Traffic assignment (or route choice)

EWCE 351
Transportation Engineering

Outline

- Traffic assignment overview
- Uncongested conditions
  - All-or-nothing assignment
  - Example
- Congested conditions
  - User equilibrium
  - Example

Traffic assignment

Trip Generation → Trip Distribution → Mode Choice → Route Choice

P_i, A_j → T_{ij} → T_{ijm} → T_{ijmr}

Traffic Analysis Zone

TAZ	P_i
1	47
2	66
3	110
T	223

trip production

from i

to j

T_{ij}	1	2	3	P_i
1	10	18	19	47
2	30	32	4	66
3	5	40	65	110
A_j	45	90	88	223

TAZ	A_j
1	45
2	90
3	88
T	223

trip attraction

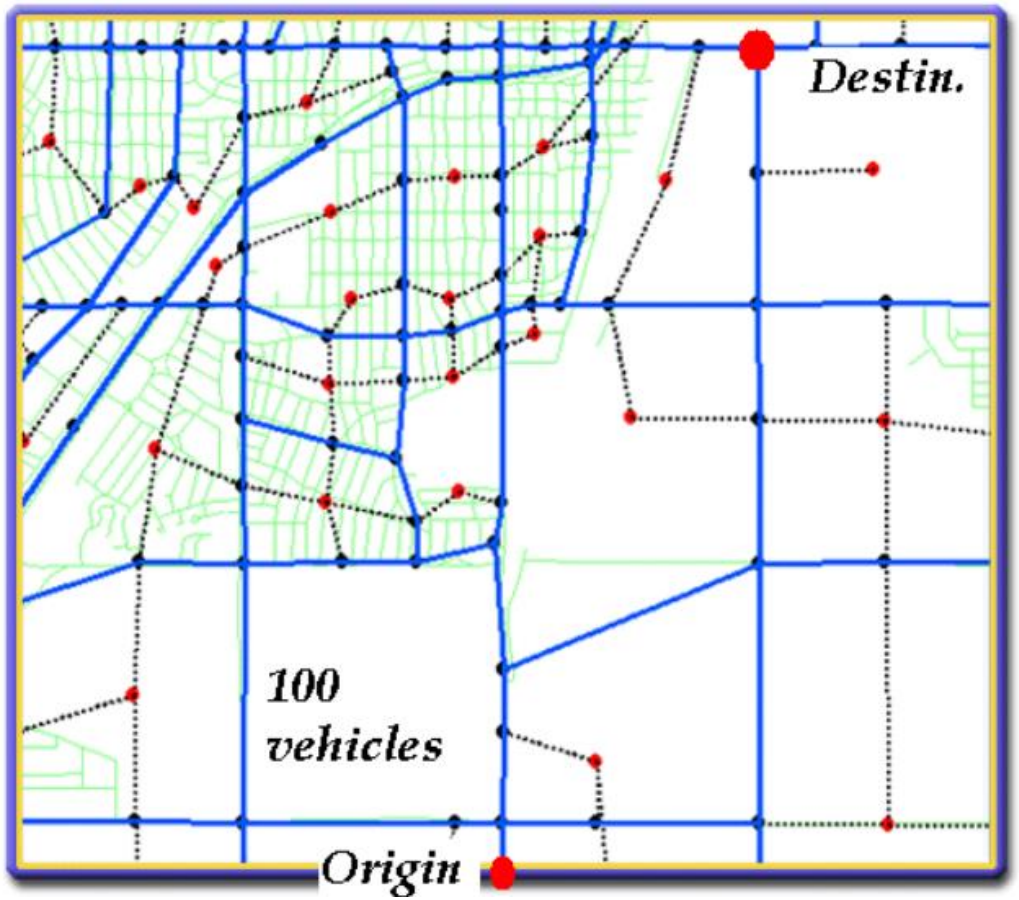
Mode	T_{32m}
car	20
bus	15
walk	5
Total	40

Path	$T_{32car,r}$
R_1	5
R_2	12
R_3	3
Total	20

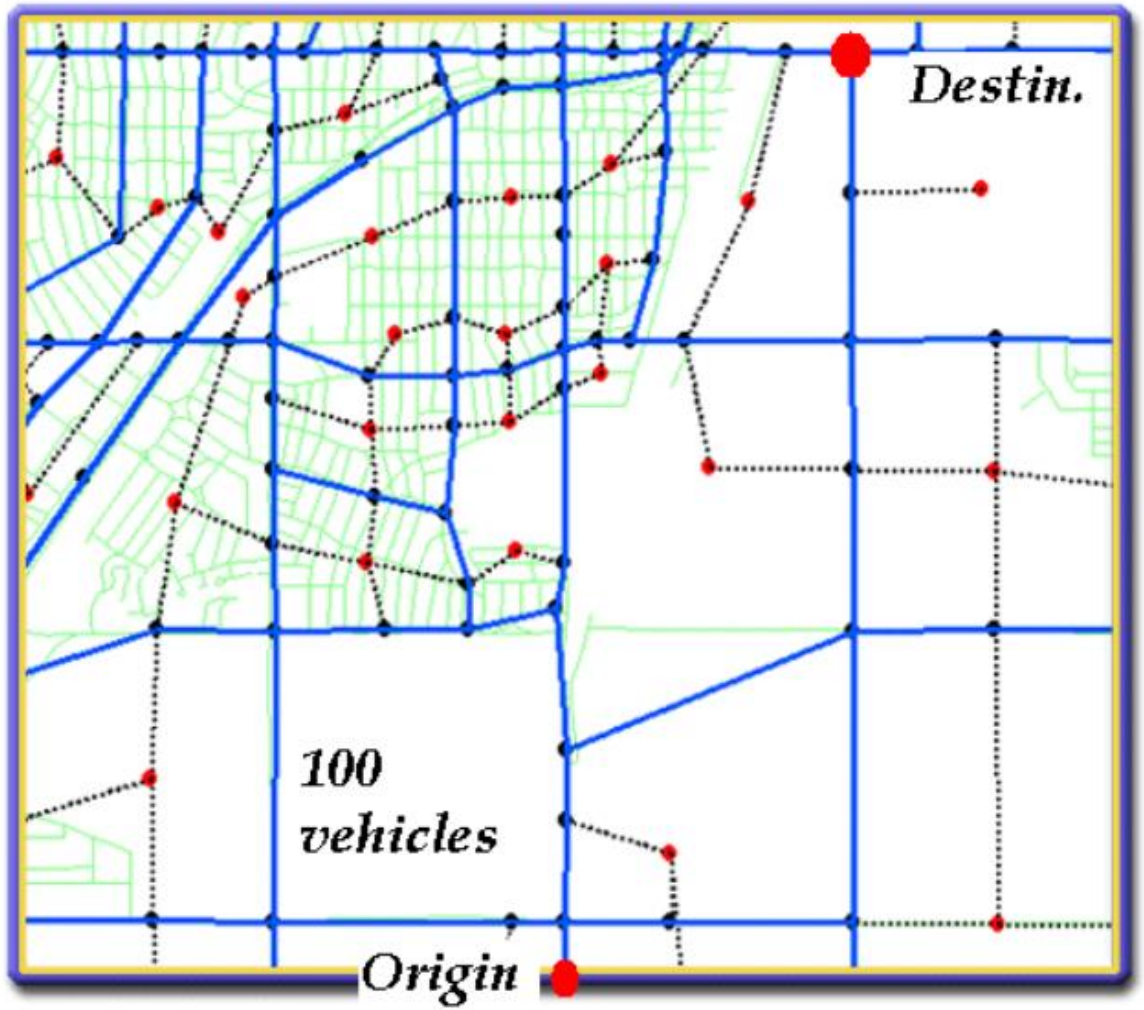
Traffic assignment (or route choice)

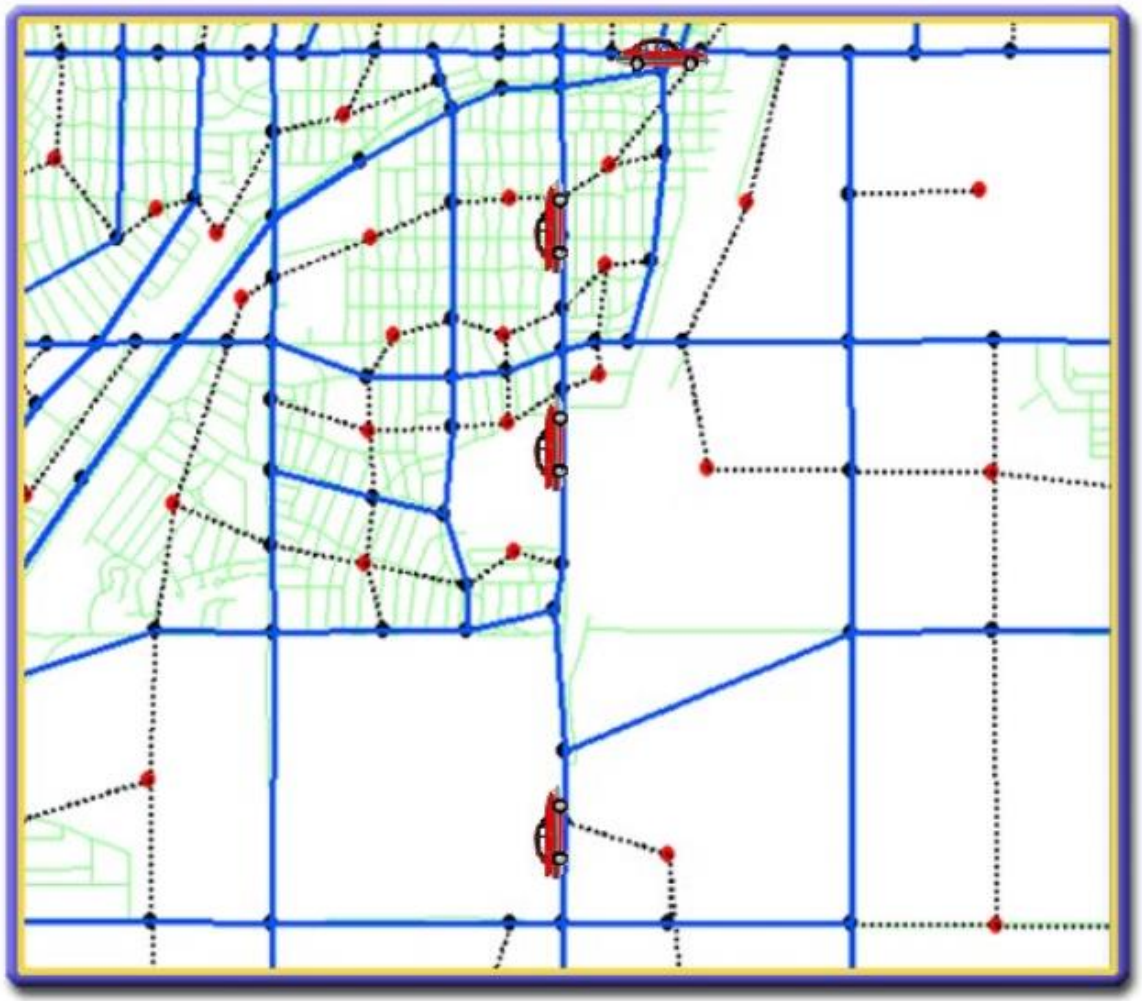
- Purpose
  - Determine the selection of routes by travelers between origins and destinations
  - Result of traffic assignment / route choice is traffic flow (vehicles per hour) on specific routes/links
    - One of the ultimate goals of the travel demand modeling/forecasting process
- Challenge
  - How to model & predict user behavior?
  - What assumptions do we make?
  - What factors affect travelers' route choices?

How do you get there?

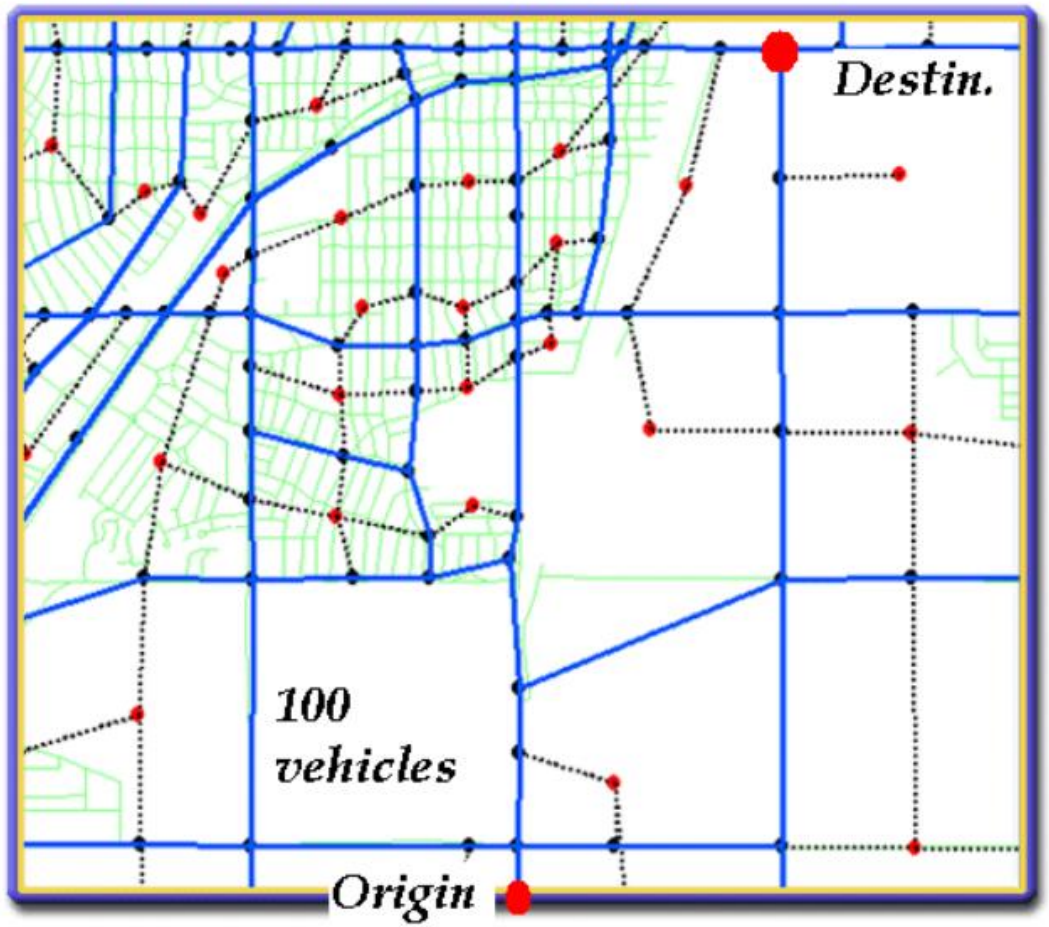


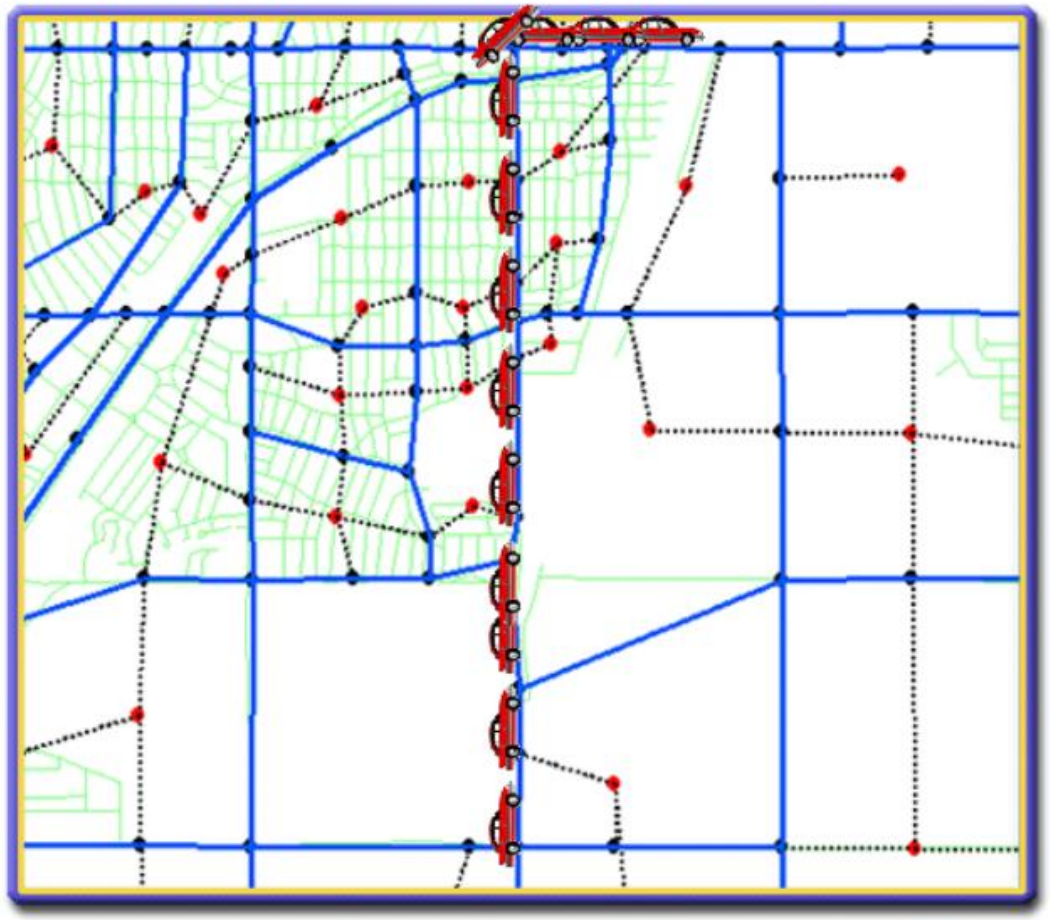
Shortest / fastest path



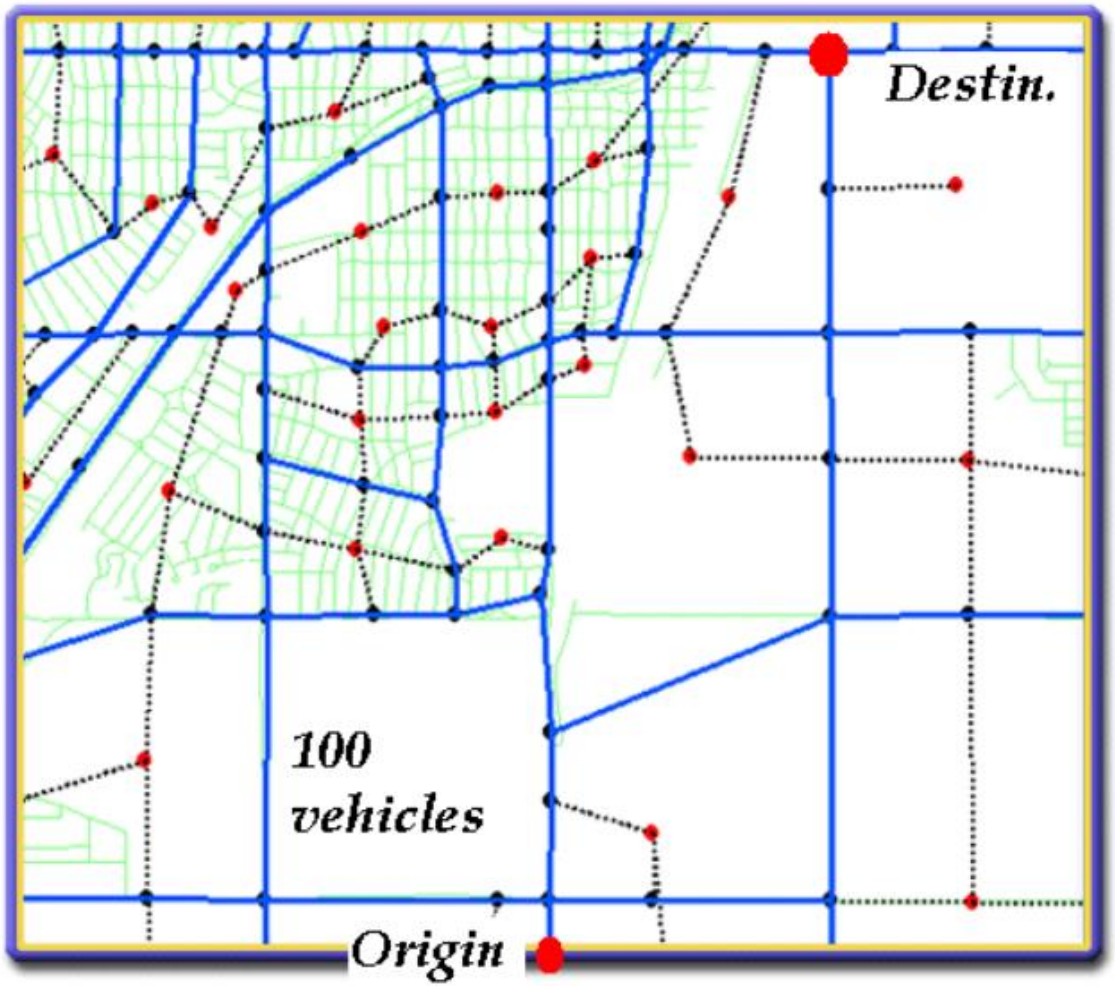


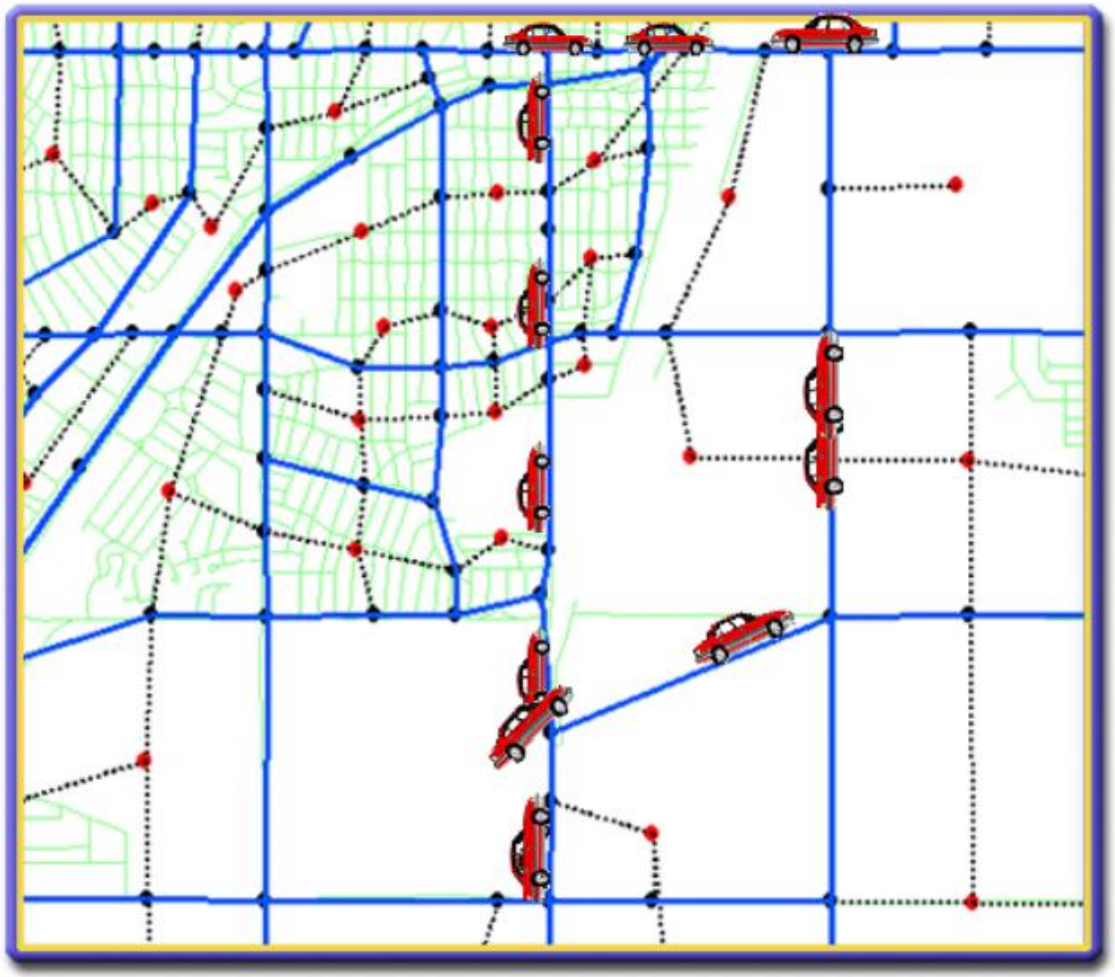
What if there is congestion?





Some detour to second shortest path





Uncongested condition

All-or-nothing (AON) assignment

All-or-nothing (AON) assignment

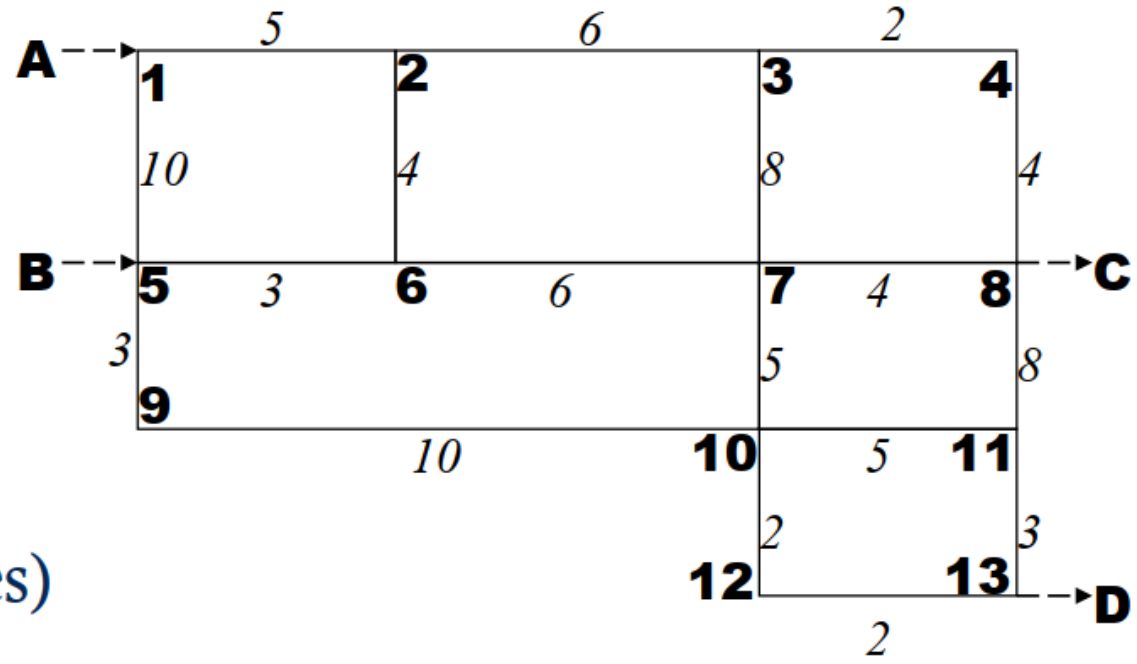
- Assumptions
  - People choose the shortest path to travel from an origin to destination
  - Roads are uncongested – the travel times/costs for each link of a roadway are a constant over the time-horizon of interest

- Procedure
  - Determine the shortest path between each origin-destination (OD) pair
  - Assign all trips between the OD pair to this path

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AON Example

- A,B,C, and D are zone centroids (origins and destinations of trips)
- 1,2,3,..,13 are nodes of a highway network
- The stretch of roadway between any pair of adjacent nodes is a roadway link (e.g., 1-2, 5-6, 3-7, 8-11, etc.)
- The numbers in italics on the links are the travel times on the links (e.g.: the travel time on link 1-2 is 5 minutes)

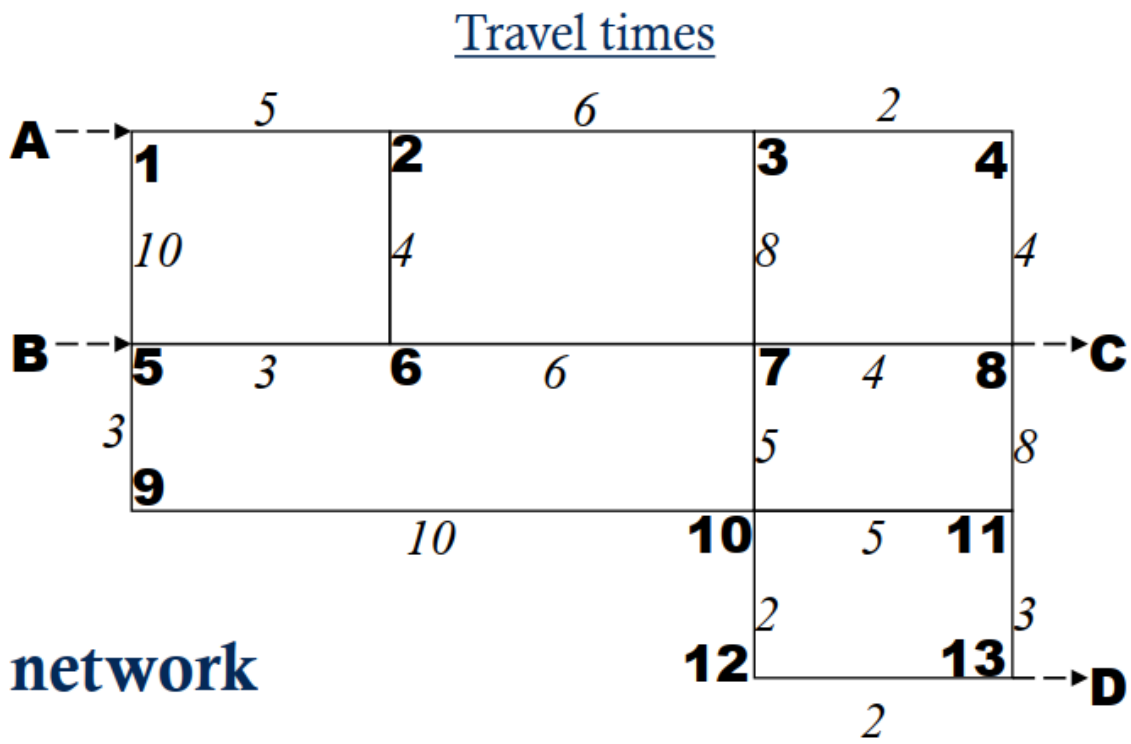


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AON Example

- The following is the demand (OD matrix)

Origin \ Destination	A	B	C	D
A	0	0	400	200
B	0	0	300	100
C	0	0	0	0
D	0	0	0	0

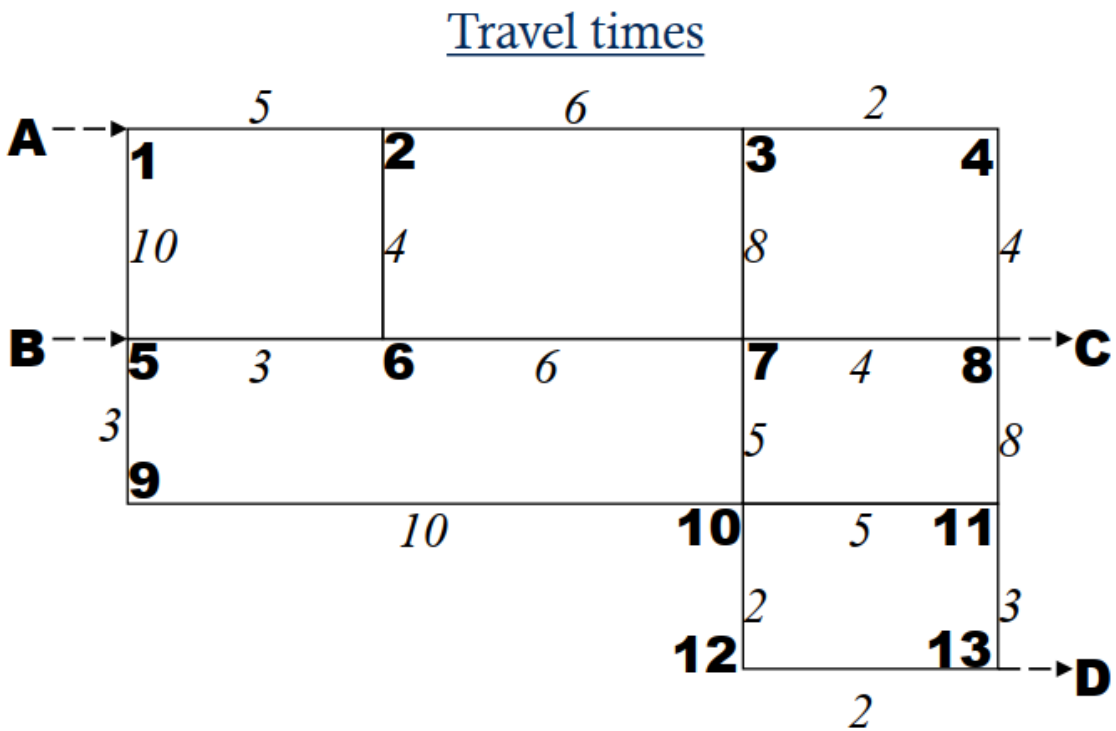


- **Determine the link flows on the highway network**

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AON Example

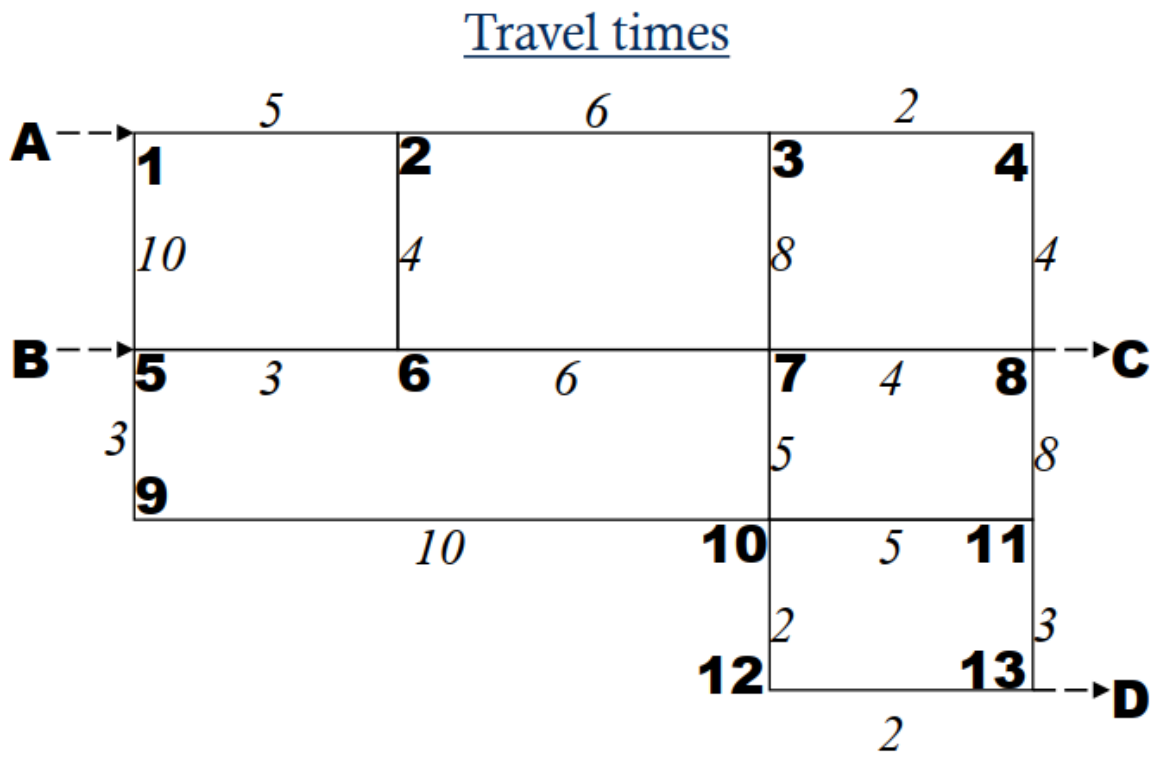
- Step 1:
  - Find the shortest paths from
    - A to C
    - A to D
    - B to C
    - B to D



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AON Example

- Step 1:
  - Find the shortest paths from
    - A to C
      - 1-2-3-4-8 → 17 minutes
    - A to D
      - 1-2-6-7-10-12-13 → 24 minutes
    - B to C
      - 5-6-7-8 → 13 minutes
    - B to D
      - 5-9-10-12-13 → 17 minutes



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AON Example

- Step 2:
  - Assign all demand between a zonal pair to the shortest path between zones
    - A to C
      - 1-2-3-4-8 → 17 minutes
    - A to D
      - 1-2-6-7-10-12-13 → 24 minutes
    - B to C
      - 5-6-7-8 → 13 minutes
    - B to D
      - 5-9-10-12-13 → 17 minutes

Origin \ Destination	A	B	C	D
A	0	0	400	200
B	0	0	300	100
C	0	0	0	0
D	0	0	0	0

Link	Flow	Link	Flow
1 to 2		7 to 8	
2 to 3		5 to 9	
3 to 4		7 to 10	
1 to 5		8 to 11	
2 to 6		9 to 10	
3 to 7		10 to 11	
4 to 8		10 to 12	
5 to 6		11 to 13	
6 to 7		12 to 13	

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AON Example

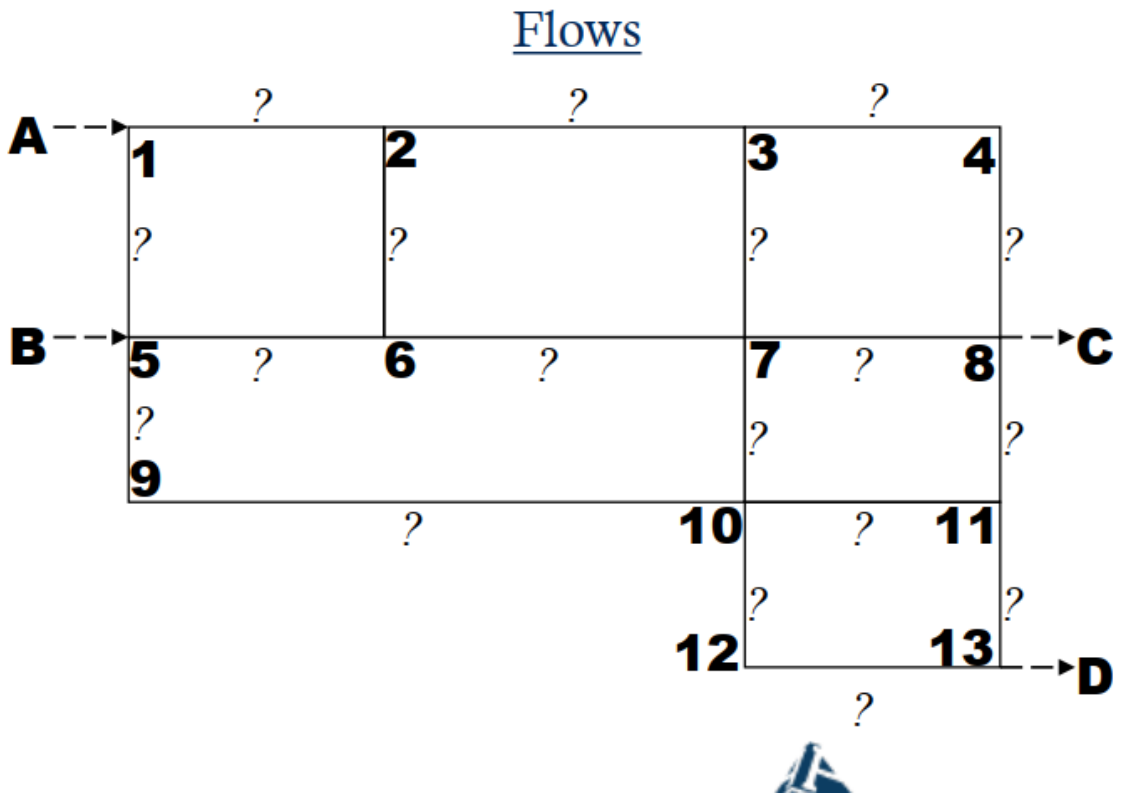
- Step 2:
  - Assign all demand between a zonal pair to the shortest path between zones
    - A to C
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    - B to D
      - 5-9-10-12-13 → 17 minutes

Origin \ Destination	A	B	C	D
A	0	0	400	200
B	0	0	300	100
C	0	0	0	0
D	0	0	0	0

Link	Flow	Link	Flow
1 to 2	600	7 to 8	300
2 to 3	400	5 to 9	100
3 to 4	400	7 to 10	200
1 to 5	0	8 to 11	0
2 to 6	200	9 to 10	100
3 to 7	0	10 to 11	0
4 to 8	400	10 to 12	300
5 to 6	300	11 to 13	0
6 to 7	500	12 to 13	300

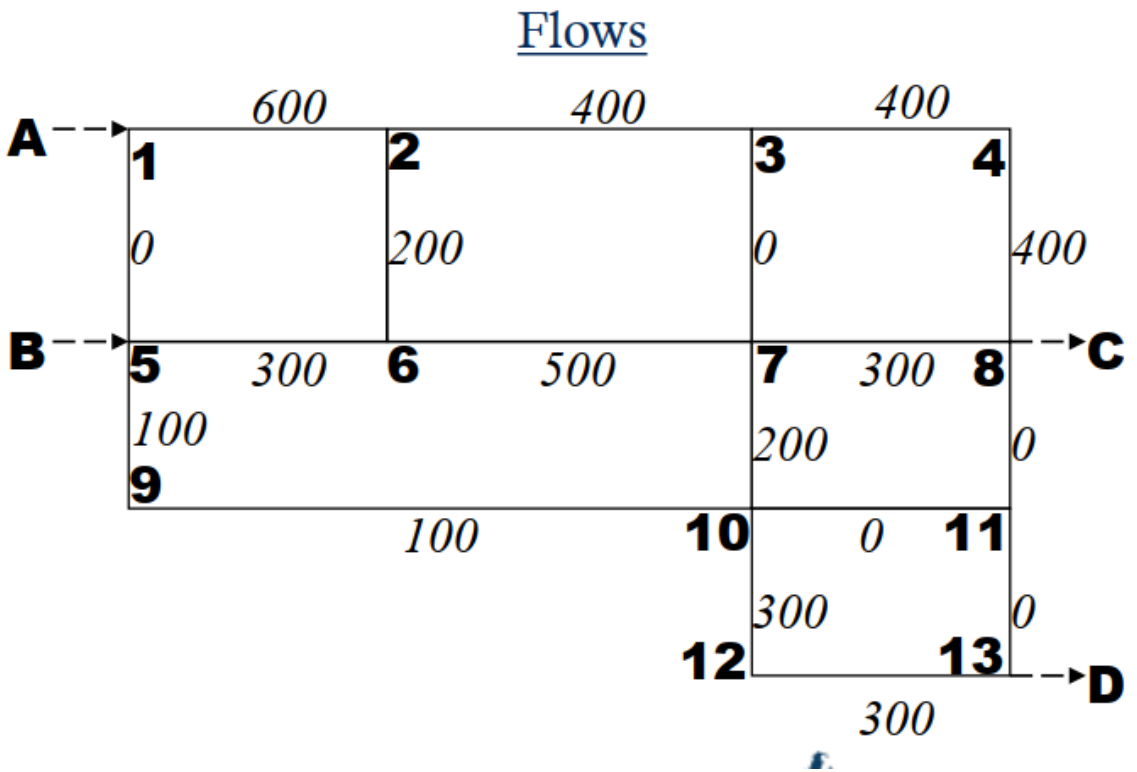
AON Example

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    - A to C
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AON Example

- Step 2:
  - Assign all demand between a zonal pair to the shortest path between zones
    - A to C
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    - B to C
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    - B to D
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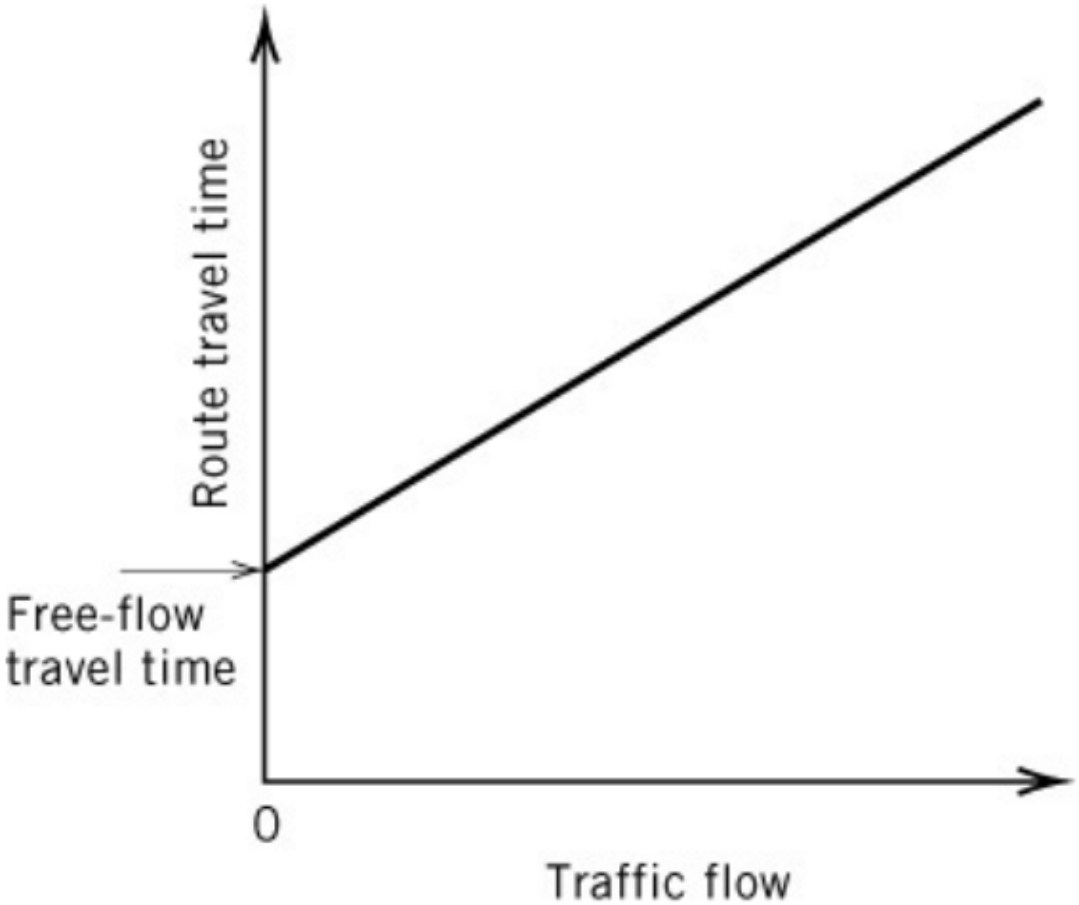
AON Summary

- Relatively simple approach
  - However determination of the shortest paths require computational time
  - There are systematic procedures for determination of the shortest paths for large-scale networks: e.g., The Dijkstra's algorithm
- Can lead to unreasonable assignment of flows when there are multiple paths between an OD pair with small differences in their travel times
- Assumes un-congested conditions: the travel times/costs for each link of a roadway are a constant over the time-horizon of interest irrespective of the number of vehicles on that link

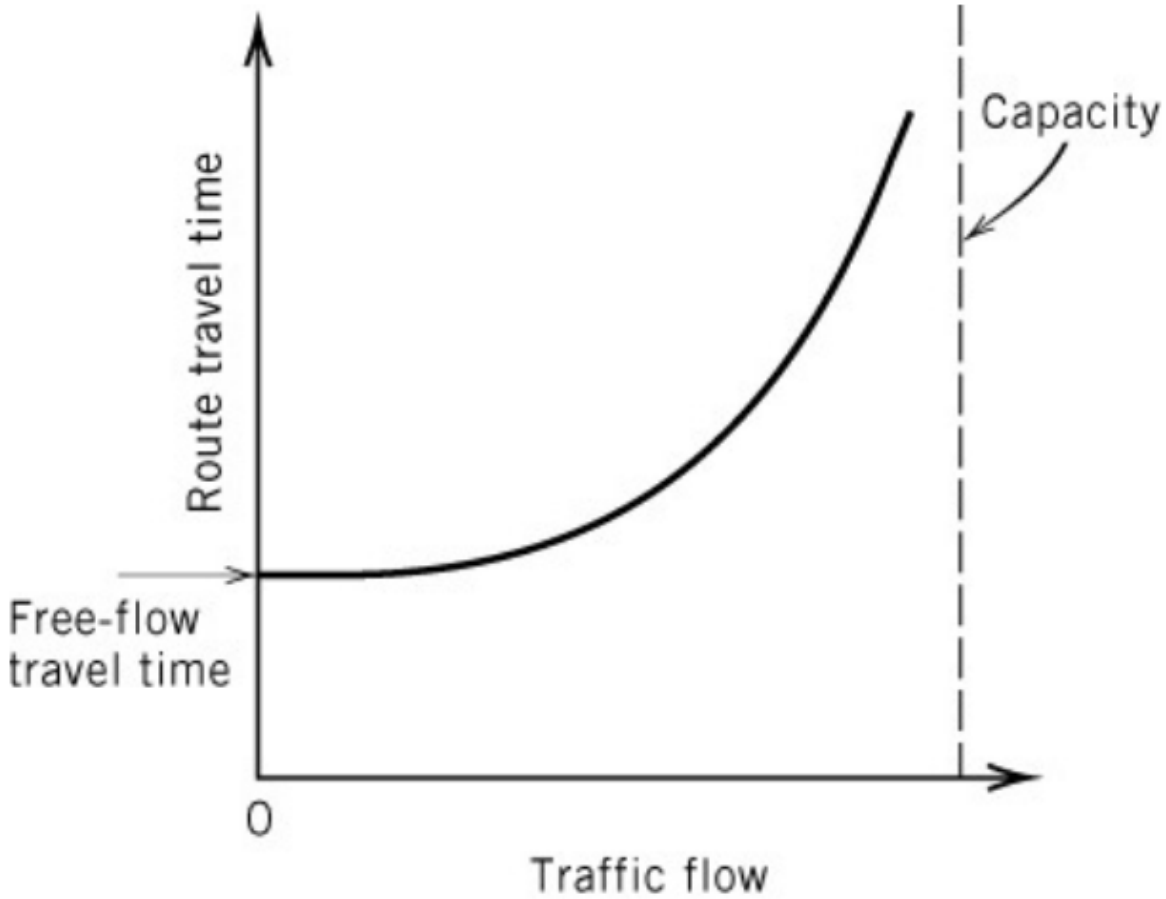
Congested condition

User equilibrium

Highway performance function



Linear travel time-traffic flow relationship



Nonlinear travel time-traffic flow relationship

Bureau of Public Roads (BPR) function

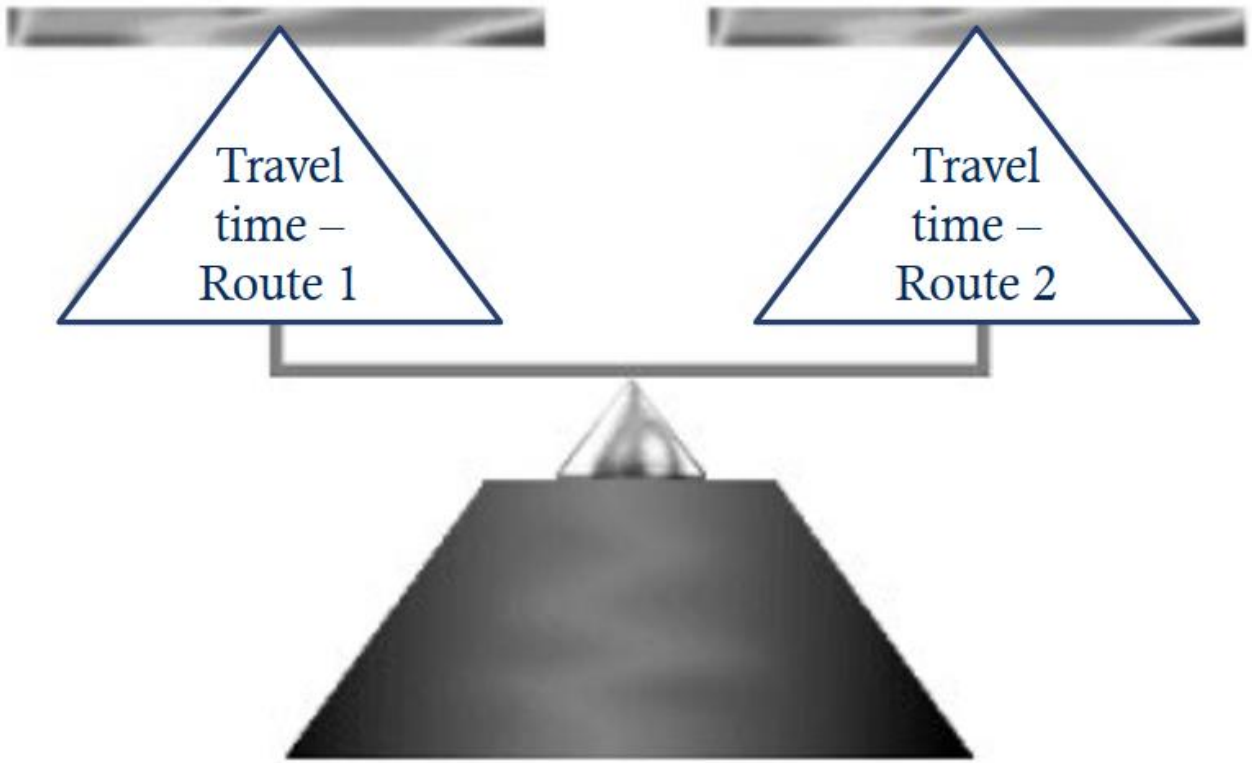
- Link congestion/performance or volume-delay or function
  - t or $S_a(v_a)$
    - Average travel time on a link
  - t_0 or t_a
    - Free flow travel time on a link
  - V or v_a
    - Volume of traffic on a link, vehicles per time
  - Q or c_a
    - Capacity of a link, vehicles per time
  - α and β
    - Parameters, $\alpha = 0.15$ and $\beta = 4$

$$t = t_0 \left[1 + \alpha \left(V/Q\right)^{\beta}\right]$$

$$S_a(v_a) = t_a \left(1 + 0.15 \left(\frac{v_a}{c_a}\right)^4\right)$$

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Equilibrium



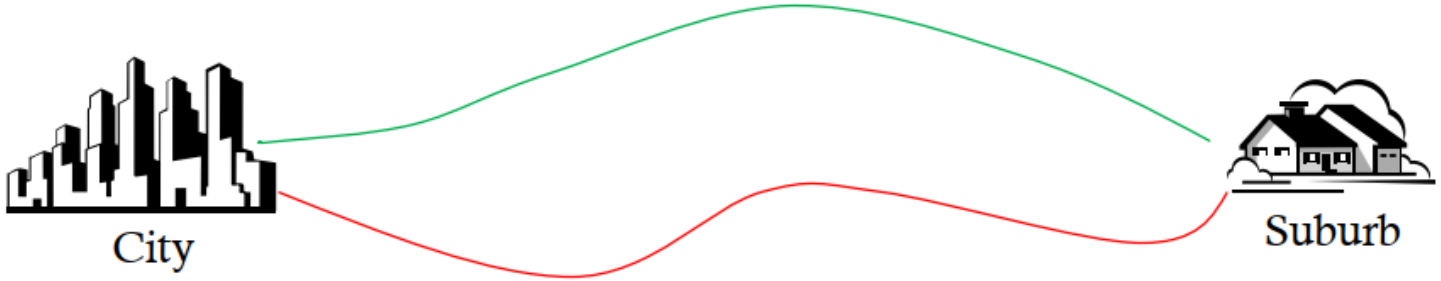
User equilibrium (UE) assignment

- Assumptions
  - Travelers will select routes based only on route travel times
  - Travelers know the travel times on all the available routes (full information)
- Wardrop's definition:
  - "The travel time between a specified origin and destination
    on all used routes
    is equal, and less than or equal
    to the travel time that would be experienced by a traveler on any unused route"
- User equilibrium exists when individual travelers cannot improve their travel times by unilaterally changing routes

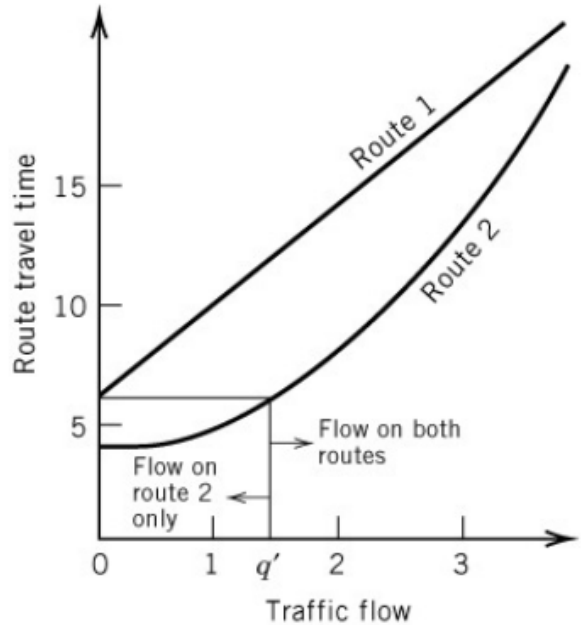
UE assignment

- Wardrop's principles
  - First principle (User Equilibrium)
    - Under equilibrium conditions traffic arranges itself in congested networks in such a way that no individual trip maker can reduce his path costs by switching routes.
  - Second principle (System Optimal)
    - Under social optimal conditions traffic should be arranged in congested networks in such a way that the average (or total) travel cost is minimized.

UE Example

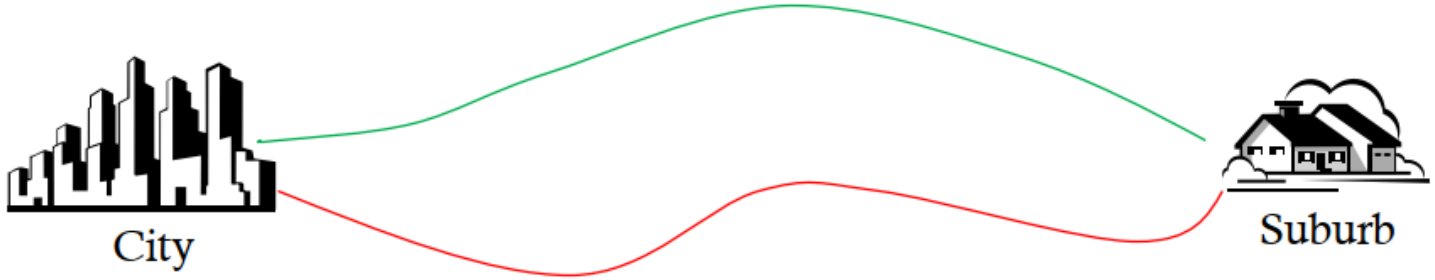


- Assume:
  - Two routes connect a city and a suburb.
  - During the peak-hour morning commute, a total of 4,500 vehicles travel from the suburb to the city.
  - The travel time functions for these routes are given:
    - $TT_1(x_1) = 6 + 4x_1$
    - $TT_2(x_2) = 4 + (x_2)^2$
    - where TT(x) is travel time in minutes and x is traffic volume in 1000 vehicles.



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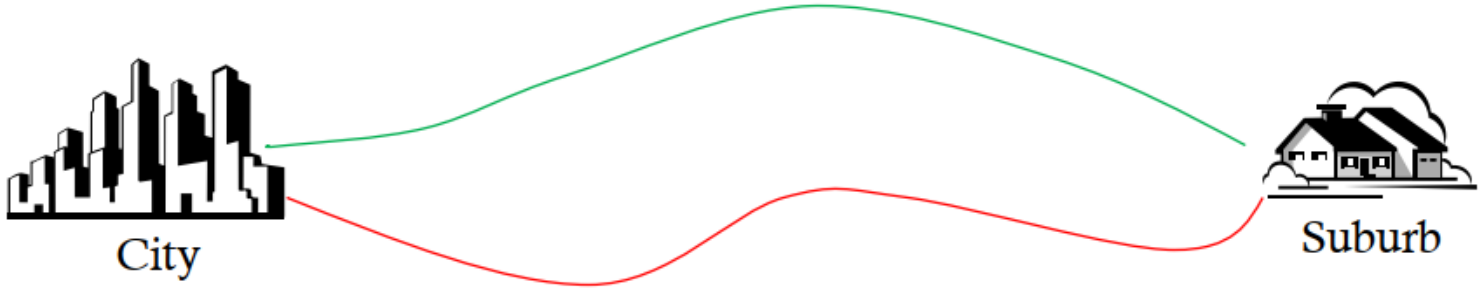
UE Example



- Step 1:
  - Write equations
    - $TT_1(x_1) = 6 + 4x_1$
    - $TT_2(x_2) = 4 + (x_2)^2$
    - Flow constraint: $x_1 + x_2 = 4.5$

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UE Example



- Step 2:
  - Check if both routes will be used
    - All-or-nothing assignment on Route 1 ($x_1 = 4.5$, $x_2 = 0.0$)
      - $TT_1(4.5) = 6 + 4(4.5) = 24$ minutes
      - $TT_2(4.5) = 4 + (0.0)^2 = 4$ minutes

      } If all the traffic is on Route 1 then Route 2 is the desirable choice

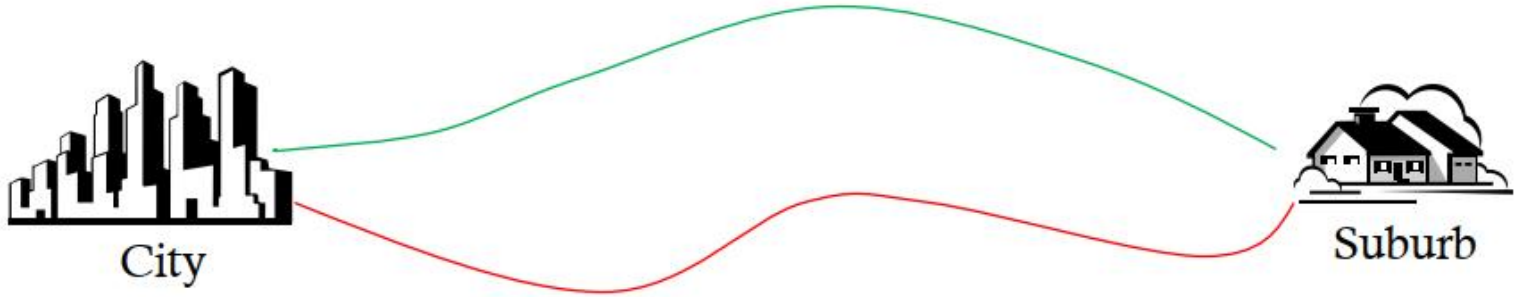
    - All-or-nothing assignment on Route 2 ($x_1 = 0.0$, $x_2 = 4.5$)
      - $TT_1(4.5) = 6 + 4(0.0) = 6$ minutes
      - $TT_2(4.5) = 4 + (4.5)^2 = 24.25$ minutes

      } If all the traffic is on Route 1 then Route 2 is the desirable choice

  - Therefore…both routes will be used

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UE Example



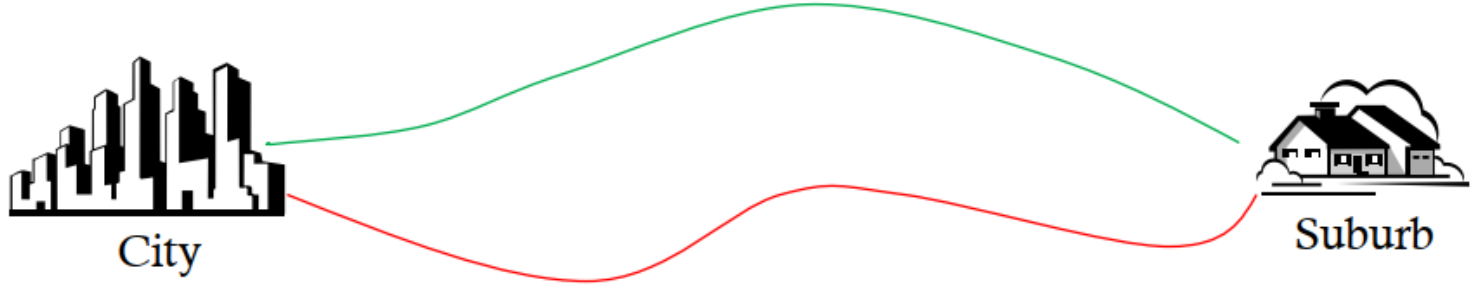
- Step 3:
  - Equate travel times and solve
    - Apply Wardrop's 1st principle requirements.
    - All routes used will have equal times, and ≤ those on unused routes.
    - Hence, if flows are distributed between Route 1 and Route 2, then both must be used, and travel times will be equal.

  - At user equilibrium:
    - $TT_1(x_1) = TT_2(x_2)$ → $6 + 4x_1 = 4 + (x_2)^2$
    - Flow constraint: $x_1 + x_2 = 4.5$

  - Two unknowns + two equations → solve for x_1 and x_2!

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UE Example



- Step 3:
  - Solve for x_1 and x_2
    - $x_1 + x_2 = 4.5 \rightarrow x_2 = 4.5 - x_1$
    - $6 + 4x_1 = 4 + (x_2)^2 \rightarrow 6 + 4x_1 = 4 + (4.5 - x_1)^2 \rightarrow 6 + 4x_1 = 4 + 20.25 - 9x_1 + x_1^2$
      $\rightarrow x_1^2 - 13x_1 + 18.25 = 0$
    - ...using quadratic formula...
    - $x_1 = 1.6$ or $11.4 \rightarrow$ only 1.6 is reasonable (< 4.5) $\rightarrow$ **x_1 = 1,600 vehicles/hour**
    - $x_2 = 4.5 - x_1 = 4.5 - 1.6 = 2.9$ $\rightarrow$ **x_2 = 2,900 vehicles/hour**

  - Check for equal travel times
    - $TT_1(1.6) = 6 + 4(1.6) = 12.4$ minutes
    - $TT_2(2.9) = 4 + (2.9)^2 = 12.4$ minutes

} Good!

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Other traffic assignment methods

1. All-Or-Nothing (AON) Assignment
2. Capacity-Restrained (CR) Assignment
3. Incremental Assignment
4. Method of Successive Averages (MSA)
5. Convex Combinations Method (also known as the Frank-Wolfe (FW) Algorithm)

Wardrop's principles

- First principle (User Equilibrium)
  - Under equilibrium conditions traffic arranges itself in congested networks in such a way that no individual trip maker can reduce his path costs by switching routes.

- Second principle (System Optimal)
  - Under social optimal conditions traffic should be arranged in congested networks in such a way that the average (or total) travel cost is minimized.

- Are these principles compatible? Does the first lead to the second?

Wardrop's principles

Nash equilibrium
(https://en.wikipedia.org/wiki/Nash_equilibrium)

- First principle (User Equilibrium)
  - Focused on the individual, individual optimum, user equilibrium
  - Useful for explaining (selfish) behavior
- Second principle (System Optimal)
  - Focused on the entire system, system optimum, social equilibrium
  - Useful for managing/planning (social) travel
- How to move: first principle → second principle?
  - Congestion charging, road pricing, other interventions

Braess' paradox

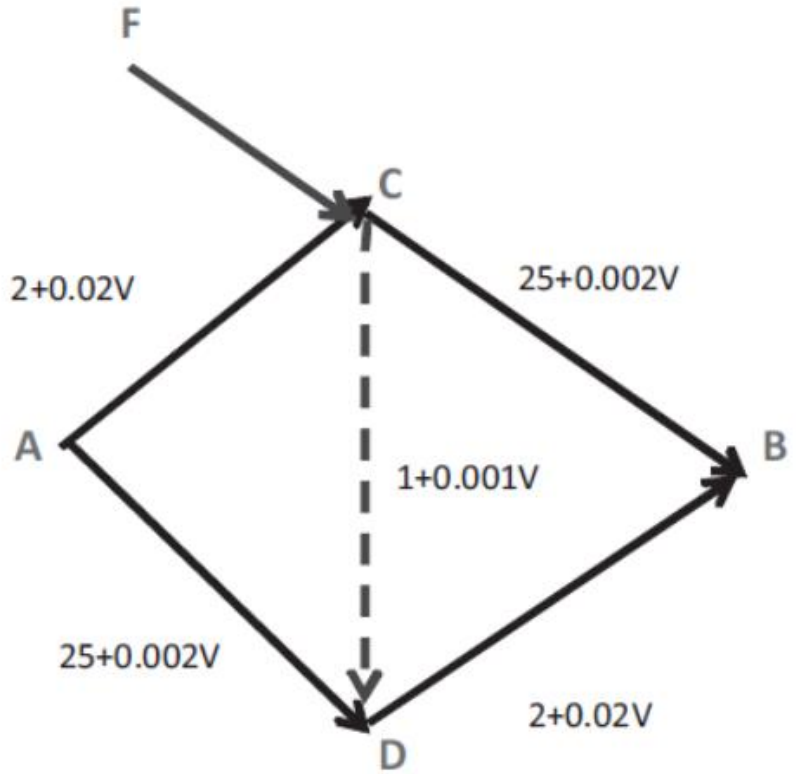


Figure 10.8 A simple network to illustrate Braess's Paradox

- Adding network capacity might actually make everyone worse off.
- Example
  - Initial conditions
    - 1000 car trips A→B
    - Only options: A–C–B, A–D–B
  - New conditions
    - Add new high-speed & high-capacity link C–D
    - What happens?

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Dynamics & randomness

Stochastic user equilibrium
Dynamic traffic assignment

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Stochastic user equilibrium (SUE)

- All-or-nothing (AON) assumes fixed and known travel times
- User equilibrium (UE) assumes congestion-restrained but known travel times
- Stochastic user equilibrium (SUE) assumes variation in congestion-restrained travel times, due to…
  - Different "perceptions" of travel times
    - Actual different perceived travel times
    - Lack of knowledge, and guesses about expected travel times
    - Travel times being truly unknowable in advance
- → add error term to volume-delay function

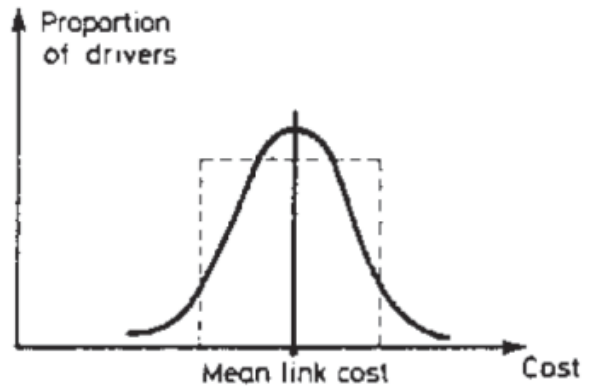


Figure 10.4 Distribution of perceived costs on a link

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Stochastic user equilibrium (SUE)

- Problems for application
  - Assumption of the error term distribution
    - Uniform? Normal? Something else?
  - Difficulty converging on a solution (efficiently)
    - UE faster and approximately equal in congested networks
- Methods
  - Monte Carlo simulation
  - Logit-like proportions
- → Not frequently used in practice

Dynamic traffic assignment (DTA)

- Unrealistic assumptions thus far
  - Perfect information
    - Full knowledge of current and future conditions
    - → Even with GPS/Waze/other tools, cannot know the future for certain
  - Separability
    - Link travel time affected by traffic flow on that link (and nothing else)
    - → Downstream nodes and links can affect upstream traffic, bottlenecks
  - Steady state
    - Traffic flows are constant during a time period
    - → Traffic flows change day-by-day, hour-by-hour, and surges / platoons

Dynamic traffic assignment (DTA)

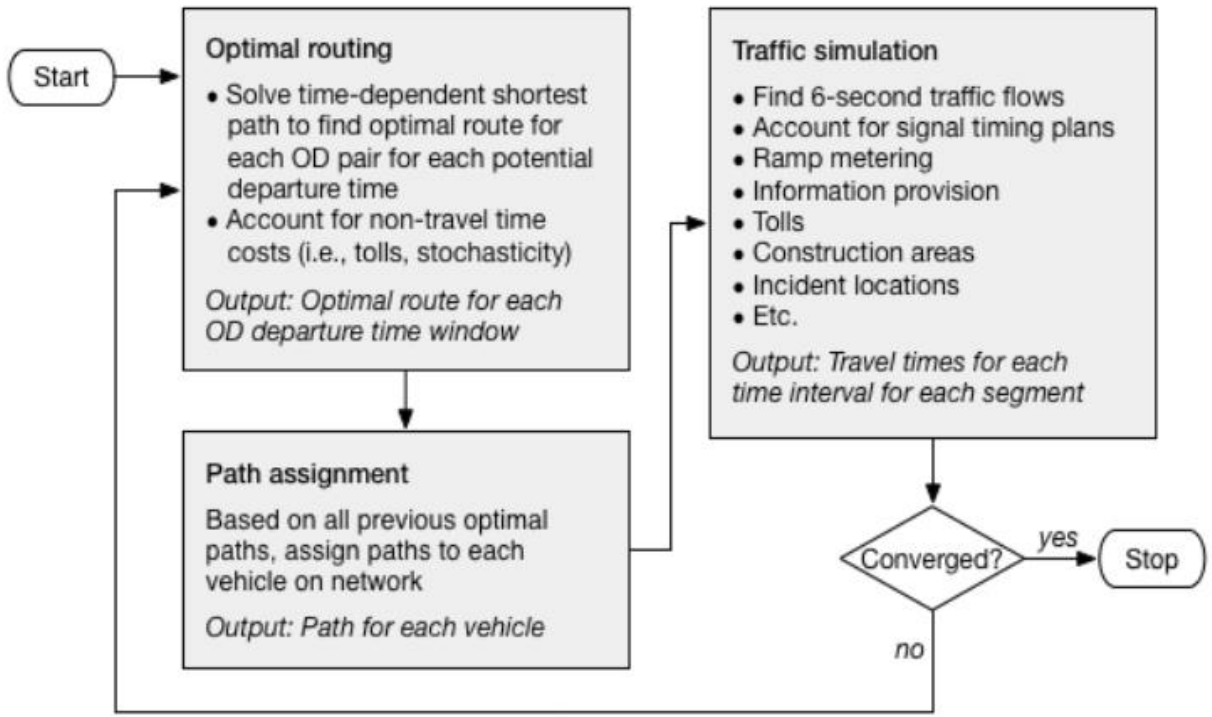
- Requirements
  - *Positivity*: non-negative flows on links, paths, trip matrices, and costs
  - *Conservation*: satisfy flow conservation requirements
  - *FIFO*: "First In, First Out" traffic behavior
  - *Minimum travel time*: flows do not propagate instantaneously
  - *Finite clearing time*: no queues left at the end of the modeling period; infinite delays do not occur (as a standard queueing model might suggest)
  - *Capacity*: strict capacity constraint, cannot be exceeded (even for a short time)
  - *Causality*: delays now affected by the present or the past, not the future

Dynamic traffic assignment (DTA)

- Wardrop's first principle (user equilibrium)
  - Under equilibrium conditions traffic arranges itself in congested networks in such a way that no individual trip maker can reduce his path costs *by switching routes*.
- Extension 1
  - Under equilibrium conditions in networks *where congestion varies over time* traffic arranges itself so that *at each instant* the costs incurred by drivers on those routes that are used are equal and no greater than those on any unused route.
- Extension 2
  - Under equilibrium conditions in networks where congestion varies over time and *travelers can choose their time of travel*, traffic arranges itself so that the total cost associated with travel on those route that are used by travelers *at the time when they are used*, are equal and no greater than those on any route *at a time when it is not used*.

Dynamic traffic assignment (DTA)

- How does DTA work?
  - A middle ground between static macroscopic traffic assignment and microscopic traffic simulation models (e.g., PTV Vissim)
  - Use same network/zonal structure, but…
  - Split time into discrete time intervals or time-slices



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Dynamic traffic assignment (DTA)

- DYNASMART: https://mctrans.ce.ufl.edu/featured/dynasmart/
- Dynus-T: https://dynust.net/
- TransDNA: https://www.caliper.com/transdna/transdna-dta-software.htm
- Dynameq: https://www.inrosoftware.com/en/products/dynameq/
- Cube Avenue: https://www.citilabs.com/software/cube/cube-avenue/
- and others…

Questions?