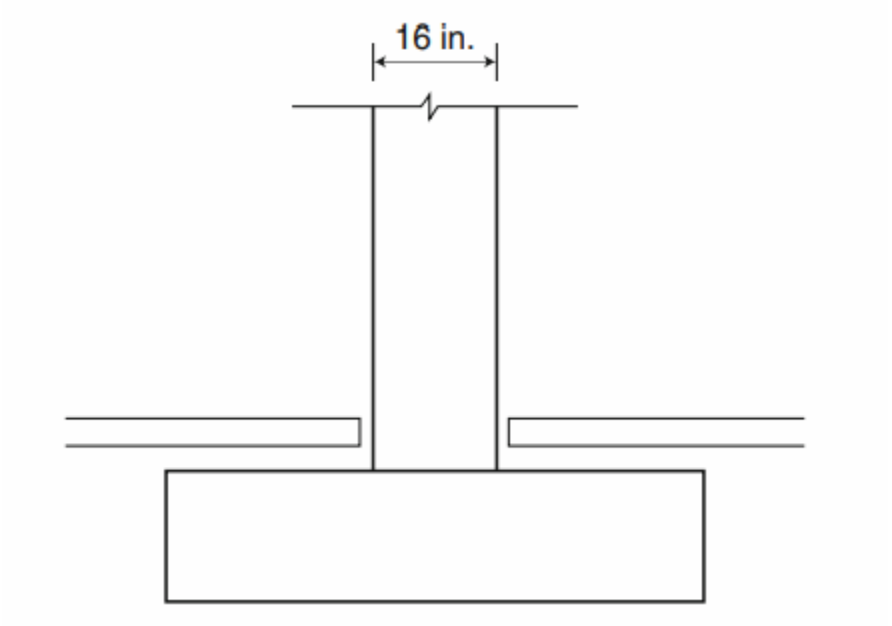
PROBLEM 01

Design a square reinforced concrete footing for the **16 inch square interior concrete column** shown below. The dead load is 200 kip, the floor live load is 80 kip, and the roof live load is 20 kip. **The allowable net soil pressure (bearing capacity) is 3,500 psf. Use $f_c' = 3{,}000$ psi** for the footing, **$f_c' = 4{,}000$ psi** for the column, and $f_y = 60{,}000$ psi. Assume the column has 4 #7 bars. Concrete is normal weight.

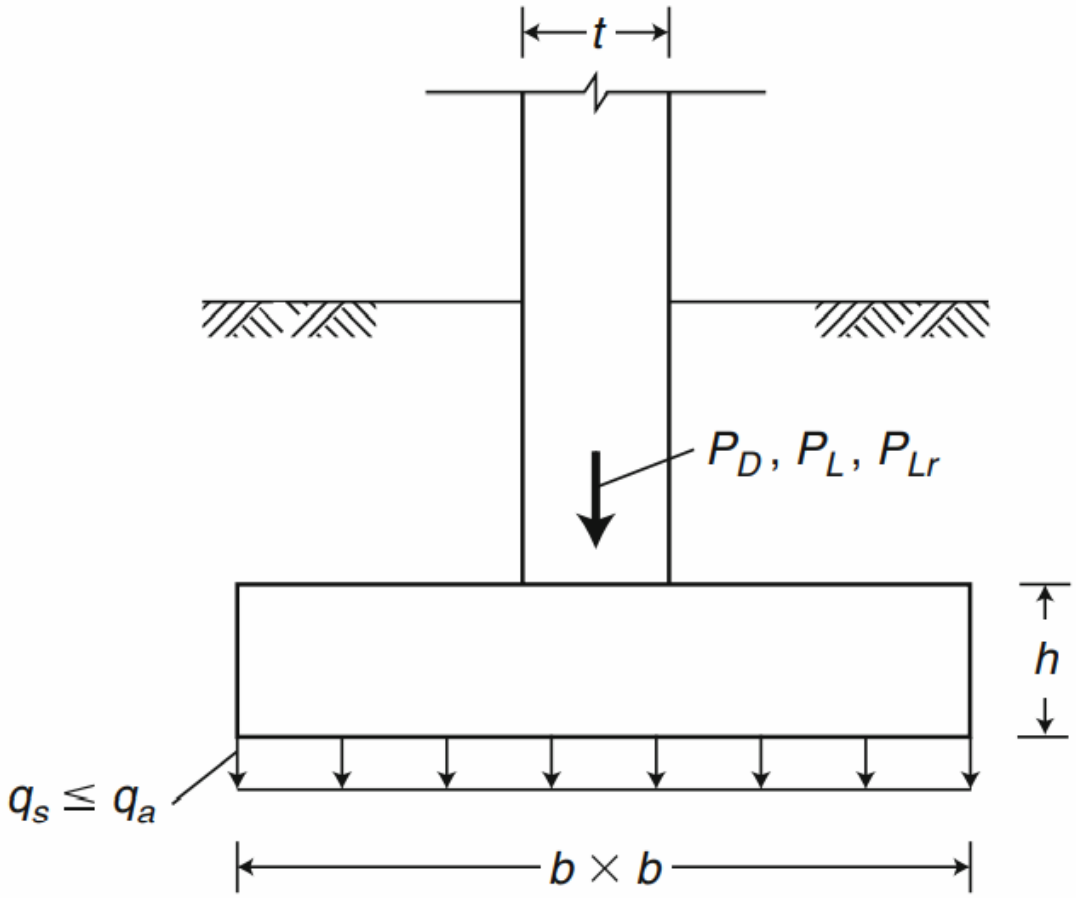


SOLUTION

The footing is sized such that the pressure on the soil (q_s) is less than the soil bearing capacity (q_a). P_D, P_L, P_{Lr}, and P_T are the applied service dead, floor live, roof live and total loads, respectively.

$$P_T = P_D + P_L + P_{Lr}$$

$$q_s = \frac{P_T}{A} \leq q_a$$



FOOTING PRESSURE ON SUPPORTING SOIL

Step 1. Determine the required size of the footing.

$$P_T = P_D + P_L + P_{Lr} = 200 + 80 + 20 = 300 \text{ kip}$$

$$A_{\text{required}} = \frac{P_T}{q_a} = \frac{300}{3.5} = 85.7 \text{ ft}^2$$

$$b_{\text{required}} = \sqrt{85.7} = 9.26 \text{ ft}$$

Round b to the nearest even inch and select $b = 9'-4''$.

Estimate the approximate footing thickness. In a square or rectangular footing, a reasonable estimate of the required thickness is about **one-half of the larger overhanging (O.H.) length of the footing**. Therefore, for a square footing, the estimated required thickness is:

$$h = 0.5(\text{O.H.}) = 0.5\left(\frac{b-t}{2}\right)$$

The minimum thickness commonly used for column spread footings is 12 inch.

Step 2. Estimate the footing thickness.

The estimated depth of the footing is:

$$h_{\text{est}} = \frac{1}{2}\left[\frac{(9 \times 12 + 4) - 16}{2}\right] = 24 \text{ in.}$$

Step 3. Calculate and check the shear.

The factored Load, P_u, is:

$$P_u = 1.2P_D + 1.6P_L + 0.5P_{Lr} \quad (\text{if } P_L \geq 1.83P_{Lr})$$

or

$$P_u = 1.2P_D + 1.6P_{Lr} + 1.0P_L \quad (\text{if } P_L < 1.83P_{Lr})$$

P_D is the total dead load. P_L and P_{Lr} are the applied service floor and roof live loads, respectively.

The factored pressure on the footing from the soil, q_u, is:

$$q_u = \frac{P_u}{b^2} \tag{7.22}$$

Footings apply pressure on their supporting soil. This pressure has to be limited to a certain allowable level (soil bearing capacity). There is also **a reaction** from the soil acting on the footing when it presses the soil. The true theoretical distribution of the reaction pressure from the soil on the footing depends on the type of supporting soil.

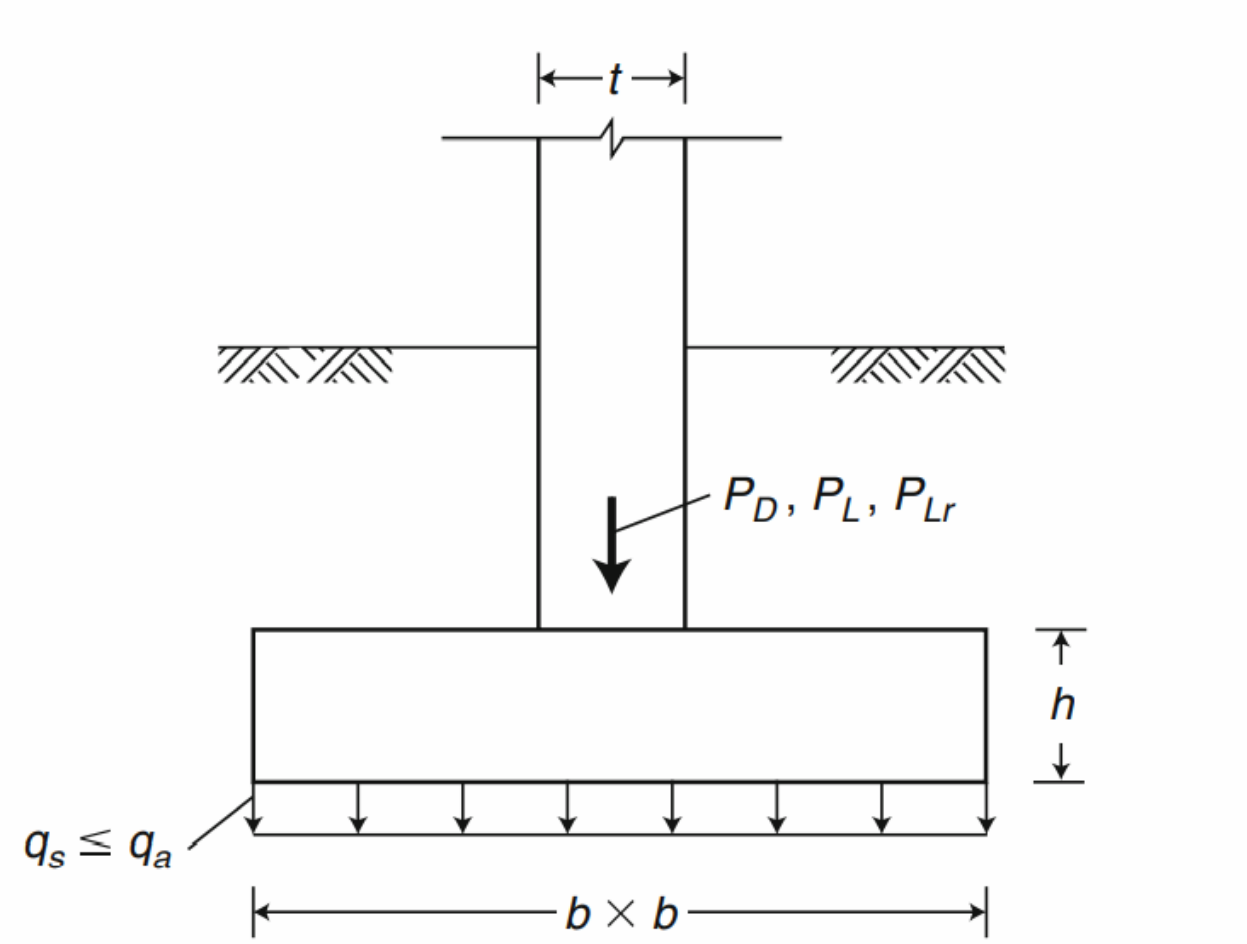
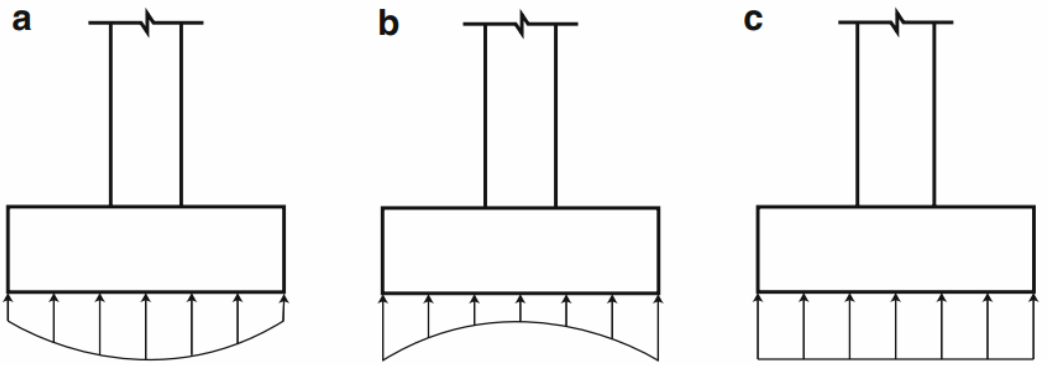
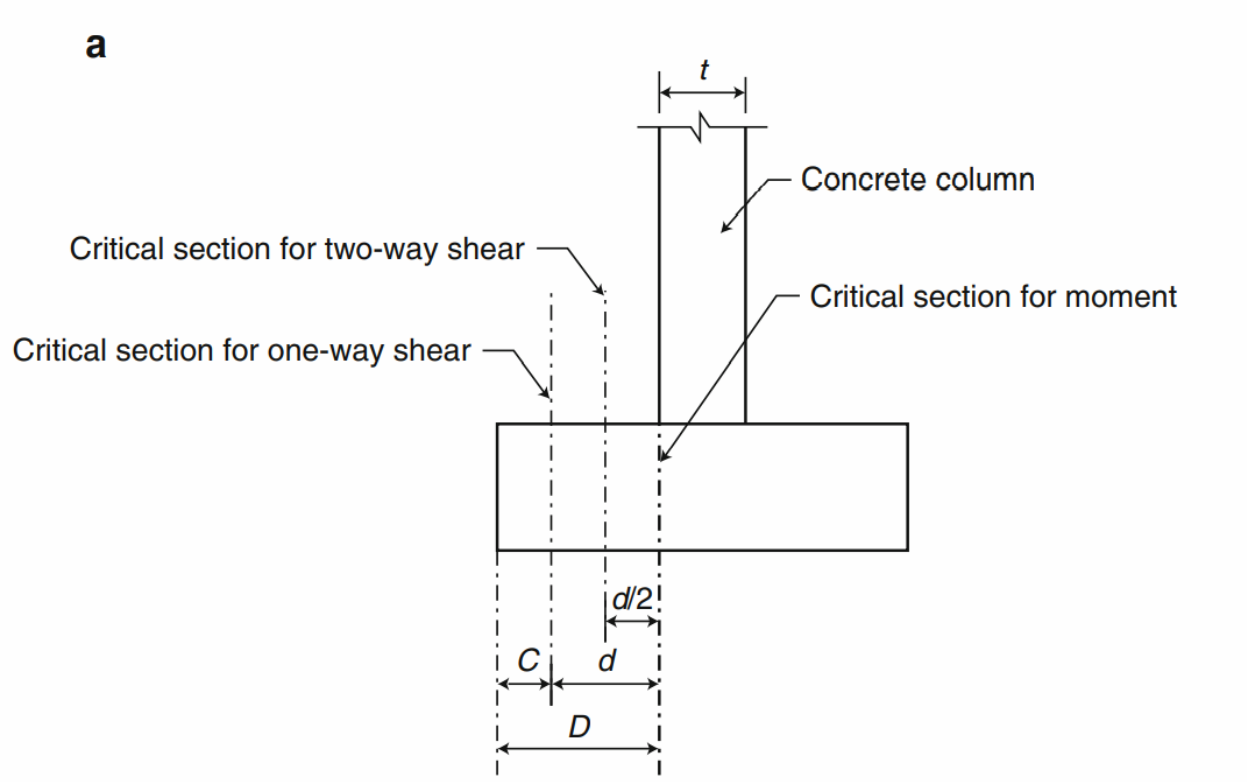


Figure below shows typical pressure distributions for different types of soil. The pressures are larger under the center of footing and smaller along the edges if the soil is sandy. This is because the sand along the edges does not have good lateral support and can easily move laterally as shown in Figure (a). The shape of the theoretical distribution of pressures in clays is shown in Figure (b).

In practice, an average uniform soil pressure distribution, like the one shown in Figure (c), is assumed. This is much simpler and the results have been proven to provide adequate and safe designs.



Soil pressure distributions: (**a**) sandy soil, (**b**) clayey soil, and (**c**) design assumption



Step 3. Calculate and check the shear.
Since $1.83P_{Lr} = 1.83(20) = 36.6\ k < P_L = 80\ k$, the factored load, P_u, is:

$$P_u = 1.2 \times 200 + 1.6 \times 80 + 0.5 \times 20$$

$$P_u = 378 \text{ kip}$$

The factored pressure on the footing from the soil is:

$$q_u = \frac{P_u}{b^2}$$

$$q_u = \frac{378}{9.33^2}$$

$$q_u = 4.34 \text{ ksf}$$

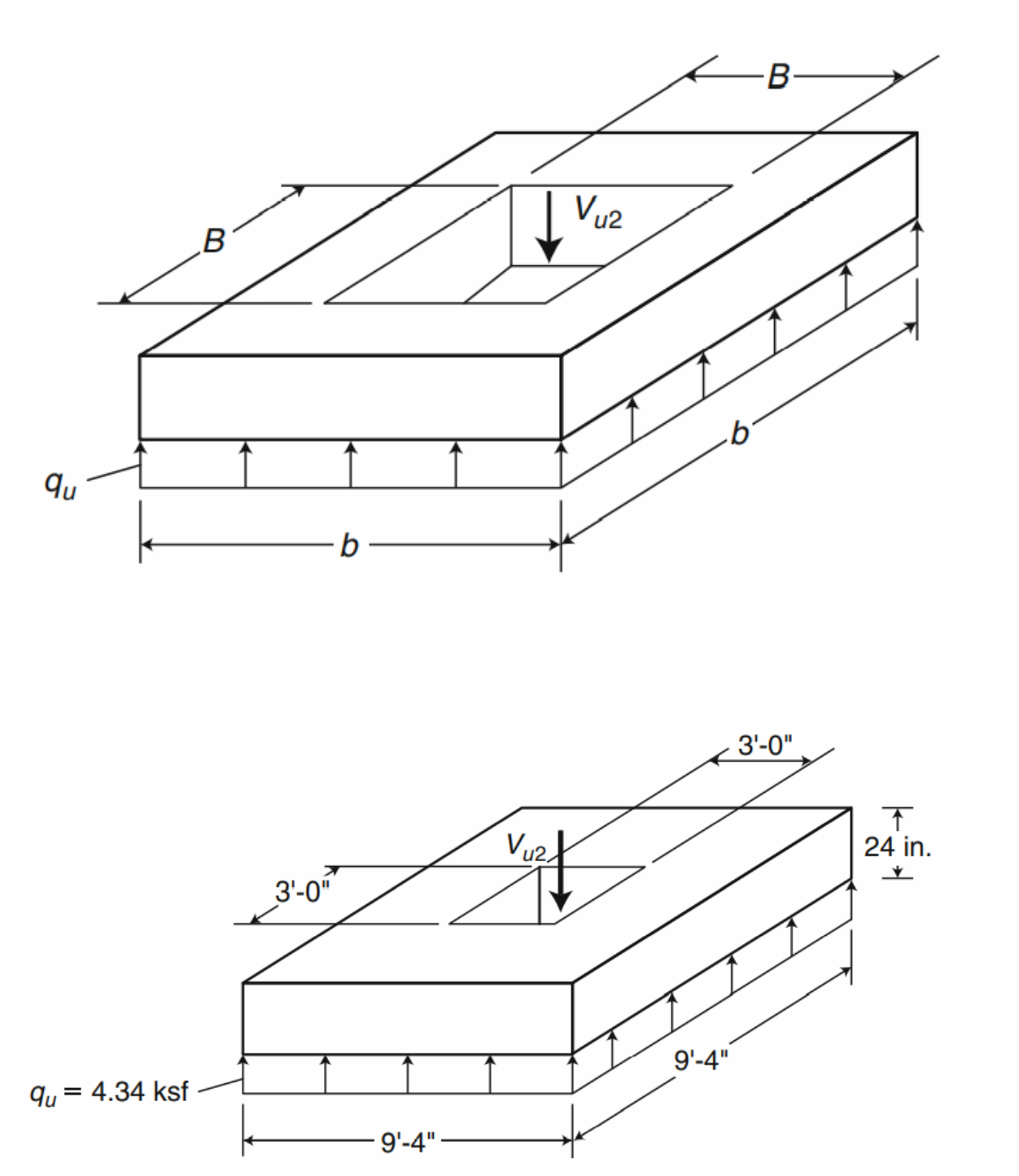


FIGURE 1: TWO WAY SHEAR IN THE SPREAD FOOTING

Step 3a. Check the two-way shear.

The average effective depth, d, is:

$$d = h - \text{cover} - \text{estimated diameter of bar}$$

$$d = 24 - 3 - 1 = 20 \text{ in.}$$

The critical section for the two-way shear is at a distance $d/2$ from the face of the concrete column. Therefore, one side of the critical section, B, is:

$$B = t + d = 16 + 20 = 36 \text{ in.} = 3'\text{–}0''$$

The shear at the critical sections, V_{U2}, is shown in Figure 1, and is calculated as follows:

$$V_{u2} = q_u\left(b^2 - B^2\right)$$

$$V_{u2} = 4.34\left(9.33^2 - 3^2\right)$$

$$V_{u2} = 339 \text{ kip}$$

The nominal two-way shear strength of the concrete, V_{c2}, is:

$$V_{c2} = \min\left\{\left(2+\frac{4}{\beta}\right)\lambda\sqrt{f_c'}\,b_o d,\ \ \left(\frac{\alpha_s d}{b_o}+2\right)\lambda\sqrt{f_c'}\,b_o d,\ \ 4\lambda\sqrt{f_c'}\,b_o d\right\}$$

$\beta = 16/16 = 1.0$

$\alpha_s = 40$ (column in the center of footing)

$b_o = 4B = 4 \times 36 = 144$ in.

$\lambda = 1.0$

$$V_{c2} = \min\left\{(2+4)(1.0)\sqrt{3{,}000}\ (144)(20)/1{,}000,\ \left(\frac{40\times 20}{144}+2\right)(1.0)\right.$$
$$\left.\sqrt{3{,}000}\ (144)(20)/1{,}000,\ \ 4(1.0)\sqrt{3{,}000}\ (144)(20)/1{,}000\right\}$$

$V_{c2} = \min\{946 \text{ kip}, 1{,}192 \text{ kip}, 631 \text{ kip}\} = 631$ kip

$\phi V_{c2} = 0.75(631) = 473 \text{ kip} > 339 \text{ kip} \quad \therefore$ ok

The two-way shear capacity of this footing is acceptable. The estimated depth could be reduced, as the shear capacity is about 40 % more than the applied shear force. However, we will conservatively continue with the assumed depth of 24 in.

Step 3b. Check one-way shear.

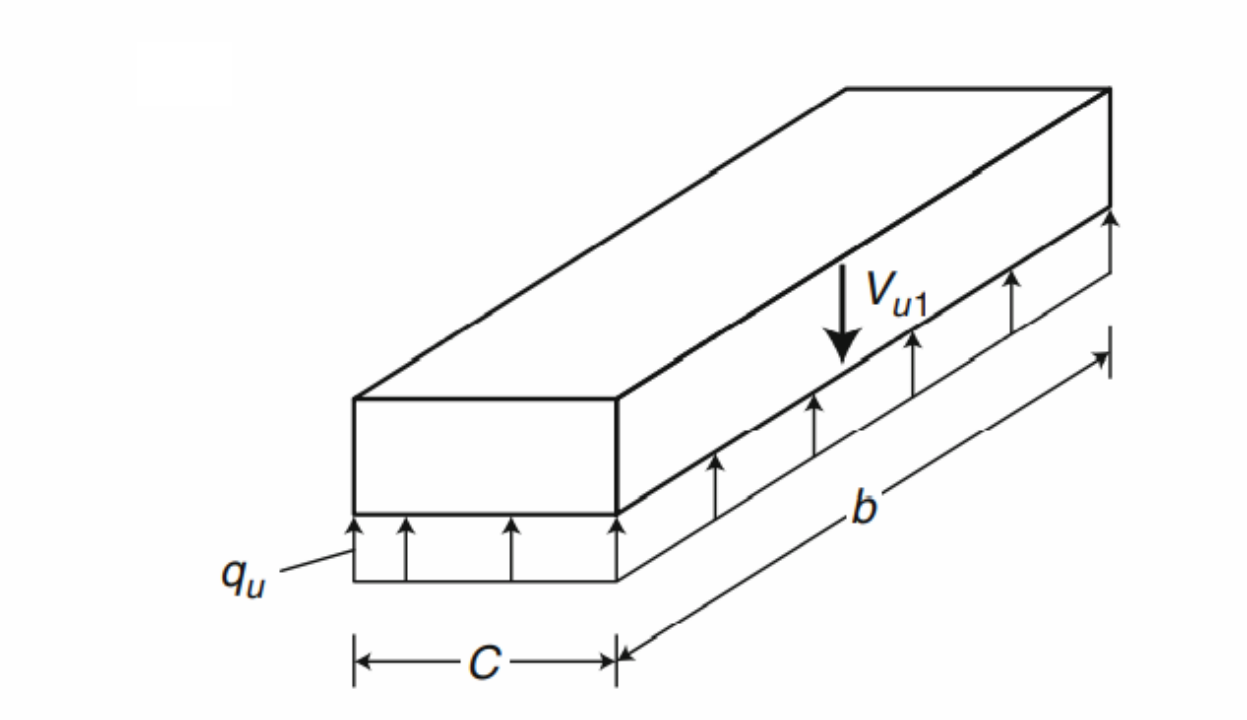
The critical section for one-way shear is at a distance, d, from the face of the column:

$$C = \frac{b-t}{2} - d$$

$$C = \frac{9.33 \times 12 - 16}{2} - 20$$

$C = 28$ in. $= 2'-4'' = 2.33$ ft

If we cut the footing at the critical section, as shown in Figure 2, the shear at this location, V_{u1}, is:



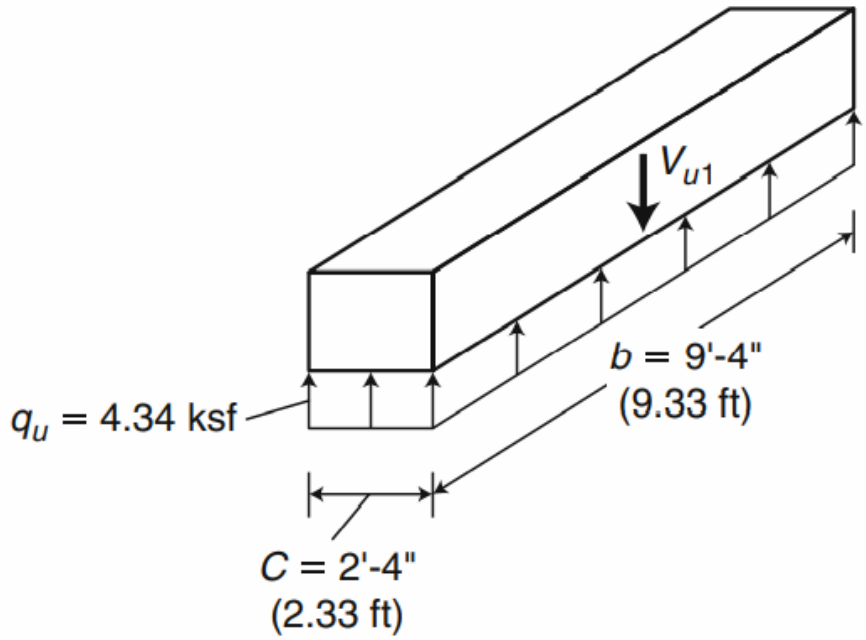


FIGURE 2: ONE WAY SHEAR IN THE SPREAD FOOTING

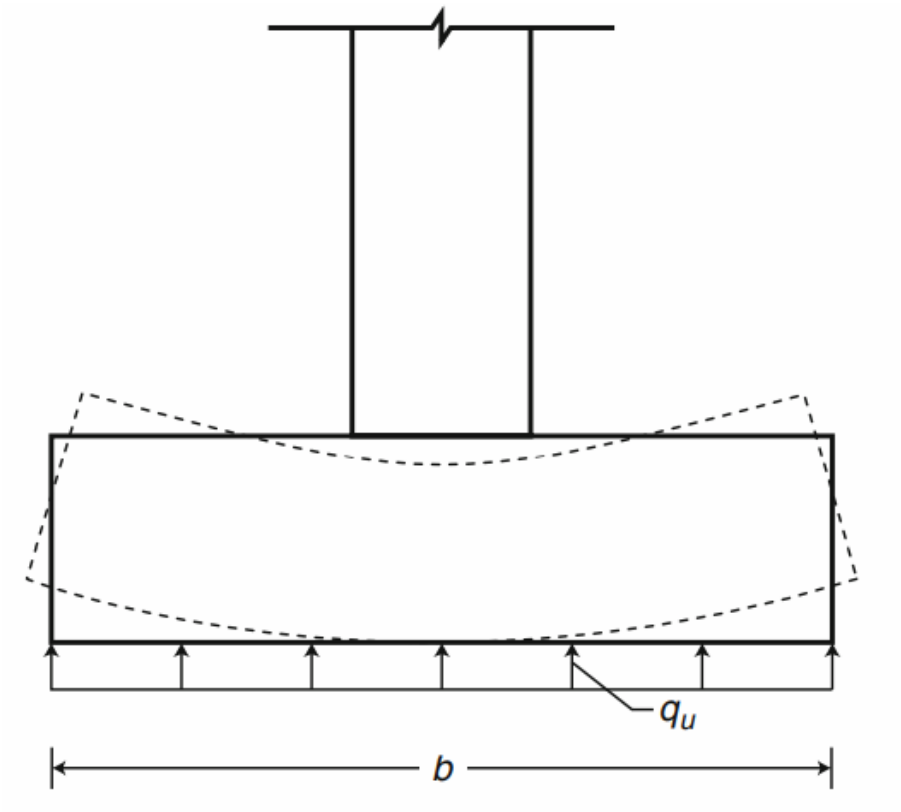
$$V_{u1} = q_u bC$$
$$V_{u1} = 4.34(9.33)(2.33)$$
$$V_{u1} = 94.5 \text{ kip}$$

The nominal one-way shear strength of concrete, V_{c1}, is:

$$V_{c1} = 2\lambda\sqrt{f_c'}bd$$
$$V_{c1} = 2(1.0)\sqrt{3{,}000}\ (9.33 \times 12)(20)/1{,}000$$
$$V_{c1} = 245.3 \text{ kip}$$
$$\phi V_{c1} = 0.75(245.3) = 184 \text{ kip} > 94.5 \text{ kip} \quad \therefore \text{ ok}$$

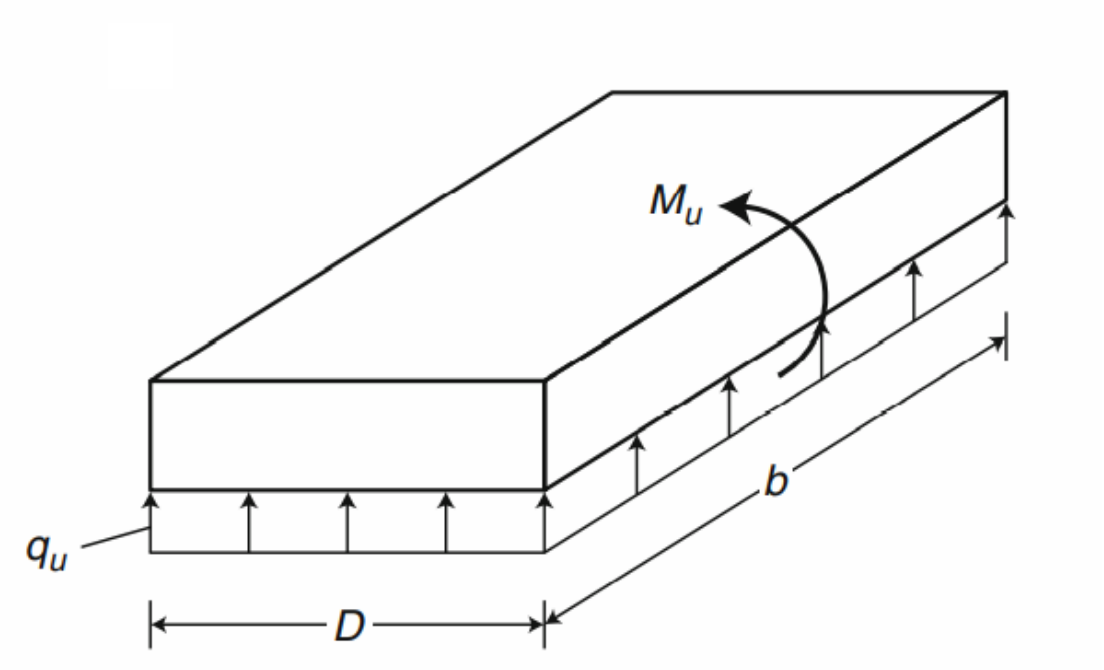
Therefore, the footing has enough capacity against the one-way shear.

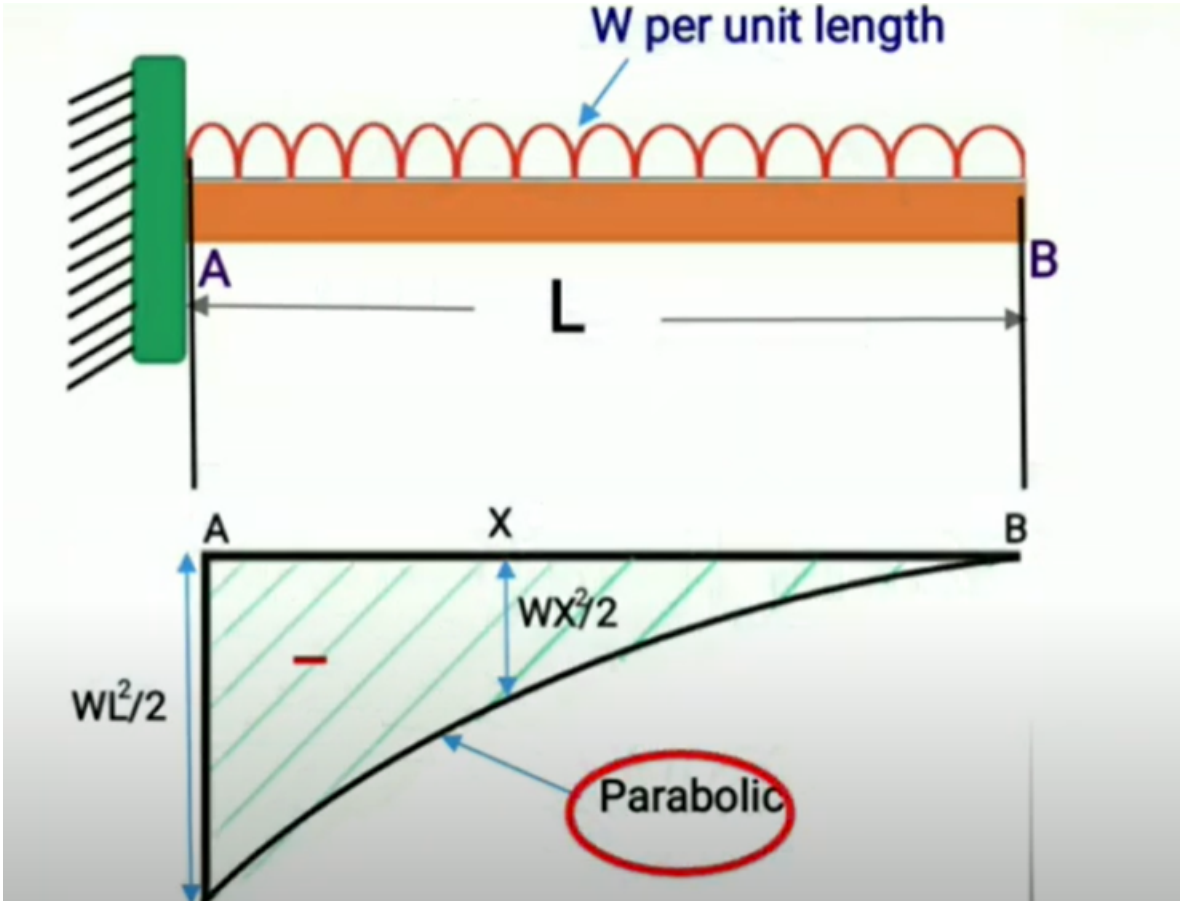
Determination of Moment in Footing at Critical Section



The footing bends in both directions like a dish when subjected to soil pressure from below. Therefore, we can consider bending of the footing from one side and find the moment at the critical section. Cutting the footing at the critical section for moment, as shown in Figure below, and setting the sum of moments to zero, we can calculate the moment at the critical section as follows:

$$M_u = q_u b(D)(D/2) = \frac{q_u b D^2}{2}$$


M_u
q_u
b
D


W per unit length
A
B
L
A
X
B
WX²/2
WL²/2
–
Parabolic

Step 4. Determine the required reinforcement.

To calculate the required reinforcement, we first calculate the bending moment at the critical section for moment (the face of the column):

$$D = \frac{b - t}{2} = \frac{9.33 \times 12 - 16}{2}$$

$$D = 48 \text{ in.} = 4.00 \text{ ft}$$

The moment at the critical section as shown in Figure 7.31 is:

$$M_u = q_u b D^2/2$$

$$M_u = 4.34(9.33)(4.0)^2/2$$

$$M_u = 324 \text{ ft-kip}$$

$$R = \frac{12{,}000 M_u}{bd^2}$$

$$R = \frac{12{,}000(324)}{(9.33 \times 12)(20)^2} = 87.0 \text{ psi}$$

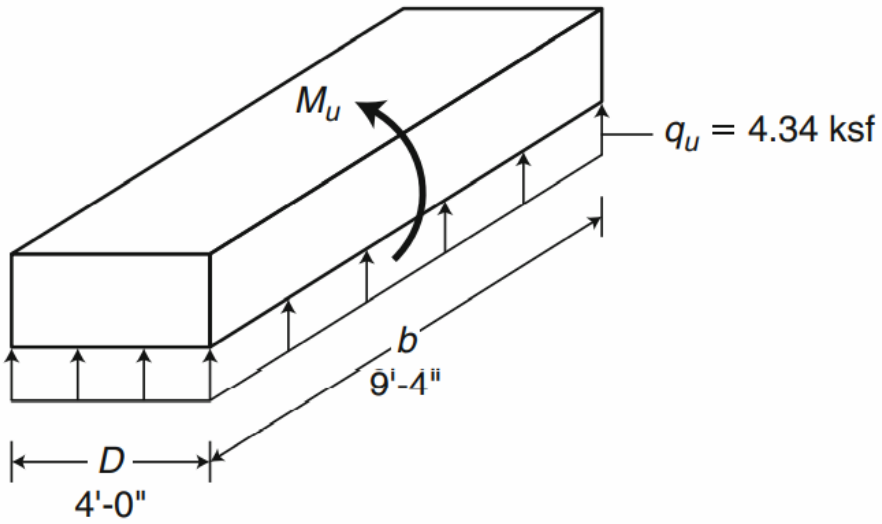


FIGURE 3: BENDING MOMENT IN THE SPREAD FOOTING

From Table A2.6a $\left(f_c' = 3{,}000 \text{ psi},\ f_y = 60{,}000 \text{ psi}\right) \rightarrow \rho = 0.0017$ (conservatively) and the required area of steel, A_s, is:

$$A_s = \rho bd$$

$$A_s = 0.0017(9.33 \times 12)(20)$$

$$A_s = 3.8 \text{ in.}^2$$

$$A_{s,\min} = 0.0018\,bh$$

$$A_{s,\min} = 0.0018(9.33 \times 12)(24)$$

$$A_{s,\min} = 4.84 \text{ in.}^2 > 3.8 \text{ in.}^2$$

Therefore, the required area of steel is:

$$A_s = 4.84 \text{ in.}^2$$

Check the bar development length by using Table A3.3 and the applicable modification factors of Table A3.1. The bar length from the critical section for moment has to be more than ℓ_d. Otherwise, the bars have to be hooked at their ends. Thus:

$$D - 3 \text{ in. (cover)} \geq \ell_d$$

Note that we can use (3.64) and the corresponding modification factors of Table A3.1 instead of using Table A3.3 to obtain the bar development length. This method usually results in a smaller required development length.

Table A2.9 $\rightarrow \therefore$Use 9 #7 bars each way ($A_s = 5.40$ in.^{2})
Check the bars development length.

From Table A3.2, because cover >0.875 in. and clear space $>$ 2(0.875 in.), condition A is applicable, and from Table A3.3:

$$\ell_d = 48 \text{ in. (\#7 bars)}$$

From Table A3.1, ℓ_d can be reduced by $\frac{A_{s,\text{required}}}{A_{s,\text{ provided}}}$, therefore :

$$\ell_d = 48 \times \frac{3.8}{5.40} = 33.8 \text{ in.}$$

The bar length measured from the critical section for moment is:

$$D - 3 \text{ in.} = 48 \text{ in.} - 3 \text{ in.} = 45 \text{ in.} > 33.8 \text{ in.} \quad \therefore \text{ ok}$$

We can use following formulae for reinforcement calculation:

$$A_s = \frac{M}{\phi f_y \left(d - \frac{a}{2}\right)}$$

$$A_{s,\min} = \frac{3\sqrt{f'_c}}{f_y} bd$$

$$A_{s,\min} = \frac{200}{f_y} bd$$

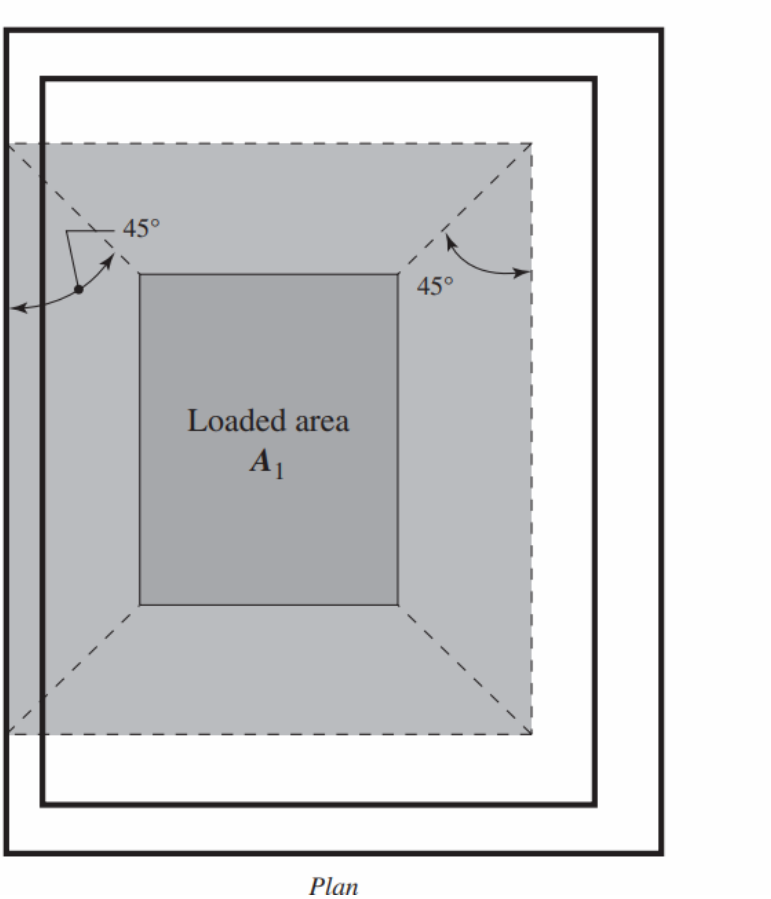
BEARING CAPACITY OF COLUMN AT BASE

The loads from the column act on the footing at the base of the column, on an area equal to the area of the column cross section. Compressive forces are transferred to the footing directly by bearing on the concrete. Forces acting on the concrete at the base of the column must not exceed the bearing strength of concrete as specified by the ACI Code, Section 22.8.3.2:

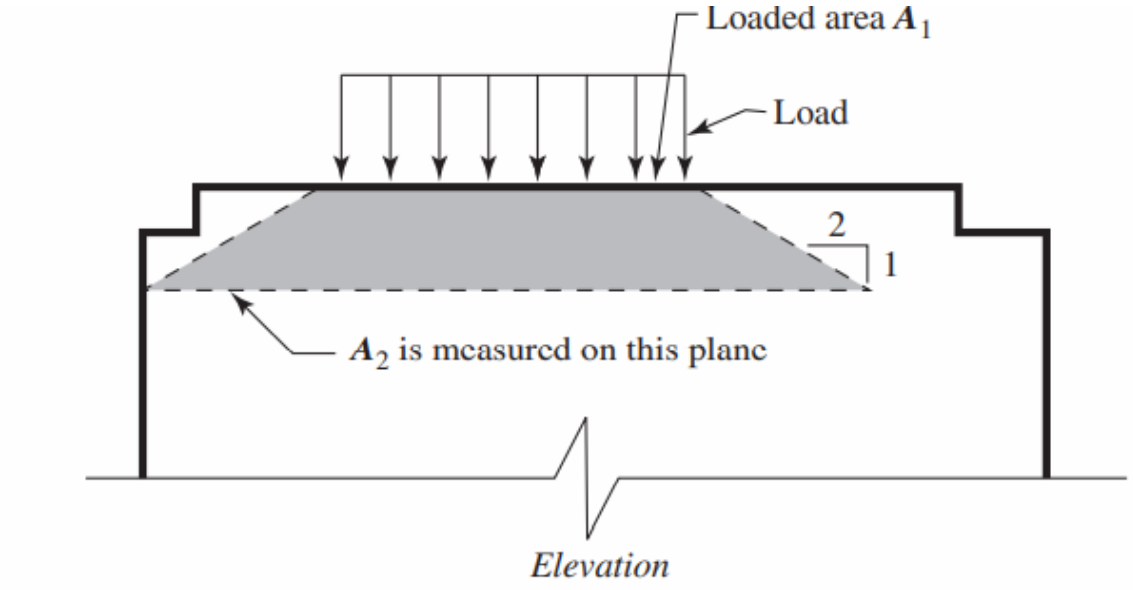
Bearing Strength N_1 = …… (1)

Where is 0.65 and A_1 is the bearing area of the column. The value of the bearing strength given in Equation (1) may be multiplied by a factor for bearing on footings when supporting surface is wider on all sides than the loaded area. Here A_2 is the area of the part of the supporting footing that is geometrically similar to and concentric with the loaded area as shown in Figure below. Because , the factor is greater than unity, indicating that the allowable bearing strength is increased because of the lateral support from the footing area surrounding the column base. The modified bearing strength is

If the factored force, P_u, is greater than either N_1 or N_2 reinforcement must be provided to transfer the excess force. This is achieved by providing dowels or extending the column bars into the footing. The excess force is $P_{ex} = P_u - N_1$ and the area of the dowel bars is $A_{sd} = (P_{ex}/f_y) \geq 0.005\ A_1$, where A_1 is the area of the column section. At least four bars should be used at the four corners of the column. If the factored force is less than either N_1 or N_2, then minimum reinforcement must be provided. The ACI Code, Section 16.3.4.1, indicates that the minimum area of the dowel reinforcement is at least $0.005A_g$ (and not less than four bars), where A_g is the gross area of the column section. The minimum reinforcement requirements apply also to the case when the factored forces are greater than N_1 and N_2. The dowel bars may be placed at the four corners of the column and extended in both the column and footing. The dowel diameter shall not exceed the diameter of the longitudinal bars in the columns by more than 0.15 in. This requirement is necessary to ensure proper action between the column and footing. The development length of the dowels must be checked to determine proper transfer of the compression force into the footing.



Plan



Elevation

AREA OF BEARING STRESS. COURTESY OF ACI CODE, SECTION 22.8.3

Step 5. Determine the required dowel bars.

$$N_1 = \phi(0.85 f_c' A_1); \quad (f_c')_{\text{column}} = 4.0 \text{ ksi}$$

$$N_1 = 0.65[0.85(4.0)(16 \times 16)] = 565.8 \text{ kip}$$

$$N_2 = \min\left\{\phi(0.85 f_c' A_g)\sqrt{\frac{A_2}{A_1}},\ 2\phi(0.85 f_c' A_1)\right\}; \quad (f_c')_{\text{footing}} = 3.0 \text{ ksi}$$

$$N_2 = \min\left\{0.65\left[0.85(3.0)(16 \times 16)\sqrt{\frac{112 \times 112}{16 \times 16}}\right],\right.$$

$$\left. \times\ 2(0.65)[0.85(3.0)(16 \times 16)]\right\}$$

$$N_2 = \min\{2{,}970,\ 848.6\} = 848.6 \text{ kip}$$

$$N_{\text{bearing}} = \min\{N_1, N_2\} = 565.8 \text{ kip}$$

$$P_u = 378 \text{ kip} < 565.8 \text{ kip}$$

$\therefore$ Use minimum area for dowels.

$$A_{sd} = 0.005\, A_g = 0.005\,(16 \times 16) = 1.28 \text{ in.}^2$$

(Table A2.9 $\rightarrow$ use 4 #6 ($A_s = 1.76$ in.^{2})

It has to be noted that for practical purposes the dowel bar size is usually selected to match the column main reinforcements, which for a 16 in. × 16 in. column it is expected to be larger than #6 bars. We have, however, selected #6 bars here for consistency and clarity in the solution.

The required development length in the footing from Table A3.6 (compression bars) for $f_c' = 3{,}000$ psi is:

$$\ell_{dc} = 17 \text{ in. } (\#6 \text{ bars})$$

Adjusting the dowel length using Table A3.5:

$$\ell_{\text{dowel}} = \ell_{dc}\left(\frac{A_{s,\text{ required}}}{A_{s,\text{ provided}}}\right) = 17\left(\frac{1.28}{1.76}\right) = 13 \text{ in.}$$

This is the length of dowel to be extended in the footing. From Table A3.6 the development length for #7 main reinforcements in the column ($f_c' = 4{,}000$ psi) is $\ell_{dc} = 17$ in.

The required lap splice length, ℓ_{sc}, for #6 dowels in the column is (Chapter 3 Section 3.4.5):

$$\ell_{sc} = 0.0005 f_y d_b \geq 12 \text{ in.}$$
$$= 0.0005(60{,}000)(0.75) = 22.5 \text{ in.} \simeq 23 \text{ in.}$$

We use the larger of 17 and 23 in., which is 23 in. for the dowel extension in the column.

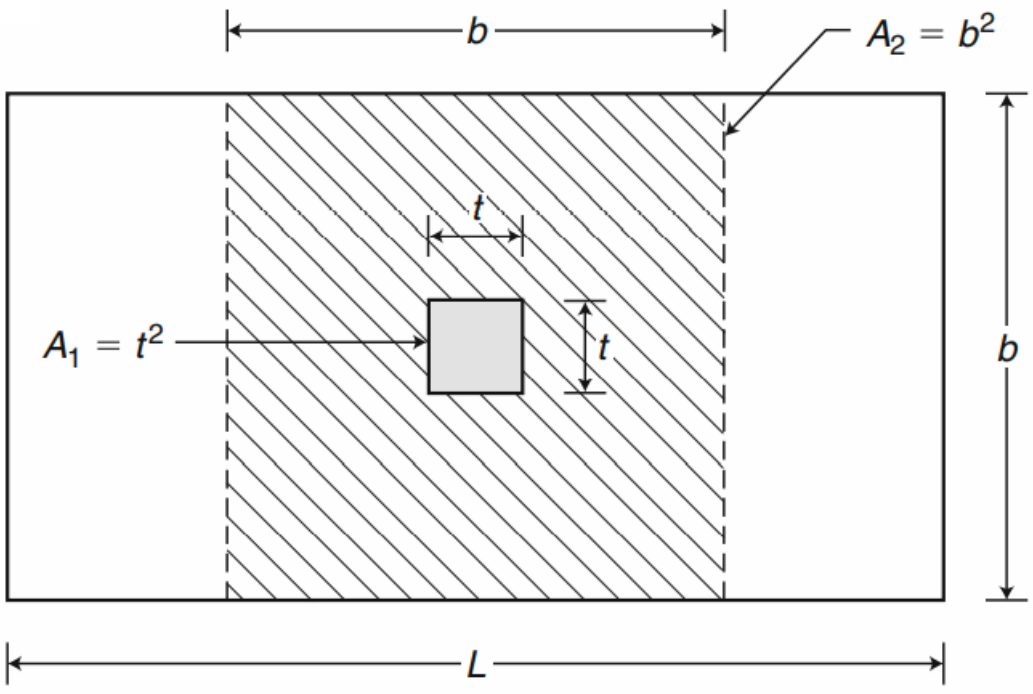
Step 5. Determine the required dowel bars.

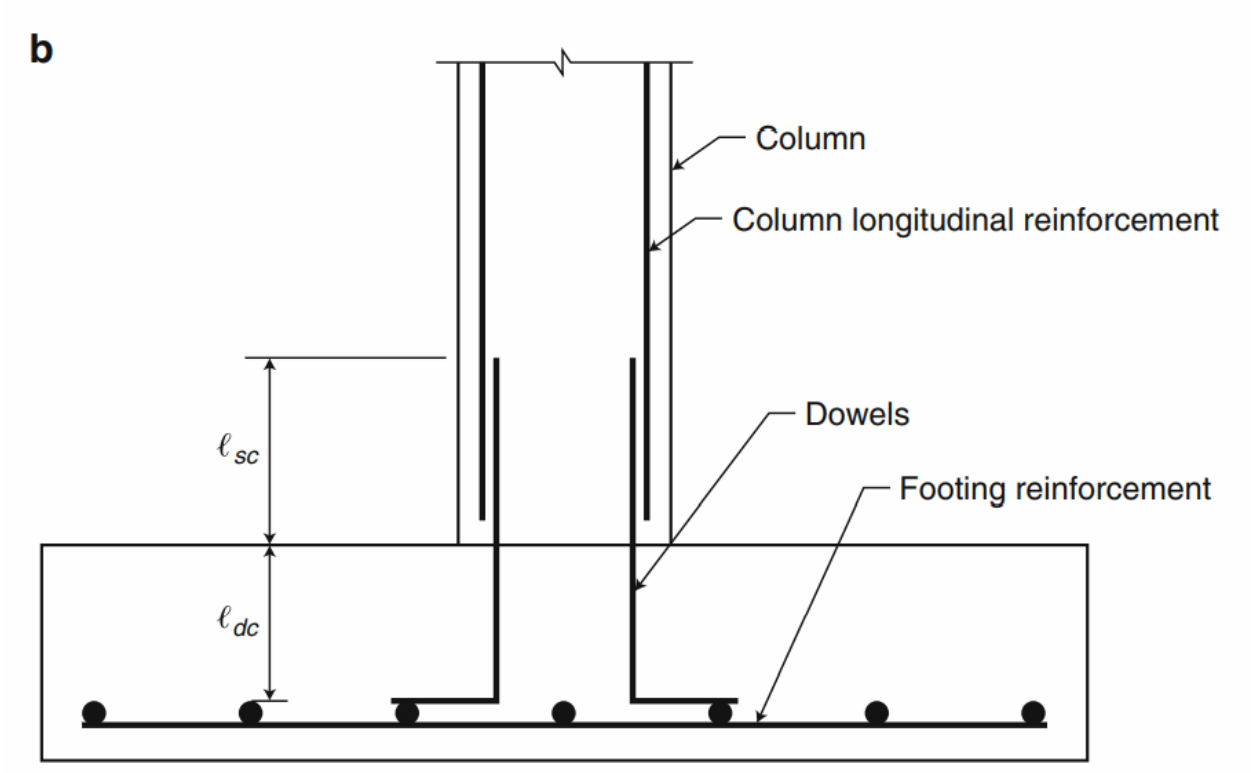
The column at its base transfers the load to the footing on an area equal to the column's cross-sectional area (A_g). This generates a bearing pressure that the footing must resist.

The bearing capacity of the concrete at the column footprint, N_{bearing}, is given by the ACI Code (Section 22.8.3.2):

$$N_1 = \phi(0.85f_c'A_1) \tag{7.32}$$

$$N_2 = \min\left\{\phi(0.85f_c'A_g)\sqrt{\frac{A_2}{A_1}},\ 2\phi(0.85f_c'A_1)\right\} \tag{7.33}$$





Dowel reinforcements: (a) bearing areas A1 and A2, and (b) dowel bars between the column and footing

$$N_{\text{bearing}} = \min\{N_1, N_2\} \tag{7.34}$$

where

ϕ	= 0.65 (ACI Code, Section 21.2.1)
A_1	= column bearing area, which for a column directly bearing on the footing is equal to A_g of the column
A_2	= area of the part of the footing that is geometrically similar to, and concentric with the column bearing area, A_1 (see Figure 7.27a)
N_1	= bearing capacity of the column
N_2	= bearing capacity of the footing

N_{bearing} = bearing capacity of the concrete at the base of the footing
f'_c = the specified strength of concrete in the column, when evaluating N_1; and the specified strength of concrete in the footing, when evaluating N_2

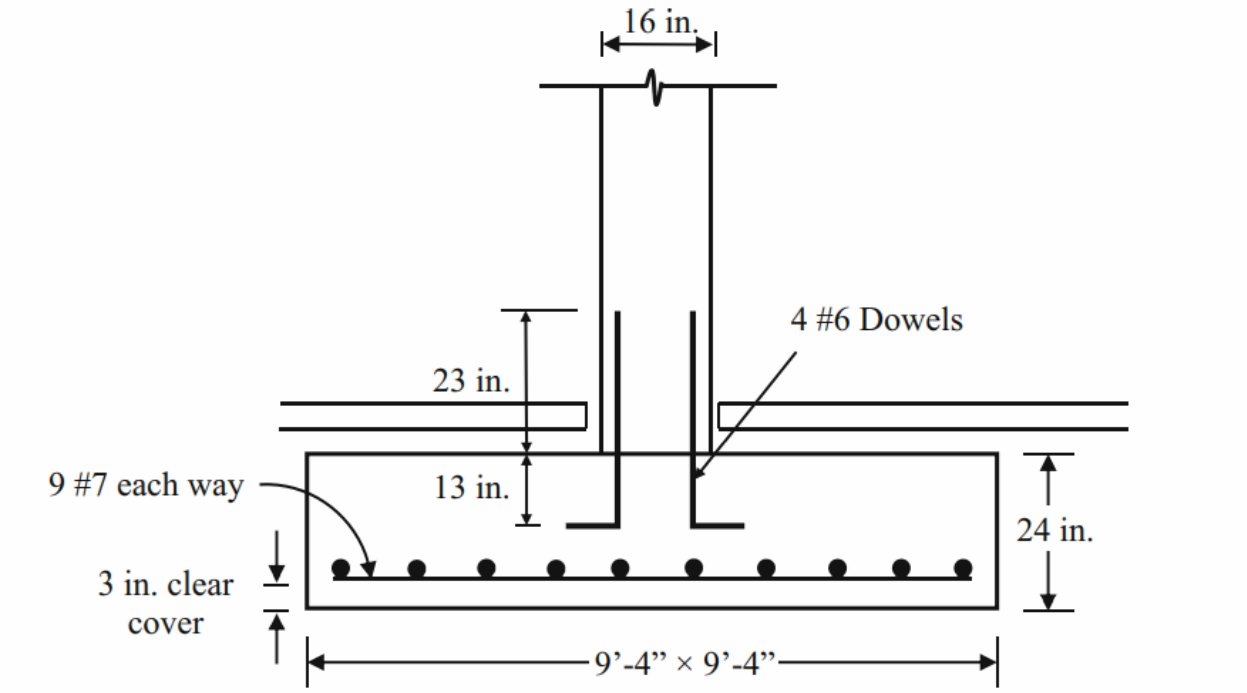
Equation (7.33) is used when the supporting surface is wider on all sides than the loaded area.

Because the compressive strength of concrete in columns is usually larger than it is in footings, we must compute both $\boldsymbol{N}_1$ and $\boldsymbol{N}_2$ to determine N_{bearing}. Dowel bars must resist the difference between the load transferred from the column to the footing, P_u, and the bearing capacity of concrete, N_{bearing}. The required area of these bars, A_{sd}, is calculated as follows:

$$P_d = P_u - N_{\text{bearing}} \tag{7.35}$$

$$A_{sd} = \max\left\{\frac{P_d}{f_y},\ 0.005\,A_g\right\} \tag{7.36}$$

The ACI Code, Section 16.3.4.1 requires a minimum amount of dowel reinforcement equal to $0.005A_g$ (A_g is the gross area of column) to transfer the loads from the column to the footing. This is in the form of a minimum of four bars placed at the corners of the column. A minimum development length for the dowels in compression equal to ℓ_{dc} is required. This minimum length has to be provided from the column bearing area extending into the column and the footing, as shown in Figure 7.27b. The dowels are commonly hooked and tied to the footing main reinforcements for ease of construction. The dowels have to be lap spliced in compression to the column reinforcement based on requirements given in Chapter 3. The length of the dowel in the column is the larger of the compression lap splice for the dowel bars (ℓ_{sc}), as given in Section 3.4.5 of Chapter 3, and the development length for the compression reinforcements in the column (ℓ_{dc}). Use Table A3.6 to obtain the development length for compression bars, ℓ_{dc}, and Table A3.5 to obtain the applicable modification factors.


16 in.
4 #6 Dowels
23 in.
13 in.
9 #7 each way
24 in.
3 in. clear
cover
9'-4" × 9'-4"