20.11.2022

Transportation Planning Overview

EWCE 351
Transportation Engineering

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What do you want to learn?

22.06.2025

- How do you plan a road? Behind the scenes. Planning process?
- How cities grow, land use changes, context of Dhaka
- Many modes working together efficiently
- Travel demand forecasting, travel time optimization
- Encouraging active transportation
- Issue unique to Dhaka, Rickshaw, slow congested traffic

Transportation Planning

22.06.2025

What is transportation planning?

- the process of
  - looking at current state of transportation,
  - designing for future transportation needs,
  - with the elements of budgets, goals and policies.



https://cdn.pixabay.com/photo/2014/01/04/12/34/street-238458_1280.jpg

Why plan?

- Anticipate/guide the future
- Inform decision making
- Moral obligation?

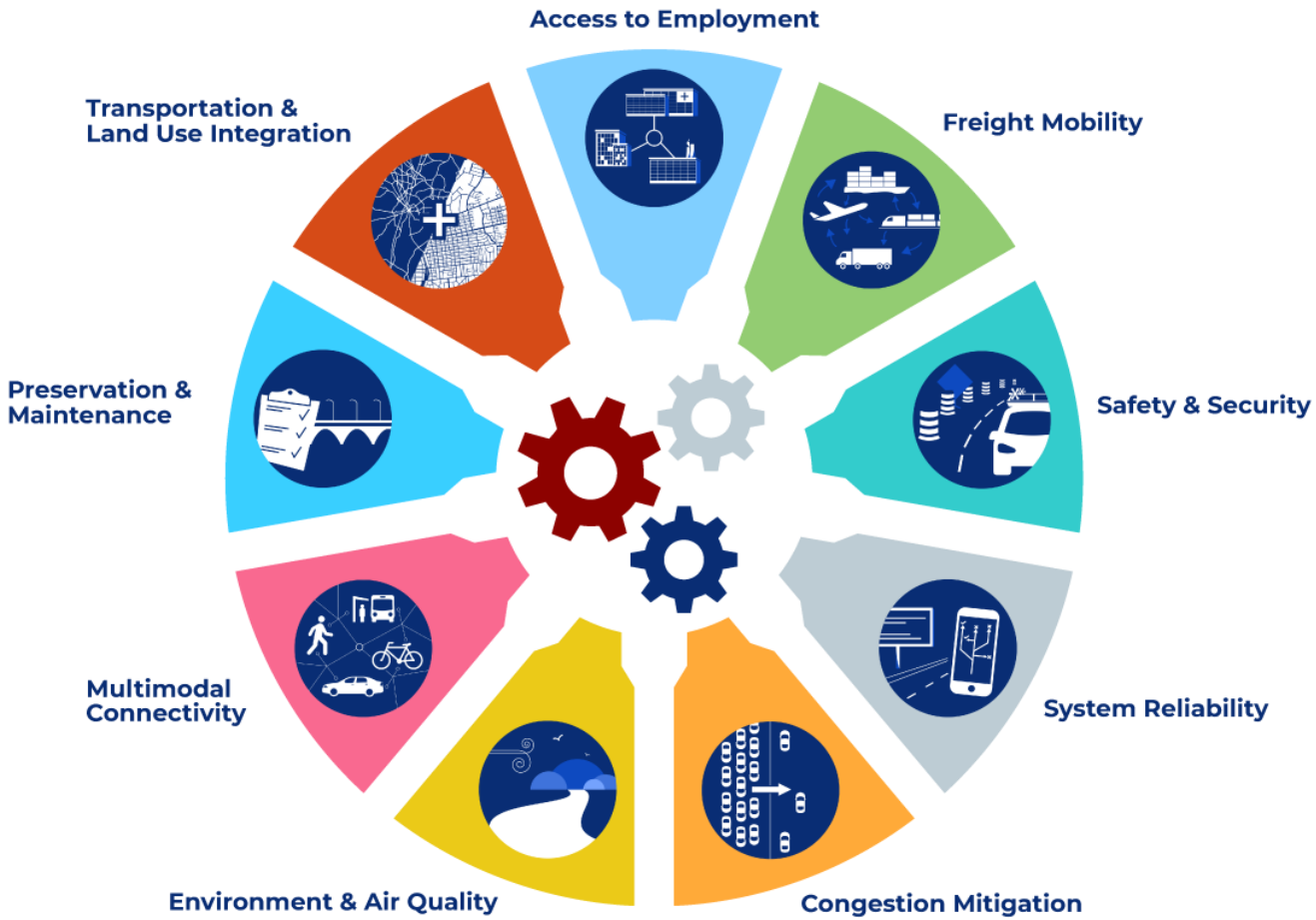
Transportation planning process



Source: FHWA Briefing Book (2015), https://www.fhwa.dot.gov/planning/publications/briefing_book/

Transportation planning goals

22.06.2025



Source: https://planrva.org/transportation/what-is-transportation-planning/

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Efficiency

22.06.2025



https://www.shareable.net/10-ways-vancouver-created-a-greener-more-efficient-transportation-system/

Equity

22.06.2025



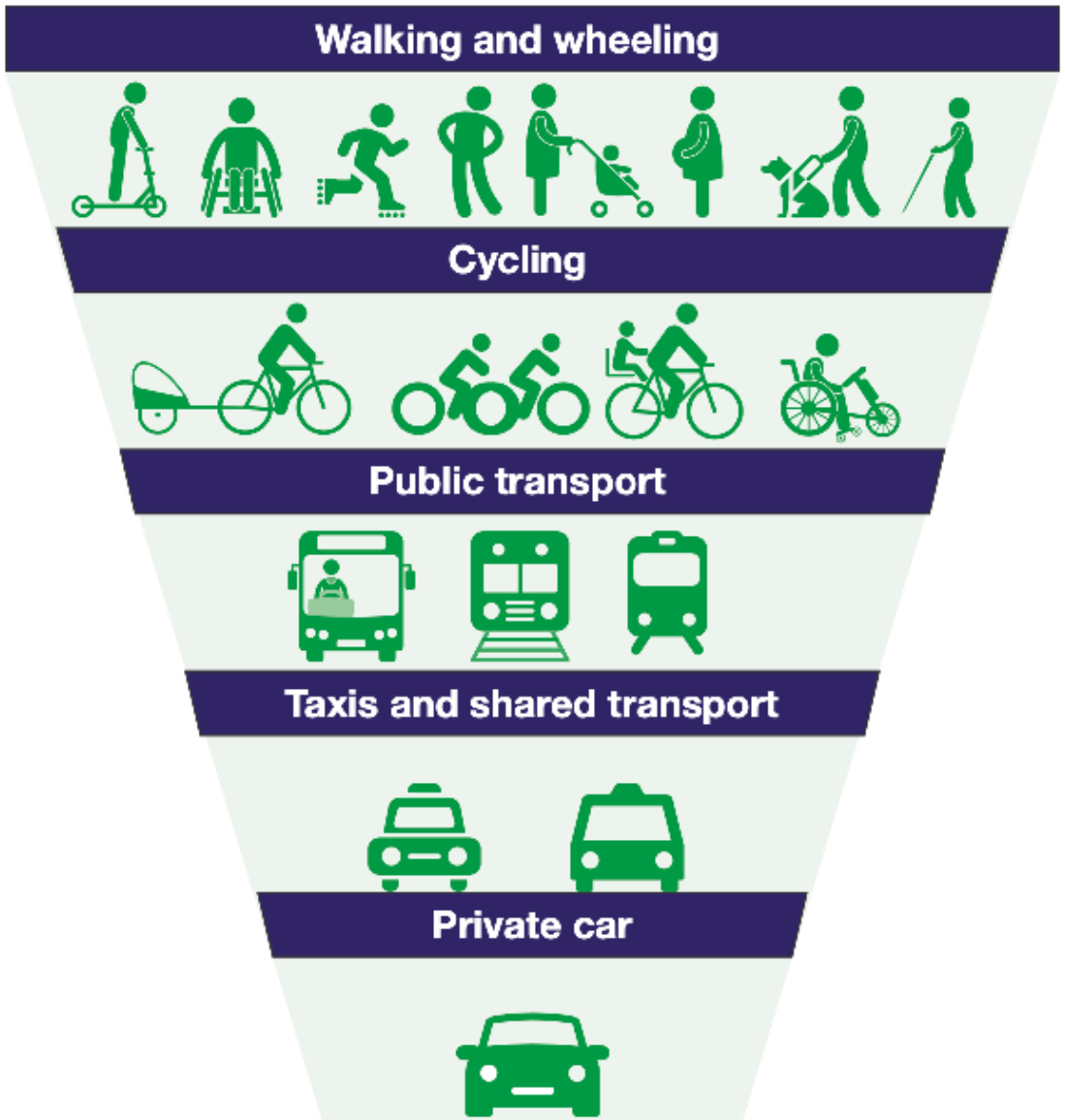
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Better Environment

22.06.2025

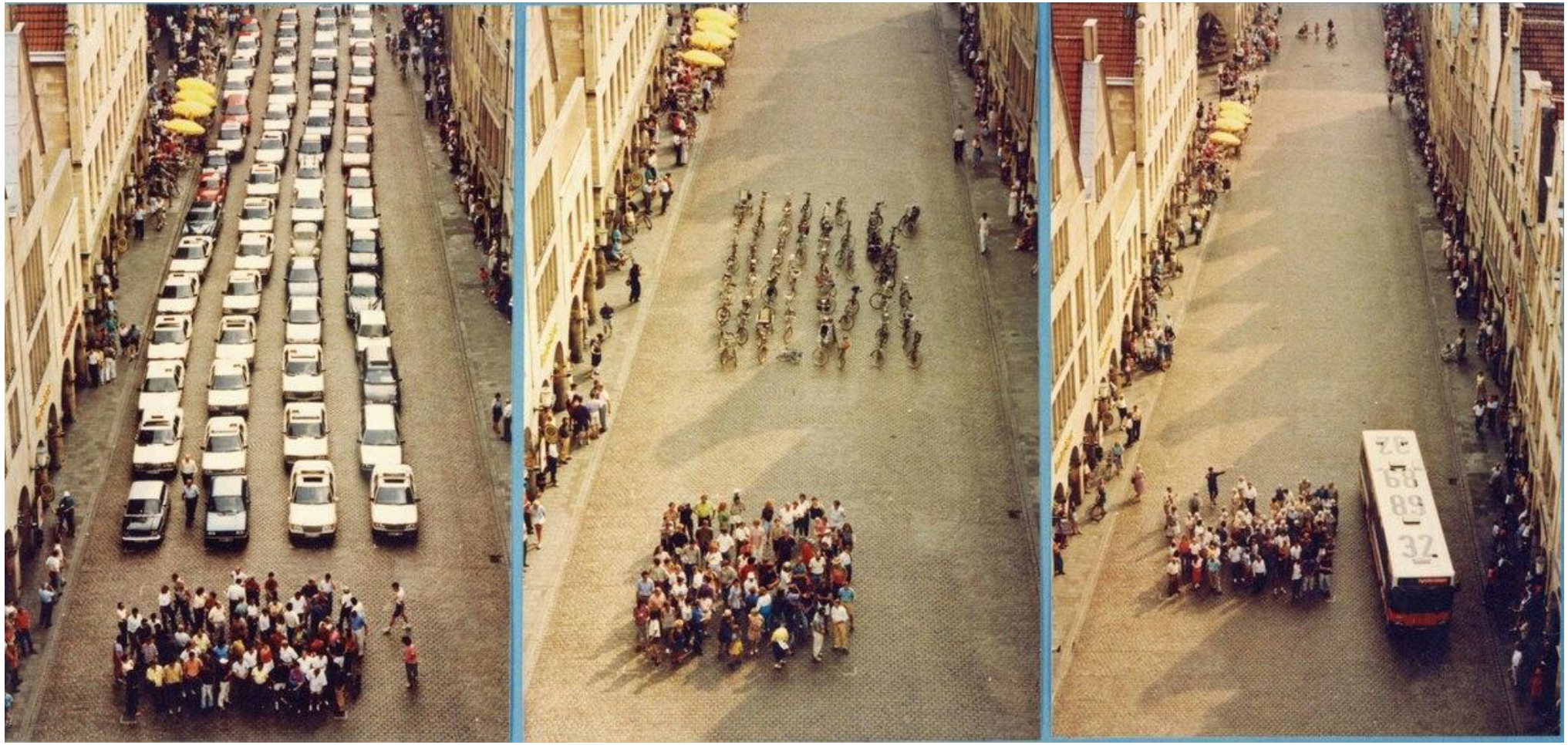
- air pollution and noises caused by automobiles should be minimized
  - by promoting public transport use and controlling the traffic demand.
  - by applying stricter vehicle emission standards.
- In addition, traffic safety should be enhanced and the number of accident victims should be minimized
  - through the enforcement of laws and regulations, intensive public campaigns, and training and education of drivers as well as general public.
  - Improvement of traffic facilities through engineering design would also contribute to the reduction of traffic accidents.


MOBILITY INSTEAD OF DIRTY TRAFFIC
tion fields on the road to a smart and sustainable city
solar rooftops for local energy generation
battery research and production facility
demand-driven traffic control
attractive traffic route for bikes an pedestrian
tramways and electric buses
self-driving cars extended public transport
charging stations for individual vehicles


Walking and wheeling
Cycling
Public transport
Taxis and shared transport
Private car

Road Space Requirement

22.06.2025

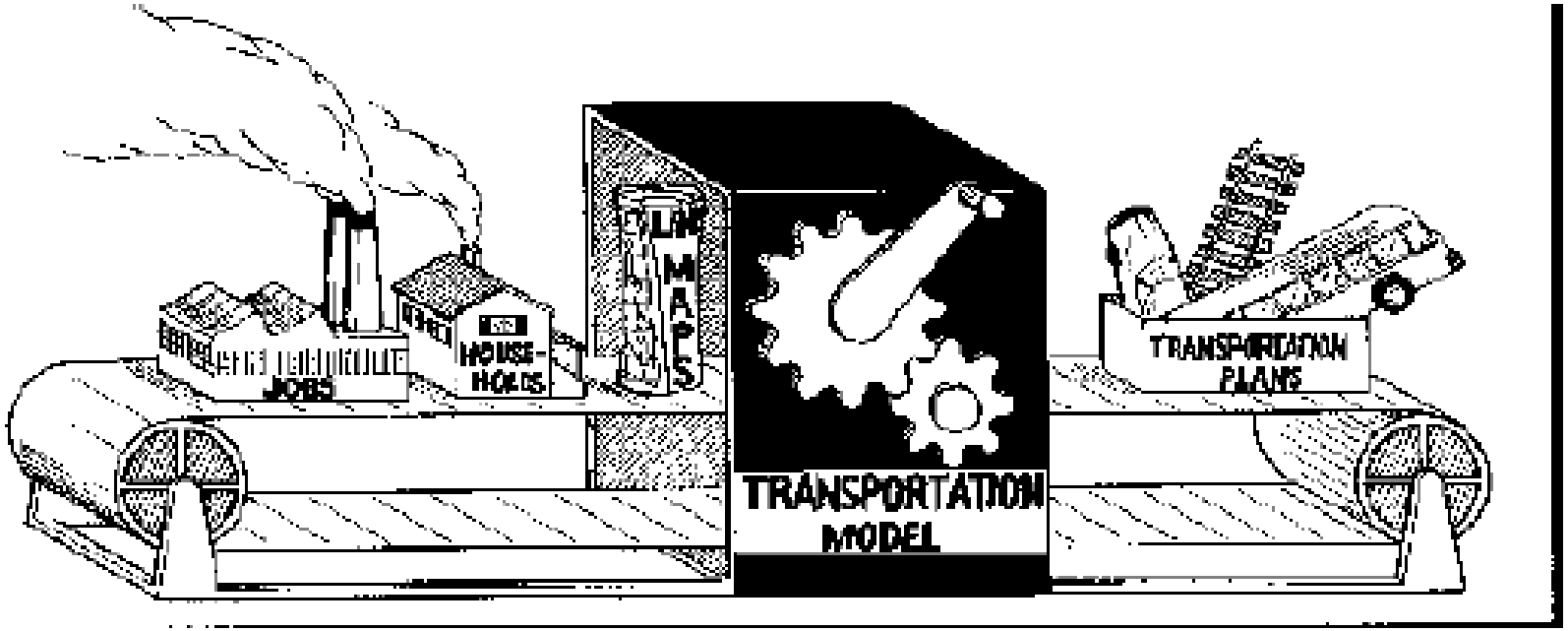


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Transportation demand forecasting

Trip-based model sequence

1. Generation
2. Distribution
3. Mode choice
4. Assignment

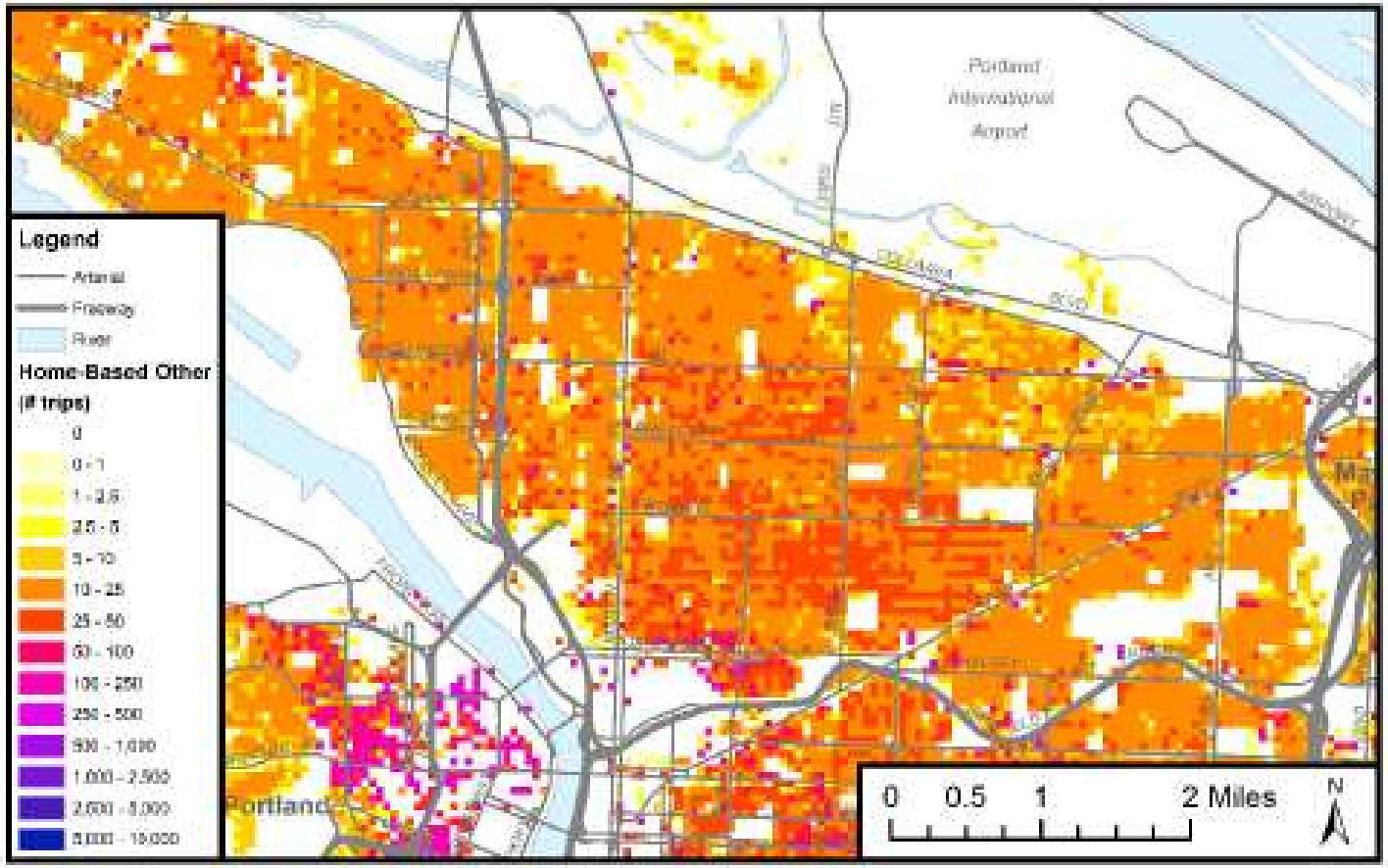


Source: Beimborn(1995), A Transportation Modeling Primer
https://www4.uwm.edu/cuts/primer.htm

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Trip generation

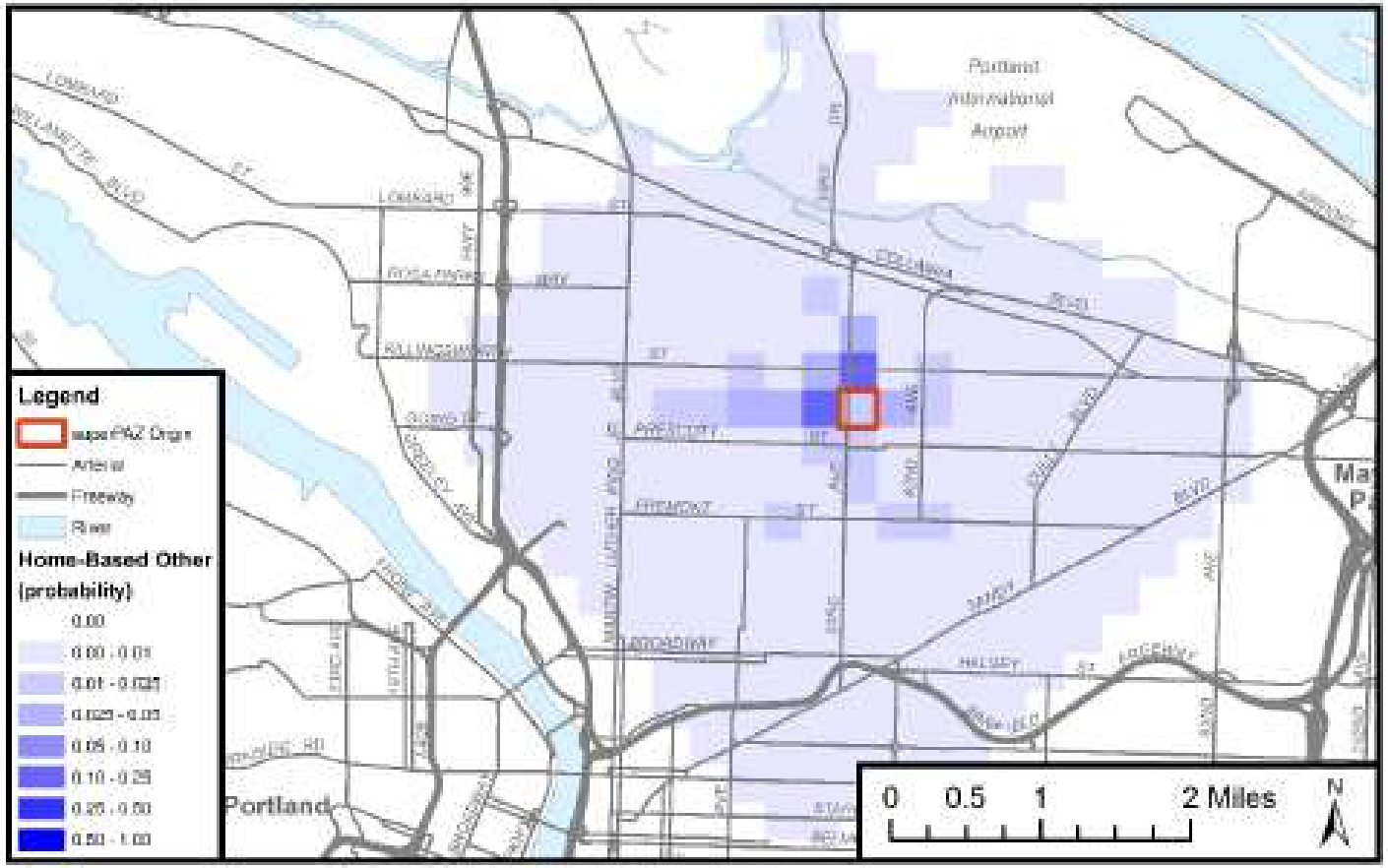
- How many trips
- Where do they start?



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Trip distribution

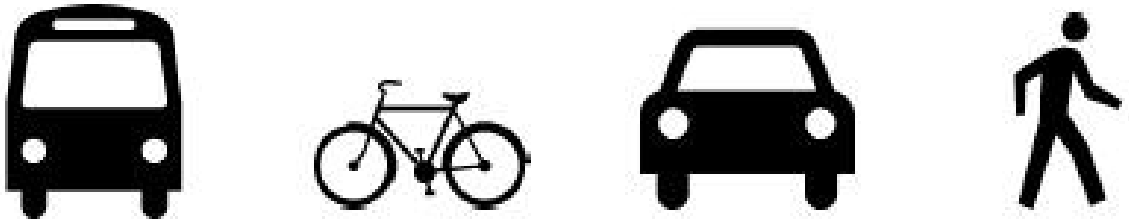
- How many trips?
- Where do they go?



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Mode choice

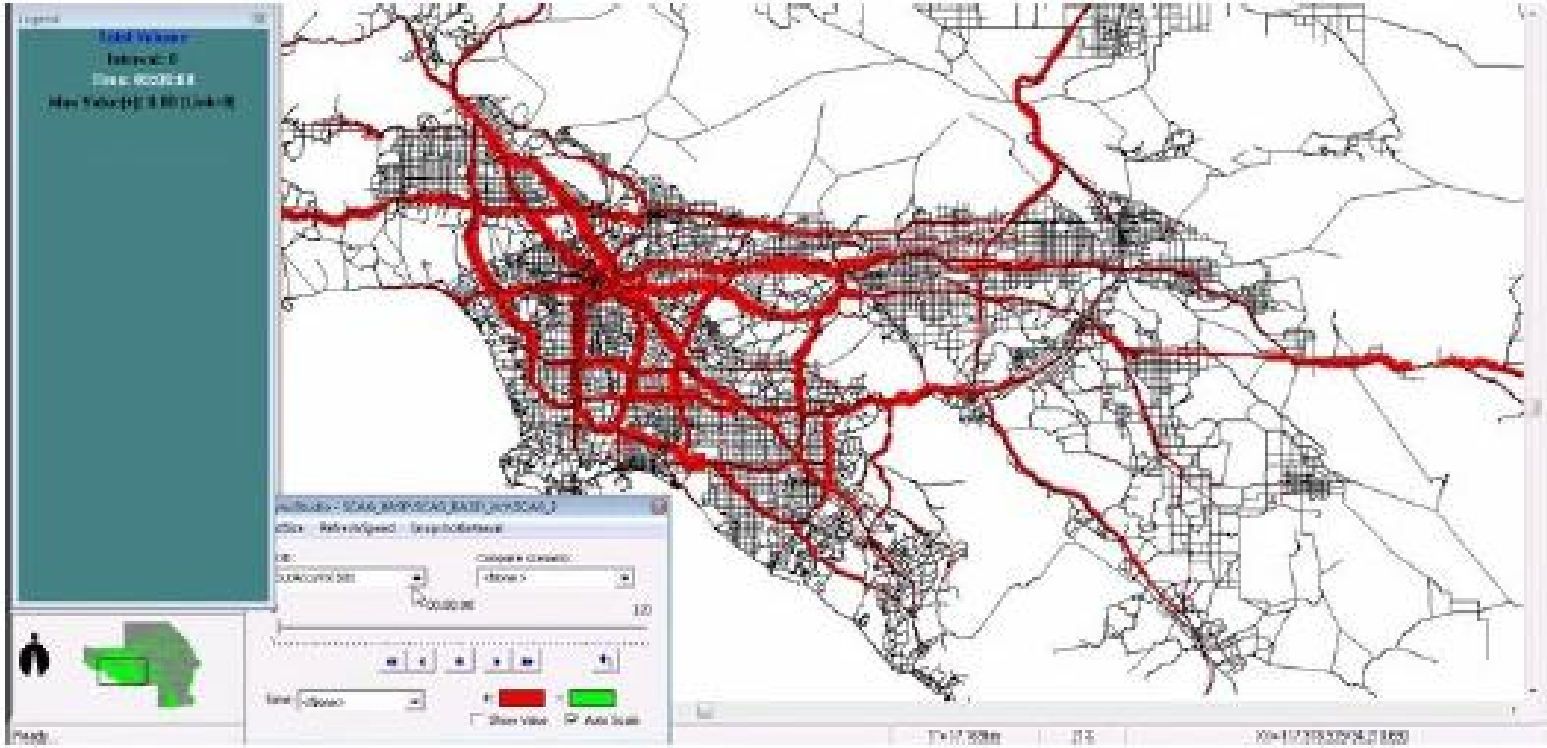
- How do they get there?
- What means of transportation?





Traffic assignment

- How do they get there?
- By what route?



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Questions?

09.07.2025

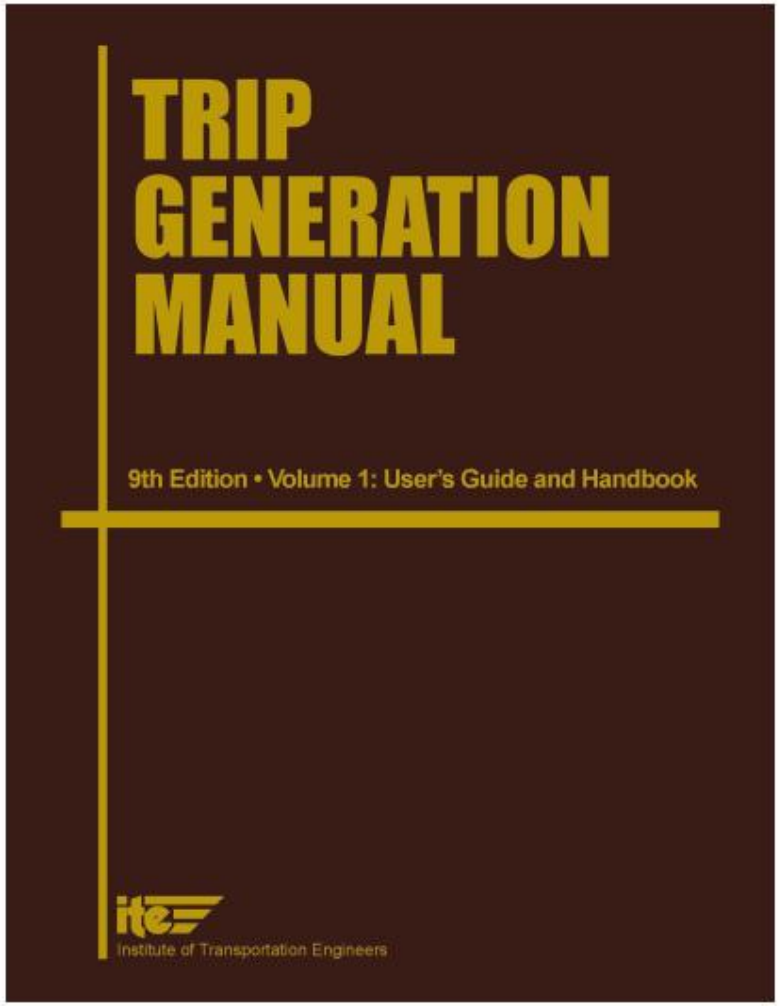
Trip Generation

EWCE 351
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Trip generation (1)

- How many trips come to & leave from a particular establishment?
  - By time of day
  - (By mode)
- ITE Trip Generation Manual
  - Handbook: how to use it
  - Data plots: trips generated, by
    - Land use
    - Time of day
    - Site size



Research and Development Center
(760)

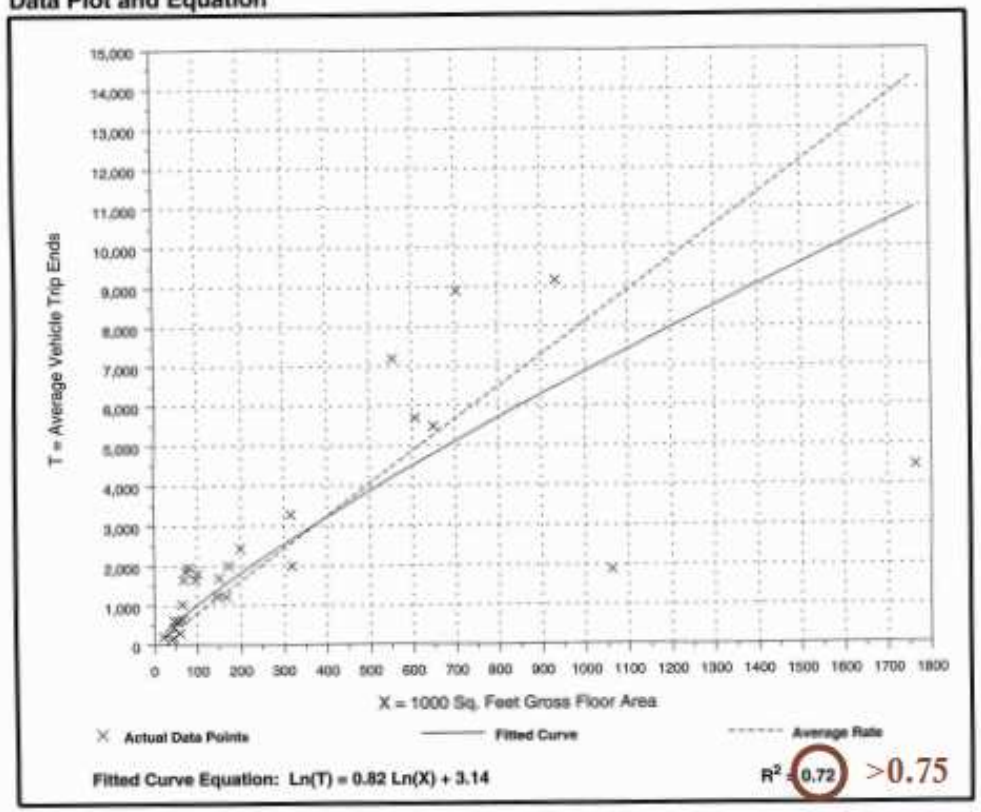
Average Vehicle Trip Ends vs: 1000 Sq. Feet Gross Floor Area
On a: Weekday

Number of Studies: 28 >20
Average 1000 Sq. Feet GFA: 308
Directional Distribution: 50% entering, 50% exiting

Trip Generation per 1000 Sq. Feet Gross Floor Area

Average Rate	Range of Rates	Standard Deviation
8.11	1.78 - 24.95	5.85

Data Plot and Equation



Fitted Curve Equation: Ln(T) = 0.82 Ln(X) + 3.14

R^2 = 0.72 >0.75

Trip Generation, 8th Edition 1326 Institute of Transportation Engineers

Trip generation

- Example application
  - USU proposed engineering research building at Innovation Campus
  - 4 stories, 200ft × 100ft floors
- How many trips generated…
  - Using average rate?
    ≈ 650 trips/weekday
  - Using fitted curve?
    ≈ 840 trips/weekday

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Trip generation – Example

- Method 1: Average rate
  - 8.11 vehicle trip ends per 1000 sq. ft. gross floor area
  - 4 stories × (200 ft × 100 ft / story) = 4 × 200 × 100 = 80,000 sq. ft. → X = 80 (1000 sq. ft.)
  - Trips = 8.11 × 80 = 648.8
  - → 649 vehicle trip ends
- Method 2: Equation
  - ln(Trips) = 0.82 ln(X) + 3.14, where X in 1000 sq. ft. gross floor area
  - → X = 80 (1000 sq. ft.)
  - Trips = exp(0.82 ln(80) + 3.14) = exp(3.59 + 3.14) = exp(6.73) = 839.88
  - → 840 vehicle trip ends

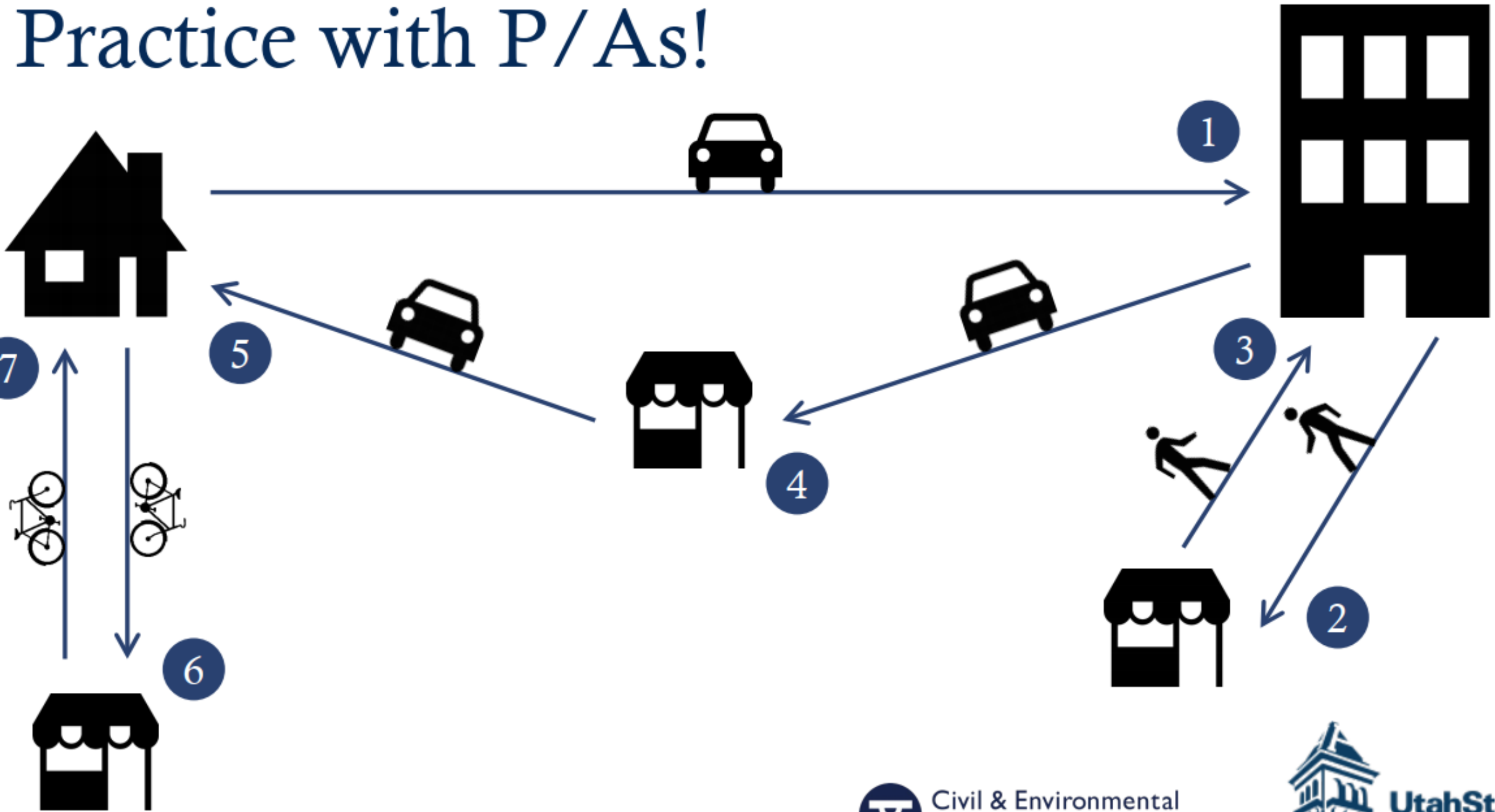
Definitions

- *Trip ends*: the two end points of a trip
- OD: Origins & destinations
  - *Trip origin*: where a trip begins
  - *Trip destination*: where a trip ends
- PA: Productions & attractions
  - *Trip production*: where the need/desire for a trip is demanded
  - *Trip attraction*: where the need/desire for a trip is satisfied
- Why the distinction?
  - (Sometimes) easier to predict travel demand

Definitions

- *Trip purposes*: classifications of trips by "purpose" (~activity)
- *Home-based*: involves home at one end
  - *Home-based work* (HBW): a trip between home and work
  - *Home-based school* (HBSch): a trip between home and school
  - *Home-based shopping* (HBShop): a trip between home and shopping
  - *Home-based recreation* (HBRec): a trip between home and recreational activities
  - *Home-based other* (HBO): a trip between home and other activities
  - **The production end is always home**
- *Non-home-based*: doesn't begin/end at home
  - *Non-home-based work* (NHBW): a trip between work and other non-home activities
  - *Non-home-based non-work* (NHBNW): a trip between two non-home/work activities
  - **The production end is always the origin end**

Practice with P/As!



20.11.2022

Trip generation in travel demand models

- Trip productions → *Cross-classification*
  - Characterize households by 1+ attributes
  - Calculate average trip rate for each household type (from HHTS!)
  - Multiply # households by type by average trip rate & sum

- Trip attractions → *Linear equation*
  - Estimate a linear regression model on destination attributes (from HHTS!)
  - Characterize amount of activity in each zone
  - Plug amounts into linear equation and calculate

Equations

Trip productions

$$P_i^p = \sum_k P\,rate_{pk} * h_{ik} \qquad (4\text{-}7)$$

where:

P_i^p = Number of trip ends produced for purpose p in zone i;
$Prate_{pk}$ = The production trip rate for purpose p per household for category k; and
h_{ik} = The number of households in category k in zone i.

Trip attractions

$$A_i^p = \sum_k A\,rate_{pk} * v_{ik} \qquad (4\text{-}8)$$

where:

A_i^p = Attraction of trip ends for purpose p in zone i;
$Arate_{pk}$ = Rate of attraction trip ends for purpose p per unit of variable k; and
v_{ik} = Value of variable k in zone i.

What factors affect trip generation rates?

- Trip productions
  - # households
  - # people
  - # workers
  - # children
  - # vehicles
  - Household income
- Trip attractions
  - # jobs (employment)

Example: Trip productions

TABLE 5. HBW Production Rates

Workers	Rate
1	1.38325222
2	2.39110122
3+	3.88667372

TABLE 6. HBshop Production Rates

	Workers			
HHsize	0	1	2	3+
1	0.65370595	0.36543758		
2	1.4747858	0.96459839	0.66841305	
3	1.4397819	1.1695282	0.93650663	1.0063395
4+	1.7925876	1.8066825	1.5106965	1.2347277

TABLE 8. HBoth Production Rates

HHsize	all household members work	some household members do not work
1	0.54391065	0.89368165
2	1.2416304	1.628105
3	1.4489857	2.2256102
4+		3.4876336

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UtahState University

Metro. (2015). *Trip-Based Travel Demand Model: Methodology Report*.

Example: Trip productions

- TAZ 123 has:
  - 50 HHs with 0 workers
  - 100 HHs with 1 worker
  - 200 HHs with 2 workers
  - 50 HHs with 3+ workers
- HBW trip production rates
  - 0 workers: 0 trips / HH
  - 1 worker: 1.38 trips / HH
  - 2 workers: 2.39 trips / HH
  - 3 workers: 3.88 trips / HH
- How many HBW trip productions are generated by TAZ 123?
  - 0 worker: $50 \times 0 = 0$
  - 1 worker: $100 \times 1.38 = 138$
  - 2 worker: $200 \times 2.39 = 478$
  - 3+ worker: $50 \times 3.88 = 194$
  - Total = $0 + 138 + 478 + 194 = 810$
  - → 810 HBW trip productions

Example: Trip attractions

Trip Attraction Rates by Trip Type							
	Total Employment	Retail	Office	Government	Industrial	Agriculture	Households
Downtown Boise TAZs (1-27)							
Home Base Work	1.25	Not applicable					
Home Base Shop	Not applicable	1.10	Not applicable				
Home Base Social	Not applicable		0.90	0.25	Not applicable		0.30
Home Base Other	Not applicable	0.70	0.80	0.25	0.25	0.25	0.70
Non-Home Base	Not applicable	1.60	1.10	0.50	0.50	0.50	0.70
All Other Ada County TAZs (28-399)							
Home Base Work	1.35	Not applicable					
Home Base Shop	Not applicable	5.80	Not applicable				
Home Base Social	Not applicable		0.90	0.50	Not applicable		0.30
Home Base Other	Not applicable	2.60	0.70	0.25	0.70	0.25	0.70
Non-Home Base	Not applicable	4.50	1.00	0.25	0.70	0.25	0.70

Example: Trip attractions

- TAZ 123 (in Downtown) has:
  - 400 households
  - 600 jobs
    - 400 office jobs
    - 200 government jobs
- Trip attraction rates
  - HBW: 1.25 trips / job
  - HBSocial: 0.90 trips / office job
    + 0.25 trips / government job
    + 0.30 trips / household

- How many HBW and HBSocial trip attractions are generated by TAZ 123?

  - HBW: $1.25 \times 600 = 750$
    → 750 HBW trip attractions

  - HBSocial: 0.90×400
    $+ 0.25 \times 200 + 0.30 \times 400$
    $= 360 + 50 + 120 = 530$
    → 530 HBSocial trip attractions

Things to consider

- What factors affect trip generation rates?
- Are trip rates stable? Across time? Between regions?
- What would happen to trip generation rates with…
  - Rise in e-commerce?
  - Drone-based deliveries?
  - New out-of-home activities?
  - Autonomous vehicles?
  - Virtual reality in-the-home?

Trip distribution

CEE 5240/6240

Urban and Regional Transportation Planning

30 January 2019

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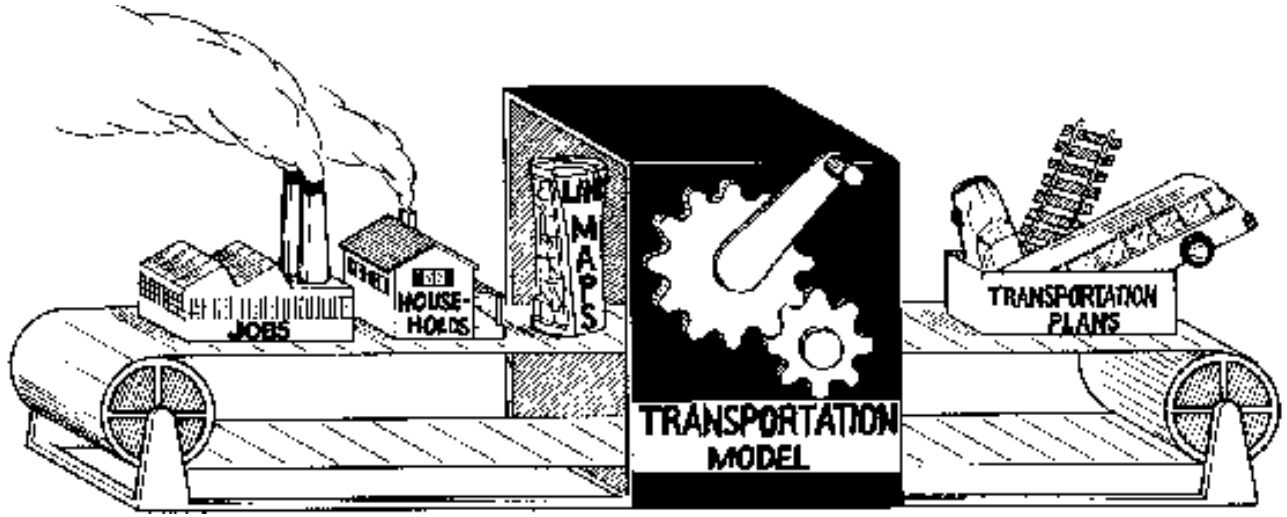
Outline

- Trip generation review
- Trip distribution overview
- Trip distribution methods
  - Growth factor model
  - Gravity model
  - Destination choice model
- Homework 5

Trip distribution

Trip-based models

1. Generation
2. Distribution
3. Mode choice
4. Assignment



Source: Beimborn (1995), A Transportation Modeling Primer
https://www4.uwm.edu/cuts/primer.htm

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Trip generation outputs

- Transportation analysis zones (TAZs)
- Trip productions vs. trip attractions
- Trip purposes
  - Home-based …
  - Non-home based…
- Methods: cross-classification, linear regression, etc.

	Trip productions			*Trip attractions*		
TAZ	**HBW**	**HBO**	**NHB**	**HBW**	**HBO**	**NHB**
1	50	75	25	5	10	25
2	87	34	112	0	0	65
3	41	42	49	35	56	53
4	0	0	236	187	134	201
5	62	57	38	14	16	54
…						

Trip distribution

- Linking trip productions & attractions
  - Productions ←→ Attractions
- Creating a matrix of trip productions & attractions
  - → PA trip matrix
- Converting to a matrix of trip origins & destinations
  - → OD trip matrix

	HBW trips				
TAZ (P\A)	**1**	**2**	**3**	**…**	**Total**
1	1	0	11		50
2	2	0	18		87
3	0	0	6		41
…					
Total	5	0	35		

Trip distribution

- Notation
  - Production zones i
  - Attraction zones j
  - Total trips P_i produced in zone i and attracted to all zones
  - Total trips A_j attracted to zone j and produced in all zones
  - Trips T_{ij} produced in zone i and attracted to zone j

	HBW trips				
TAZ (P\A)	**$j = 1$**	**$j = 2$**	**$j = 3$**	**…**	**Total**
$i = 1$	T_{11}	T_{12}	T_{13}		P_1
$i = 2$	T_{21}	T_{22}	T_{23}		P_2
$i = 3$	T_{31}	T_{32}	T_{33}		P_3
…					
Total	A_1	A_2	A_3		

Trip distribution

- Constraints
  - Row sums should equal trip productions
    - $\sum_j T_{ij} = P_i$
  - Column sums should equal trip attractions
    - $\sum_i T_{ij} = A_j$

	HBW trips				
TAZ (P\A)	***j* = 1**	***j* = 2**	***j* = 3**	**…**	**Total**
***i* = 1**	T_{11}	T_{12}	T_{13}		P_1
***i* = 2**	T_{21}	T_{22}	T_{23}		P_2
***i* = 3**	T_{31}	T_{32}	T_{33}		P_3
…					
Total	A_1	A_2	A_3		

- *Singly constrained*: if only one of these conditions (usually P_i) holds
- *Doubly constrained*: if both of these conditions hold

What factors affect trip distribution?

- What do you consider when you choose a destination?

- Impedance / level-of-service (LOS)
  - Travel time or (generalized) cost

- Attractiveness
  - # trip attractions
  - Employment by type

Trip distribution methods

- Growth factor model
  - Fratar/Furness method
  - Iterative fitting of growth factors
- Gravity model
  - Calibration → Friction factors
  - Application
- Destination choice model
  - Disaggregate approach
  - Multinomial logit modeling

Growth factor model

- Given a base-year PA or OD trip matrix, and limited information about future trips, apply growth factor(s) to base-year trip matrix to create future-year trip matrix.

- Rarely used in practice anymore
  - Cannot account for changes in impedance

Base Year

P \ A	z1	z2	z3	Total
z1	5	50	95	150
z2	45	5	100	150
z3	50	95	5	150
Total	100	150	200	450

Forecast Year

P \ A	z1	z2	z3	Total
z1	?	?	?	??
z2	?	?	?	??
z3	?	?	?	??
Total	??	??	??	???

Growth factor model – Simple

- Given:
  - Base-year PA trip matrix from survey data
  - Uniform area-wide growth of 20% ($\tau = 1.20$)

$$T'_{ij} = \tau \times T_{ij}$$

Base Year

P \ A	z1	z2	z3	Total
z1	5	50	95	150
z2	45	5	100	150
z3	50	95	5	150
Total	100	150	200	450

Forecast Year

P \ A	z1	z2	z3	Total
z1	6	60	120	186
z2	60	6	120	186
z3	60	120	6	186
Total	126	186	246	558

Growth factor model – Singly constrained

- Given:
  - Base-year PA trip matrix from survey data
  - Future-year production zone-specific growth factors (or productions)

$$T'_{ij} = \tau_i \times T_{ij}$$

Base Year

P \ A	z1	z2	z3	Total	τ_i
z1	5	50	95	150	??
z2	45	5	100	150	??
z3	50	95	5	150	??
Total	100	150	200	450	--

Forecast Year

P \ A	z1	z2	z3	Goal
z1	?	?	?	180
z2	?	?	?	165
z3	?	?	?	255
Total	??	??	??	600

Growth factor model – Doubly constrained

- Given:
  - Base-year PA trip matrix from survey data
  - Future-year production & attraction zone-specific growth factors

$$T'_{ij} = \tau_i \times T_{ij}$$

$$\neq$$

$$T'_{ij} = \tau_j \times T_{ij}$$

Base Year

P \ A	z1	z2	z3	Total	τ_i
z1	5	50	95	150	??
z2	45	5	100	150	??
z3	50	95	5	150	??
Total	100	150	200	450	--
τ_j	??	??	??	--	--

Forecast Year

P \ A	z1	z2	z3	Goal
z1	?	?	?	180
z2	?	?	?	165
z3	?	?	?	255
Goal	195	165	240	600

???

Fratar/Furness method

- Iteratively calculate growth factors τ_i and τ_j until they approach 1.
  - Step 1:

$$\tau_i = P^G{}_i \div P^B{}_i \longrightarrow T^1{}_{ij} = \tau_i \times T^B{}_{ij} \longrightarrow A^1{}_j$$

Base Year (B)

P \ A	z1	z2	z3	Total	Goal	τ_i
z1	5	50	95	150	180	1.20
z2	45	5	100	150	165	1.10
z3	50	95	5	150	255	1.70
Total	100	150	200	450	--	--
Goal	195	165	240	--	600	--

Forecast Year (1)

P \ A	z1	z2	z3	Total
z1	6.0	60.0	114.0	180
z2	49.5	5.5	110.0	165
z3	85.0	161.5	8.5	255
Total	140.5	227.0	232.5	600

Fratar/Furness method

- Iteratively calculate growth factors τ_i and τ_j until they approach 1.
  - Step 2:

$$\tau_j = A^G{}_j \div A^1{}_j \longrightarrow T^2{}_{ij} = \tau_j \times T^1{}_{ij} \longrightarrow P^2{}_i$$

Forecast Year (1)

P \ A	z1	z2	z3	Total	Goal
z1	6	60	114	180	180
z2	49.5	5.5	110	165	165
z3	85	161.5	8.5	255	255
Total	140.5	227	232.5	600	--
Goal	195	165	240	--	600
τ_j	1.39	0.73	1.03	--	--

Forecast Year (2)

P \ A	z1	z2	z3	Total
z1	8.3	43.6	117.7	169.6
z2	68.7	4.0	113.5	186.3
z3	118.0	117.4	8.8	244.1
Total	195	165	240	600

Fratar/Furness method

- Iteratively calculate growth factors τ_i and τ_j until they approach 1.
  - Step 1: $\tau_i = P^G_i \div P^2_i \longrightarrow T^3_{ij} = \tau_i \times T^2_{ij} \longrightarrow A^3_j$

Forecast Year (2)

P \ A	z1	z2	z3	Total	Goal	τ_i
z1	8.3	43.6	117.7	169.6	180	1.06
z2	68.7	4.0	113.5	186.3	165	0.89
z3	118.0	117.4	8.8	244.1	255	1.04
Total	195	165	240	600	--	--
Goal	195	165	240	--	600	--

Forecast Year (3)

P \ A	z1	z2	z3	Total
z1	8.8	46.3	124.9	180
z2	60.9	3.5	100.6	165
z3	123.2	122.6	9.2	255
Total	192.9	172.4	234.6	600

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Fratar/Furness method

- Iteratively calculate growth factors τ_i and τ_j until they approach 1.
  - Step 2:

$$\tau_j = A^G{}_j \div A^3{}_j \longrightarrow T^4{}_{ij} = \tau_j \times T^3{}_{ij} \longrightarrow P^4{}_i$$

Forecast Year (3)

P \ A	z1	z2	z3	Total	Goal
z1	8.8	46.3	124.9	180	180
z2	60.9	3.5	100.6	165	165
z3	123.2	122.6	9.2	255	255
Total	192.9	172.4	234.6	600	--
Goal	195	165	240	--	600
τ_j	1.01	0.96	1.02	--	--

Forecast Year (4)

P \ A	z1	z2	z3	Total
z1	8.9	44.3	127.7	181.0
z2	61.5	3.4	102.9	167.8
z3	124.5	117.3	9.4	251.2
Total	195	165	240	600

Fratar/Furness method

- Iteratively calculate growth factors τ_i and τ_j until they approach 1.
  - Step 1:

$$\tau_i = P^G{}_i \div P^4{}_i \longrightarrow T^5{}_{ij} = \tau_i \times T^4{}_{ij} \longrightarrow A^5{}_j$$

Forecast Year (4)

P \ A	z1	z2	z3	Total	Goal	τ_i
z1	8.9	44.3	127.7	181.0	180	0.99
z2	61.5	3.4	102.9	167.8	165	0.96
z3	124.5	117.3	9.4	251.2	255	1.01
Total	195	165	240	600	--	--
Goal	195	165	240	--	600	--

Forecast Year (5)

P \ A	z1	z2	z3	Total
z1	8.9	44.1	127.1	180
z2	60.5	3.3	101.2	165
z3	126.4	119.1	9.5	255
Total	195.8	166.5	237.8	600

Fratar/Furness method

- Iteratively calculate growth factors τ_i and τ_j until they approach 1.
  - Step 2: $\tau_j = A^G{}_j \div A^5{}_j \longrightarrow |\tau_j - 1| < 0.01$ or 0.001

Forecast Year (5)

P \ A	z1	z2	z3	Total	Goal
z1	8.9	44.1	127.1	180	180
z2	60.5	3.3	101.2	165	165
z3	126.4	119.1	9.5	255	255
Total	195.8	166.5	237.8	600	--
Goal	195	165	240	--	600
τ_j	1.00	0.99	1.01	--	--

Done!?

Growth factor model

- Advantages?
  - Simple
  - No impedance/LOS information needed

- Disadvantages?
  - No volumes to and from new zones
  - Not good for looking at policy effects
  - Ignores congestion (and any LOS)

Gravity model

- Given
  - a base-year PA or OD trip matrix
  - information about future trip productions & attractions, and
  - information on zone-to-zone impedance
- Iteratively develop/apply friction factors to create future-year trip matrix
- Still commonly used in practice
  - Practice heading toward destination choice models

Base Year

P \ A	z1	z2	z3	Total
z1	5	50	95	150
z2	45	5	100	150
z3	50	95	5	150
Total	100	150	200	450

Forecast Year

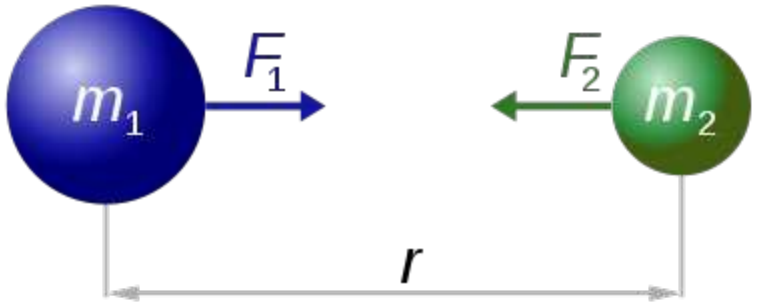
P \ A	z1	z2	z3	Goal
z1	?	?	?	180
z2	?	?	?	165
z3	?	?	?	255
Goal	195	165	240	600

Gravity model

- Analogous to Newton's law of gravity:
  - $T_{ij} = \alpha \times P_i \times A_j \div (d_{ij})^2$

- Directly proportional to…
  - Trips P_i produced in zone i
  - Trips A_j attracted to zone j

- Inversely proportional to…
  - Impedance d_{ij} between zones i and j



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

Image: https://en.wikipedia.org/wiki/Newton%27s_law_of_universal_gravitation#/media/File:NewtonsLawOfUniversalGravitation.svg

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Gravity model

$$T_{ij} = \alpha \times P_i \times A_j \div (d_{ij})^2$$

- Generalization

$$T_{ij} = \alpha_{ij} \times P_i \times A_j \times f(c_{ij})$$

$$T_{ij} = (M_i \times N_j) \times P_i \times A_j \times f(c_{ij})$$

- Directly proportional to…
  - Trips P_i produced in zone i
  - Trips A_j attracted to zone j

$$T_{ij} = (M_i \times P_i) \times (N_j \times A_j) \times f(c_{ij})$$

$$T_{ij} = m_i \times n_j \times f(c_{ij})$$

- Inversely proportional to…
  - Generalized cost function $f(c_{ij})$ between zones i and j

Gravity model

$$T_{ij} = (M_i \times P_i) \times (N_j \times A_j) \times f(c_{ij})$$

- Singly constrained

$$M_i = 1 \div \sum_j (A_j \times f(c_{ij})) \qquad N_j = 1$$

- Doubly constrained

$$M_i = 1 \div \sum_j (N_j \times A_j \times f(c_{ij}))$$

$$N_j = 1 \div \sum_i (M_i \times P_i \times f(c_{ij}))$$

Friction factors

$$f(c_{ij}) = F_{ij}$$

- Parametric (mathematical equation)
  - Power $$f(c_{ij}) = F_{ij} = (c_{ij})^{y}$$
  - Exponential $$f(c_{ij}) = F_{ij} = \exp(z \times c_{ij})$$
  - Gamma $$f(c_{ij}) = F_{ij} = x \times (c_{ij})^{y} \times \exp(z \times c_{ij})$$

$$y < 0 \qquad z < 0$$

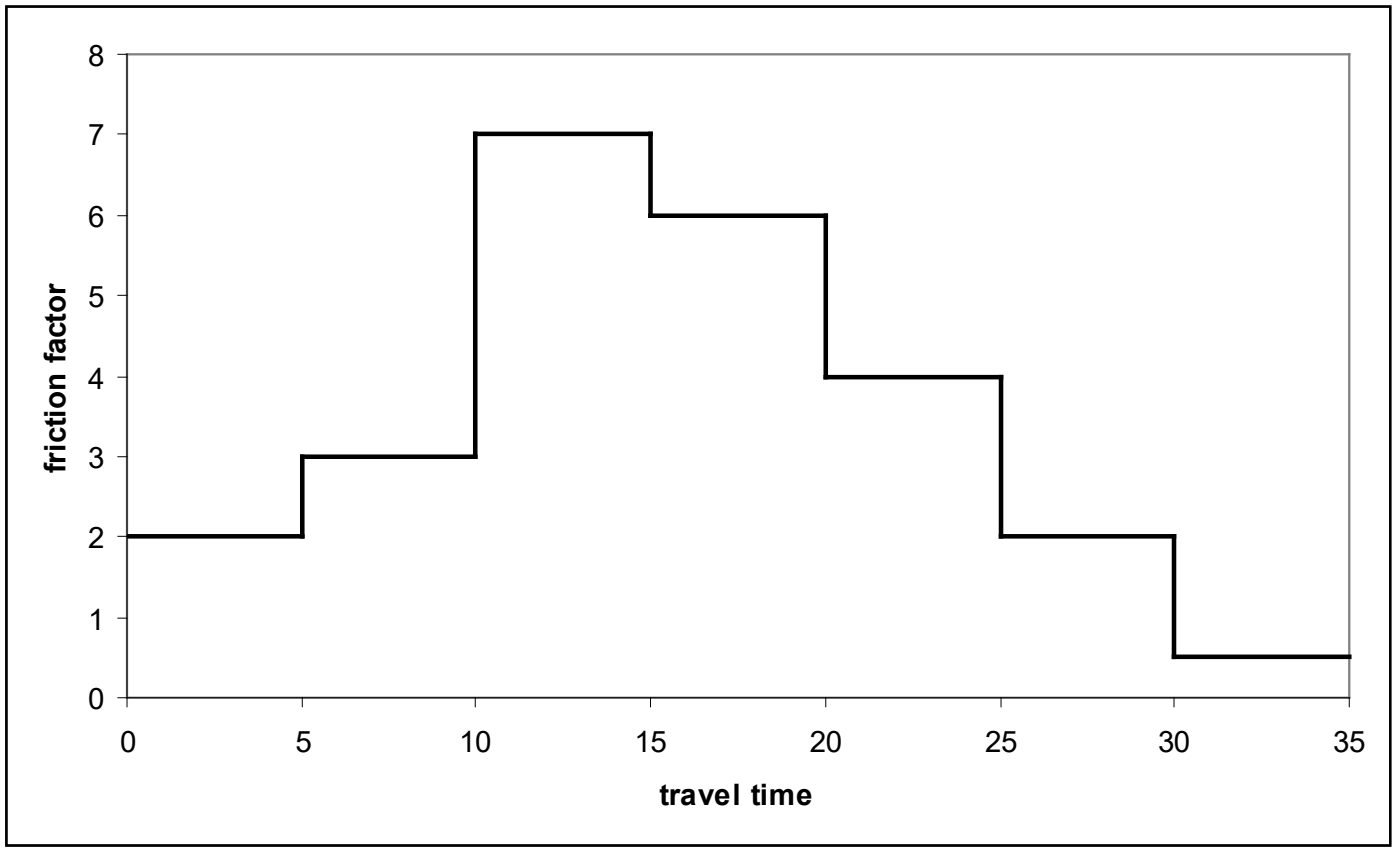
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Friction factors

$$f(c_{ij}) = F_{ij}$$

- Non-parametric (look-up table)

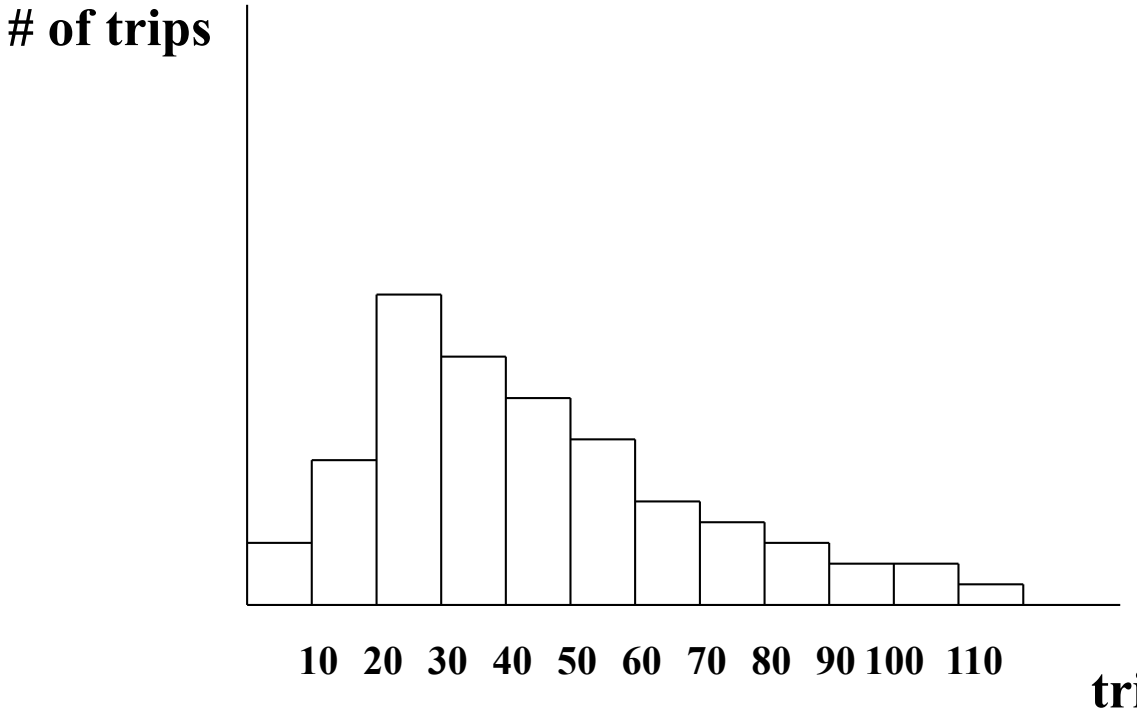


$$F_{ij} = \begin{cases} F_1 & \text{if } 0 \le t_{ij} \le 5 \\ F_2 & \text{if } 5 \le t_{ij} \le 10 \\ & \\ F_7 & \text{if } 30 \le t_{ij} \le 35 \end{cases}$$

Friction factors

$$f(c_{ij}) = F_{ij}$$

- Used to match observed trip length distribution



Time Bin	Predicted	Observed	FF
1	799.9964	800	0.798501
2	2639.926	2640	1.512652
3	1520.033	1520	1.083482
4	905.0445	905	0.476607
Total	**5865**	**5865**	

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Gravity model calibration

- Given
  - Initial PA trip matrix T_{ij}, P_i, A_j
  - Generalized cost (or impedance) matrix
  - Gravity model form
  - Friction factor functional form

Base Year

P \ A	z1	z2	z3	Total
z1	5	50	95	150
z2	45	5	100	150
z3	50	95	5	150
Total	100	150	200	450

Impedance

TT	z1	z2	z3
z1	5	16	13
z2	16	7	20
z3	13	20	2

$$T_{ij} = P_i \times A_j \times F_{ij} \div \sum_j (A_j \times F_{ij})$$

F_{ij} = lookup table

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Gravity model calibration

- Determine the friction factors (F_{ij})
  - 0. Assume initial friction factors $F_{ij} = 1$
  - 1. Create friction factor matrix
  - 2. Calculate denominator sum $\sum_j (A_j \times F_{ij})$
  - 3. Apply formula to calculate $T_{ij} = P_i \times A_j \times F_{ij} \div \sum_j (A_j \times F_{ij})$

Impedance

TT	z1	z2	z3
z1	5	16	13
z2	16	7	20
z3	13	20	2

FF matrix (1)

TT	z1	z2	z3
z1	1.00	1.00	1.00
z2	1.00	1.00	1.00
z3	1.00	1.00	1.00

1

Base Year (1)

P \ A	z1	z2	z3	Total	$\sum_j$
z1	33.3	50.0	66.7	150	450.0
z2	33.3	50.0	66.7	150	450.0
z3	33.3	50.0	66.7	150	450.0
Total	100.0	150.0	200.0	450	--

2

3

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Gravity model calibration

- Determine the friction factors (F_{ij})
  - 4. Calculate estimated trip length distribution
  - 5. Update friction factors $FF_{new} = FF_{old} \times (\text{Obs} \div \text{Est})$
  - Repeat steps 1. through 5. until FFs don't change

Friction Factors (2)

TT	Est.	Obs.	FF
1-10	150	15	0.10
11-15	100	145	1.45
16-20	200	290	1.45

4

5

FF matrix (2)

TT	z1	z2	z3
z1	0.10	1.45	1.45
z2	1.45	0.10	1.45
z3	1.45	1.45	0.10

1

Base Year (2)

P \ A	z1	z2	z3	Total	$\sum_j$
z1	2.9	63.0	84.1	150	517.5
z2	48.3	5.0	96.7	150	450.0
z3	56.9	85.3	7.8	150	382.5
Total	108.1	153.3	188.6	450	--

2

3

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Gravity model calibration

- Determine the friction factors (F_{ij})
  - Repeat steps 1. through 5. until FFs don't change

Friction Factors (3)

TT	Est.	Obs.	FF
1-10	15.7	15	0.10
11-15	140.9	145	1.49
16-20	293.3	290	1.43

4

5

FF matrix (3)

TT	z1	z2	z3
z1	0.10	1.43	1.49
z2	1.43	0.10	1.43
z3	1.49	1.43	0.10

1

Base Year (3)

P \ A	z1	z2	z3	Total	$\sum_j$
z1	2.7	61.7	85.6	150	522.9
z2	48.4	4.8	96.8	150	444.3
z3	58.4	84.2	7.5	150	383.3
Total	109.5	150.7	189.8	450	--

2

3

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Gravity model calibration

- Determine the friction factors (F_{ij})
  - Repeat steps 1. through 5. until FFs don't change

Friction Factors (4)

TT	Est.	Obs.	FF
1-10	15.0	15	0.10
11-15	144.0	145	1.50
16-20	291.0	290	1.43

4

5

FF matrix (4)

TT	z1	z2	z3
z1	0.10	1.43	1.50
z2	1.43	0.10	1.43
z3	1.50	1.43	0.10

1

Base Year (4)

P \ A	z1	z2	z3	Total	$\sum_j$
z1	2.7	61.3	86.0	150	524.3
z2	48.4	4.8	96.8	150	442.8
z3	58.8	83.8	7.4	150	383.6
Total	109.9	149.9	190.2	450	--

2

3

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Gravity model calibration

- Determine the friction factors (F_{ij})
  - Repeat steps 1. through 5. until FFs don't change

$$| FF_{new} - FF_{old} | < 0.01 \text{ or } 0.001$$

Done!?

Friction Factors (4)

TT	Est.	Obs.	FF
1-10	15.0	15	0.10
11-15	144.0	145	1.50
16-20	291.0	290	1.43

Friction Factors (5)

TT	Est.	Obs.	FF
1-10	15.0	15	0.10
11-15	144.7	145	1.51
16-20	290.3	290	1.43

4

5

Gravity model calibration

- A few notes and warnings
  - Things that are slightly different from the Fratar/Furness method

- 1. When calculating trips using $T_{ij} = P_i \times A_j \times F_{ij} \div \sum_j (A_j \times F_{ij})$ …
  - Always use the original P_i and A_j values, NOT the new totals

- 2. When updating the friction factors using $FF_{new} = FF_{old} \times (\text{Obs} \div \text{Est})$
  - Always multiply by the old FF values, DON'T just divide observed by estimated

Gravity model application

- Given
  - Future trip productions & attractions P_i , A_j
  - Future generalized cost / impedance matrix
  - Calibrated friction factors F_{ij}
- Calculate future $T_{ij} = P_i \times A_j \times F_{ij} \div \sum_j (A_j \times F_{ij})$

Impedance

TT	z1	z2	z3
z1	7	18	16
z2	18	11	20
z3	16	20	5

Friction Factors

TT	Est.	Obs.	FF
1-10	15	15	0.10
11-15	145	145	1.51
16-20	290	290	1.43

Forecast Year

P \ A	z1	z2	z3	Goal
z1	?	?	?	180
z2	?	?	?	165
z3	?	?	?	255
Goal	195	165	240	600

Gravity model application

- Apply calibrated model $T_{ij} = P_i \times A_j \times F_{ij} \div \sum_j (A_j \times F_{ij})$
  - 1. Create friction factor matrix
  - 2. Calculate denominator sum $\sum_j (A_j \times F_{ij})$
  - 3. Apply formula to calculate T_{ij}

Impedance

TT	z1	z2	z3
z1	7	18	16
z2	18	11	20
z3	16	20	5

Forecast Year

P \ A	z1	z2	z3	Total	$\sum_j$
z1	6.0	70.9	103.1	180	598.7
z2	52.6	47.7	64.7	165	871.2
z3	131.8	111.6	11.6	255	538.8
Total	190.4	230.2	179.4	600	--

Friction Factors

TT	z1	z2	z3
z1	0.10	1.43	1.43
z2	1.43	1.51	1.43
z3	1.43	1.43	0.10

1 2 3

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Gravity model

- Advantages?
  - Takes into account impedance/LOS information
  - Can accommodate new zones

- Disadvantages?
  - Requires restrictive assumptions (FFs)
  - Not fully behaviorally consistent

Destination choice model

- Based in microeconomic (& psychological) consumer behavior theories
  - Discrete choice model

- People choose an alternative (destination) in order to maximize utility
  - Destination choice utility = f(impedance, destination attractiveness)

$$U_{ij} = (\beta_{TT} \times TT_{ij}) + \beta_{Emp} \times \ln(\sum_s \beta_s \times Emp_{js}) + \varepsilon_{ij}$$

Destination choice model

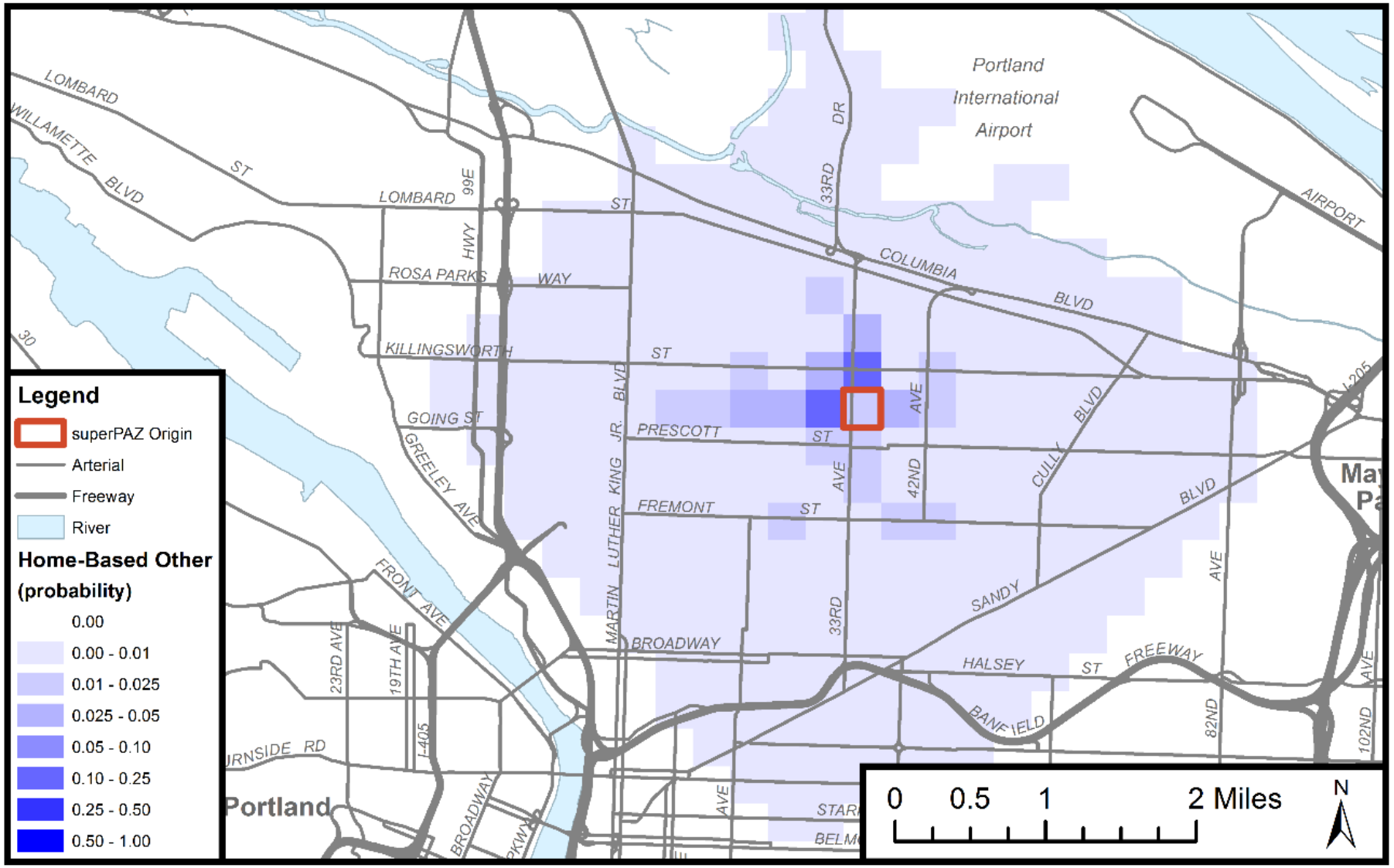
- Multinomial logit model
  - Assumes error terms are independently & identically distributed Gumbel (an approximation of the normal distribution)

  - Probability of choosing destination zone j

$$P(j \mid i) = \exp(U_{ij}) \div \sum_k \exp(U_{ik})$$

$$U_{ij} = (\beta_{TT} \times TT_{ij}) + \beta_{Emp} \times \ln(\sum_s \beta_s \times Emp_{js}) + \varepsilon_{ij}$$

Destination choice model



Destination choice model

- Advantages?
  - Takes into account impedance/LOS information
  - Can account for specific attraction information
  - More consistent with behavioral theory

- Disadvantages?
  - More complex to understand (probabilities)
  - More involved to estimate and apply
  - Only singly constrained (productions)

WFRC trip distribution

Exhibit 39: Doubly- Constrained Gravity Model Equations

$$T_{ij} = P_i * \frac{A_j * f(d_{ij})}{\sum_{all\ zones\ z} A_z * f(d_{iz})}$$

Where:

T_{ij}	=	the forecast flow produced by zone *i* and attracted to zone *j*
P_i	=	the forecast number of trips produced by zone *i*
A_j	=	the forecast number of trips attracted to zone *j*
d_{ij}	=	the impedance between zone *i* and zone *j*
$f(d_{ij})$	=	the friction factor between zone *i* and zone *j*

7.1.2 Friction Factors

The distribution model uses the generalized costs to look up the trip impedance between zones. These trip impedance look up vales are called friction factors. The distribution model's friction factors for HBW, HBSch, HBShp, HB0th, NHBW, NHBNW, IX and XI trip purposes were calibrated to match trip-length frequencies from the 2012 HH travel survey.

Friction factors for the SH_LT, SH_MD and SH_HV truck purposes were based on a gamma function, exhibiting the following functional form:

$$A \cdot x^B \cdot e^{Cx}$$

Where:

e = the exponential function

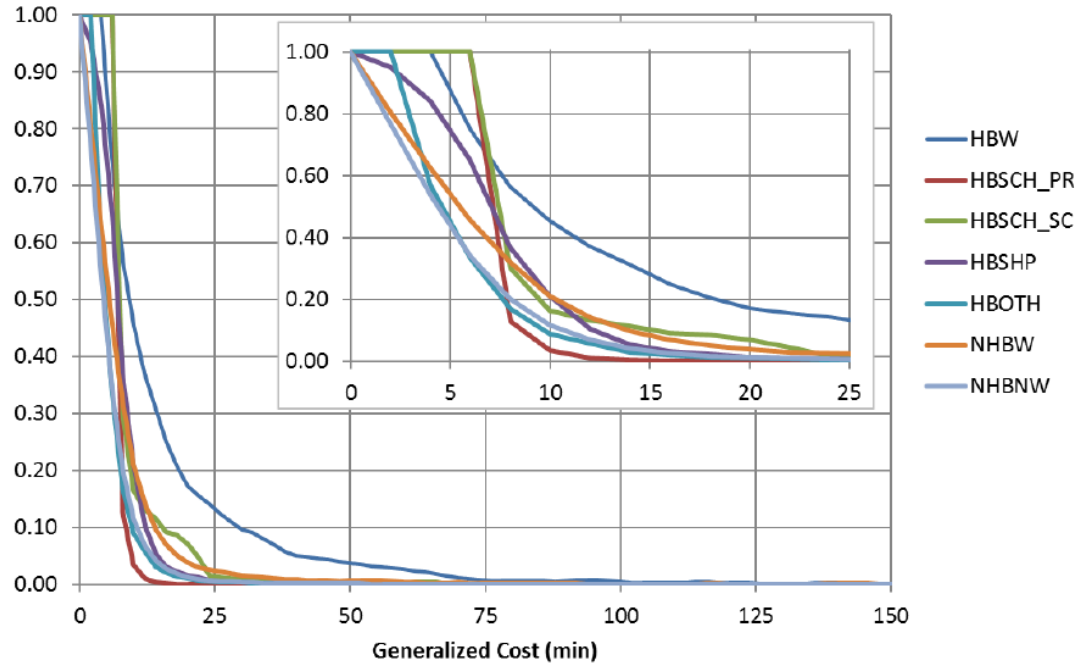
x = generalized cost (expressed in minutes)

A, B & C = purpose specific parameters

The purpose specific parameters were set to the following:

	A	B	C
SH_LT	1.0	-0.20	-0.080
SH_MD	1.0	-0.18	-0.050
SH_HV	1.0	-0.18	-0.035

Exhibit 40. Friction Factors – Person Trips



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WFRC HBW destination choice

Exhibit 44. Logit Model Formulation

$$P_i(k) = \frac{\exp(U_{k|i})}{\sum_{j \in D} \exp(U_{j|i})}$$

where:

$P_i(k)$ is the probability of selecting attraction , given production zone

$j \in D$ are the unique alternatives (attractions) in the sample set

U_j is the utility of selecting an attraction zone, given production zone

Exhibit 45: Utility of Each Destination

$$U_{j|i} = \beta_0 \times \beta_1 \times accessibility_{j|i} + \beta_2 \times quality_{j|i} + \ln(\beta_3 \times quantity_{j|i})$$

7.2.1 Destination Choice Model Estimation

In version 8 of the model, the destination choice model was not updated based on the most recent data. The data used for model estimation come from two sources, a 1992 home interview survey and a 2002 transit onboard. Both surveys were expanded to represent the 2002 population in the region. The estimation data set used auto and non-motorized trips from the home interview survey and transit trips from the onboard. Estimation weights were computed by purpose, mode, and household auto ownership level (0, 1, 2+). Illogical observations, e.g. drive alone trips in a 0 auto household, and observations not connected in the path builder were dropped. The estimation file contains 7,024 usable records, 5,111 are from the home interview survey, and 1,913 are from the onboard survey.

The data were grouped into two income categories, low and high household income. Low income was defined as those earning less than $25,000 in 2002 dollars. This definition corresponds to the lowest three income categories in the onboard survey. The income categories for the 1992 home interview survey were inflated into 2002 dollars using the consumer price index. After inflation, the low income definition included the two lowest income categories.

The primary accessibility variable in the destination choice model is the mode choice logsum variable. This variable is the primary separation measure.

Exhibit 43: Model Flow Diagram with Destination Choice

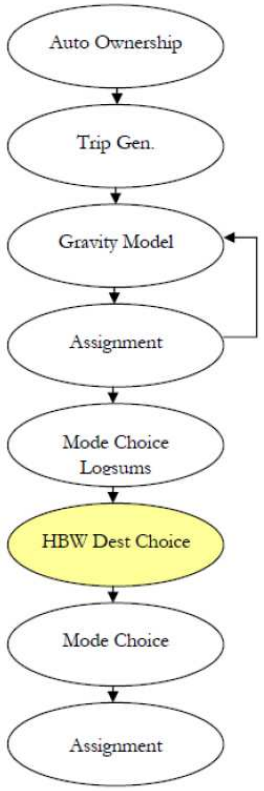


Exhibit 46. HBW Destination Choice Model Estimation Results

Variable	Coefficient
Logsum - 0 Auto	0.311
Logsum - 1 Auto, Low Income	0.212
Logsum - 1 Auto, High Income	0.212
Logsum - 2 Auto, Low Income	0.399
Logsum - 2 Auto, High Income	0.428
Highway Distance	-0.089
Retail Ratio - 0 Auto	0.458
Retail Ratio - 1 Auto, High Income	-0.524
Retail Ratio - 2 Auto, High Income	-0.718
Industrial Ratio - 1 Auto, Low Income	-1.269
Industrial Ratio - 1 Auto, High Income	-0.921
Industrial Ratio - 2 Auto, High Income	-0.459

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Questions?

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Homework 5

- Gain experience estimating and applying trip generation models.
  - Apply a doubly-constrained growth factor model
  - Calibrate a singly-constrained gravity model
  - Apply a singly-constrained gravity model
  - Describe advantages/disadvantages of trip distribution methods and factors/variables included in trip distribution models

Mode Choice

EWCE 351
Transportation Engineering

Mode choice

- What portion & how many passenger trips between origin *i* & destination *j* are made by each of the following modes?
  - Motorized
    - Automobile
      - By occupancy: drive-alone, two-person, three+-person
      - By facility usage: general purpose, toll lanes, etc.
    - Public transit
      - By type: local bus, express bus, bus rapid transit, light rail, commuter rail
      - By access/egress mode: walk, drive
  - Non-motorized
    - Walk
    - Bicycle

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What factors affect your choice of mode?

- Mode availability
- Trip purpose
- Level-of-service
  - Travel time: in-vehicle, wait/transfer, reliability
  - Trip cost: fuel, parking, tolls
  - Route attributes: pleasant scenery, turns, stops, terrain
- Traveler characteristics: income, etc.
- Area characteristics: density, etc.

How might you estimate mode choices?

- Heuristics
  - Based on trip distances & travel times
  - e.g., walking for all trips < 0.5 miles; driving for all trips > 3 miles
- Fixed percentages
  - Based on existing mode shares (from data collection)
  - e.g., 7% walk, 2% bicycle, 14% transit, 77% auto
- Combination:
  - Based on existing mode shares for trips of varying distance/duration
  - e.g., for trips < 0.5 miles: 75% walk, 10% bike, 0% transit, 15% auto

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Mode choice model

- Probabilistic
- Non-linear regression on independent variables
  - Level-of-service, traveler characteristics, area characteristics
- Dependent variable is not mode choice but unobserved mode-specific variable:
  - "Utility"
- Assumptions based on error distributions & equation functional form:
  - Multinomial logit model
  - Nested logit model

} Discrete choice models

Discrete choice behavioral theory

- Foundations
  - Mathematical psychology
  - Microeconomics
- Four elements
  - Decision maker
  - Alternatives
  - Attributes of alternatives
  - Decision rule

Discrete choice behavioral theory

- Alternatives
  - Choice set: categorical, mutually exclusive, complete
  - Feasible and known
  - e.g., drive alone, carpool, bus, bicycle, walk

- Attributes of alternatives
  - Characteristics of the alternatives
  - e.g., travel time, cost, comfort, etc.

Discrete choice behavioral theory

- Decision maker
  - Rational actor: aware of alternatives and their attributes
  - Consistent and transitive preferences
    - Will make same choice under same situation
    - Alternatives can be ranked in order of preference
- Decision rule
  - How alternative is chosen from choice set of alternatives
  - Dominance
  - Satisfaction
  - **Utility maximization**

Utility maximization

- Utility (U)
  - Measure of satisfaction from consumption decision (i.e., consuming travel)
  - Latent variable: we cannot observe, we can only infer
  - No natural scale or fixed zero point: only differences matter

- Decision rule
  - Select the alternative that provides the highest (most positive) utility

Utility maximization

- Example

Alternative	Time	Cost
Car	5 minutes	$5.00
Bus	15 minutes	$2.00

- Utility function: $U_k = (-0.2) \times \text{time}_k + (-1) \times \text{cost}_k$
- Which alternative will be chosen?
  - $U_{car} = (-0.2) \times (5) + (-1) \times (5) = (-1) + (-5) = -6$
  - $U_{bus} = (-0.2) \times (15) + (-1) \times (2) = (-3) + (-2) = -5$
  - $(U_{bus} = -5) > (U_{car} = -6)$ → choose bus

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Utility maximization

- Example

Alternative	Time	Cost
Car	5 minutes	$5.00
Bus	15 minutes	$2.00

- Utility function: $U_k = (-0.5) \times \text{time}_k + (-1) \times \text{cost}_k$
- Which alternative will be chosen?
  - $U_{car} = (-0.5) \times (5) + (-1) \times (5) = (-2.5) + (-5) = -7.5$
  - $U_{bus} = (-0.5) \times (15) + (-1) \times (2) = (-7.5) + (-2) = -9.5$
  - $(U_{car} = -7.5) > (U_{bus} = -9.5)$ → choose car

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Utility maximization

- Really analyzing a three-alternative problem
- Example

Alternative	Time	Cost	Benefit
Car to shop	5 minutes	$5.00	Yes
Bus to shop	15 minutes	$2.00	Yes
Stay at home	0 minutes	$0.00	No

- Utility function: $U_k = (-0.5) \times \text{time}_k + (-1) \times \text{cost}_k + (20) \times \text{benefit}_k$
- Which alternative will be chosen?
  - $U_{car} = (-0.5) \times (5) + (-1) \times (5) + (20) \times (1) = (-2.5) + (-5) + (20) = 12.5$
  - $U_{bus} = (-0.5) \times (15) + (-1) \times (2) + (20) \times (1) = (-7.5) + (-2) + (20) = 10.5$
  - $U_{home} = (-0.5) \times (0) + (-1) \times (5) + (20) \times (0) = (0) + (0) + (0) = 0.0$
  - $U_{car} > U_{bus} > U_{home}$ → choose car

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Random utility maximization (RUM)

- Analyst/modeler/planner cannot see inside travelers' heads
  - Decision-makers still assumed to choose highest utility alternative
- Must treat utility as an unobserved random variable
- Sources of randomness:
  - Unobserved attributes
    - We may not be able to measure all of the attributes considered
  - Measurement errors
    - Attributes measured are imperfect approximations of attributes considered
  - Unobserved taste variation
    - People have different preferences, make different tradeoffs between attributes

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Random utility maximization

- Answer is probabilistic, not deterministic
- Split utility equation into measured and unmeasured elements $U_k = V_k + \varepsilon_k$
  - Deterministic/systematic/observable/measurable utility

$$V_k = \sum_m \beta_m \times X_{mk}$$

  - Stochastic/random/unobservable/unmeasurable utility

$$\varepsilon_k \qquad with\ expected\ mean = 0$$

- Mathematical form of the model depends on assumptions about the error term ε_k

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Multinomial logit (MNL) model

$$P_n(k \mid C_n) = P_{kn} = \exp(V_k) \div \sum_{l \text{ in } Cn} \exp(V_l)$$

- For two alternatives
  - $P_{1n} = \exp(V_1) \div [\exp(V_1) + \exp(V_2)]$
  - $P_{2n} = \exp(V_2) \div [\exp(V_1) + \exp(V_2)]$

- For three alternatives
  - $P_{1n} = \exp(V_1) \div [\exp(V_1) + \exp(V_2) + \exp(V_3)]$
  - $P_{2n} = \exp(V_2) \div [\exp(V_1) + \exp(V_2) + \exp(V_3)]$
  - $P_{3n} = \exp(V_3) \div [\exp(V_1) + \exp(V_2) + \exp(V_3)]$

Why "logit"?

- Definitions
  - Probability p — the likelihood of choosing alternative k (0–1)
  - Odds = $p_1 \div (1 - p_1)$ — the ratio of probability(yes) to probability(no)
  - Log-odds = ln (odds) — the log of the odds (a.k.a., the "logit")

- $p = \exp(V_1) \div (\exp(V_1) + \exp(V_2)) = \exp(V_{diff}) \div (\exp(V_{diff}) + 1)$
- $$\begin{aligned} \text{Odds} &= [\exp(V_{diff}) \div (\exp(V_{diff}) + 1)] \div [1 - \{\exp(V_{diff}) \div (\exp(V_{diff}) + 1)\}] \\ &= [\exp(V_{diff}) \div (\exp(V_{diff}) + 1)] \div [1 \div (\exp(V_{diff}) + 1)] \\ &= \exp(V_{diff}) \div 1 = \exp(V_{diff}) \end{aligned}$$
- Log-odds = $\ln(\exp(V_{diff})) = V_{diff}$ = logit

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MNL model example

Alternative	Time	Cost
Car	5 minutes	$5.00
Bus	15 minutes	$2.00

$$V_k = (-0.2) \times \text{time}_k + (-1) \times \text{cost}_k$$

- Probabilities of choosing each alternative
  - $V_{car} = (-0.2) \times (5) + (-1) \times (5) = (-1) + (-5) = -6$
  - $V_{bus} = (-0.2) \times (15) + (-1) \times (2) = (-3) + (-2) = -5$
  - $\exp(V_{car}) = e^{-6} = 0.00248$
  - $\exp(V_{bus}) = e^{-5} = 0.00674$
  - $\text{Prob(car)} = 0.00248 \div (0.00248 + 0.00674) = 0.00248 \div 0.00922 = 0.269$
  - $\text{Prob(bus)} = 0.00674 \div (0.00248 + 0.00674) = 0.00674 \div 0.00922 = 0.731$

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MNL model example

Alternative	Time	Cost
Car	5 minutes	$5.00
Bus 1 (blue)	15 minutes	$2.00
Bus 2 (red)	15 minutes	$2.00

$$V_k = (-0.2) \times \text{time}_k + (-1) \times \text{cost}_k$$

- Probabilities of choosing each alternative
  - $V_{car} = -6$, $V_{bus1} = -5$, $V_{bus2} = -5$
  - $\exp(V_{car}) = 0.00248$, $\exp(V_{bus1}) = 0.00674$, $\exp(V_{bus2}) = 0.00674$,
    $\exp(V_{car}) + \exp(V_{bus1}) + \exp(V_{bus2}) = 0.00248 + 0.00674 + 0.00674 = 0.01595$
  - Prob(car) = $0.00248 \div 0.01595$ $= 0.155$
  - Prob(bus1) = Prob(bus2) = $0.00674 \div 0.01595$ $= 0.422$

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MNL model example

Alternative	Time	Cost
Car 1 (Uber)	5 minutes	$5.00
Car 2 (Lyft)	5 minutes	$5.00
Bus	15 minutes	$2.00

$$V_k = (-0.2) \times \text{time}_k + (-1) \times \text{cost}_k$$

- Probabilities of choosing each alternative
  - $V_{car1} = -6,\ V_{car2} = -6,\ V_{bus} = -5$
  - $\exp(V_{car1}) = 0.00248,\ \exp(V_{car2}) = 0.00248,\ \exp(V_{bus}) = 0.00674,$
    $\exp(V_{car}) + \exp(V_{bus1}) + \exp(V_{bus2}) = 0.00248 + 0.00248 + 0.00674 = 0.01170$
  - $\text{Prob(car1)} = \text{Prob(car2)} = 0.00248 \div 0.01170 = 0.212$
  - $\text{Prob(bus)} = 0.00674 \div 0.01170 = 0.576$

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Independence from irrelevant alternatives (IIA)

- Example 1: car and bus alternatives
  - 27% car, 73% bus
- Example 2: car and two (nearly identical) bus alternatives
  - Red bus / blue bus paradox
  - 16% car, 84% bus (total)
- Example 3: bus and two (nearly identical) car alternatives
  - Let's call this the "Uber/Lyft" paradox
  - 42% car (total), 58% bus
- We might expect car and bus mode shares to remain ~ like in Example 1

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Independence from irrelevant alternatives (IIA)

- In general, for a given decision-maker n,
  the ratio of the choice probabilities of two alternatives P_{kn} / P_{ln}
  is unaffected by addition, subtraction, or changes
  to any other alternative m.

- Why?
  - Independently distributed errors ε_k
  - Unobserved utility for both bus/car alternatives is likely highly correlated
    (e.g., similar comfort levels, crowding, user preferences, etc.)

- How to avoid?
  - Nested logit model

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Nested logit model

- Assign alternatives to nests, select (or estimate) a nesting parameter m
  - m is based on (1 – correlation of alternatives within nest)
  - m ranges from near 0 (completely similar, like one alternative) to near 1 (completely different, like red/blue buses or Uber/Lyft paradoxes)
- We must calculate conditional probabilities (within each nest)
  - $P_n(k \mid C_{mn}) = P_{kmn} = \exp(V_k \div m) \div \sum_{l \text{ in } Cmn} \exp(V_l \div m)$
- Inclusive value: value of nest's maximum expected utility
  - $IV_m = m \times \ln[\sum_{l \text{ in } Cmn} \exp(V_l \div m)]$
- Upper level probabilities
  - $P_n(m \mid C_n) = P_{mn} = \exp(IV_m) \div \sum_{l \text{ in } Cn} \exp(IV_l)$

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Nested logit model

Alternative	Time	Cost
Car	5 minutes	$5.00
Bus 1 (blue)	15 minutes	$2.00
Bus 2 (red)	15 minutes	$2.00

$$V_k = (-0.2) \times \text{time}_k + (-1) \times \text{cost}_k$$

Buses 1 & 2 in nest with $m = 0.15$ (correlation = 0.85)

- Probabilities of choosing each alternative
  - $IV_{\text{bus}} = m_{\text{bus}} \times \ln[\exp(V_{\text{bus1}} \div m_{\text{bus}}) + \exp(V_{\text{bus2}} \div m_{\text{bus}})]\}]$
  - → $= 0.15 \times \ln[\exp(-5 \div 0.15) + \exp(-5 \div 0.15)]\}]$
  - → $= 0.15 \times \ln[3.34\text{E-}15 + 3.34\text{E-}15] = 0.15 \times -32.64 = -4.896$
  - $\text{Prob(car)} = \exp(V_{\text{car}}) \div [\exp(V_{\text{car}}) + \exp(IV_{\text{bus}})]$
  - → $= 0.00248 \div [0.00248 + 0.00748] = 0.00248 \div 0.00996 = 0.249$
  - $\text{Prob(buses)} = \exp(IV_{\text{bus}}) \div [\exp(V_{\text{car}}) + \exp(IV_{\text{bus}})] = \ldots = 0.751$

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Types of independent variables

- Variables that vary across alternatives
  - Level-of-service variables: travel time, cost, crowding, comfort, etc.
  - → Can be included in every alternative's utility equation
  - → Can have generic (same for all alternatives) or alternative-specific coefficients
- Variables that do not vary across alternatives
  - Traveler characteristics: income, attitudes, etc.
  - → Can be included in every alternative's utility equation EXCEPT one (base)
  - → Must have alternative-specific coefficients
  - → Can be interacted with variables that vary across alternatives

Traffic assignment (or route choice)

EWCE 351
Transportation Engineering

Outline

- Traffic assignment overview
- Uncongested conditions
  - All-or-nothing assignment
  - Example
- Congested conditions
  - User equilibrium
  - Example

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Traffic assignment

Trip Generation → Trip Distribution → Mode Choice → Route Choice

P_i, A_j → T_{ij} → T_{ijm} → T_{ijmr}

Traffic Analysis Zone

TAZ	P_i
1	47
2	66
3	110
T	223

trip production

from i

to j

T_{ij}	1	2	3	P_i
1	10	18	19	47
2	30	32	4	66
3	5	40	65	110
A_j	45	90	88	223

Mode	T_{32m}
car	20
bus	15
walk	5
Total	40

Path	$T_{32car,r}$
R_1	5
R_2	12
R_3	3
Total	20

TAZ	A_j
1	45
2	90
3	88
T	223

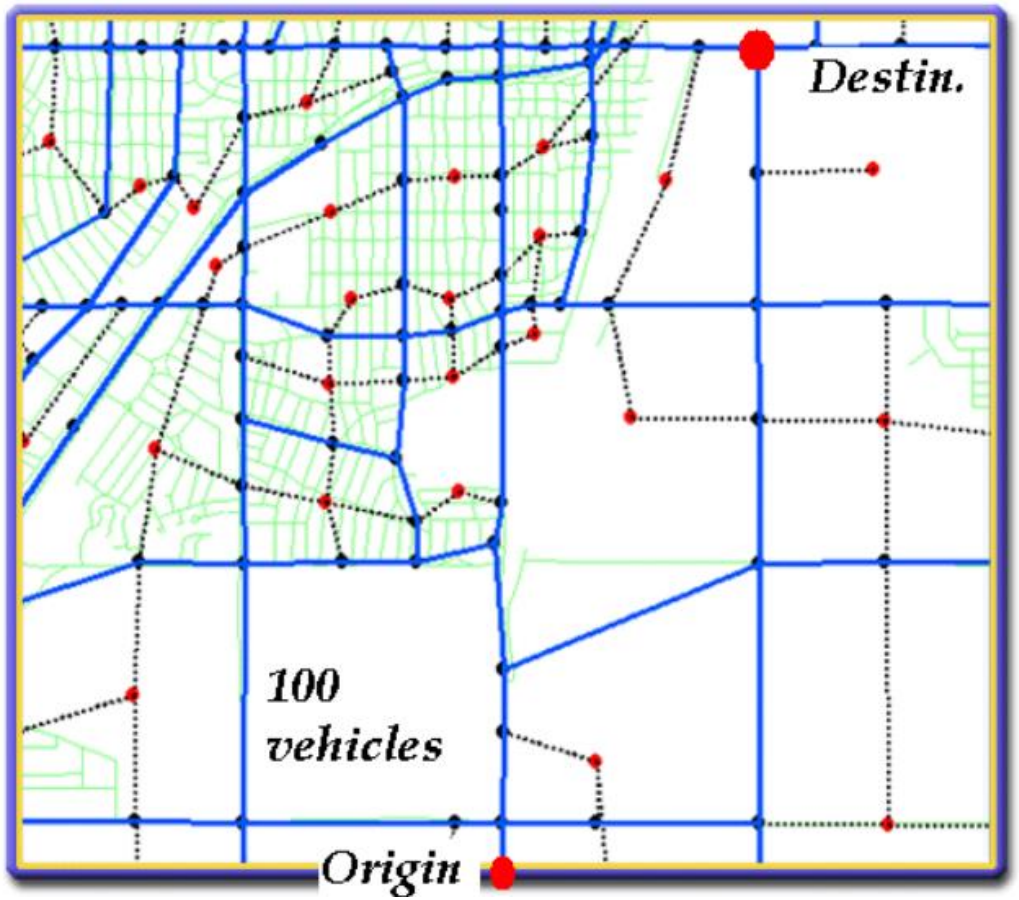
trip attraction

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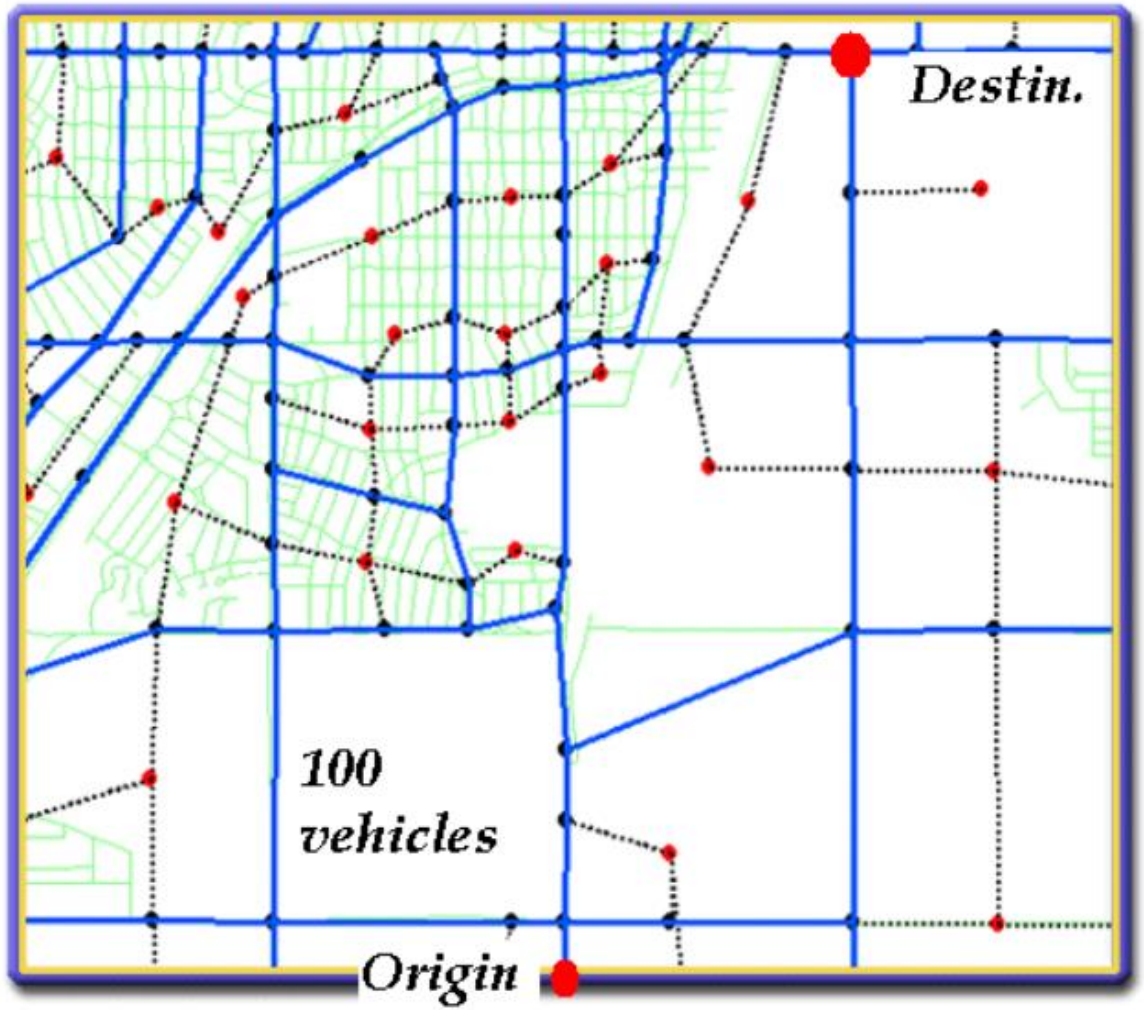
Traffic assignment (or route choice)

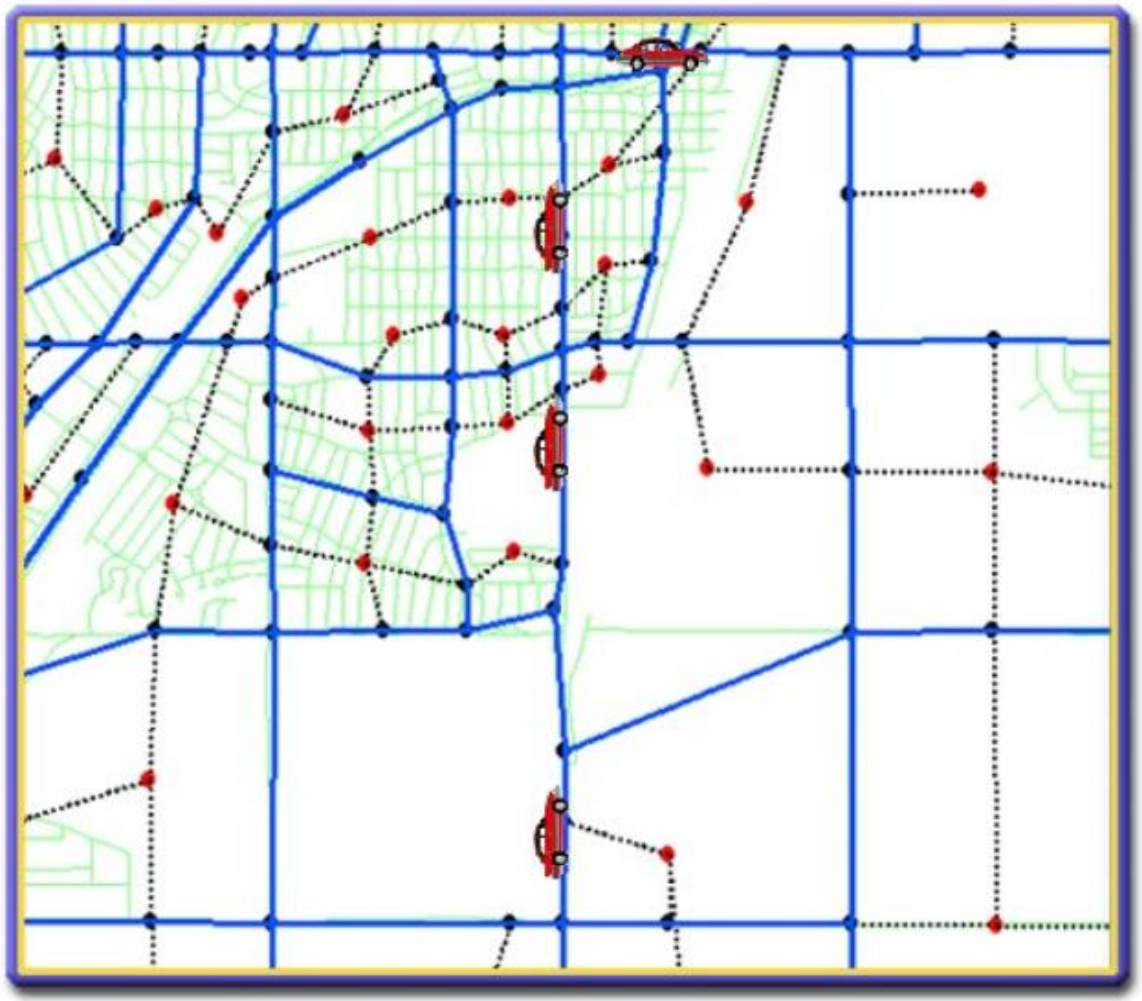
- Purpose
  - Determine the selection of routes by travelers between origins and destinations
  - Result of traffic assignment / route choice is traffic flow (vehicles per hour) on specific routes/links
    - One of the ultimate goals of the travel demand modeling/forecasting process
- Challenge
  - How to model & predict user behavior?
  - What assumptions do we make?
  - What factors affect travelers' route choices?

How do you get there?

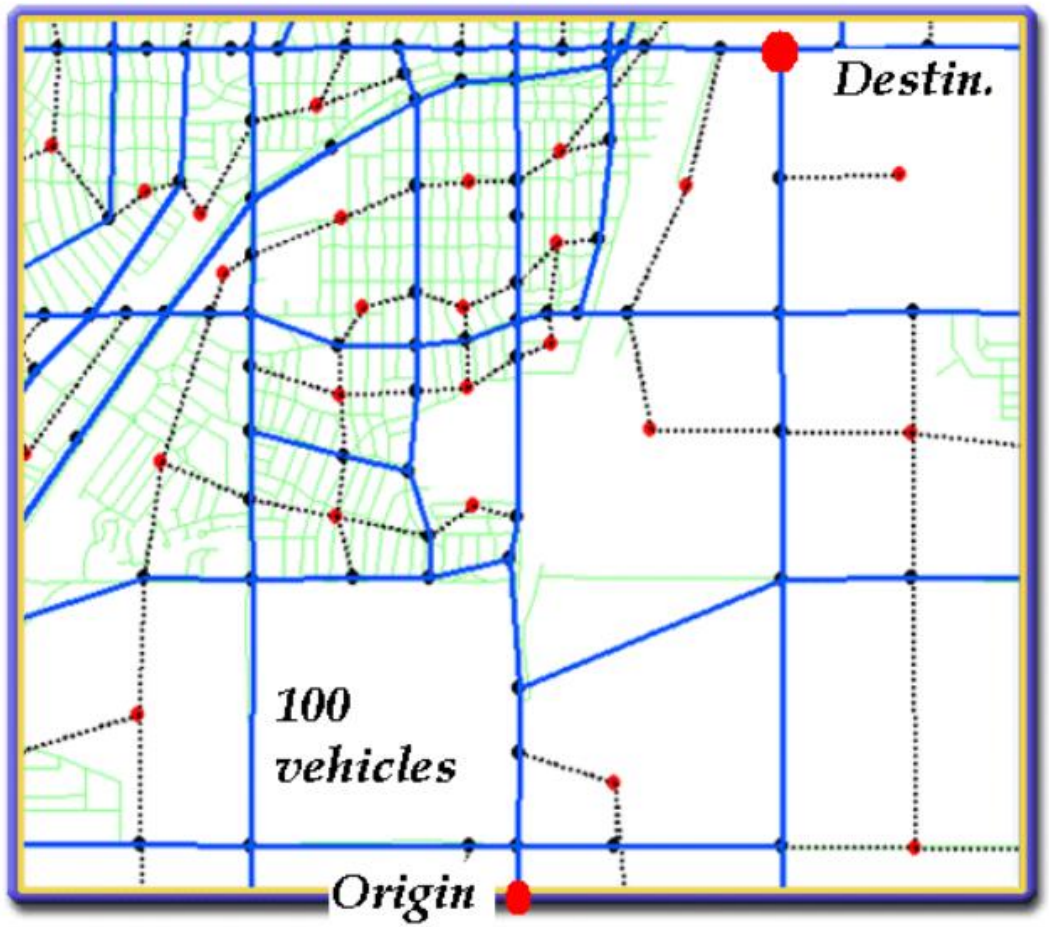


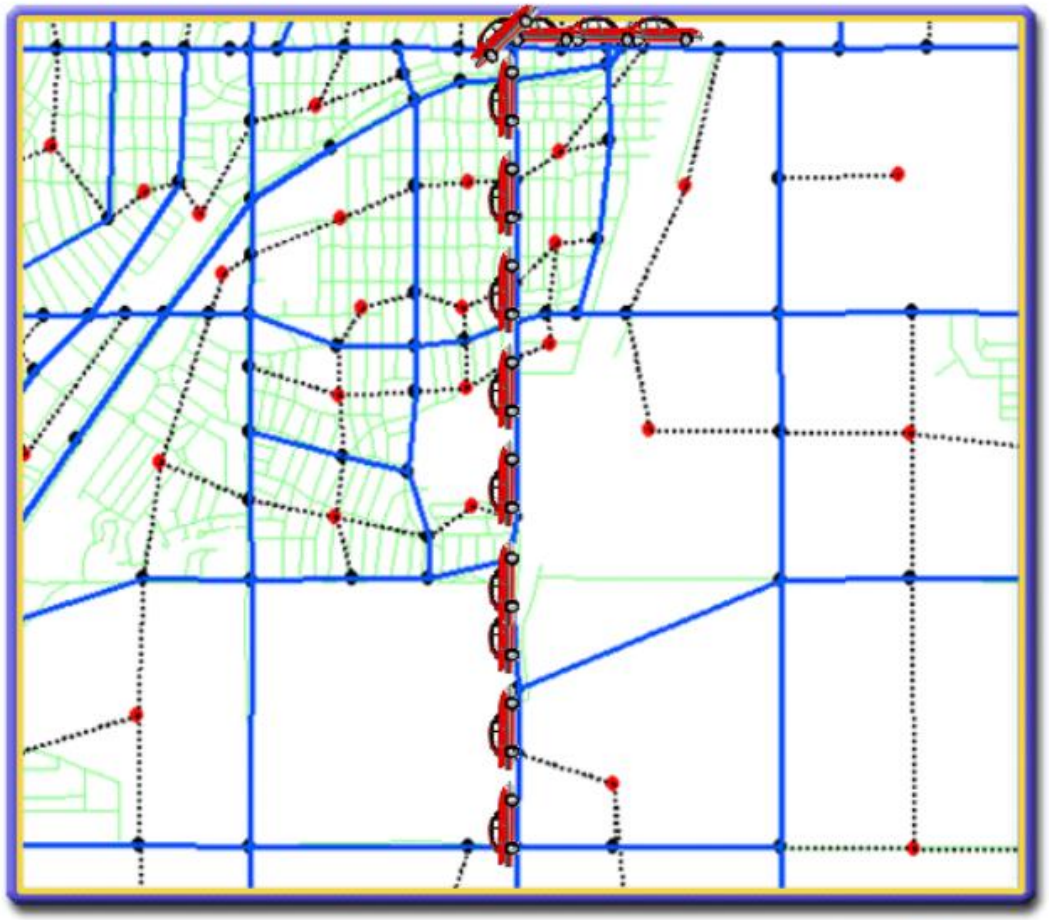
Shortest / fastest path



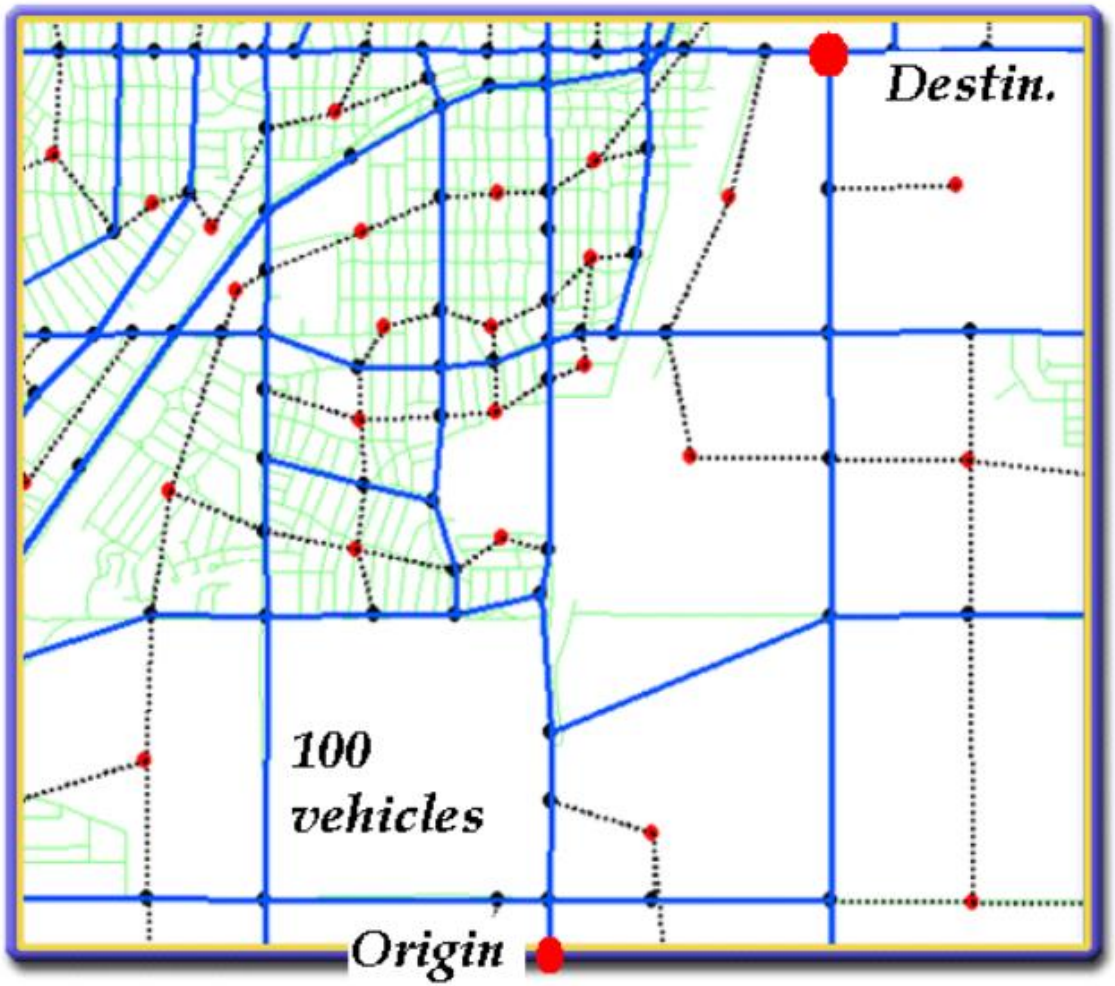


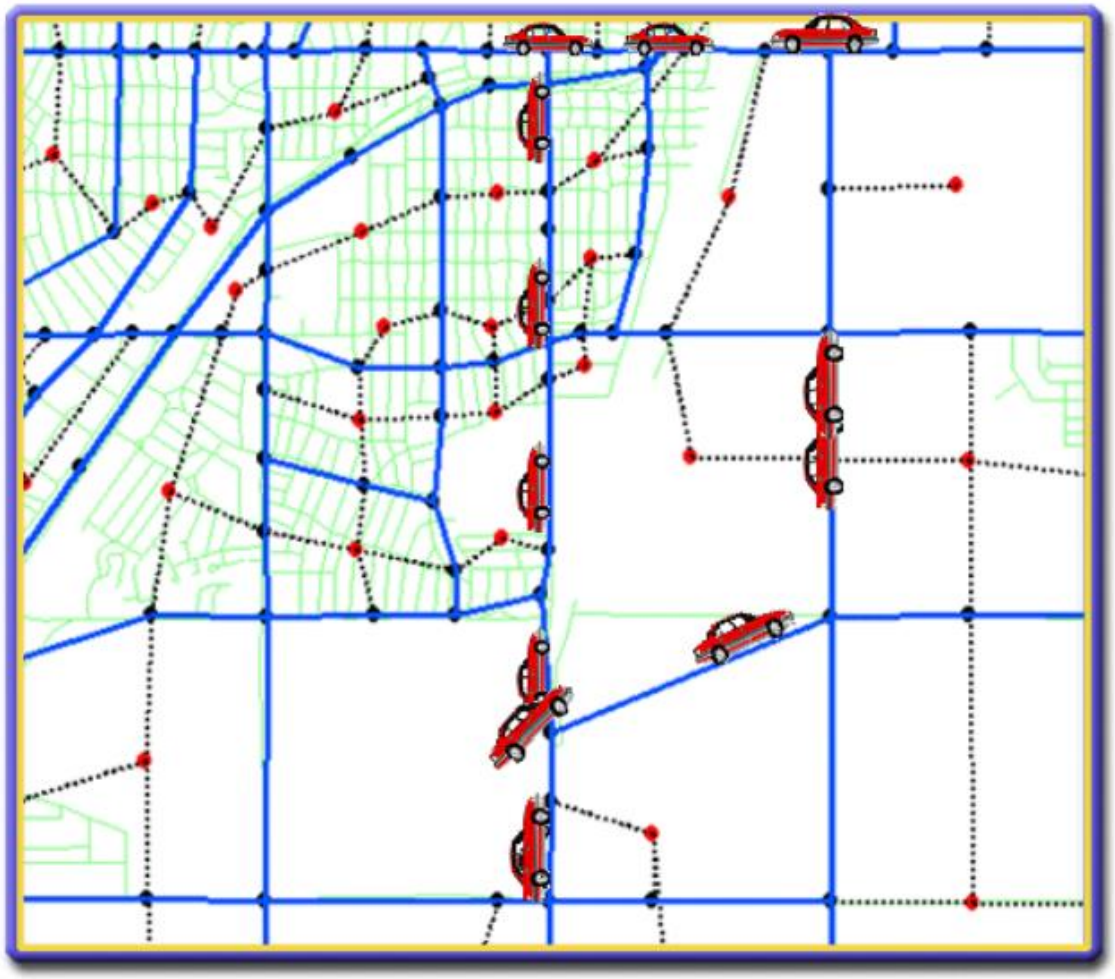
What if there is congestion?





Some detour to second shortest path





Uncongested condition

All-or-nothing (AON) assignment

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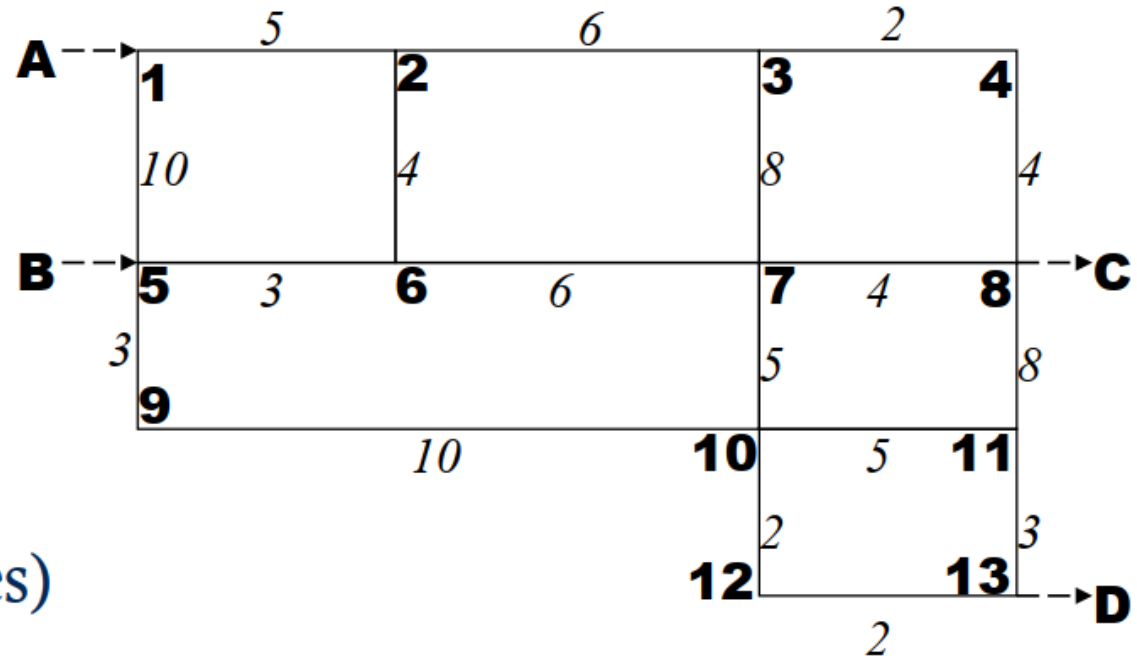
All-or-nothing (AON) assignment

- Assumptions
  - People choose the shortest path to travel from an origin to destination
  - Roads are uncongested – the travel times/costs for each link of a roadway are a constant over the time-horizon of interest

- Procedure
  - Determine the shortest path between each origin-destination (OD) pair
  - Assign all trips between the OD pair to this path

AON Example

- A,B,C, and D are zone centroids (origins and destinations of trips)
- 1,2,3,..,13 are nodes of a highway network
- The stretch of roadway between any pair of adjacent nodes is a roadway link (e.g., 1-2, 5-6, 3-7, 8-11, etc.)
- The numbers in italics on the links are the travel times on the links (e.g.: the travel time on link 1-2 is 5 minutes)



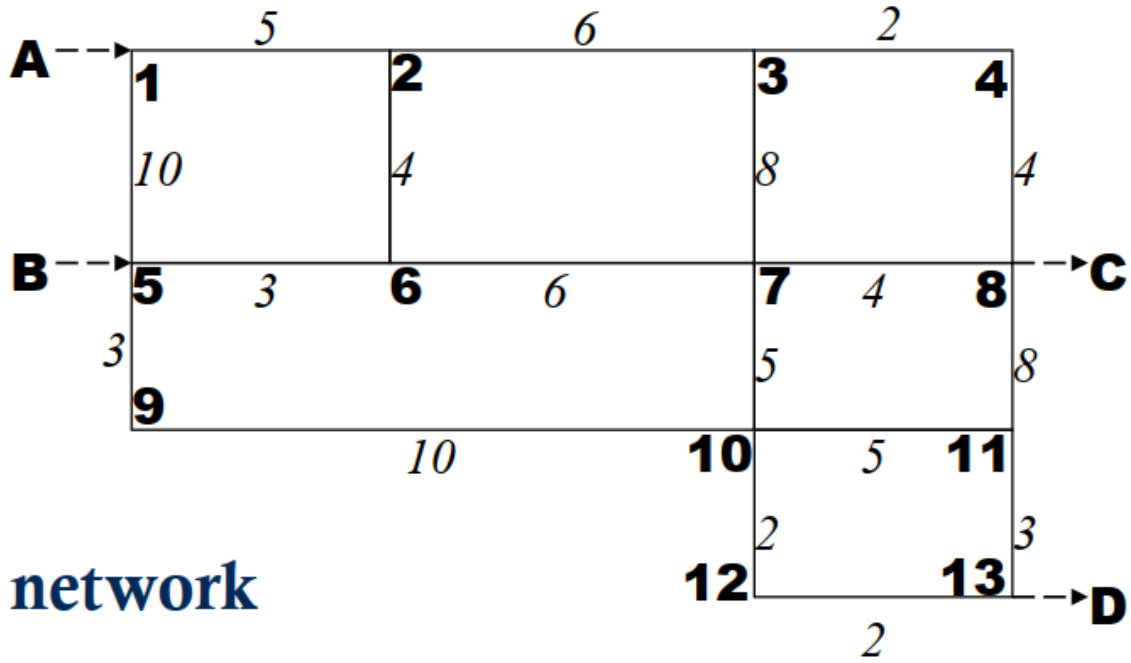
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AON Example

- The following is the demand (OD matrix)

Origin \ Destination	A	B	C	D
A	0	0	400	200
B	0	0	300	100
C	0	0	0	0
D	0	0	0	0

Travel times

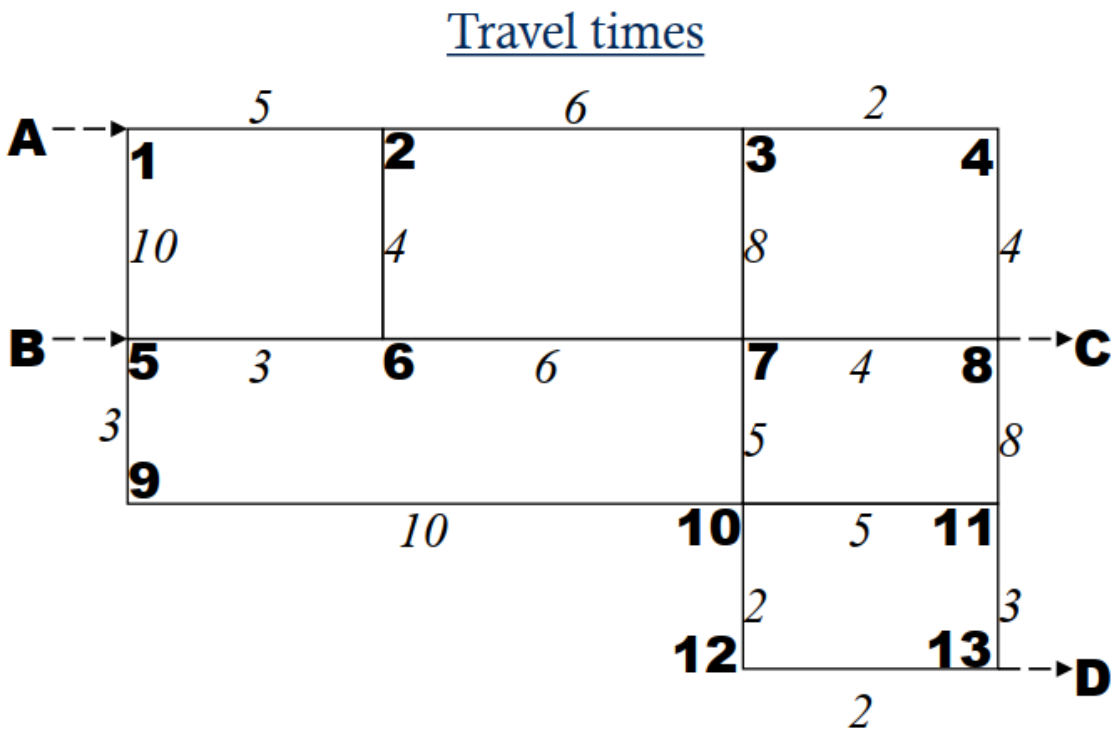


- **Determine the link flows on the highway network**

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AON Example

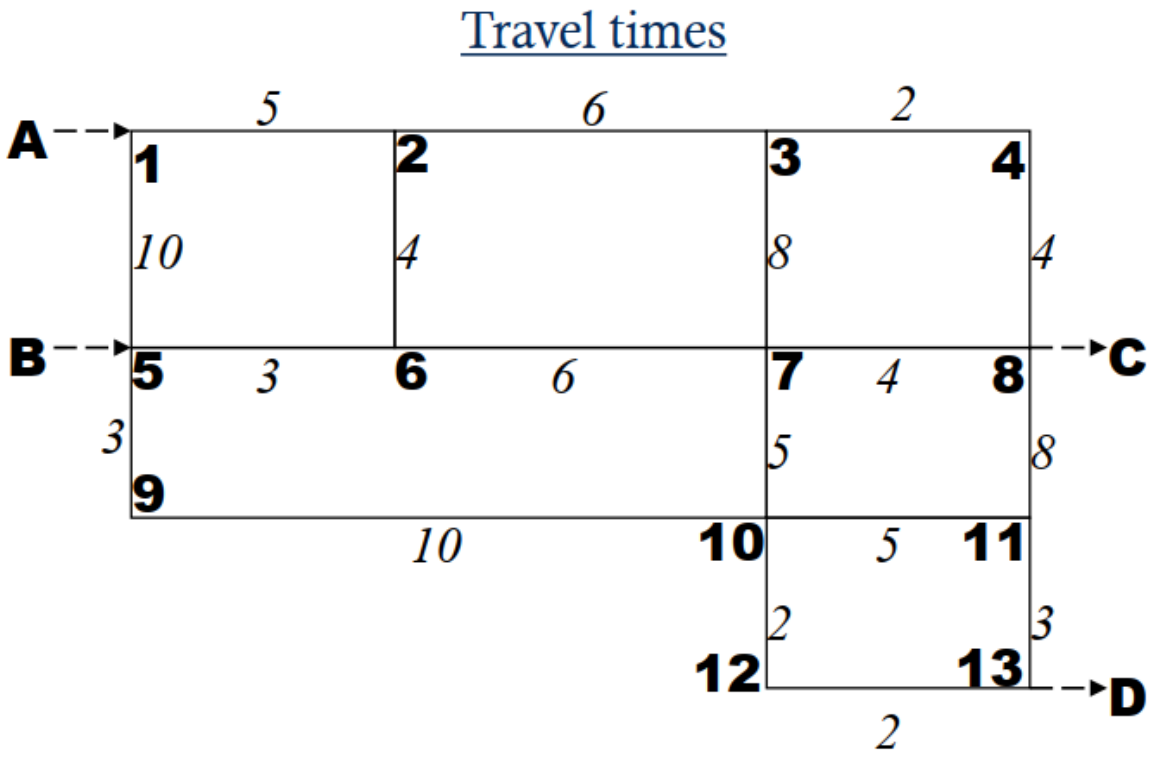
- Step 1:
  - Find the shortest paths from
    - A to C
    - A to D
    - B to C
    - B to D



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AON Example

- Step 1:
  - Find the shortest paths from
    - A to C
      - 1-2-3-4-8 → 17 minutes
    - A to D
      - 1-2-6-7-10-12-13 → 24 minutes
    - B to C
      - 5-6-7-8 → 13 minutes
    - B to D
      - 5-9-10-12-13 → 17 minutes



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AON Example

- Step 2:
  - Assign all demand between a zonal pair to the shortest path between zones
    - A to C
      - 1-2-3-4-8 → 17 minutes
    - A to D
      - 1-2-6-7-10-12-13 → 24 minutes
    - B to C
      - 5-6-7-8 → 13 minutes
    - B to D
      - 5-9-10-12-13 → 17 minutes

Origin \ Destination	A	B	C	D
A	0	0	400	200
B	0	0	300	100
C	0	0	0	0
D	0	0	0	0

Link	Flow	Link	Flow
1 to 2		7 to 8	
2 to 3		5 to 9	
3 to 4		7 to 10	
1 to 5		8 to 11	
2 to 6		9 to 10	
3 to 7		10 to 11	
4 to 8		10 to 12	
5 to 6		11 to 13	
6 to 7		12 to 13	

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AON Example

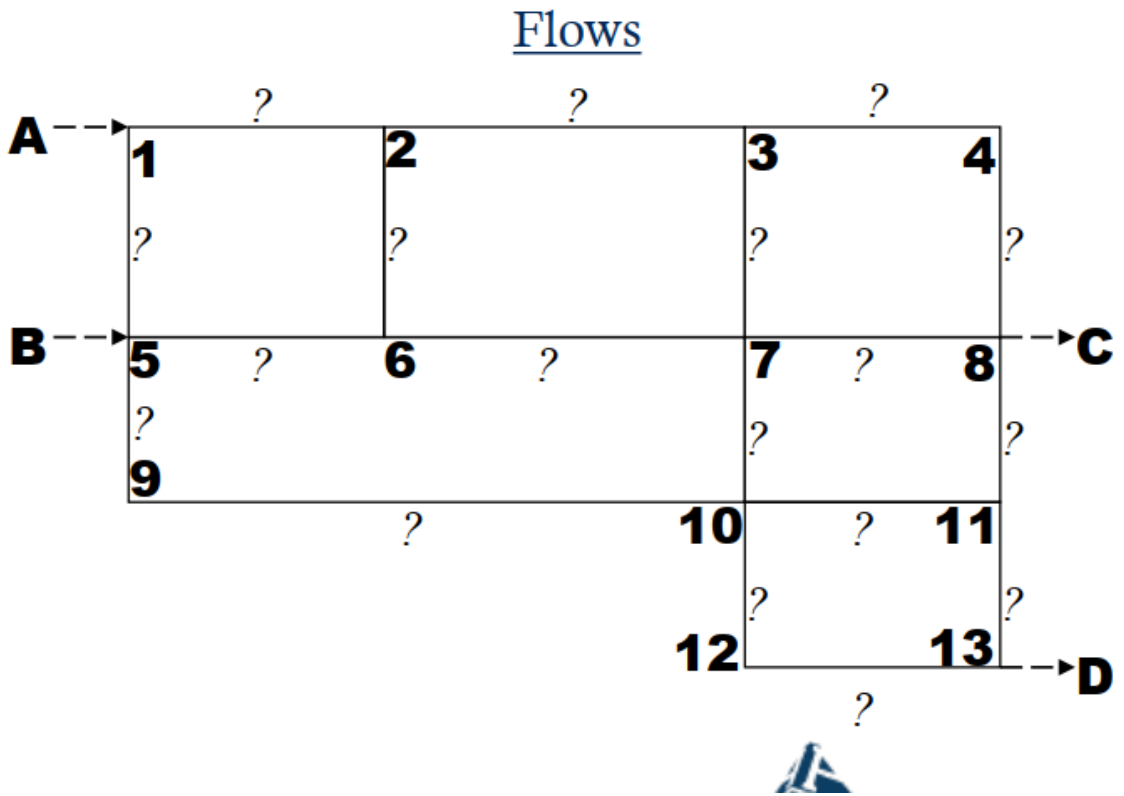
- Step 2:
  - Assign all demand between a zonal pair to the shortest path between zones
    - A to C
      - 1-2-3-4-8 → 17 minutes
    - A to D
      - 1-2-6-7-10-12-13 → 24 minutes
    - B to C
      - 5-6-7-8 → 13 minutes
    - B to D
      - 5-9-10-12-13 → 17 minutes

Origin \ Destination	A	B	C	D
A	0	0	400	200
B	0	0	300	100
C	0	0	0	0
D	0	0	0	0

Link	Flow	Link	Flow
1 to 2	600	7 to 8	300
2 to 3	400	5 to 9	100
3 to 4	400	7 to 10	200
1 to 5	0	8 to 11	0
2 to 6	200	9 to 10	100
3 to 7	0	10 to 11	0
4 to 8	400	10 to 12	300
5 to 6	300	11 to 13	0
6 to 7	500	12 to 13	300

AON Example

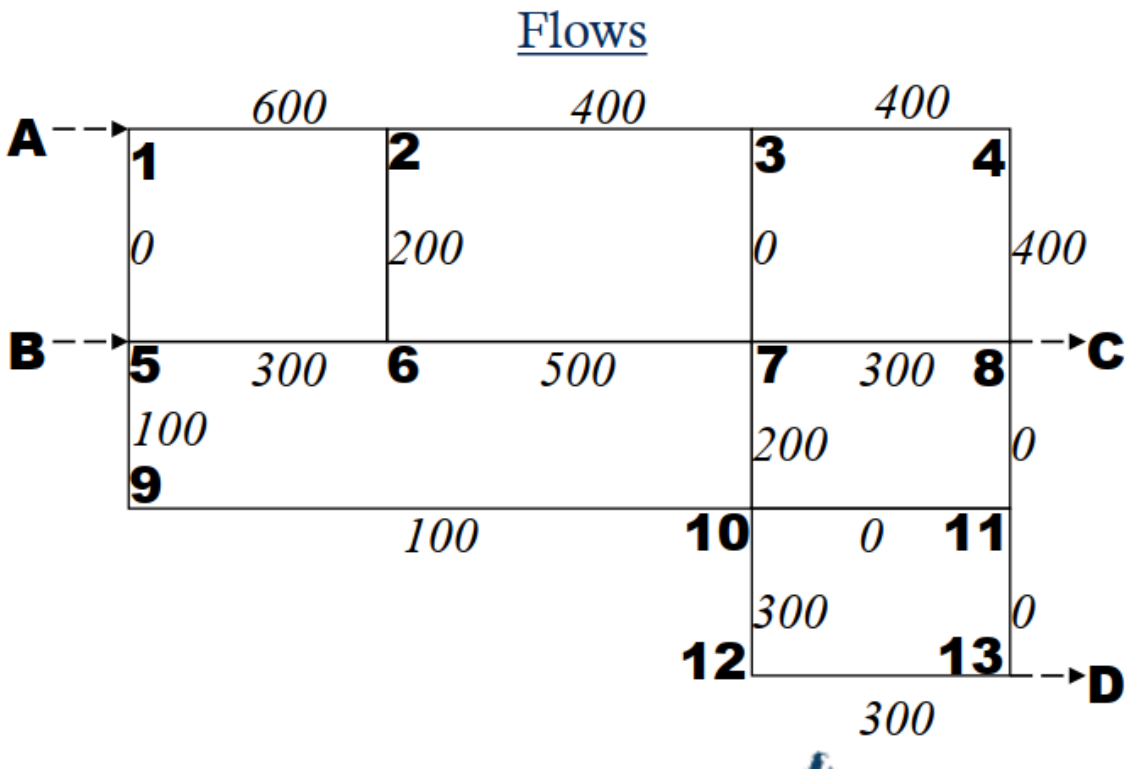
- Step 2:
  - Assign all demand between a zonal pair to the shortest path between zones
    - A to C
      - 1-2-3-4-8 → 17 minutes
    - A to D
      - 1-2-6-7-10-12-13 → 24 minutes
    - B to C
      - 5-6-7-8 → 13 minutes
    - B to D
      - 5-9-10-12-13 → 17 minutes



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AON Example

- Step 2:
  - Assign all demand between a zonal pair to the shortest path between zones
    - A to C
      - 1-2-3-4-8 → 17 minutes
    - A to D
      - 1-2-6-7-10-12-13 → 24 minutes
    - B to C
      - 5-6-7-8 → 13 minutes
    - B to D
      - 5-9-10-12-13 → 17 minutes



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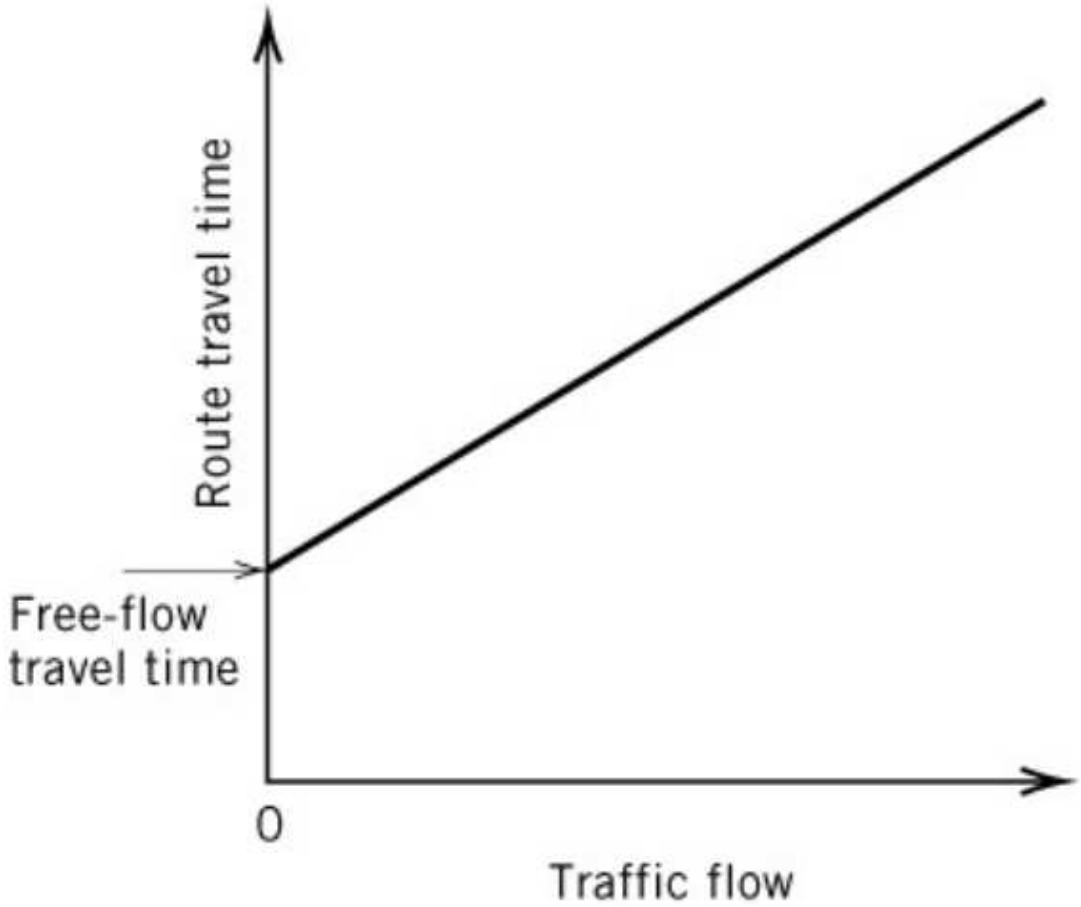
AON Summary

- Relatively simple approach
  - However determination of the shortest paths require computational time
  - There are systematic procedures for determination of the shortest paths for large-scale networks: e.g., The Dijkstra's algorithm
- Can lead to unreasonable assignment of flows when there are multiple paths between an OD pair with small differences in their travel times
- Assumes un-congested conditions: the travel times/costs for each link of a roadway are a constant over the time-horizon of interest irrespective of the number of vehicles on that link

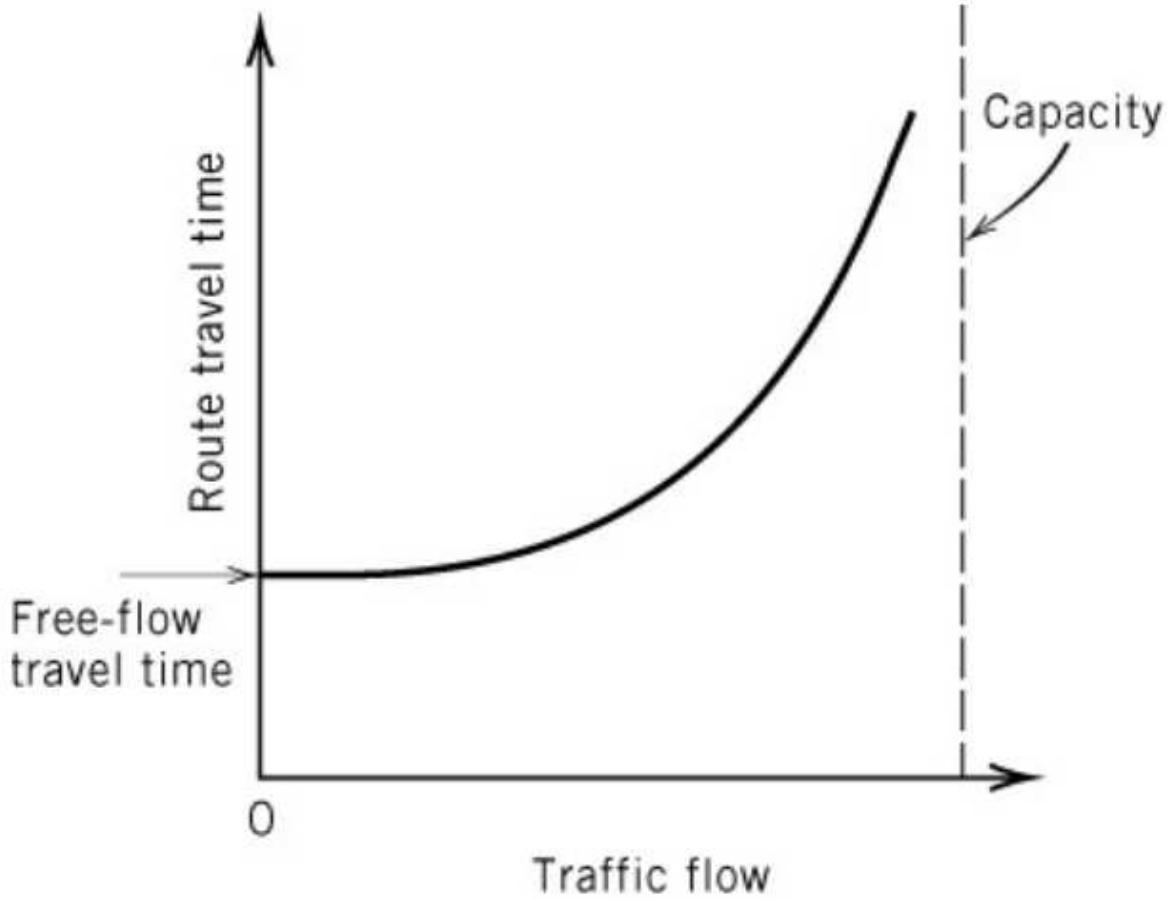
Congested condition

User equilibrium

Highway performance function



Linear travel time-traffic flow relationship



Nonlinear travel time-traffic flow relationship

Bureau of Public Roads (BPR) function

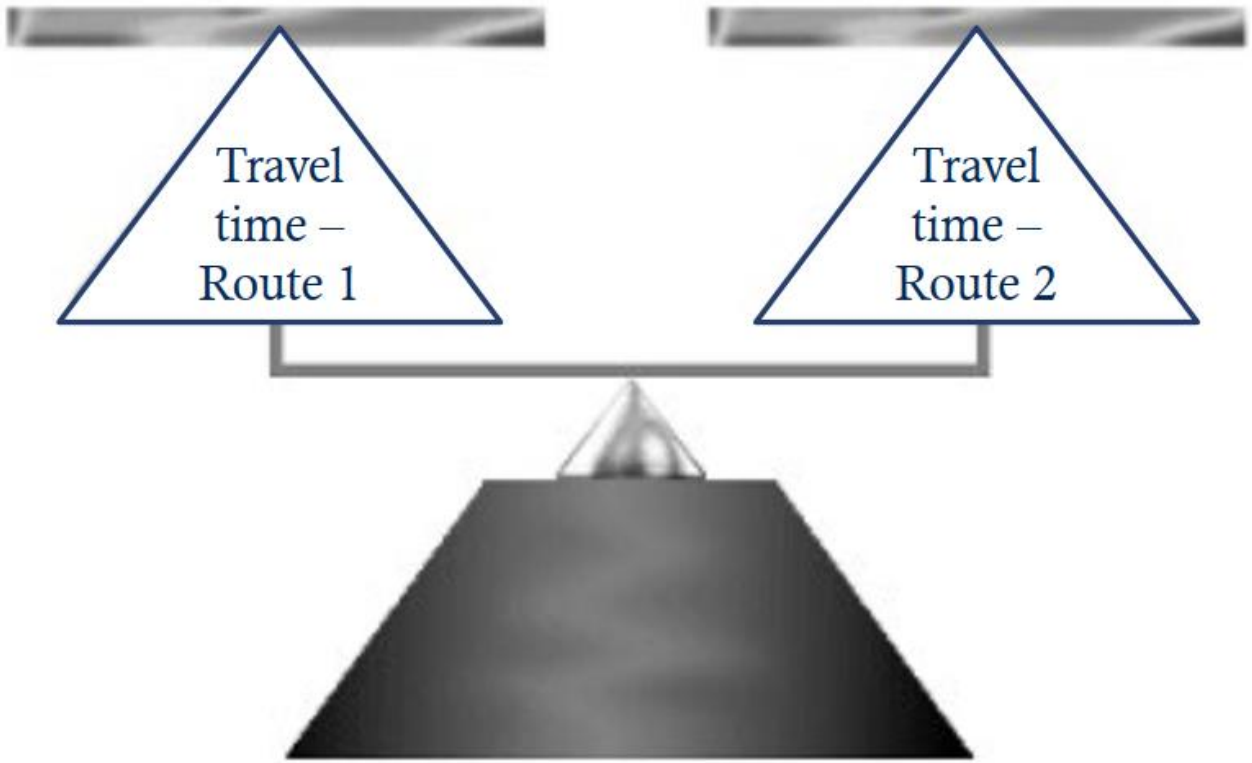
- Link congestion/performance or volume-delay or function
  - t or $S_a(v_a)$
    - Average travel time on a link
  - t_0 or t_a
    - Free flow travel time on a link
  - V or v_a
    - Volume of traffic on a link, vehicles per time
  - Q or c_a
    - Capacity of a link, vehicles per time
  - α and β
    - Parameters, $\alpha = 0.15$ and $\beta = 4$

$$t = t_0 \left[1 + \alpha \left(V/Q\right)^{\beta}\right]$$

$$S_a(v_a) = t_a \left(1 + 0.15 \left(\frac{v_a}{c_a}\right)^4\right)$$

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Equilibrium



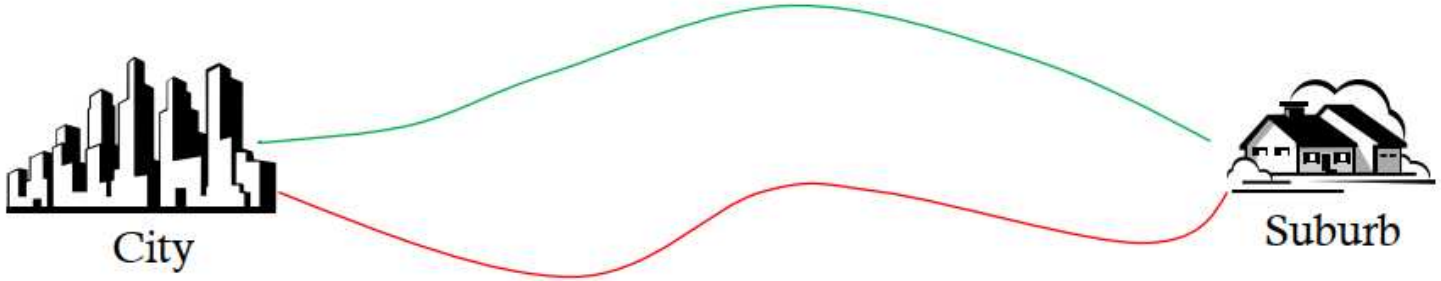
User equilibrium (UE) assignment

- Assumptions
  - Travelers will select routes based only on route travel times
  - Travelers know the travel times on all the available routes (full information)
- Wardrop's definition:
  - "The travel time between a specified origin and destination
    on all used routes
    is equal, and less than or equal
    to the travel time that would be experienced by a traveler on any unused route"
- User equilibrium exists when individual travelers cannot improve their travel times by unilaterally changing routes

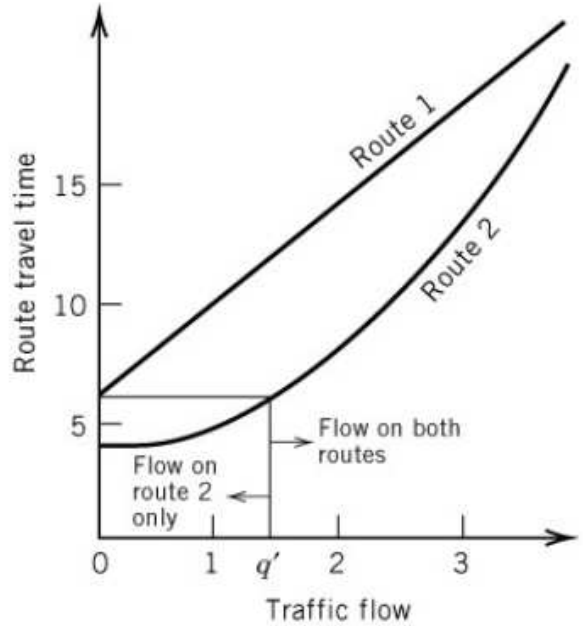
UE assignment

- Wardrop's principles
  - First principle (User Equilibrium)
    - Under equilibrium conditions traffic arranges itself in congested networks in such a way that no individual trip maker can reduce his path costs by switching routes.
  - Second principle (System Optimal)
    - Under social optimal conditions traffic should be arranged in congested networks in such a way that the average (or total) travel cost is minimized.

UE Example

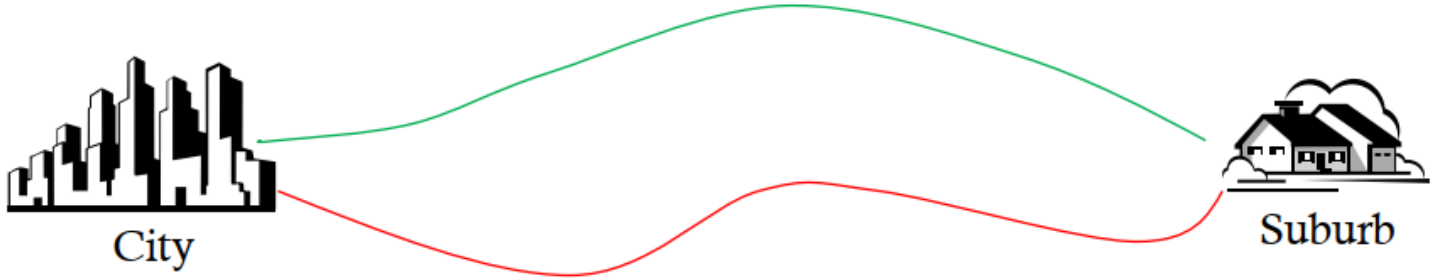


- Assume:
  - Two routes connect a city and a suburb.
  - During the peak-hour morning commute, a total of 4,500 vehicles travel from the suburb to the city.
  - The travel time functions for these routes are given:
    - $TT_1(x_1) = 6 + 4x_1$
    - $TT_2(x_2) = 4 + (x_2)^2$
    - where TT(x) is travel time in minutes and x is traffic volume in 1000 vehicles.



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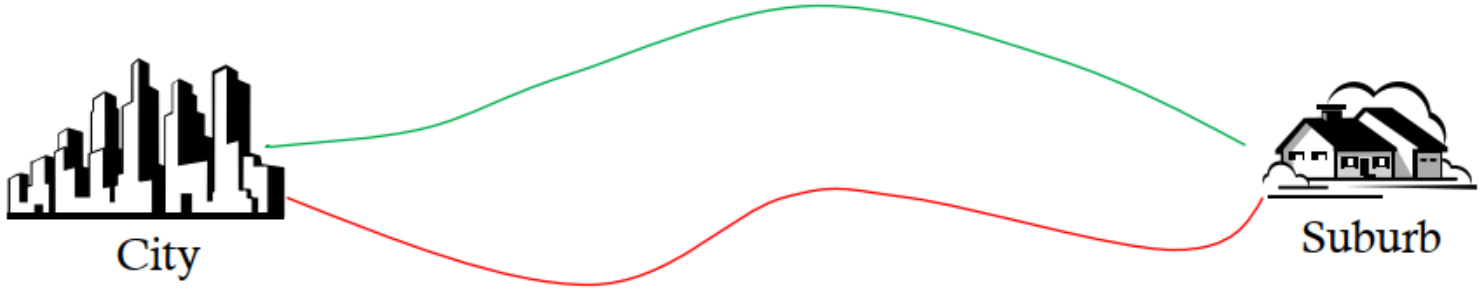
UE Example



- Step 1:
  - Write equations
    - $TT_1(x_1) = 6 + 4x_1$
    - $TT_2(x_2) = 4 + (x_2)^2$
    - Flow constraint: $x_1 + x_2 = 4.5$

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UE Example



- Step 2:
  - Check if both routes will be used
    - All-or-nothing assignment on Route 1 ($x_1 = 4.5$, $x_2 = 0.0$)
      - $TT_1(4.5) = 6 + 4(4.5) = 24$ minutes
      - $TT_2(4.5) = 4 + (0.0)^2 = 4$ minutes

      } If all the traffic is on Route 1 then Route 2 is the desirable choice

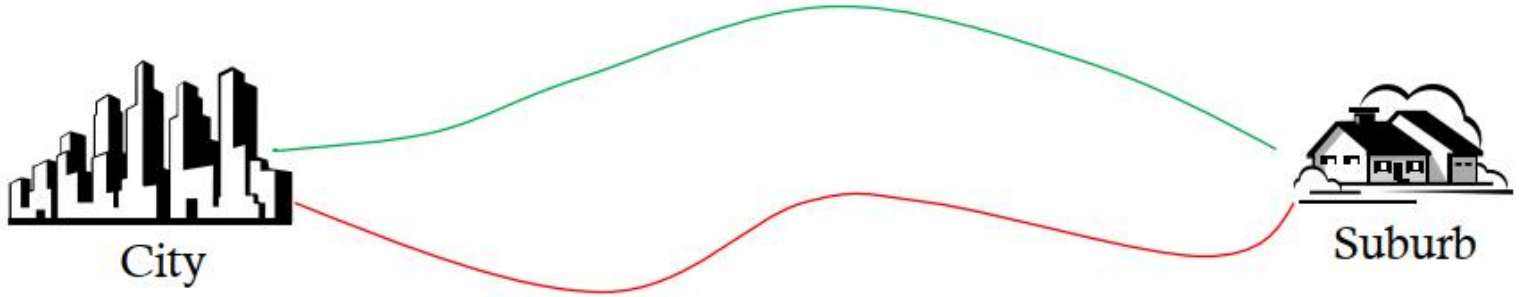
    - All-or-nothing assignment on Route 2 ($x_1 = 0.0$, $x_2 = 4.5$)
      - $TT_1(4.5) = 6 + 4(0.0) = 6$ minutes
      - $TT_2(4.5) = 4 + (4.5)^2 = 24.25$ minutes

      } If all the traffic is on Route 1 then Route 2 is the desirable choice

  - Therefore…both routes will be used

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UE Example



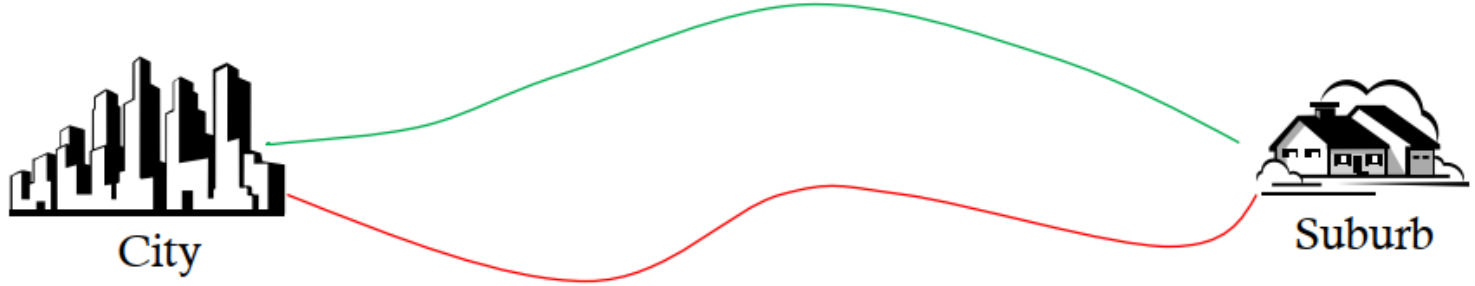
- Step 3:
  - Equate travel times and solve
    - Apply Wardrop's 1st principle requirements.
    - All routes used will have equal times, and ≤ those on unused routes.
    - Hence, if flows are distributed between Route 1 and Route 2, then both must be used, and travel times will be equal.

  - At user equilibrium:
    - $TT_1(x_1) = TT_2(x_2)$ → $6 + 4x_1 = 4 + (x_2)^2$
    - Flow constraint: $x_1 + x_2 = 4.5$

  - Two unknowns + two equations → solve for x_1 and x_2!

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UE Example



- Step 3:
  - Solve for x_1 and x_2
    - $x_1 + x_2 = 4.5 \rightarrow x_2 = 4.5 - x_1$
    - $6 + 4x_1 = 4 + (x_2)^2 \rightarrow 6 + 4x_1 = 4 + (4.5 - x_1)^2 \rightarrow 6 + 4x_1 = 4 + 20.25 - 9x_1 + x_1^2$
      $\rightarrow x_1^2 - 13x_1 + 18.25 = 0$
    - ...using quadratic formula...
    - $x_1 = 1.6$ or $11.4 \rightarrow$ only 1.6 is reasonable (< 4.5) $\rightarrow$ **x_1 = 1,600 vehicles/hour**
    - $x_2 = 4.5 - x_1 = 4.5 - 1.6 = 2.9$ $\rightarrow$ **x_2 = 2,900 vehicles/hour**

  - Check for equal travel times
    - $TT_1(1.6) = 6 + 4(1.6) = 12.4$ minutes
    - $TT_2(2.9) = 4 + (2.9)^2 = 12.4$ minutes

} Good!

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Other traffic assignment methods

1. All-Or-Nothing (AON) Assignment
2. Capacity-Restrained (CR) Assignment
3. Incremental Assignment
4. Method of Successive Averages (MSA)
5. Convex Combinations Method (also known as the Frank-Wolfe (FW) Algorithm)

Wardrop's principles

- First principle (User Equilibrium)
  - Under equilibrium conditions traffic arranges itself in congested networks in such a way that no individual trip maker can reduce his path costs by switching routes.

- Second principle (System Optimal)
  - Under social optimal conditions traffic should be arranged in congested networks in such a way that the average (or total) travel cost is minimized.

- Are these principles compatible? Does the first lead to the second?

Wardrop's principles

Nash equilibrium
(https://en.wikipedia.org/wiki/Nash_equilibrium)

- First principle (User Equilibrium)
  - Focused on the individual, individual optimum, user equilibrium
  - Useful for explaining (selfish) behavior
- Second principle (System Optimal)
  - Focused on the entire system, system optimum, social equilibrium
  - Useful for managing/planning (social) travel
- How to move: first principle → second principle?
  - Congestion charging, road pricing, other interventions

Braess' paradox

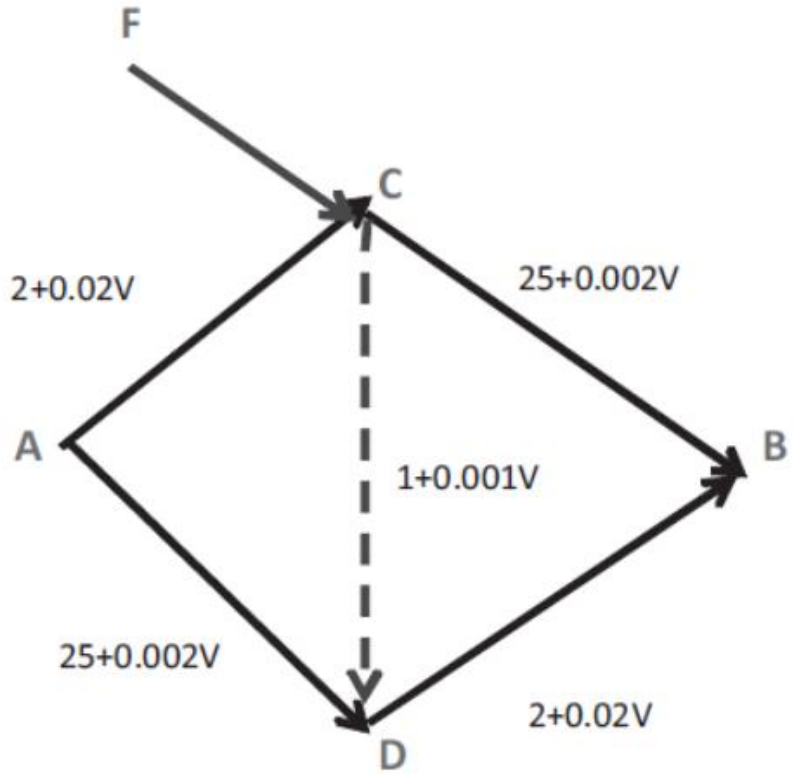


Figure 10.8 A simple network to illustrate Braess's Paradox

- Adding network capacity might actually make everyone worse off.
- Example
  - Initial conditions
    - 1000 car trips A→B
    - Only options: A–C–B, A–D–B
  - New conditions
    - Add new high-speed & high-capacity link C–D
    - What happens?

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Dynamics & randomness

Stochastic user equilibrium
Dynamic traffic assignment

Stochastic user equilibrium (SUE)

- All-or-nothing (AON) assumes fixed and known travel times
- User equilibrium (UE) assumes congestion-restrained but known travel times
- Stochastic user equilibrium (SUE) assumes variation in congestion-restrained travel times, due to…
  - Different "perceptions" of travel times
    - Actual different perceived travel times
    - Lack of knowledge, and guesses about expected travel times
    - Travel times being truly unknowable in advance
- → add error term to volume-delay function

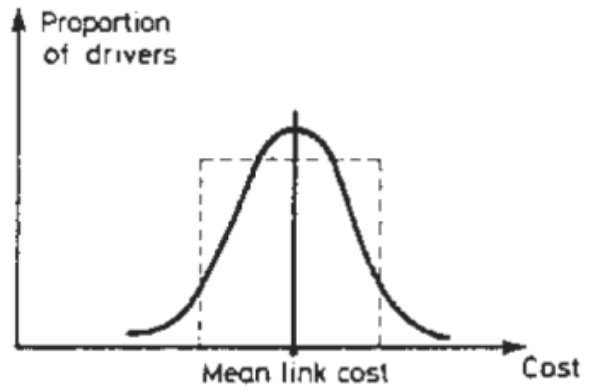


Figure 10.4 Distribution of perceived costs on a link

Stochastic user equilibrium (SUE)

- Problems for application
  - Assumption of the error term distribution
    - Uniform? Normal? Something else?
  - Difficulty converging on a solution (efficiently)
    - UE faster and approximately equal in congested networks
- Methods
  - Monte Carlo simulation
  - Logit-like proportions
- → Not frequently used in practice

Dynamic traffic assignment (DTA)

- Unrealistic assumptions thus far
  - Perfect information
    - Full knowledge of current and future conditions
    - → Even with GPS/Waze/other tools, cannot know the future for certain
  - Separability
    - Link travel time affected by traffic flow on that link (and nothing else)
    - → Downstream nodes and links can affect upstream traffic, bottlenecks
  - Steady state
    - Traffic flows are constant during a time period
    - → Traffic flows change day-by-day, hour-by-hour, and surges / platoons

Dynamic traffic assignment (DTA)

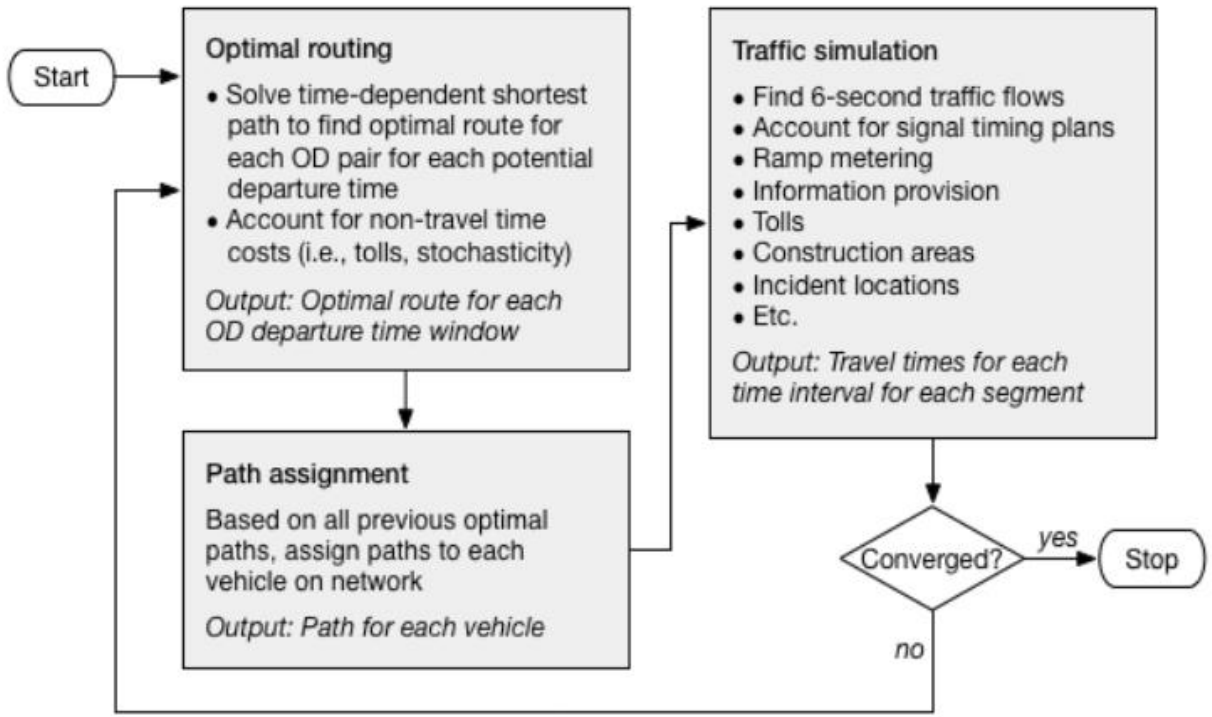
- Requirements
  - *Positivity*: non-negative flows on links, paths, trip matrices, and costs
  - *Conservation*: satisfy flow conservation requirements
  - *FIFO*: "First In, First Out" traffic behavior
  - *Minimum travel time*: flows do not propagate instantaneously
  - *Finite clearing time*: no queues left at the end of the modeling period; infinite delays do not occur (as a standard queueing model might suggest)
  - *Capacity*: strict capacity constraint, cannot be exceeded (even for a short time)
  - *Causality*: delays now affected by the present or the past, not the future

Dynamic traffic assignment (DTA)

- Wardrop's first principle (user equilibrium)
  - Under equilibrium conditions traffic arranges itself in congested networks in such a way that no individual trip maker can reduce his path costs *by switching routes*.
- Extension 1
  - Under equilibrium conditions in networks *where congestion varies over time* traffic arranges itself so that *at each instant* the costs incurred by drivers on those routes that are used are equal and no greater than those on any unused route.
- Extension 2
  - Under equilibrium conditions in networks where congestion varies over time and *travelers can choose their time of travel*, traffic arranges itself so that the total cost associated with travel on those route that are used by travelers *at the time when they are used*, are equal and no greater than those on any route *at a time when it is not used*.

Dynamic traffic assignment (DTA)

- How does DTA work?
  - A middle ground between static macroscopic traffic assignment and microscopic traffic simulation models (e.g., PTV Vissim)
  - Use same network/zonal structure, but…
  - Split time into discrete time intervals or time-slices



Dynamic traffic assignment (DTA)

- DYNASMART: https://mctrans.ce.ufl.edu/featured/dynasmart/
- Dynus-T: https://dynust.net/
- TransDNA: https://www.caliper.com/transdna/transdna-dta-software.htm
- Dynameq: https://www.inrosoftware.com/en/products/dynameq/
- Cube Avenue: https://www.citilabs.com/software/cube/cube-avenue/
- and others…

Questions?