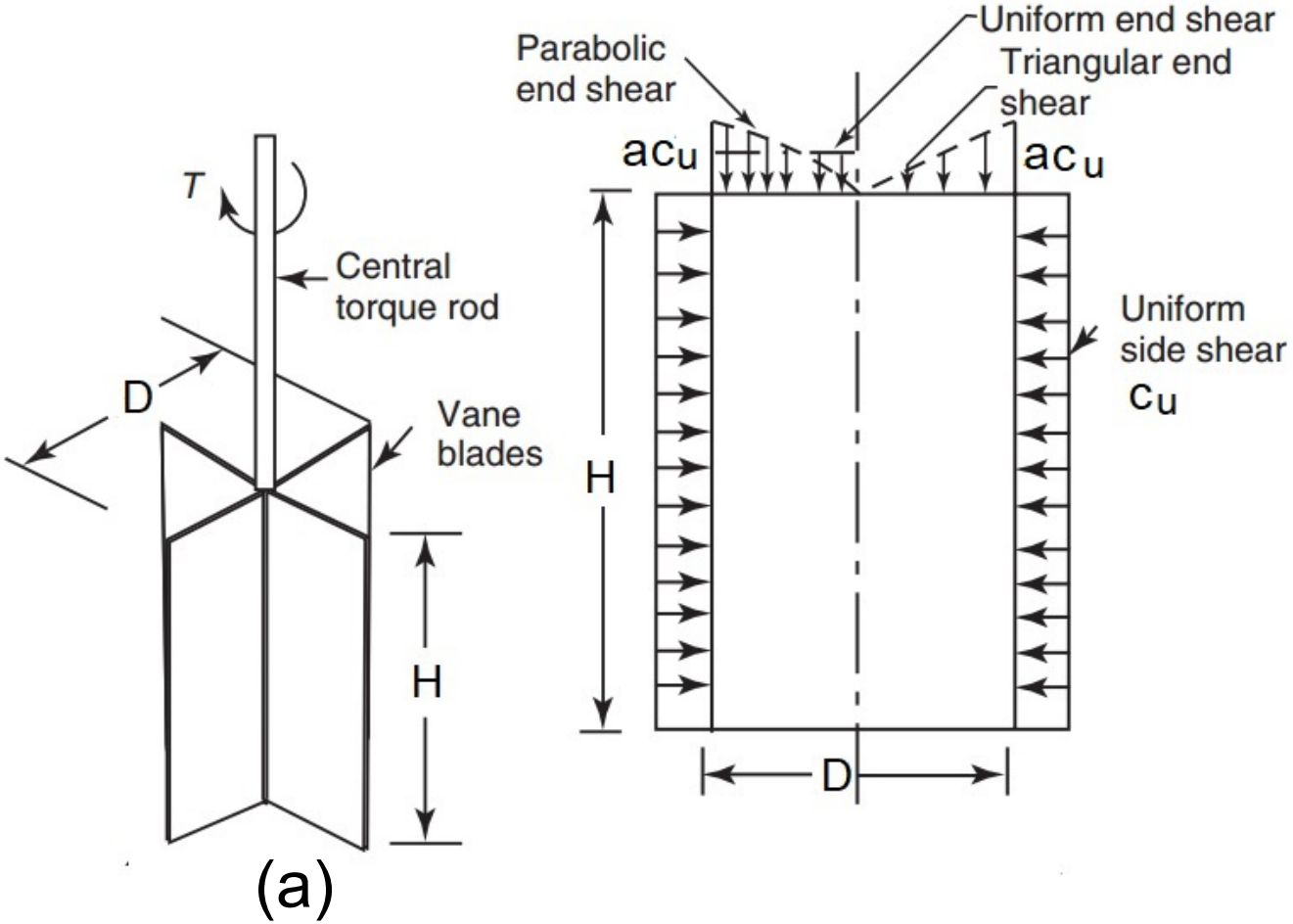


Fig. 6.54 (a) Blade dimensions & rotation; (b) Shear stress distribution.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Vane Shear Test

Let the applied torque be T which is resisted by the moments due shear stress mobilized along the cylinder side (M_{side}) and on the two edges $(M_{top}\ and\ M_{bottom}\ or\ M_{edge})$ of the failure surfaces. Since the test is carried out relatively fast, undrained conditions can be assumed and the shear stress at failure is the same as the undrained shear strength, c_u.

The total torque T is thus the sum of the opposing moments, Fig. 6.54b. Mathematically, it can be written as:

$$T = M_{side} + M_{top} + M_{bottom} = M_{side} + 2M_{edge} \qquad (6.19)$$

6.7.8 Determination of Shear Strength Parameters

Vane Shear Test

A cylinder of soil of height, H and diameter, D is considered that will resist the torque, T until the soil fails (Figs. 6.54a & b). If the assumption of full shear mobilization at the edges are not valid, there could be various assumptions regarding the distribution of mobilized shear strength at the edges. The possible distributions are indicated in Fig. 6.54(b).

Accordingly, an expression for resisting torque can be obtained by integrating the resulting moment for the relevant stress distribution diagram of the edges and adding the value for the moment due to surface resistance.

6.7.8 Determination of Shear Strength Parameters

Vane Shear Test

By taking moments about the shaft axis and considering $r =$ *any radial distance;* $\frac{D}{2} =$ *radius of blades.*

$$M_{side} = (\pi DH) \times c_u \times \frac{D}{2} = \frac{\pi D^2 H c_u}{2} \qquad \textbf{(6.20)}$$

And, $M_{edge} = \int_0^{D/2} (2\pi r \times dr \times r)(ac_u) = 2\pi ac_u \int_0^{D/2} r^2 dr$

$$\Rightarrow M_{edge} = 2\pi ac_u \left[\frac{r^3}{3}\right]_0^{D/2} = 2\pi ac_u \left[\frac{D^3}{24}\right] = \frac{\pi ac_u D^3}{12} \qquad \textbf{(6.21)}$$

$$\therefore \quad T = \frac{\pi D^2 H c_u}{2} + 2 \times \frac{\pi ac_u D^3}{12}$$

$$\Rightarrow \quad T = \pi c_u \left(\frac{D^2 H}{2} + \frac{aD^3}{6}\right) \qquad \textbf{(6.22)}$$

6.7.8 Determination of Shear Strength Parameters

Vane Shear Test

Equation (6.22) is the general governing equation to estimate the undrained shear strength of fine grained soil from vane shear test. This is referred to as Calding's equation. Here, a is the coefficient defining the value of shear stress at centroidal distance of the area of stress distribution from centre of a circular plane in terms of diameter and depends on the distribution of stresses on the top and bottom circular edges. The values of a are:

$$a = 1 \; for \; uniform \; distribution \; of \; stresses$$

$$a = \frac{2}{3} \; for \; triangular \; distribution \; of \; stresses$$

$$a = \frac{3}{5} \; for \; parabolic \; distribution \; of \; stresses$$

6.7.8 Determination of Shear Strength Parameters

Vane Shear Test

Usually height h of a vane blade is 2 times the diameter d. Using this value in Eq. (6.12), we get:

$$T = \pi c_u \left(\frac{D^2 \times 2D}{2} + \frac{aD^3}{6}\right) = \pi c_u D^3 \left(1 + \frac{a}{6}\right) = \frac{\pi c_u D^3 (a+6)}{6}$$

$$\Rightarrow c_u = \frac{6T}{\pi D^3 (a+6)} \qquad \textbf{(6.16)}$$

Considering a uniform distribution of failure stress c_u over the entire circular area, the value of a that is the coefficient of c_u at centroid of stress distribution area becomes 1 and we get:

$$c_u = \frac{6T}{\pi D^3 (1+6)} = \frac{6T}{\pi D^3 \times 7} = \frac{6T}{7\pi D^3} = \frac{3T}{11D^3} \qquad \textbf{(6.17)}$$

6.7.8 Determination of Shear Strength Parameters

Vane Shear Test

Researchers found that the values of c_u obtained from the field vane shear test (FVST) are too high for design purposes; its use reduces the factor of safety considerably. It is recommended to use the empirical correction factor λ with c_u in order to obtain appropriate value for design undrained shear strength. Hence,

$$Design, c_u = \lambda \times Analitical\ c_u$$

Where,

$\lambda = 1.7 - 0.54 \log PI$; PI is the plasticity index.

6.7.8 Determination of Shear Strength Parameters

Empirical Relations of Undrained Shear Strength

Undrained shear strength of normally consolidated clays varies with effective overburden pressure (i.e. with depth). Several empirical relations are available. Of them, the one given by Skempton (1957) is mentioned below.

$$\left(\frac{c_u}{\sigma_o'}\right)_{NC} = 0.11 + 0.0037 I_p \qquad \textbf{(6.18a)}$$

Where, c_u is the undrained cohesion, σ_o' is the effective over burden pressure and I_p is the plasticity index in percent. For overconsolidated clays, Ladd and Foott (1974) proposed the empirical formula:

$$\left(\frac{c_u}{\sigma_o'}\right)_{OC} = (OCR)^{0.8}\left(\frac{c_u}{\sigma_o'}\right)_{NC} \qquad \textbf{(6.18b)}$$

6.7.8 Determination of Shear Strength Parameters

Factors Affecting Angle of Internal Friction

The shear resistance of sands and gravels is made up of sliding and rolling friction plus the resistance to volume change by interlocking. Interlocking effect is important for dense sands and is the main contributing factor for the increased angle of shearing resistance in them. However, the angle of shearing resistance corresponding to the ultimate strength is found to be not much different for the dense sand and the loose sand.

The angle of shearing resistance decreases rapidly with increase in percentage of fine. A fine content of about 20 to 25 per cent may reduce the φ' value for clean sand by 5° to 7°. Table 6.7 shows the typical values for of φ'.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Factors Affecting Angle of Internal Friction

Type of sand	Round grains, uniform grain size	Angular grains, well graded
Loose sand	28.5°	34°
Dense sand	35°	46°

An empirical formula suggested by Brinch Hansen and Lundgren (1960) takes into account the influence of the major factors in determining φ' in sands and gravels.

6.7.8 Determination of Shear Strength Parameters

Factors Affecting Angle of Internal Friction

The value of φ' is expressed in the form:

$$\varphi' = 36^o + \varphi'_1 + \varphi'_2 + \varphi'_3 + \varphi'_4 \qquad \textbf{(6.28)}$$

In Eq. 6.20, $36°$ represents the φ' value for average conditions while φ'_1, φ'_1, φ'_1 and φ'_1 account for influence of grain shape, size, gradation and density index respectively, Table 6.8.

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Factors Affecting Angle of Internal Friction

Angular grain		Poorly graded soil	
Sub-angular grain		Medium uniformity	
Rounded grain		Well graded soil	
Well-rounded grain			
Sand		Loosest packing	
Fine gravel		Medium density	
Med. & coarse gravel		Densest packing	

Shear Strength of Soil

6.7.8 Determination of Shear Strength Parameters

Shear Strength of Unsaturated Soil

Generally, the shear strength of saturated soils is much discussed. However, in the most cases, unsaturated soils are encountered in the field. The phase diagram of saturated and unsaturated soils are

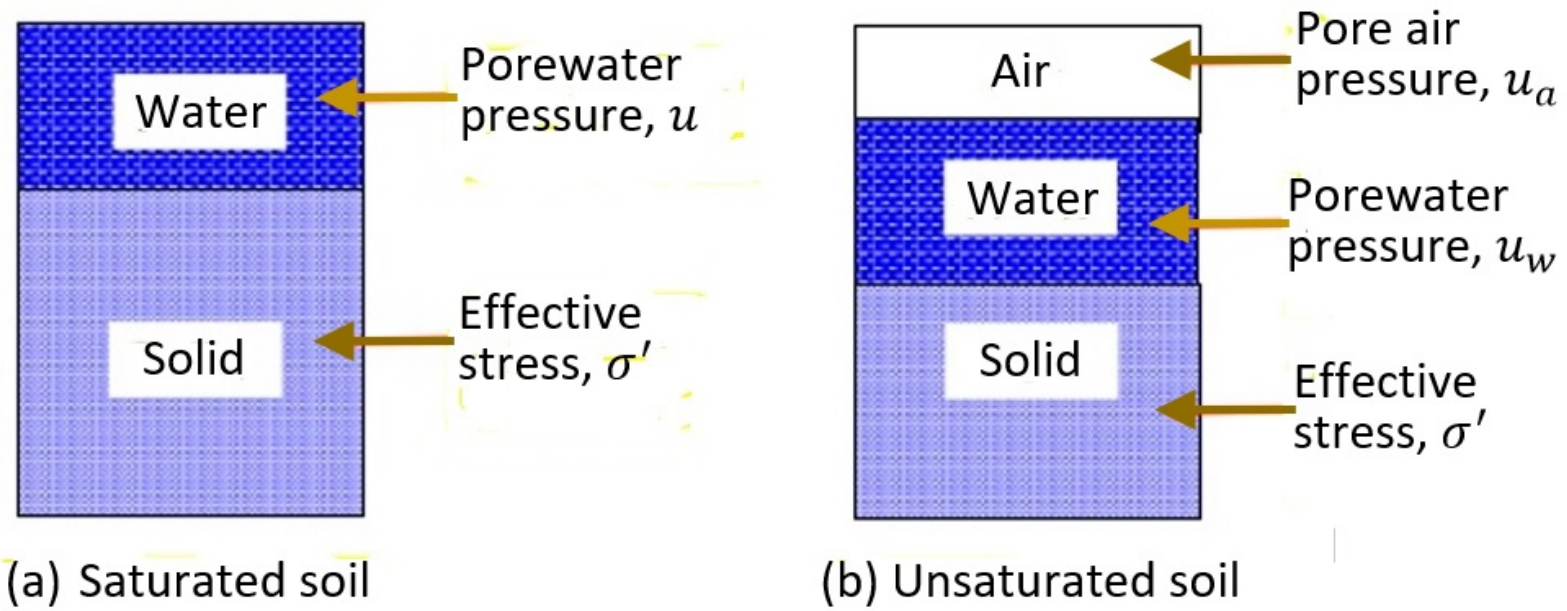


Fig. 6.55 Phase diagram for saturated and unsaturated soils

6.7.8 Determination of Shear Strength Parameters

Shear Strength of Unsaturated Soil

For unsaturated soils, the effective stress σ' can be calculated using the most general expression –

$$\sigma' = \sigma - \alpha u_w - \beta u_a \qquad (6.18)$$

Therefore, the general equation for the shear strength of a soil (unsaturated or saturated) is:

$$\tau_f = c' + \sigma' \tan \varphi' = c' + (\sigma - \alpha u_w - \beta u_a) \tan \varphi' \qquad (6.19)$$

Where, σ' is the normal effective stress on the failure plane, σ is the normal total stress on the failure plane, α is the fraction of the total area of the failure plane covered by water, β is the fraction of the plane covered by air, u_w is the water stress, u_a is the air stress, and φ' is the effective stress friction angle.

6.7.8 Determination of Shear Strength Parameters

Shear Strength of Unsaturated Soil

When the soil is saturated or when the air is occluded, $\alpha \cong 1$ and β becomes 0, as such Eq. (6.19) becomes:

$$\sigma' = \sigma - u_w \qquad \textbf{(6.20)}$$

If the air is not occluded, there is a path for the air to be connected directly to the atmosphere and the air stress is atmospheric or zero gauge pressure. Then the most general expression of the effective stress in soils covering all real cases is:

$$\tau_f = c' + (\sigma - \alpha u_w) \tan \phi' \qquad \textbf{(6.22)}$$

6.7.8 Determination of Shear Strength Parameters

Shear Strength of Unsaturated Soil

Parameter α can be estimated by taking it equal to the degree of saturation S or by using a slightly modified version of the Khalili and Khabbaz (1998) equation.

$$\alpha = S \tag{6.23a}$$

$$\alpha = \sqrt{\frac{u_{wae}}{u_w}} \tag{6.23b}$$

Where, u_{wae} is the air entry value of the water tension and u_w is the water tension. Shear strength equations other than Eq. 6.19 have been proposed, such as by Bishop (1959) and Fredlund and Rahardjo (1993).

6.7.8 Determination of Shear Strength Parameters

Shear Strength of Unsaturated Soil

Bishop (1959) proposed the following equation for shear strength of unsaturated soil.

$$\tau_f = c' + [(\sigma_n - u_a) + \chi(u_a - u_w)] \tan \varphi' \qquad \textbf{(6.24)}$$

Where,

$\sigma_n - u_a$ = **net normal stress**

$u_a - u_w$ = **matric suction**

χ = **a parameter depending on degree of saturation.**
$\chi = 1$ **for fully saturated soil and** 0 **for dry soils.**

6.7.8 Determination of Shear Strength Parameters

Shear Strength of Unsaturated Soil

Fredlund and Rahardjo (1993) modified Equation 6.24) in the following form:

$$\tau_f = c' + [(\sigma_n - u_a)\tan\varphi' + \chi(u_a - u_w)]\tan\varphi^b \qquad \textbf{(6.25)}$$

Where, φ^b is an angle indicating the rate of increase in shear strength relative to the matric suction $(u_a - u_w)$. Equation 6.25 can be reorganized as follows:

$$\tau_f = c' + \left[\sigma - \frac{\tan\varphi^b}{\tan\varphi'}\,u_w - \left(1 - \frac{\tan\varphi^b}{\tan\varphi'}\right)u_a\right]\tan\varphi' \qquad \textbf{(6.26)}$$

Comparison of Eq. 6.25 with Eq. 6.26 shows that the two equations are identical if:

6.7.8 Determination of Shear Strength Parameters

Shear Strength of Unsaturated Soil

$$\alpha = \frac{\tan \varphi^b}{\tan \varphi'} \quad and \quad \beta = 1 - \frac{\tan \varphi^b}{\tan \varphi'} \tag{6.27}$$

It is established that $\alpha + \beta = 1$; therefore, both conditions are satisfied automatically and the ratio $\frac{\tan \varphi^b}{\tan \varphi'}$ can be estimated through Eqs. (6.23a) and (6.23b).

Equations (6.24), (6.25) and (6.26) dictates that the strength of unsaturated soils is much higher than the strength of saturated soil due to matric suction. They are illustrated in Figs. 6.56(a) and 6.56(b).

6.7.8 Determination of Shear Strength Parameters

Shear Strength of Unsaturated Soil

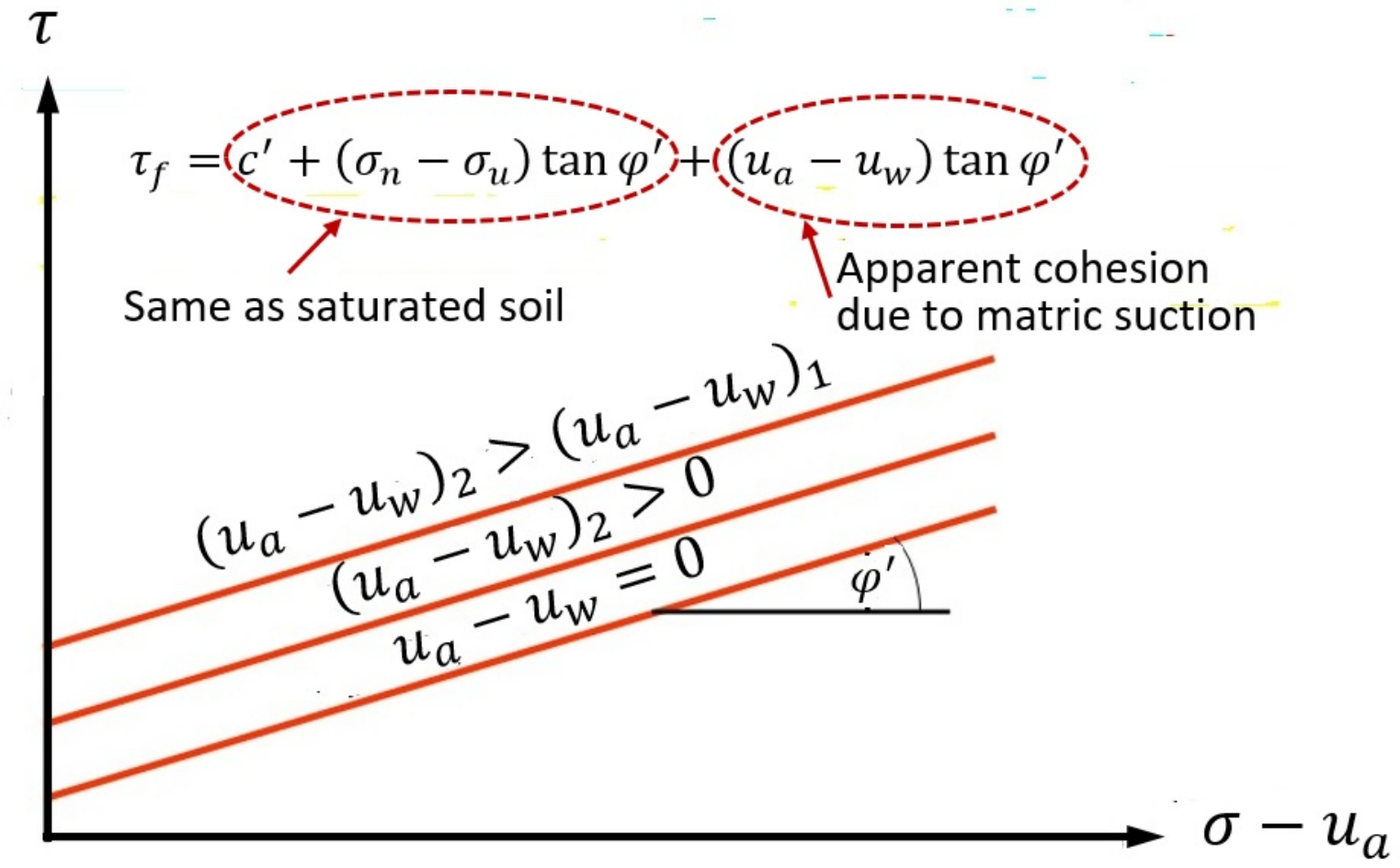


Fig. 6.56(a) Illustration of unsaturated shear strength parameters

6.7.8 Determination of Shear Strength Parameters

Shear Strength of Unsaturated Soil

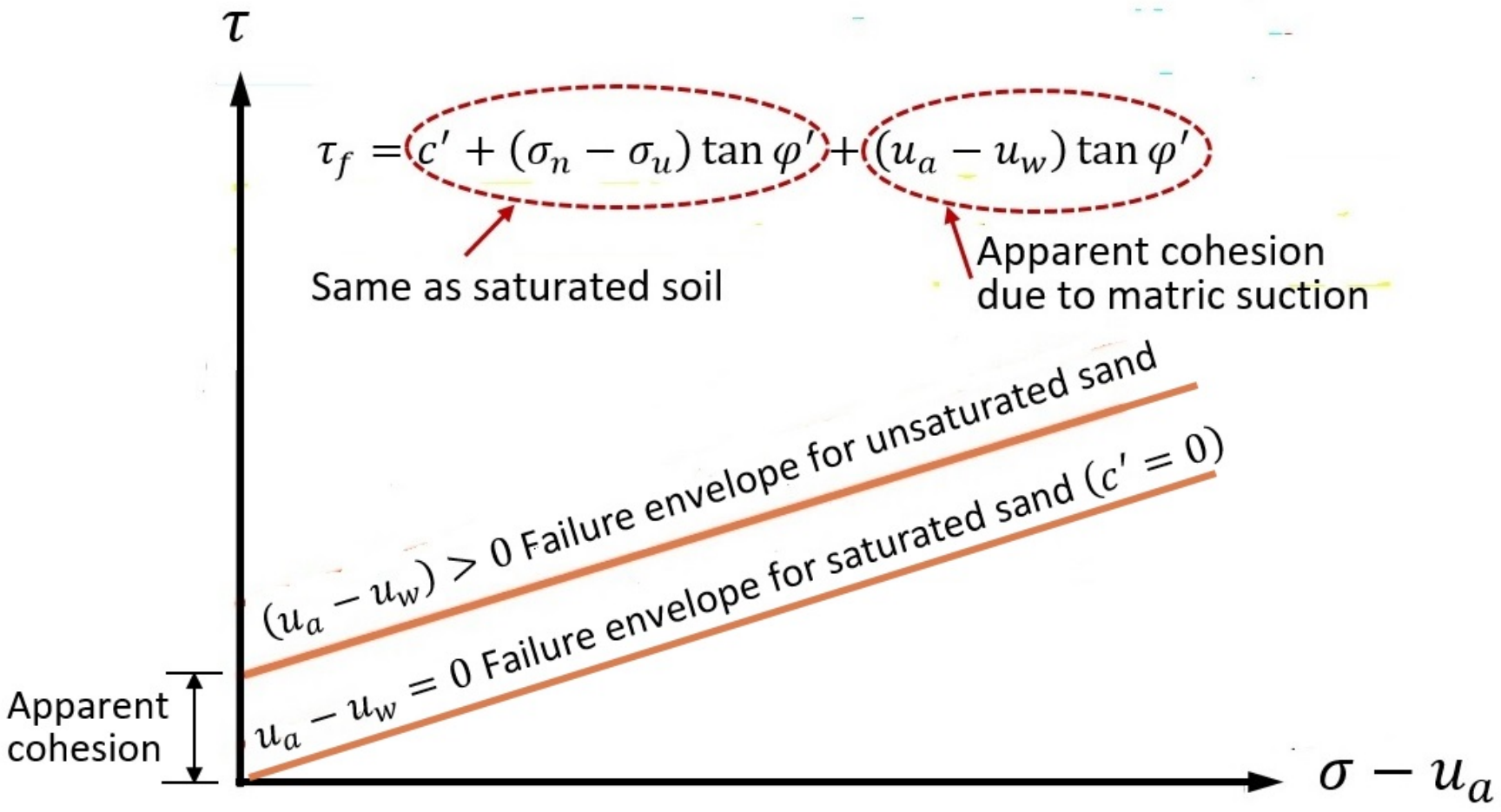


Fig. 6.56(b) Illustration of unsaturated shear strength parameters

6.7.8 Determination of Shear Strength Parameters

Stress Point and Stress Path

The analysis of shear strength of soil using the Mohr circle considers only the stress conditions at failure. However, it is also important to observe changes in shear stress from the beginning of load application until the sample fails. To do this, the stress path concept can be applied which is schematically explained in Fig. 6.57.

Instead of drawing numerous Mohr circles for each stress increment, we will use only one point that can represent the stress conditions of the Mohr circle.

6.7.8 Determination of Shear Strength Parameters

Stress Point and Stress Path

This "stress point" is located on the top of the Mohr circle (Fig. 6.57a) and it is described using the p and q parameters (Fig. 6.57b) which are defined in Equations (6.28a) and (6.28b).

$$p = \frac{\sigma_1 + \sigma_3}{2} \qquad \textbf{(6.28a)}$$

$$p = \frac{\sigma_1 + \sigma_3}{2} \qquad \textbf{(6.28b)}$$

By using the stress path approach, the values of α and m at failure can be obtained as shown in Fig. 6.57(c).

6.7.8 Determination of Shear Strength Parameters

Stress Point and Stress Path

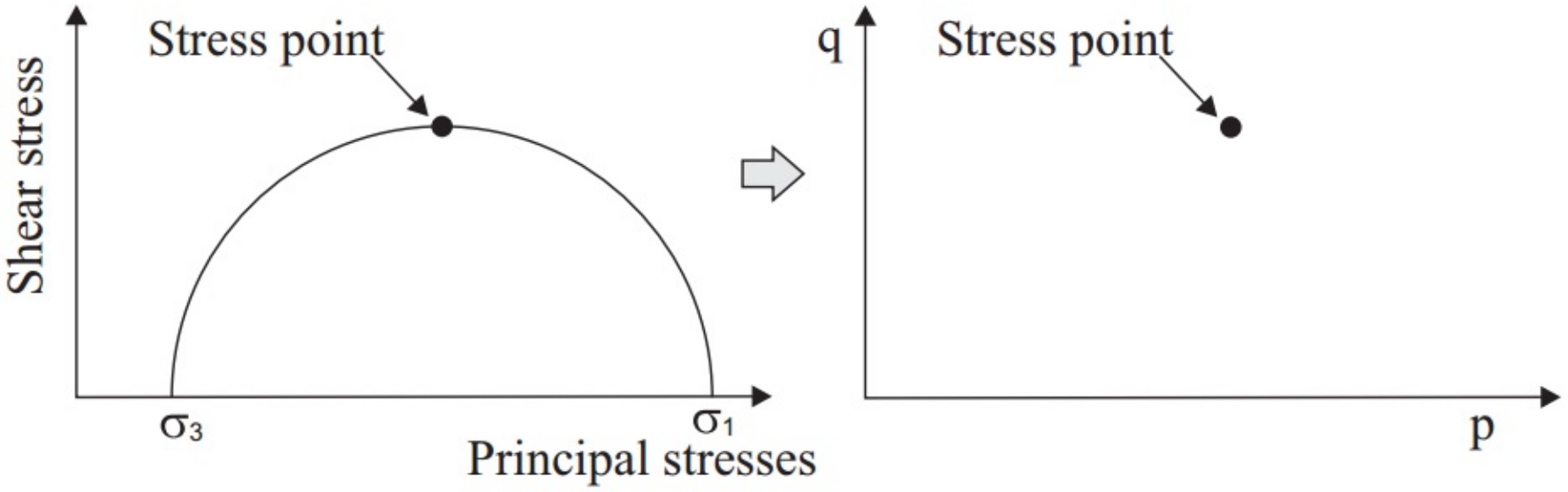


Fig. 6.57(a) Stress path concept: (a) Top point of the Mohr circle; (b) Stress point in the p vs. q space.

6.7.8 Determination of Shear Strength Parameters

Stress Point and Stress Path

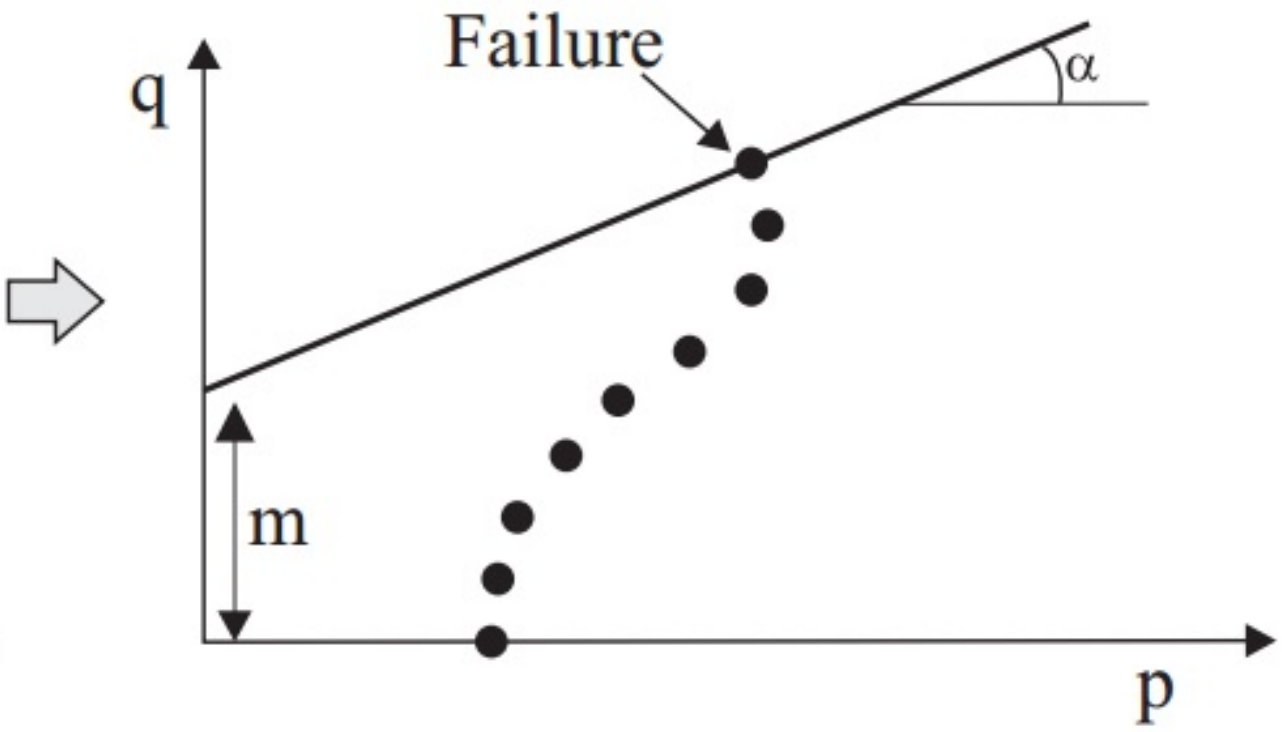


Fig. 6.57(a) Stress path concept: (c) Stress path reaching failure.

6.7.8 Determination of Shear Strength Parameters

Stress Point and Stress Path

It is to be noted that α and m are not soil friction angle (ϕ) and cohesion (c), respectively (Fig. 6.58). However, through the relationship given in Equations (6.28a) and (6.28b), the friction angle and cohesion can be found as:

$$m = c' \cos \varphi' \qquad \textbf{(6.28a)}$$

$$\varphi' = \sin^{-1}(\tan \alpha) \qquad \textbf{(6.28b)}$$

It is to be noted that the stress path is commonly used for effective stress conditions, i.e., p' and q'.

6.7.8 Determination of Shear Strength Parameters

Stress Point and Stress Path

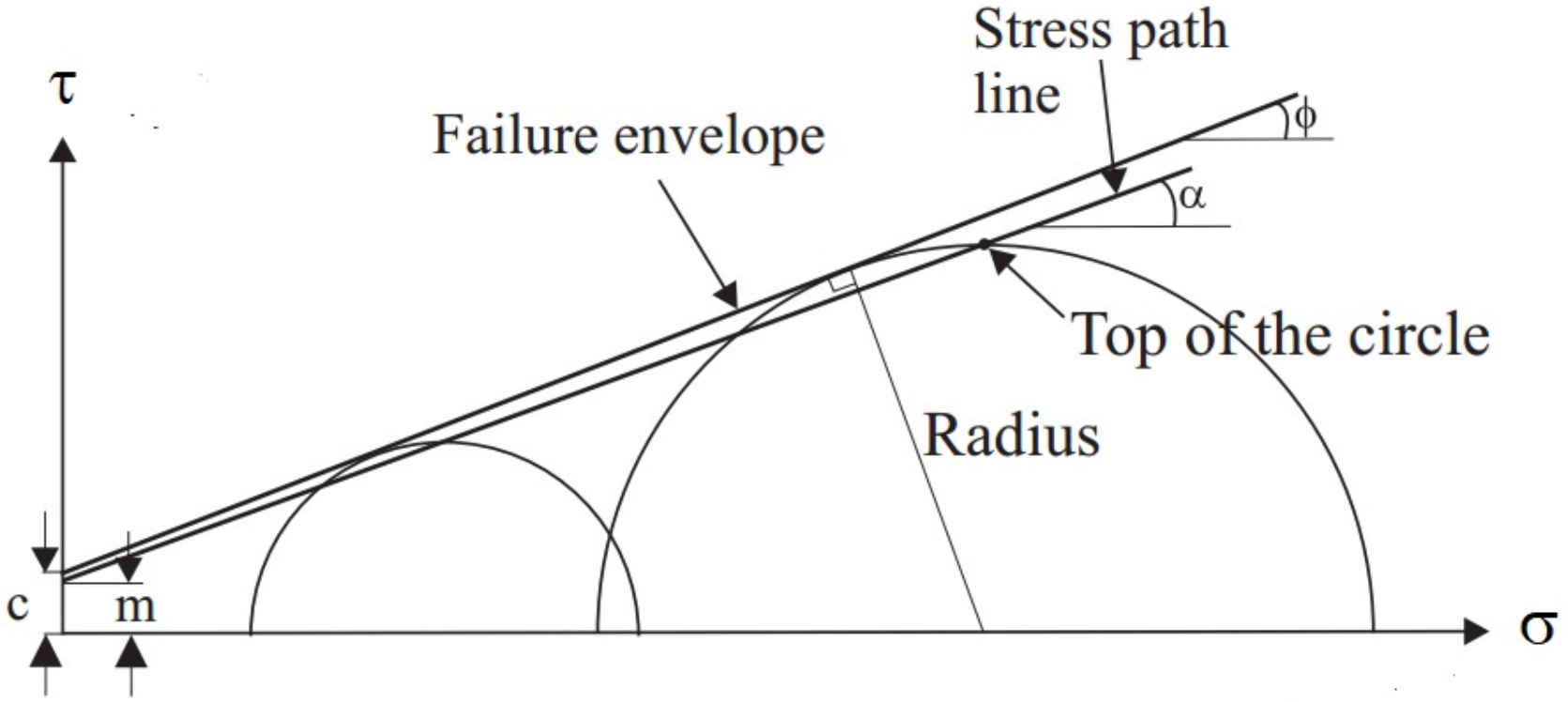


Fig. 6.58 Difference between stress path and Mohr-Coulomb failure criterion.

6.7.8 Determination of Shear Strength Parameters

Stress Point and Stress Path

Shear Strength Parameters

The shear strength parameters c and φ , thus, may be evaluated from test results in three ways.

1. **Graphically, by Mohr's circles**
2. **Semi-graphically, by stress paths**
3. **Analytically, by means of stress path coordinates.**