EWCE343: Foundation Engineering

L3/T2: 3.00 Credit Hours 3
Hours per Week

Dr. Md. Zoynul Abedin/
Brig. Md. Gazi Ferooz Rahman

Department of Civil Engineering,
MIST, Dhaka

EWCE343: Foundation Engineering



Chapters to be Studied

- Types of Foundations
- Bearing Capacity of Shallow and Deep Foundations
- Subsoil Investigation Techniques
- Settlement and Distortion of Foundations
- Design and Construction of Footings, Rafts and Piles
- Lateral Earth Pressure
- Slope Stability

EWCE343: Foundation Engineering



Chapters to be Studied

- Lateral Earth Pressure
- Subsoil Investigation
- Pile Foundation
- Slope Stability

Text & Reference Books

(i) Foundation Engineering – Peck, Hanson & Thornburn
(ii) Principles of Foundation Engineering – B.M. Das
(iii) Foundations Design: Principles and Practices - Coduto

Lateral Earth Pressure (Topics to be Covered)

- Introduction to Plastic Equilibrium in Soils
- Types of Lateral Earth Pressure & Their Definitions
- Earth Pressure at Rest
- Active and Passive Earth Pressures by Rankine and Coulomb's Theories
- Culmann's Graphical Construction for Active and Passive Earth Pressures

1.0 Introduction to Plastic Equilibrium in Soils

A mass of soil is said to be in a state of plastic equilibrium if failure is incipient or imminent at all points within the mass. This is commonly referred to as general state of plastic equilibrium and occurs in rare instances such as when tectonic forces act. Usually, however, failure may be imminent only in a small portion of the mass such as that produced by the yielding of a retaining structure in the soil mass adjacent to it. Such a situation is referred to as local state of plastic equilibrium.

1.0 Introduction to Plastic Equilibrium in Soils

The state of stress in backfill soil of a retaining structure mainly depends on the movement of structure with reference to the backfill. The backfill material is said to be in a state of elastic equilibrium when the stress involved and the corresponding strain are within **elastic limit**. This generally occurs for no or very less movement of the wall.

Further increase in stress develops shear stresses at some point in the body, reaching the shear strength of soil. Subsequent increase in stress causes a substantial increase in strain producing a condition known as **plastic flow**. The soil mass prior to onset of the plastic flow condition is said to be in a state of plastic equilibrium.

1.0 Introduction to Plastic Equilibrium in Soils

Plastic collapse occurs after the state of plastic equilibrium has been reached and the load or stress at this condition is referred to as collapse load. The determination of collapse load, adopting plasticity theory, is rather complex. However, plasticity theory provides a simplified analysis.

Rankine was the first to investigate the stress conditions associated with the states of plastic equilibrium in a semi infinite mass of homogeneous, elastic and isotropic soil mass under the influence of gravity or self weight alone.

2.0 Lateral Earth Pressure

The pressure exerted by the soil against an engineering structure or acting on a surface of surrounding soil mass is called earth pressure. Strictly speaking the two terms used in geotechnical engineering related to pressure namely pressure and stress are nearly synonymous. However, pressure may be defined as the contact force per unit area between a structure and soil whereas stress is the force per unit area within the soil mass.

One of the first steps in designing any structural member is to determine the magnitude and direction of applied load. For retaining structures, the primary applied load is the resultant of the lateral earth pressure the soil exerts on the wall.

2.0 Lateral Earth Pressure

Lateral earth pressure due to soil depends on:

- 1. Physical characteristics of soil
  - Cohesionless soil
  - Cohesive soil
  - c-φ soil
- 2. Deformation characteristics of structure
  - No deformation
  - Structure away from soil (lateral expansion)
  - Structure towards soil (lateral contraction)

2.0 Lateral Earth Pressure

Depending on the deformation characteristics of structure, lateral earth pressure may have varying magnitude; as such lateral earth pressure can be classified as

- Earth pressure at rest (no deformation)
- Active earth pressure (lateral expansion)
- Passive earth pressure (Lateral contraction)

When a soil maintains the same lateral dimension regardless of the vertical pressure, the lateral pressure exerted by the soil on the structure or the lateral stress induced within the soil mass is known as earth pressure at rest. The ratio of lateral effective stress, σ_3, to vertical effective stress, σ_1, at this condition is termed as coefficient of earth pressure at rest and designated by K_o.

2.0 Lateral Earth Pressure

When a soil expands laterally due to stresses, the maximum lateral stress at which the soil fails due to shear is called active earth pressure. The ratio of lateral stress to vertical stress at this condition is termed as coefficient of active earth pressure and designated by K_a.

When a soil contracts laterally due to stresses, the minimum lateral stress at which it fails due to shear is called passive earth pressure. The ratio of lateral stress to vertical stress at the condition is termed as coefficient of passive earth pressure and designated by K_p.

2.0 Lateral Earth Pressure

As shown in Fig. 1, generally the following relationship can be observed: $K_p > K_o > K_a$.

It is to be noted that to achieve either passive or active critical stages requires a sufficient amount of wall movement. The passive stage requires a larger wall movement than the active stage. **Typical wall movement (δ) to the wall height (H) ratio to achieve these critical stages is active approximately 0.01 for the passive case and 0.001 for the case in loose sandy soil mass.** The wall movement requirements for active and passive conditions for different backfill soils are presented in Table 1.

2.0 Lateral Earth Pressure

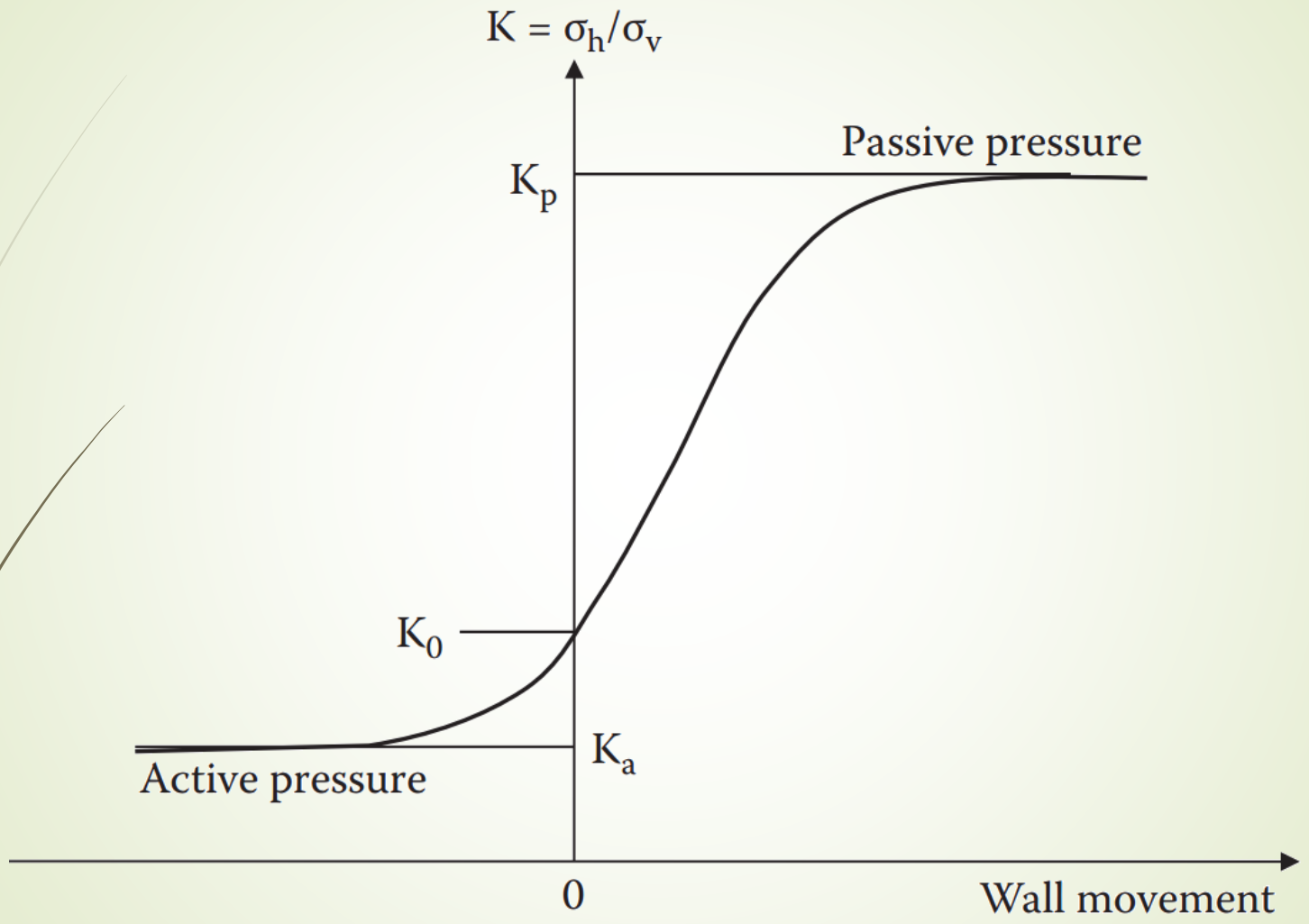


Fig. 1 Coefficient of lateral earth pressure K versus wall movement.

2.0 Lateral Earth Pressure

Table 1: Wall movement required to reach active & passive conditions

Soil type	Active condition	Passive condition
Dense cohesionless	0.0005H	0.0020H
Loose cohesionless	0.002H	0.0060H
Stiff cohesive	0.010H	0.020H
Soft cohesive	0.020H	0.040H
H = Height of the wall; Cohesionless include sands and gravels; Cohesive soils are those with significant clay contents.		

2.0 Lateral Earth Pressure

In general the ratio of lateral stress to vertical stress is known as coefficient of earth pressure and designated as K. The coefficient of lateral earth pressure influences many aspects of engineering behaviour of soil. As K increases,

- Bearing capacity increases
- Slope stability decreases
- Fracture in earth dam becomes less likely
- Lateral pressure acting on retaining walls increases
- Skin friction resistance on deep foundations increases
- Foundation settlements becomes smaller
- Seismic liquefaction becomes less likely
- Soil improvement becomes more difficult

2.0 Lateral Earth Pressure

In case of rigid and unyielding retaining wall, no lateral strain occurs in the soil. The corresponding lateral stress or pressure is the earth pressure at rest. The vertical and lateral stresses, σ_v and σ_h respectively, at this condition, can related as

$$\sigma_h = K_o \sigma_v = K_o \gamma h \qquad (1)$$

A theoretical expression of K_o for an isotropic, homogeneous and elastic material can be obtained from the condition of zero lateral strain. The lateral strain, ε_h, can be expressed as

$$\varepsilon_h = E[\sigma_h - \mu(\sigma_v + \sigma_h)] \qquad (2)$$

2.0 Lateral Earth Pressure

If $\varepsilon_h = 0$, then, $E[\sigma_h - \mu(\sigma_v + \sigma_h)] = 0$

$$\Rightarrow \quad \sigma_h = \frac{\mu}{(1-\mu)}\sigma_v = K_o\sigma_v \qquad (3)$$

Where, μ is the Poisson's ratio of soil. It is not easy to determine the Poisson's ratio of insitu soil. As such a less satisfactory but workable method of measuring K_o is usually done in the laboratory using triaxial apparatus. However, it also suffers from sample disturbance, stress history and aging effects. It is conventional to use empirical methods to estimate the coefficient of earth pressure at rest. Jacky (1944) developed the following relationship for normally consolidated soils.

$$K_o = 1 - \sin\varphi' \qquad (4)$$

2.0 Lateral Earth Pressure

Where, φ' is the drained friction angle of soil. The magnitude of K_o in an overconsolidated soil is higher than that of a normally consolidated soil and somewhat more difficult to predict. It appears to be a primary function of friction angle and overconsolidation ratio (OCR). **Mayne and Kulhawy (1982) developed the following empirical relationship;**

$$K_o = (1 - \sin\varphi')OCR^{\sin\varphi'} \quad (5)$$

The plasticity index, I_p(%) has a secondary effect on K_o and **Alpan (1967) suggested the following relationship for normally consolidated clays.**

$$K_{o(NCC)} = 0.19 + 0.233\log(I_p) \quad (6)$$

2.0 Lateral Earth Pressure

For overconsolidated clay (OCC), K_o can be estimated as

$$K_{o(OCC)} = K_{o(NCC)}\sqrt{OCR} \qquad (6)$$

The value of K_o computed using the above relationships considers a level ground surface. If the ground surface behind a retaining wall is inclined upwards at an angle of β from the horizontal, a correction factor of $(1 + 0.5\tan\beta)^2$ may be used as multiplier to consider the surface inclination.

The field value of K_o depends on many factors like –

- The manner of formation of soil deposit
- The density index (angle of shearing resistance) for sand
- The stress history for clays

2.0 Lateral Earth Pressure

The value of K_o may vary between 0.2 to 6; typical values are in the range given in Table 1.

Table 1. Typical Values of Coefficient of Earth Pressure at Rest (K_o)

Type of soil	Typical value
Loose sand ($e = 0.8$): Dry	0.64
Loose sand ($e = 0.8$): Saturated	0.46
Dense sand ($e = 0.6$): Dry	0.49
Dense sand ($e = 0.6$): Saturated	0.36
Sand (compacted in layers)	0.80
Soft clay ($I_p = 30$)	0.60
Hard clay ($I_p = 9$)	0.42
Undisturbed Silty clay ($I_p = 45$)	0.57

2.0 Lateral Earth Pressure

Example: A 2.5 m tall basement wall retains a normally consolidated clay soil that has the following properties: $\varphi' = 35^o$, $\gamma = 18\,kN/m^3$, $OCR = 2$. The ground surface inclines at an angle of 15^o with the horizontal. Compute the force that acts between the wall and the soil. Also calculate the plasticity index of the soil.

4.0 Earth Pressure Theories

The magnitude of lateral earth pressure, especially the active and passive pressures, is evaluated by the application of so called lateral earth pressure theories. The problem of determining the lateral earth pressure against retaining walls is the oldest in the field of engineering.

Several investigators proposed theories, but those given by **Coulomb and Rankine** have been considered reliable in spite of some limitations. Their theories are usually considered as the **Classical Earth Pressure Theories.**

4.1 Rankine's Earth Pressure Theory

Rankine developed his theory considering the backfill consists of dry cohesionless soil. His theory was later extended by Resal (1910) and Bell (1915) to be applicable to cohesive soils. Following assumptions are made in Rankine's theory:

- Soil mass is semi infinite, homogeneous, dry and cohesionless
- Ground surface is a horizontal or inclined plane
- The face of the wall in contact with the backfill is vertical and smooth
- Conjugate relationship exists between vertical and lateral pressures

Rankine considered the equilibrium of a soil element in deriving the formula of lateral earth pressure.

4.1.1 Active Earth Pressure due to Cohesionless Soil: Rankine's Theory

The Mohr's circle of stresses may be used to determine the lateral thrust on retaining walls from backfill soil for which the shear strength may be expressed by $\tau = \sigma \tan \varphi$. The value of φ designates effective angle of internal friction.

Let us consider a vertical and smooth retaining wall, and a cohesionless backfill with a horizontal surface. At a given depth h, on an element, the vertical stress σ_1 is equal to γh. The lateral stress is designated as σ_3. Both of these stresses are principal stresses and the corresponding Mohr's circle is shown in Fig. 2. It is assumed that the wall has yielded sufficiently to satisfy active earth pressure condition, such that $\sigma_3 = K_a \gamma h = K_a \sigma_1$.

4.1.1 Active Earth Pressure due to Cohesionless Soil: Rankine's Theory

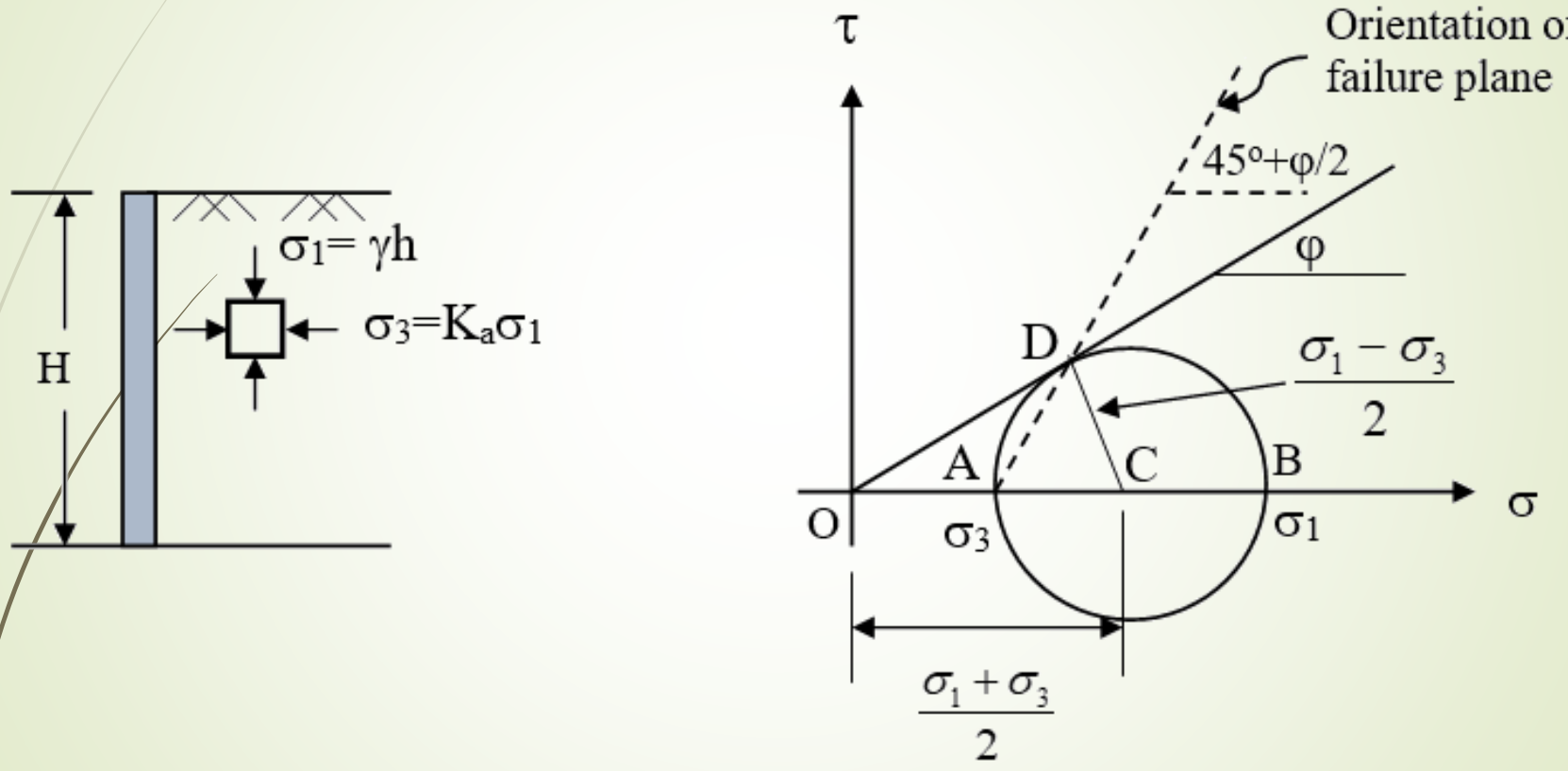


Fig. 2 Soil Element and Mohr's circle for a cohesionless soil in active condition.

4.1.1 Active Earth Pressure due to Cohesionless Soil: Rankine's Theory

Now, $K_a = \frac{\sigma_3}{\sigma_1}$

In Mohr's circle diagram, $\frac{\sigma_3}{\sigma_1} = \frac{OA}{OB} = \frac{(OC-AC)}{(OC+BC)}$

Now, AC and BC are the radius of the Mohr's circle and that are equal to CD. Replacing AC and BC by CD, we get,

$$\frac{\sigma_3}{\sigma_1} = \frac{OA}{OB} = \frac{(OC-AC)}{(OC+BC)} = \frac{(OC-CD)}{(OC+CD)}$$

Diving both numerator and denumerator by OC, we get,

$$\frac{\sigma_3}{\sigma_1} = \frac{OA}{OB} = \frac{(OC-AC)}{(OC+BC)} = \frac{(OC-CD)}{(OC+CD)} = \frac{\left(1-\frac{CD}{OC}\right)}{\left(1+\frac{CD}{OC}\right)}$$

4.1.1 Active Earth Pressure due to Cohesionless Soil: Rankine's Theory

Now, in right angle triangle ΔOCD, $\frac{CD}{OC} = \sin\varphi$

Therefore,

$$K_a = \frac{\sigma_3}{\sigma_1} = \frac{\left(1-\frac{CD}{OC}\right)}{\left(1+\frac{CD}{OC}\right)} = \frac{1-\sin\varphi}{1+\sin\varphi} = \tan^2\left(45-\frac{\varphi}{2}\right) \qquad (7)$$

4.1.2 Passive Earth Pressure due to Cohesionless Soil: Rankine's Theory

Mohr's circle of stresses may also be used to determine the lateral thrust on retaining walls in passive condition. As usual, let us consider a vertical and smooth retaining wall, and a cohesionless backfill with a horizontal surface.

At a given depth h on an element, the vertical stress is equal to γh. As passive condition prevails, shear failure of soil element will occur because of lateral contraction. That is, the lateral pressure has to be increased such that the lateral stress is considered as the major principal stress and designated as $\sigma_1$and equals to $K_p \gamma h = K_p \sigma_3$. The corresponding Mohr's circle is shown in Fig. 3.

4.1.2 Passive Earth Pressure due to Cohesionless Soil: Rankine's Theory

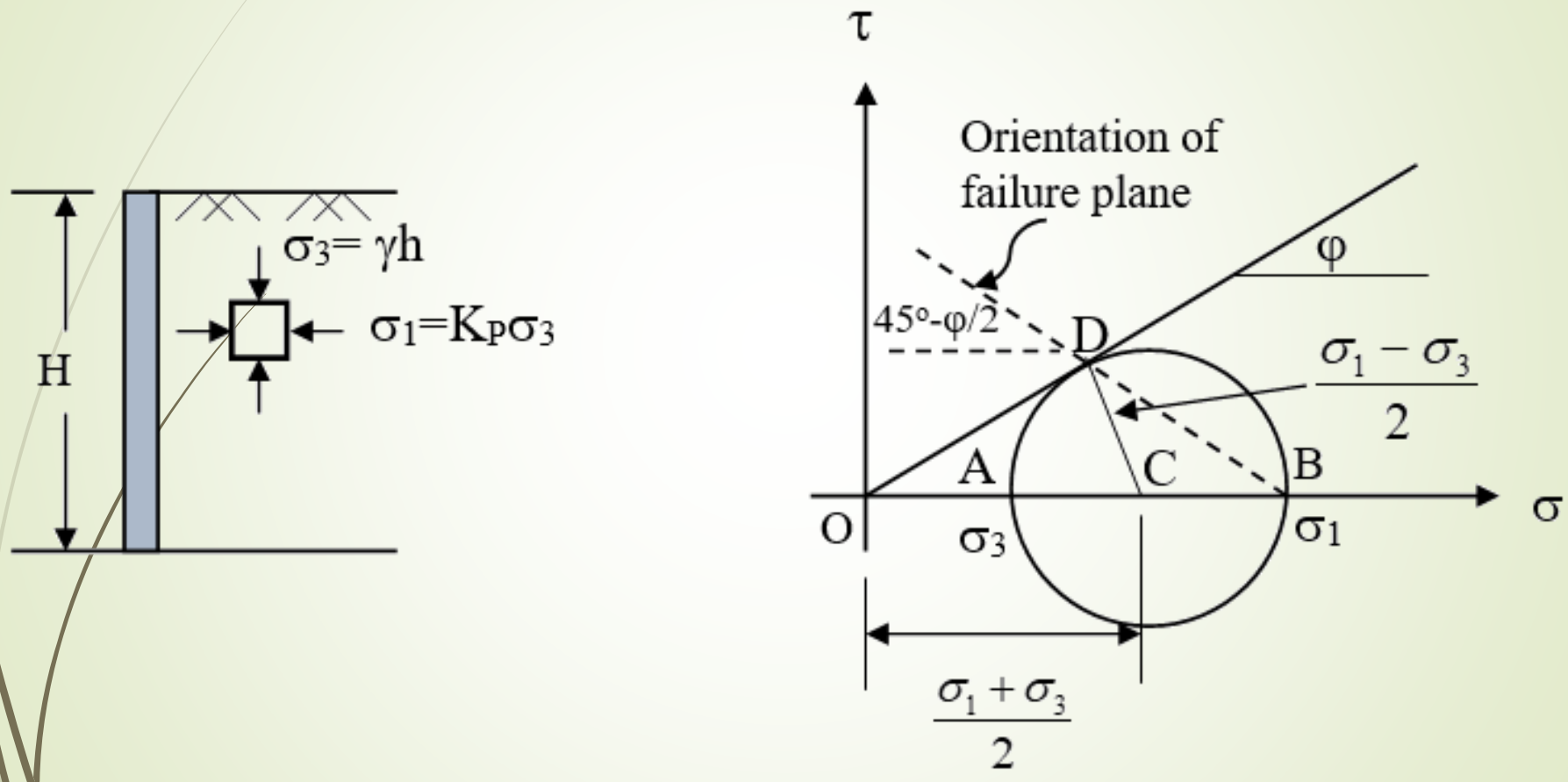


Fig. 3 Soil element and Mohr's circle for a cohesionless soil in passive condition.

4.1.2 Passive Earth Pressure due to Cohesionless Soil: Rankine's Theory

Now, $K_p = \frac{\sigma_1}{\sigma_3}$

In Mohr's circle diagram, $\frac{\sigma_1}{\sigma_3} = \frac{OB}{OA} = \frac{(OC+BC)}{(OC-AC)}$

Now, AC and BC are the radius of the Mohr's circle and both are equal to CD. Replacing AC and BC by CD, we get,

$$\frac{\sigma_1}{\sigma_3} = \frac{OB}{OA} = \frac{(OC+BC)}{(OC-AC)} = \frac{(OC+CD)}{(OC-CD)}$$

Diving both numerator and denumerator by OC, we get,

$$\frac{\sigma_1}{\sigma_3} = \frac{OB}{OA} = \frac{(OC+BC)}{(OC-AC)} = \frac{(OC+CD)}{(OC-CD)} = \frac{\left(1+\frac{CD}{OC}\right)}{\left(1-\frac{CD}{OC}\right)}$$

4.1.2 Passive Earth Pressure due to Cohesionless Soil: Rankine's Theory

Now, in right angle triangle ΔOCD, $\frac{CD}{OC} = \sin\varphi$

Therefore,

$$K_p = \frac{\sigma_1}{\sigma_3} = \frac{\left(1+\frac{CD}{OC}\right)}{\left(1-\frac{CD}{OC}\right)} = \frac{1+\sin\varphi}{1-\sin\varphi} = \tan^2\left(45+\frac{\varphi}{2}\right) \qquad (8)$$

4.1.3 Failure Planes in Active and Passive Conditions

The failure planes for active and passive conditions are shown in Figs. 2 and 3. They are presented in Fig. 4.

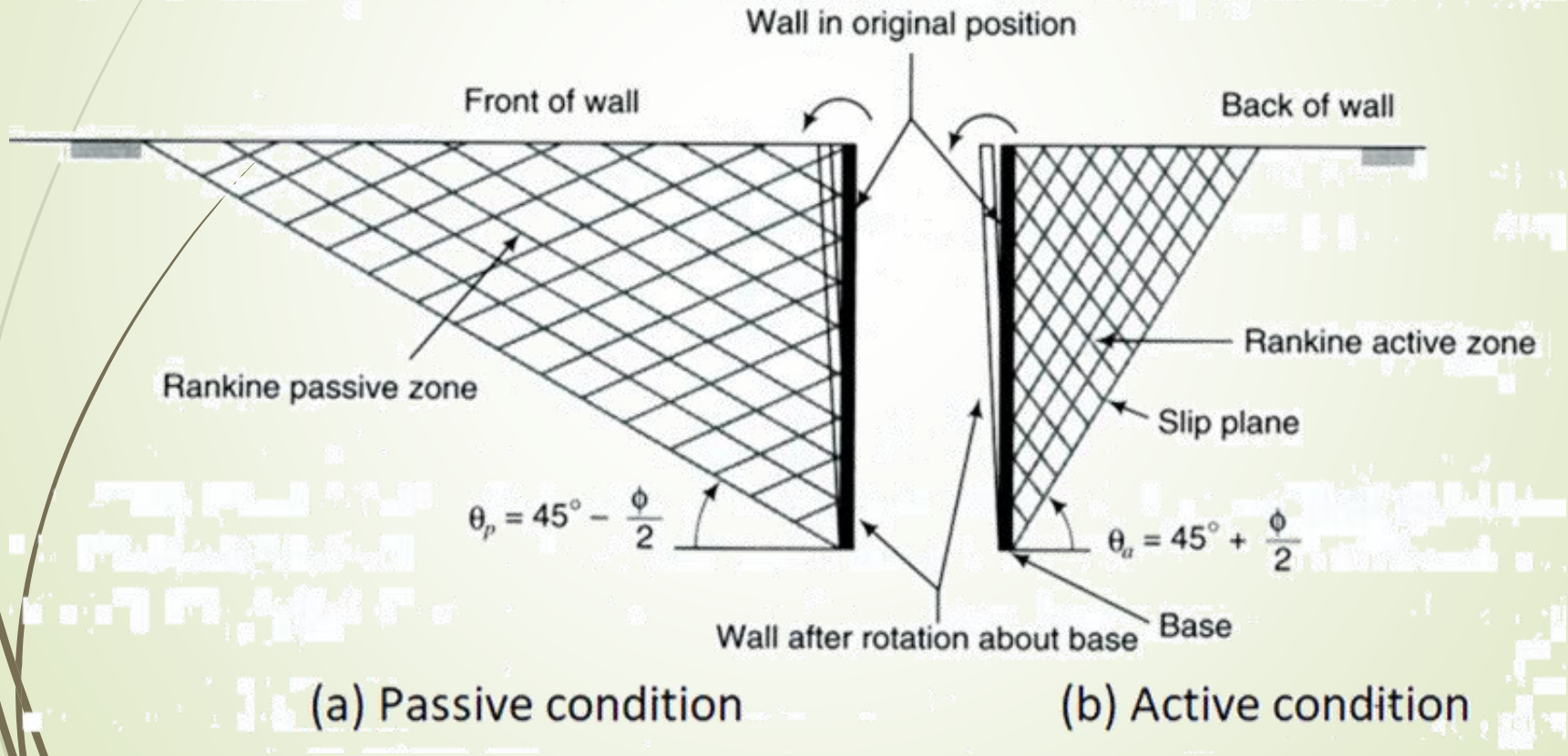


Fig. 4 Failure planes in active and passive conditions.

4.1.3 Failure Planes in Active and Passive Conditions

The failure planes for active and passive conditions are shown in Figs. 2 and 3. They are presented in Fig. 4.

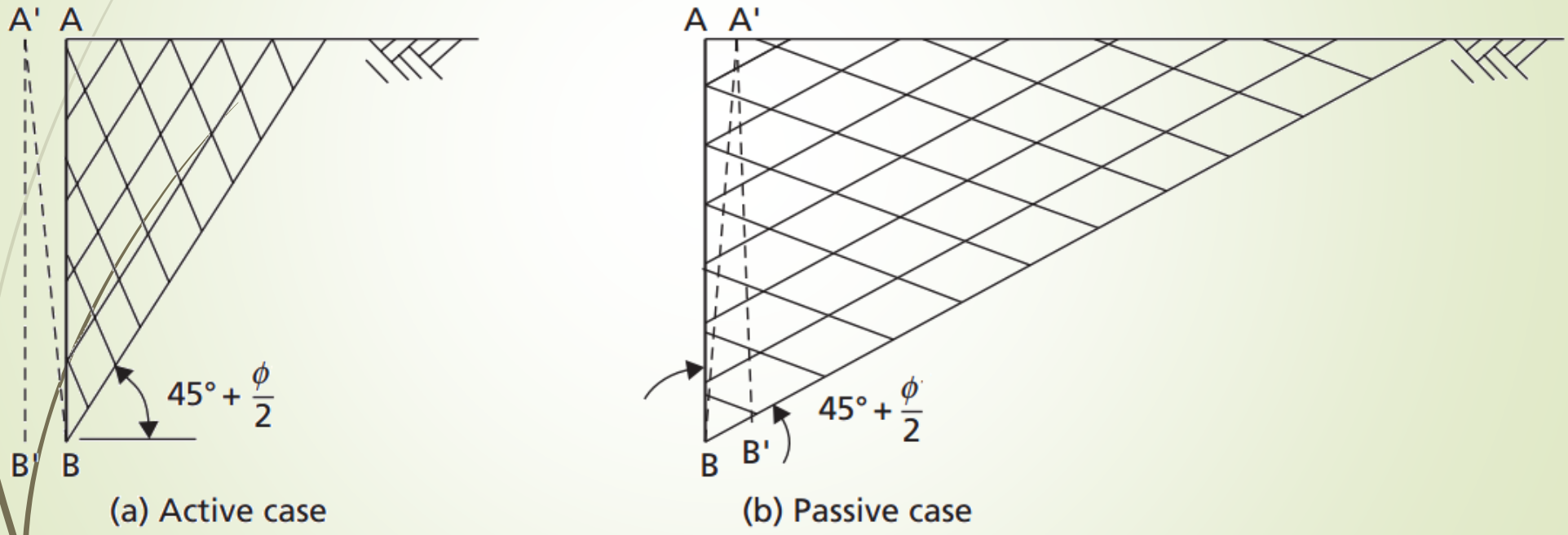


Fig. 4 Failure planes in active and passive conditions.

4.1.4 Active Earth Pressure for Partially Cohesive (c-φ) Soil: Rankine's Theory

The similar concept of Mohr's circle of stresses may be used to determine the lateral thrust on retaining walls from backfill soil for which the shear strength may be expressed by $\tau = c + \sigma \tan \varphi$. c and φ designates cohesion and angle of internal friction of the soil respectively.

A vertical and smooth retaining wall with a $c - \varphi$ backfill having a horizontal surface is assumed.

At a given depth h on an element, the vertical stress σ_1 is equal to γh. The lateral stress is designated by σ_3. Both of these are principal stresses and the corresponding Mohr's circle is shown in Fig. 5. It is also assumed that the wall has yielded sufficiently to satisfy active earth pressure condition, such that $\sigma_3 = K_a \gamma h$.

4.1.4 Active Earth Pressure for Partially Cohesive (c-φ) Soil: Rankine's Theory

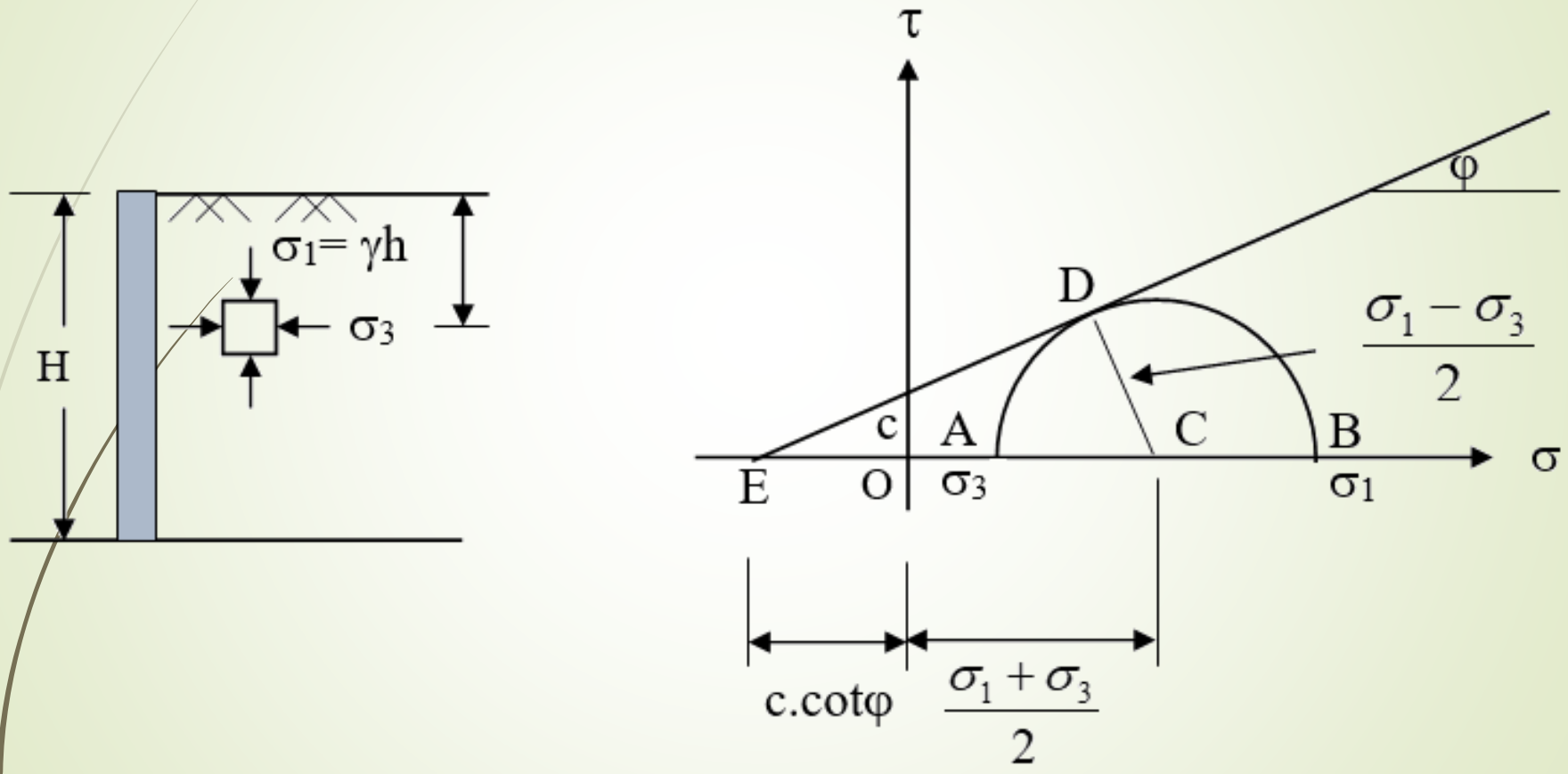


Fig. 5 Soil element and Mohr's circle for a $c - \varphi$ soil.

4.1.4 Active Earth Pressure for Partially Cohesive (c-φ) Soil: Rankine's Theory

Referring to Fig. 3,

$$\sin\varphi = \frac{CD}{EC} = \frac{CD}{EO+OC} = \frac{\frac{\sigma_1-\sigma_3}{2}}{c\cot\varphi+\frac{\sigma_1+\sigma_3}{2}}$$

$$\Rightarrow \quad (\sigma_1+\sigma_2)\sin\varphi + 2c\cot\varphi\sin\varphi = \sigma_1-\sigma_2$$

$$\Rightarrow \quad (1-\sin\varphi)\sigma_1 + (1+\sin\varphi)\sigma_3 = 2c\cos\varphi$$

$$\Rightarrow \quad \sigma_3 = \left(\frac{1-\sin\varphi}{1+\sin\varphi}\right)\sigma_1 + \frac{2c\cos\varphi}{1+\sin\varphi}$$

Now, $\frac{\cos\varphi}{1+\sin\varphi} = \frac{\sqrt{\cos^2\varphi}}{1+\sin\varphi} = \frac{\sqrt{1-\sin^2\varphi}}{1+\sin\varphi} = \frac{\sqrt{(1-\sin\varphi)(1+\sin\varphi)}}{\sqrt{(1+\sin\varphi)^2}}$

$$\Rightarrow \quad = \sqrt{\frac{(1-\sin\varphi)}{(1+\sin\varphi)}} = \sqrt{K_a}$$

$$\Rightarrow \quad \sigma_3 = p_{ac} = K_a\gamma h - 2c\sqrt{K_a} \qquad (9)$$

4.1.4 Active Earth Pressure for Partially Cohesive (c-φ) Soil: Rankine's Theory

Depth of Tension Crack and Theoretical Unsupported Height

The active pressure behind a retaining wall as expressed by Eq. (9) can be plotted as Fig. 6.

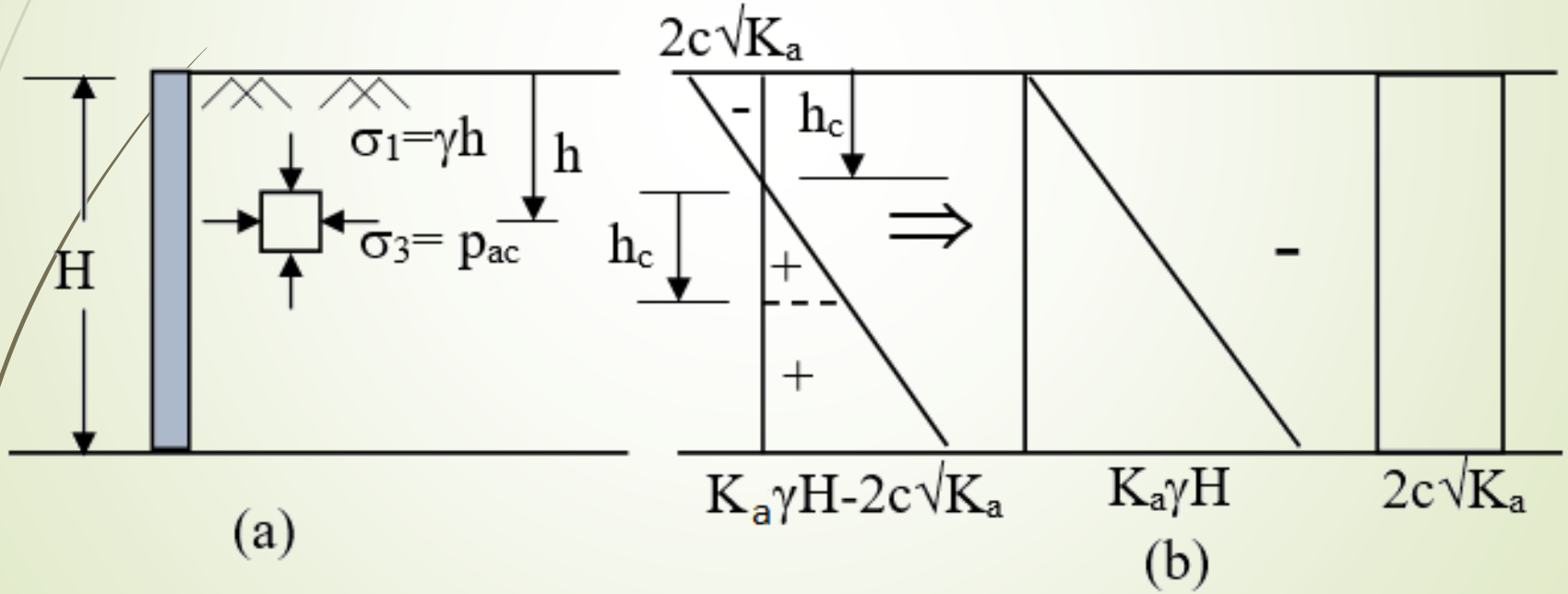


Fig. 6 Active earth pressure distribution diagram for a c-φ Soil; (a) Wall & Stresses, (b) PD diagram

4.1.4 Active Earth Pressure for Partially Cohesive (c-φ) Soil: Rankine's Theory

Depth of Tension Crack and Theoretical Unsupported Height

When $h = 0$, $p_{ac} = -2c\sqrt{K_a}$; that is negative pressure will be exerted on the retaining wall.

With the increase of depth, h, it reaches to a critical value h_c such that the lateral pressure at this depth is zero. Putting $h = h_c$, and $p_{ac} = 0$, in equation (9) we get,

$$0 = K_a \gamma h_c - 2c\sqrt{K_a}$$

$$\Rightarrow \quad h_c = \frac{2c\sqrt{K_a}}{K_a \gamma} = \frac{2c}{\gamma\sqrt{K_a}} = \frac{2c}{\gamma\sqrt{\tan^2\left(45^o - \frac{\varphi}{2}\right)}}$$

$$\Rightarrow \quad h_c = \frac{2c}{\gamma \tan\left(45^o - \frac{\varphi}{2}\right)} = \frac{2c}{\gamma}\tan\left(45^o + \frac{\varphi}{2}\right) \qquad (10)$$

4.1.4 Active Earth Pressure for Partially Cohesive (c-φ) Soil: Rankine's Theory

Depth of Tension Crack and Theoretical Unsupported Height

It means that down to a depth of h_c, negative pressure i.e. tension exists, as such it is known as **depth of tension crack**.

While considering twice the depth of h_c, the summation of the lateral pressure on the wall becomes zero. It means that, if a retaining wall has a height of $2h_c$, no lateral pressure would be exerted. Reversely, we can say that if we are to go for an excavation of the depth of $2h_c$, no retaining structure would be required. This depth is known as **theoretical unsupported height** and usually designated as (H_u). That is,

$$H_u = 2h_c = \frac{4c}{\gamma} tan\left(45^o + \frac{\varphi}{2}\right) \qquad (11)$$

4.1.4 Active Earth Pressure for Partially Cohesive (c-φ) Soil: Rankine's Theory

Depth of Tension Crack and Theoretical Unsupported Height

As the soil is weak in tension, it can resist a very small tension. Beyond this, crack develops between soil and wall. As such, this depth, h_c is known as depth of tension crack. For a purely cohesive soil where angle of internal friction of soil, $\varphi = 0$, equation (10) takes the form,

$$h_c = \frac{2c}{\gamma} \qquad (12)$$

Similarly, theoretical unsupported height H_u as given by equation (11), for a purely cohesive soil, takes the form,

$$H_u = \frac{4c}{\gamma} \qquad (13)$$

4.1.4 Active Earth Pressure for Partially Cohesive (c-φ) Soil: Rankine's Theory

Depth of Tension Crack and Theoretical Unsupported Height

As mentioned earlier, soil does not resist any tension and as such it is quite unlikely that the soil would adhere to the wall within the tension zone of depth h_c producing cracks in the soil. It is therefore commonly assumed that the active pressure is represented by the positive marked area of Fig. 4(a). Thus, the total thrust coming on the retaining wall is:

$$P_{ac} = \frac{1}{2}\left(K_a\gamma H - 2c\sqrt{K_a}\right)(H - h_c)$$

$$= \frac{1}{2}\left(K_a\gamma H - 2c\sqrt{K_a}\right)\left(H - \frac{2c}{\gamma\sqrt{K_a}}\right)$$

$$= \frac{1}{2}K_a\gamma H^2 - \frac{1}{2}K_a\gamma \mathrm{H} \times \frac{2c}{\gamma\sqrt{K_a}} - cH\sqrt{K_a} + c\sqrt{K_a} \times \frac{2c}{\gamma\sqrt{K_a}}$$

4.1.4 Active Earth Pressure for Partially Cohesive (c-φ) Soil: Rankine's Theory

Depth of Tension Crack and Theoretical Unsupported Height

Therefore,

$$P_{ac} = \frac{1}{2}K_a\gamma H^2 - cH\sqrt{K_a}\mathrm{H} - cH\sqrt{K_a} + \frac{2c^2}{\gamma}$$

$$\Rightarrow P_{ac} = \frac{1}{2}K_a\gamma H^2 - 2cH\sqrt{K_a} + \frac{2c^2}{\gamma} \qquad (14)$$

Passive Earth Pressure for Partially Cohesive Soil

Similar to that of active earth pressure, an expression for passive earth pressure can be derived for passive pressure in a $c - \varphi$ soil as:

$$p_{pc} = K_p\gamma h + 2c\sqrt{K_p} \qquad (15)$$

4.1.5 Active Earth Pressure: Cohesionless Soil: Inclined Backfill Surface

For a sloping backfill surface, it is assumed that lateral pressure is acting parallel to the ground surface. Considering rhombic soil element, Fig. 7, and conjugate stress relationship, Mohr's circle of stresses can be drawn that gives:

$$K_a = \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi}} \qquad (16a)$$

The active pressure at any depth h is given by:

$$p_a = K_a \gamma h \qquad (16b)$$

The active thrust (force/m length of wall) is given by:

$$P_a = K_a \gamma H^2 \cos\beta \qquad (16c)$$

4.1.5 Active Earth Pressure: Cohesionless Soil: Inclined Backfill Surface

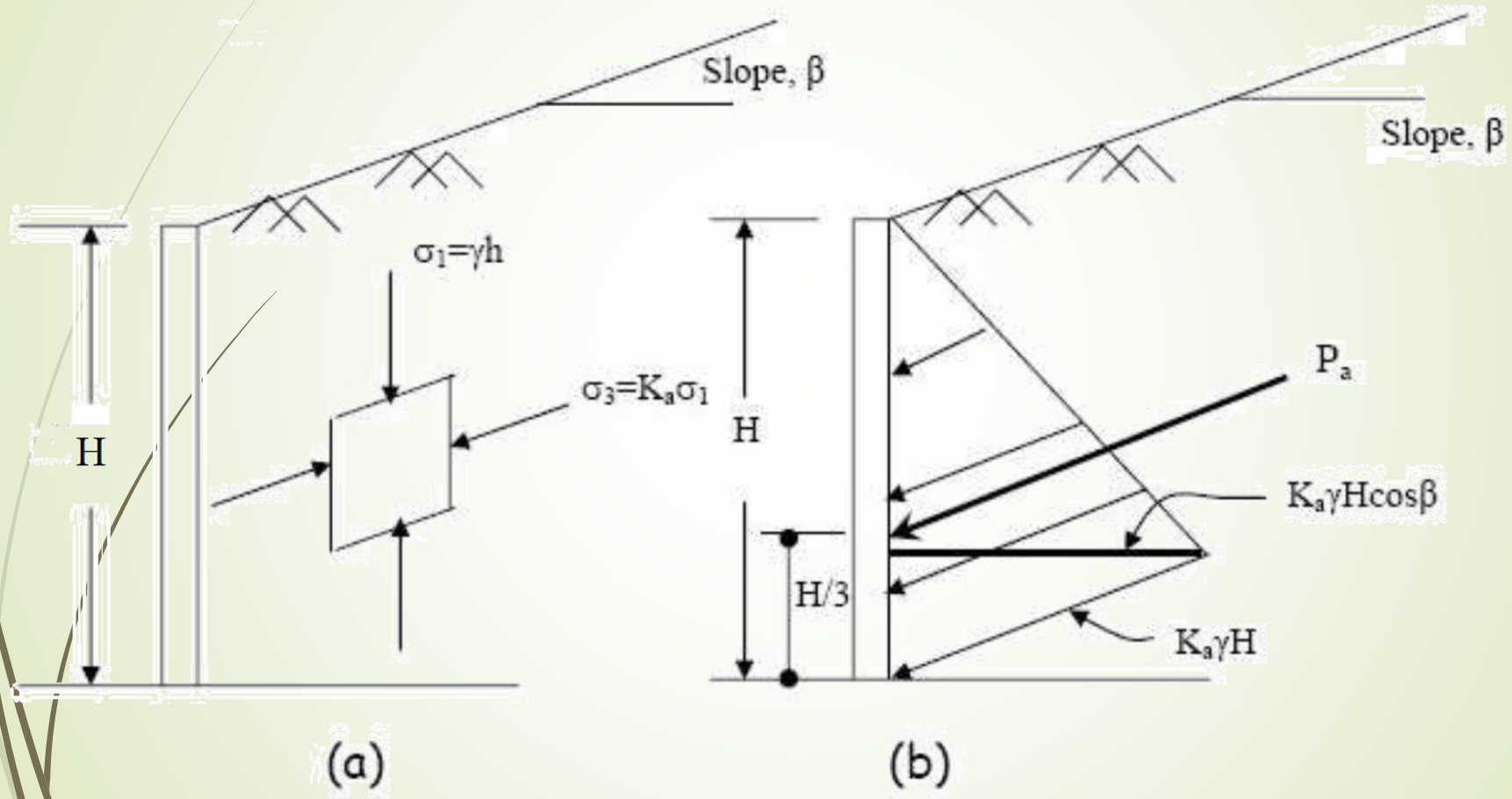


Fig. 7 Sloping backfill surface; (a) Rhombic element; (b) Pressure distribution diagram

4.1.6 Passive Earth Pressure: Cohesionless Soil: Inclined Backfill Surface

In a similar fashion to that of active condition, Mohr's circle of stresses can be drawn for passive condition that gives:

$$K_p = \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi}} \qquad (17a)$$

The passive pressure at any depth h is given by:

$$p_p = K_p \gamma h \qquad (17b)$$

The active thrust (force/m length of wall) is given by:

$$P_p = K_p \gamma H^2 \cos\beta \qquad (17c)$$

4.1.6 Active Earth Pressure: c-φ Soil: Inclined Backfill Surface

Mazindrani and Ganjali (1997) made an analytical solution of lateral earth pressure problem of cohesive backfill with sloping surface. According to them:

$$K_a = \frac{1}{\cos^2\varphi}\left[2\cos^2\beta + \frac{2c}{\gamma}\cos\varphi\sin\varphi - Y_1\right]\text{-}1 \qquad (18a)$$

and,

$$K_p = \frac{1}{\cos^2\varphi}\left[2\cos^2\beta + \frac{2c}{\gamma}\cos\varphi\sin\varphi + Y_1\right]\text{-}1 \qquad (18b)$$

Where,

$$Y_1 = \sqrt{\left[4\cos^2\beta(\cos^2\beta - \cos^2\varphi) + \left(\frac{2c}{\gamma}\right)^2\cos^2\varphi + \frac{8c}{\gamma z}\cos^2\beta\cos\varphi\sin\varphi\right]} \qquad (18c)$$

4.1.6 Active Earth Pressure: c-φ Soil: Inclined Backfill Surface

The active thrust (force/m length of wall) is given by:

$$P_a = \frac{\cos\beta}{\cos^2\varphi}\left[2\gamma H\cos^2\beta + 2c\cos\varphi\sin\varphi - Y_2\right] - \gamma H\cos\beta \qquad (19a)$$

And, the passive thrust (force/m length of wall) is:

$$P_p = \frac{\cos\beta}{\cos^2\varphi}\left[2\gamma H\cos^2\beta + 2c\cos\varphi\sin\varphi + Y_2\right] - \gamma H\cos\beta \qquad (19b)$$

Where,

$$Y_2 = \sqrt{4\cos^2\beta(\cos^2\beta - \cos^2\varphi)\gamma^2H^2 + (2c\cos\varphi)^2 + 8c\gamma H\cos^2\beta\cos\varphi\sin\varphi} \qquad (19c)$$

They also found that the depth of tension crack is independent of backfill slope and is given by the conventional formula, $\frac{2c}{\gamma}\tan\left(45^o + \frac{\varphi}{2}\right)$.

4.1.7 Active (or Passive) Pressure: Cohesionless Soil: Effect of Water Table

Lateral earth pressure concerns with the effective stress. As such, if there is a presence of water table in uniform backfill soil, in calculating the lateral pressure, the effective unit weight of the soil is to be considered. Let the water table be at a depth of H_1 and the layer height under submerged condition is H_2, Fig. 8. The Figure illustrates the calculation of active effect of water table. While calculating passive earth pressure the coefficient of active pressure (K_a) is to be replaced by coefficient of passive pressure (K_p).

However, in calculating the total pressure coming to the wall the lateral pressure due to water is also to be considered which is independent of lateral pressure coefficients.

4.1.7 Active (or Passive) Pressure: Cohesionless Soil: Effect of Water Table

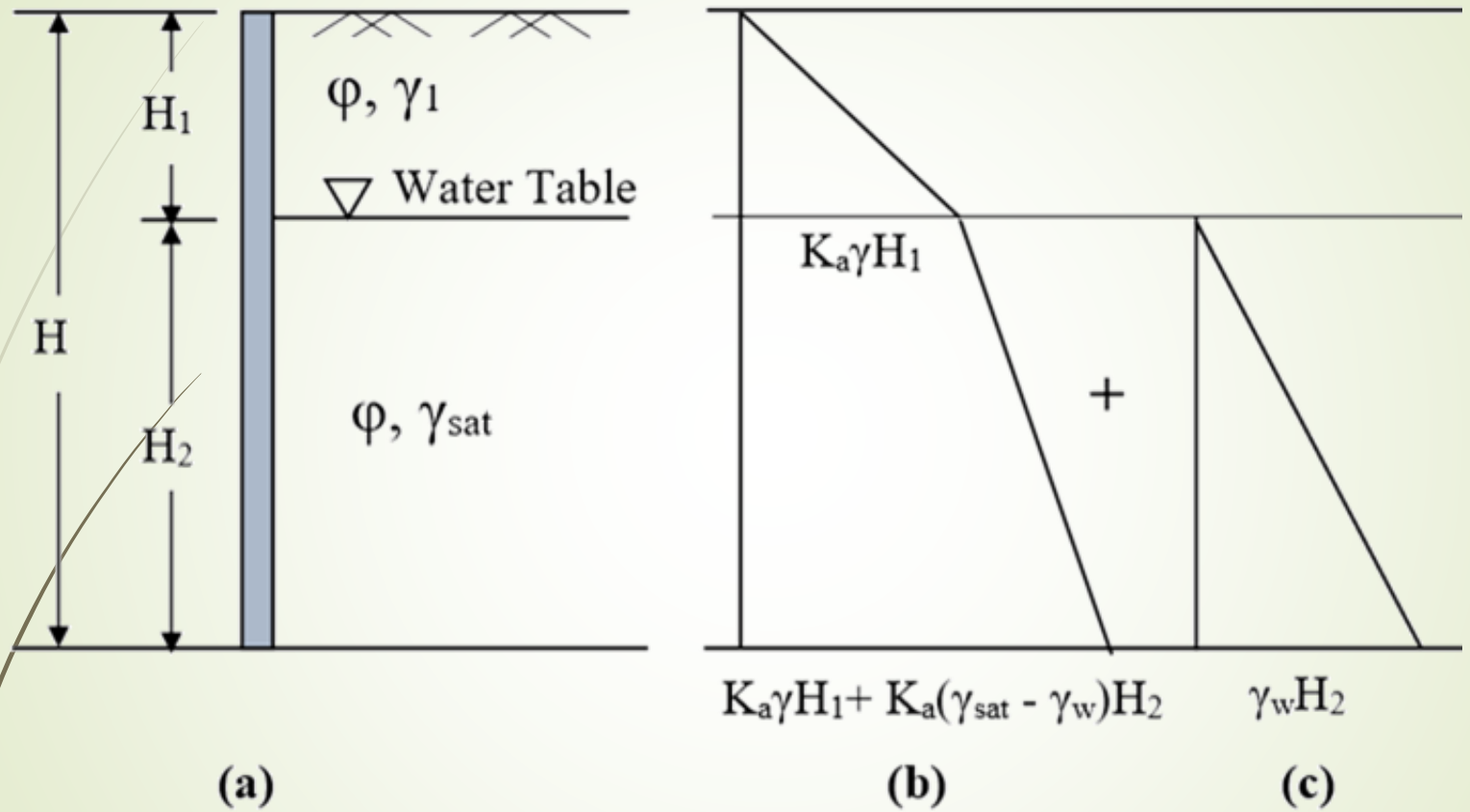


Fig. 8 Effect of water table on lateral earth pressure; (a) Wall and soil properties; (b) Active (or passive) pressure diagram; (c) Lateral water pressure diagram. *(For passive pressure, K_a is to be replaced by K_p)*

4.1.8 Active (or Passive) Pressure: Cohesionless Soil: Effect of Stratification

When a retaining structure retains stratified soil with different strength parameters (c, φ) there will be a change in lateral pressure at the interfaces of layer (just above and below the interface) because of the changes in the values of lateral earth pressure coefficients. This is illustrated in Fig. 9 for active condition. For passive condition, K_a will be replaced by K_p. In illustration of Fig. 9, the two pressure distribution lines are shown (AA' and BB') for the bottom layer pressure. Pressure distribution will follow AA' whenever $\varphi_2 > \varphi_1$. Pressure distribution will follow BB' in case of $\varphi_2 < \varphi_1$.

4.1.8 Active (or Passive) Pressure: Cohesionless Soil: Effect of Stratification

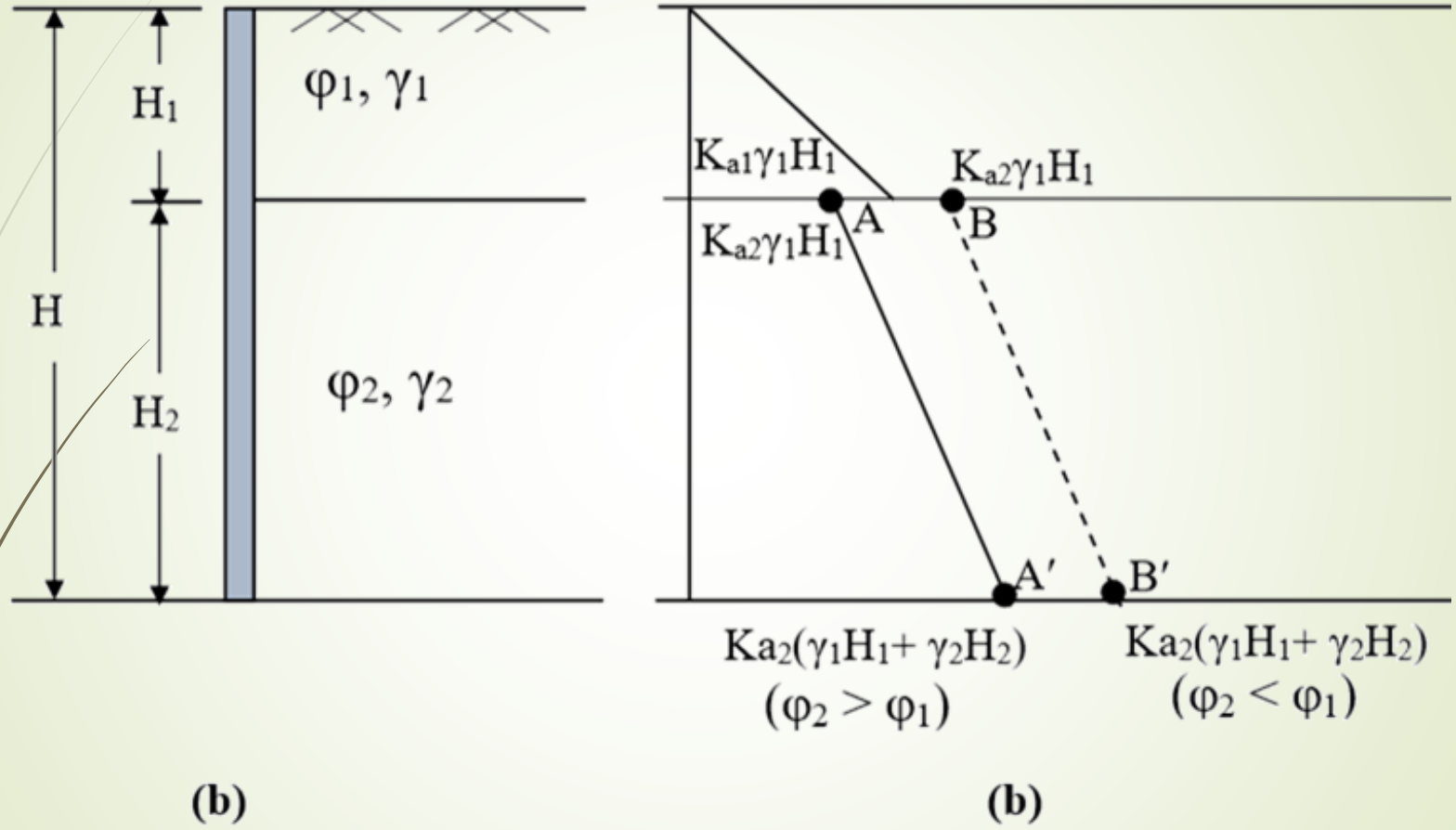


Fig. 9 Effect of layer stratification; (a) Wall and soil properties; (b) Active (or passive) pressure distribution diagram. *(For passive pressure, K_a is to be replaced by K_p)*

4.1.9 Active (or Passive) Pressure: Cohesionless Soil: Effect of Surcharge

Surcharge means additional load on the surface (or at a depth) of backfill soil. There could be various types of surcharge load, like:

(1) Uniform load

(2) Sloping load

(3) Point load

(4) Line load

(5) Strip load

(6) Ramp load

(7) Triangular load

(8) Area load

4.1.9 Active (Passive or Rest) Pressure: Cohesionless Soil: Effect of Surcharge

(1) Uniform Surcharge Load

When uniform charge q kN/m^2 would be acting on the surface of backfill soil the surcharge can be converted to equivalent height (H_e) of backfill soil that would exert equal vertical pressure q. The following relation is used.

$$H_e = \frac{q}{\gamma} \qquad (20)$$

The lateral earth pressure diagram is to be drawn from the top of the equivalent height, Fig. 10. As such, the lateral pressure at the top of the wall would be $K_a \gamma H_e (= K_a\, q)$. The pressure at the bottom is, as such, $K_a \gamma\,(\,H_e + H = K_a\, q + K_a \gamma H$. For passive condition, K_a will be replaced by K_p.

4.1.9 Active (Passive or Rest) Pressure: Cohesionless Soil: Effect of Surcharge

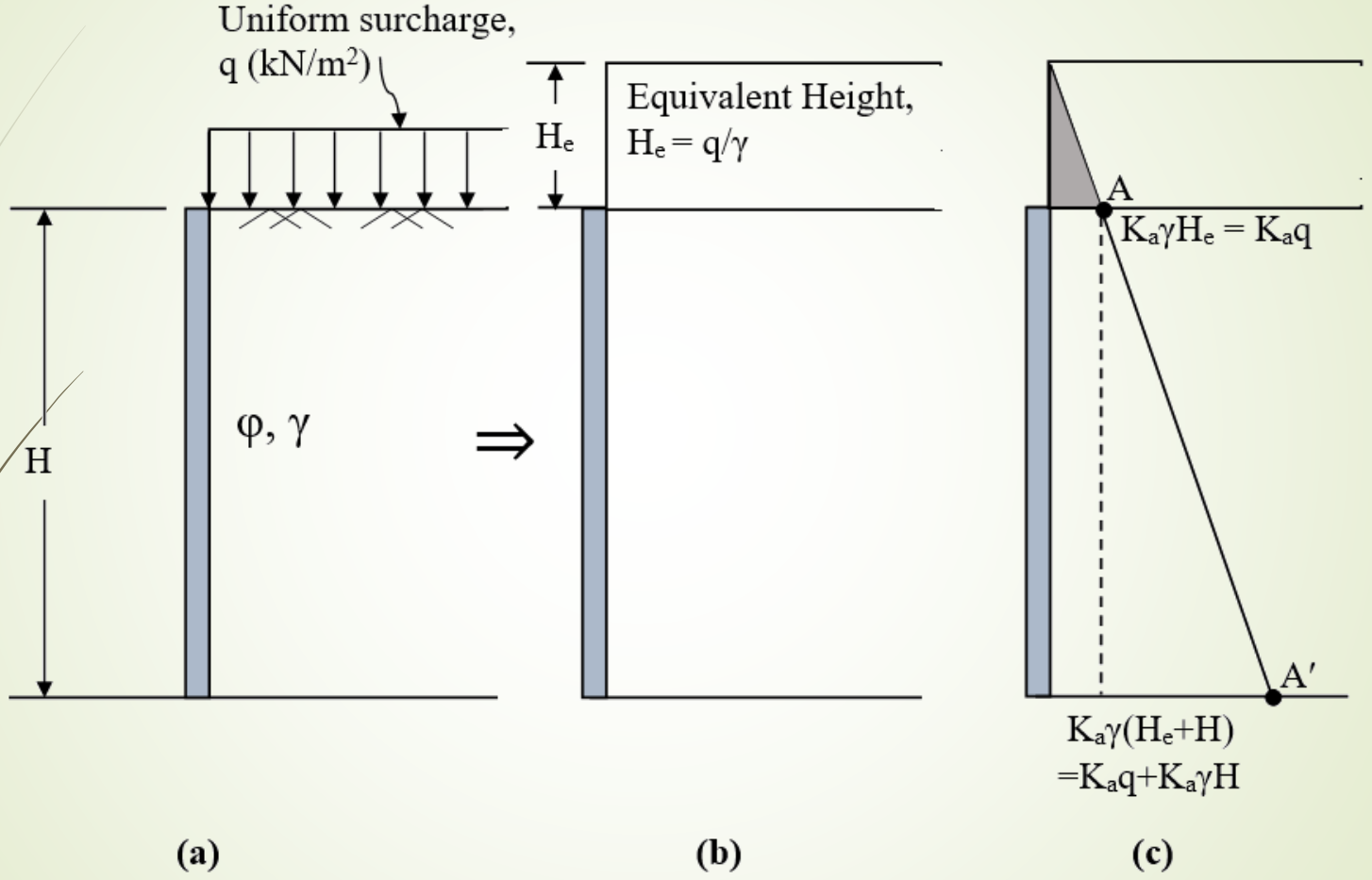


Fig. 10 Effect of uniform surcharge; (a) Wall and soil properties; (b) Equivalent height; (c) Lateral pressure diagram. *(For passive pressure, K_a is to be replaced by K_p)*

4.1.9 Active (Passive or Rest) Pressure: Cohesionless Soil: Effect of Surcharge

(2) Point Load Surcharge

Let Q_p be a point load acting on the backfill surface as shown in Fig. 11. Referring to the Figure lateral earth pressures along the vertical section at perpendicular distance of loading point Q_p are given by:

(i) For $m > 0.4$

$$\sigma_h = \frac{1.77Q_p}{H^2}\frac{n^2}{\left(m^2+n^2\right)^3} \qquad (21a)$$

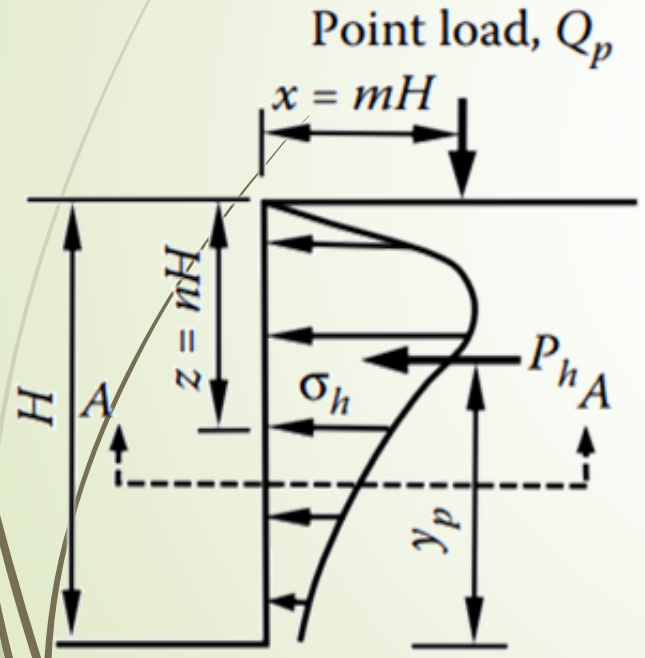
(ii) For $m \leq 0.4$

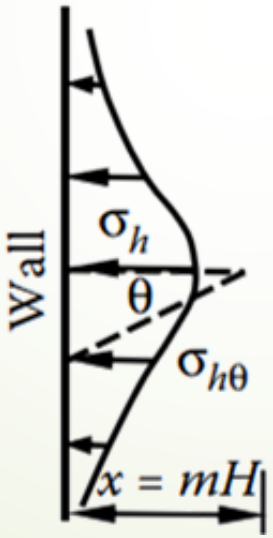
$$\sigma_h = \frac{0.28Q_p}{H^2}\frac{n^2}{\left(0.16+n^2\right)^3} \qquad (21b)$$

4.1.9 Active (Passive or Rest) Pressure: Cohesionless Soil: Effect of Surcharge

Lateral pressure at any points of either side of perpendicular is to be taken as:

$$\sigma_{h\theta} = \sigma_h cos^2(1.1\theta) \qquad (21c)$$





For $m \le 0.4$

$$\sigma_h\left(\frac{H^2}{Q_p}\right) = \frac{0.28n^2}{(0.16 + n^2)^3}$$

$$P_h = 0.69Q_p$$

For $m > 0.4$

$$\sigma_h\left(\frac{H^2}{Q_p}\right) = \frac{1.77m^2n^2}{(m^2 + n^2)^3}$$

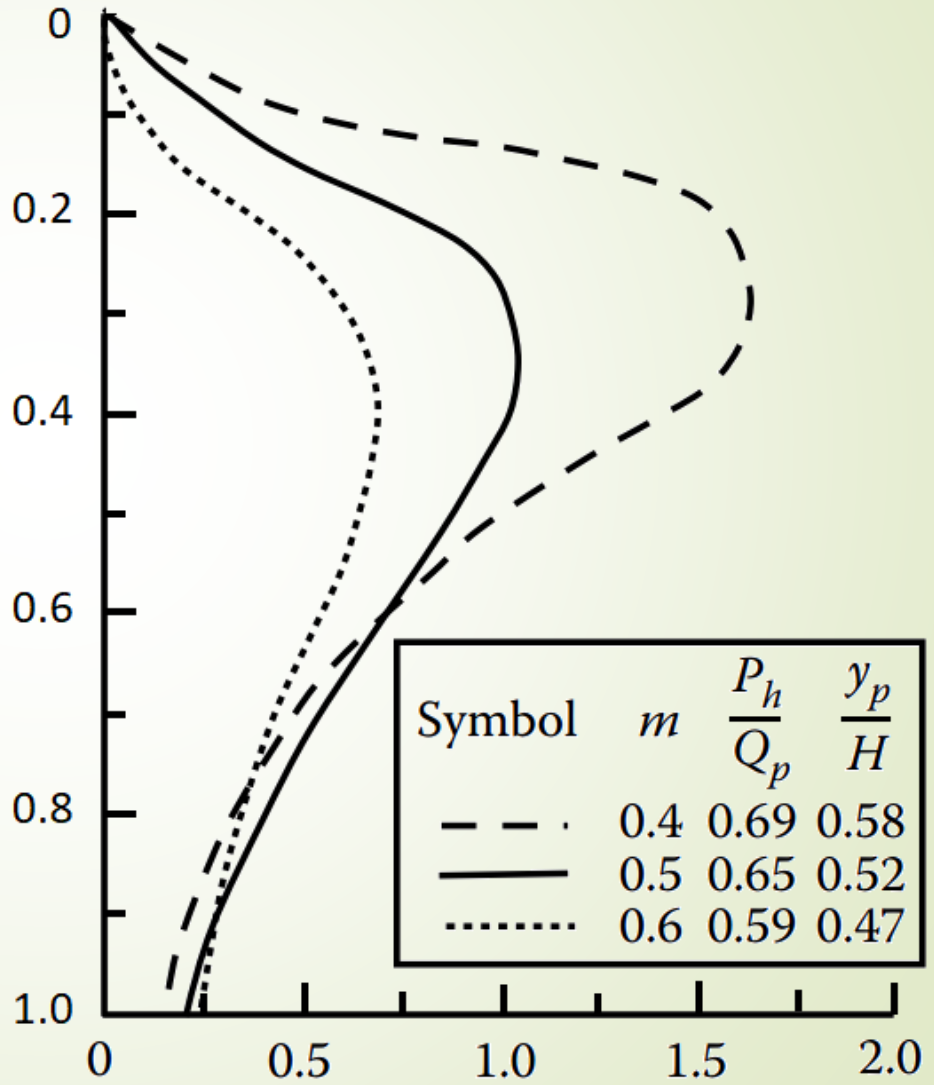
$$P_h = 0.48Q_p\left[\frac{m\,(1 - m^2)}{(1 + m^2)^2} + \tan^{-1}\left(\frac{1}{m}\right)\right]$$

$\sigma_{h\theta} = \sigma_h \cos^2(1.1\,\theta)$

Fig. 11 Lateral pressure due to point load; (a) Vertical section; (b) Horizontal section A-A.

4.1.9 Active (Passive or Rest) Pressure: Cohesionless Soil: Effect of Surcharge

Fig. 11(c). Lateral pressure distribution diagram due to vertical point load, Q_p .



4.1.9 Active (Passive or Rest) Pressure: Cohesionless Soil: Effect of Surcharge

(3) Line Load Surcharge

A concrete block wall, conduit laid on the surface or the wide strip load may be considered as a series of parallel line loads. Referring to Fig. 12, where Q_l is the load in kN/m,

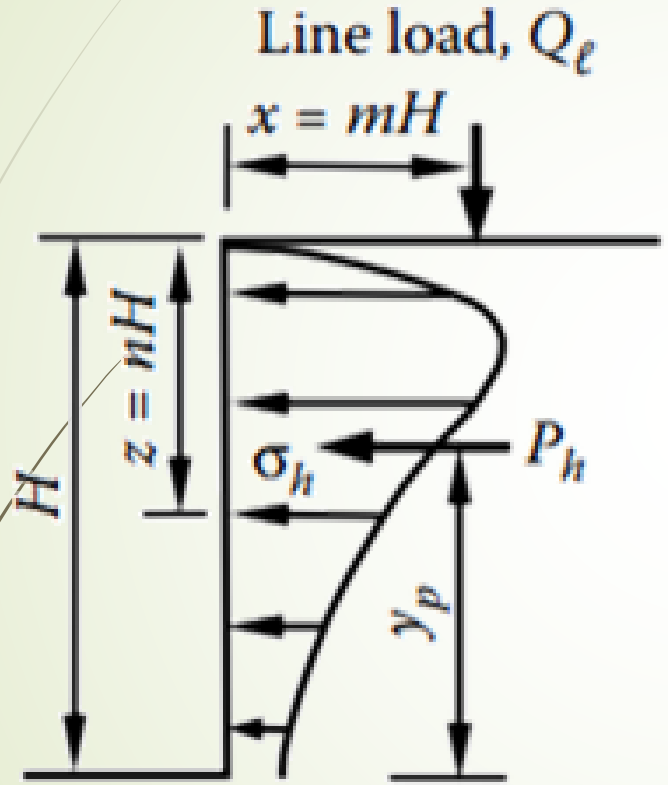
(i) For $m > 0.4$

$$p_h = \frac{4Q_l}{\pi H}\frac{m^2 n}{\left(m^2+n^2\right)^2} \qquad (22a)$$

(ii) For $m \leq 0.4$

$$p_h = \frac{Q_l}{H}\frac{0.203n}{\left(0.16+n^2\right)^2} \qquad (22b)$$

4.1.9 Active (Passive or Rest) Pressure: Cohesionless Soil: Effect of Surcharge



(a)

For $m \le 0.4$

$$\sigma_h\left(\frac{H}{Q_\ell}\right) = \frac{0.20n}{(0.16 + n^2)^2}$$

$$P_h = 0.55\, Q_\ell$$

For $m > 0.4$

$$\sigma_h\left(\frac{H}{Q_\ell}\right) = \frac{1.28m^2n}{(m^2 + n^2)^2}$$

$$P_h = \frac{0.64Q_\ell}{(1 + m^2)}$$

Fig. 12 (a) Lateral pressure due to vertical line load.

4.1.9 Active (Passive or Rest) Pressure: Cohesionless Soil: Effect of Surcharge

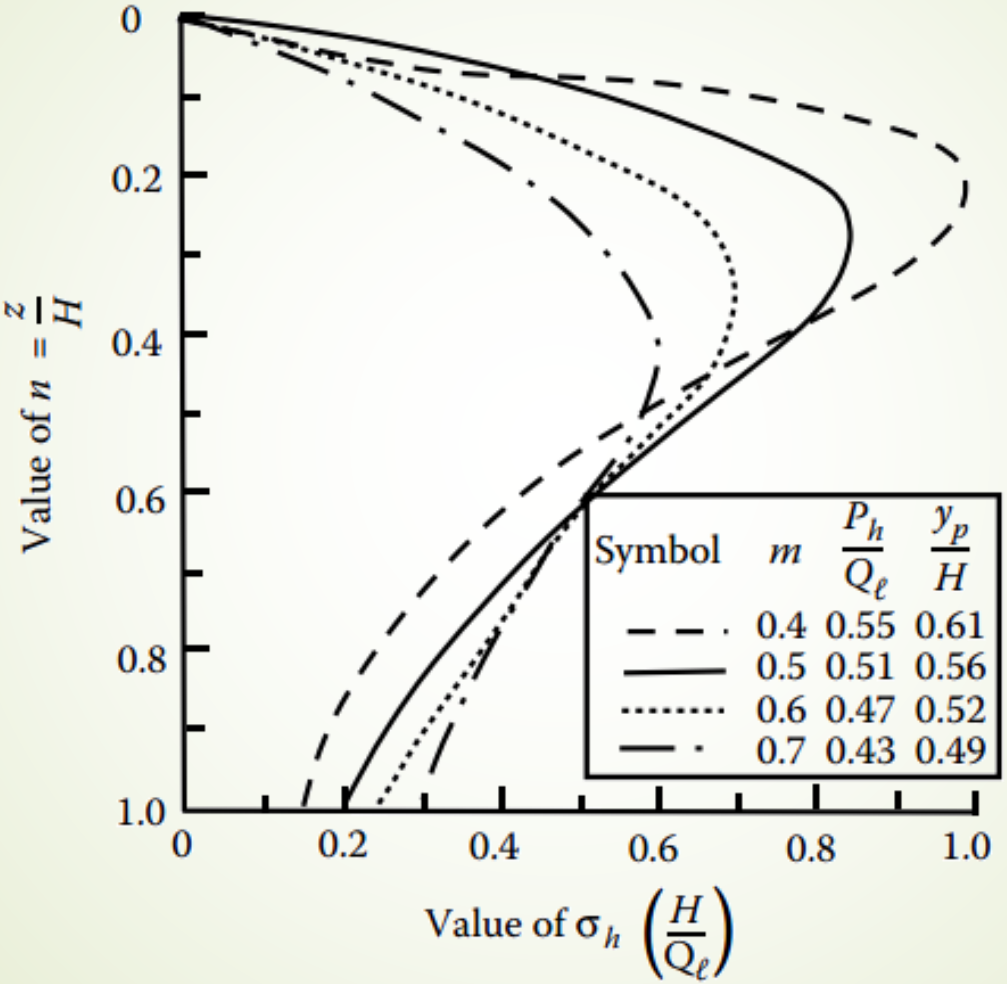


Fig. 12 (b) Lateral pressure distribution diagram due to vertical line load.

4.1.9 Active (Passive or Rest) Pressure: Cohesionless Soil: Effect of Surcharge

(4) Strip Load Surcharge

A strip load is a load intensity with a finite width , such as a highway, rail line or earth embankment which is parallel to the retaining structure. The application of load is shown in Fig. 13. The lateral pressure at any depth z is given by:

$$\sigma_h = \frac{2q}{\pi}(\beta - \sin\beta \cos 2\alpha) \qquad (23a)$$

Total lateral pressure per unit length of the wall due to strip loading may be expressed as

$$P_h = \frac{q}{90}[H(\alpha_2 - \alpha_1)] \qquad (22b)$$

4.1.9 Active (Passive or Rest) Pressure: Cohesionless Soil: Effect of Surcharge

Where,

$$\alpha_1 = \tan^{-1}\left(\frac{A}{H}\right) \quad \text{and} \quad \alpha_2 = \tan^{-1}\left(\frac{A+B}{H}\right)$$

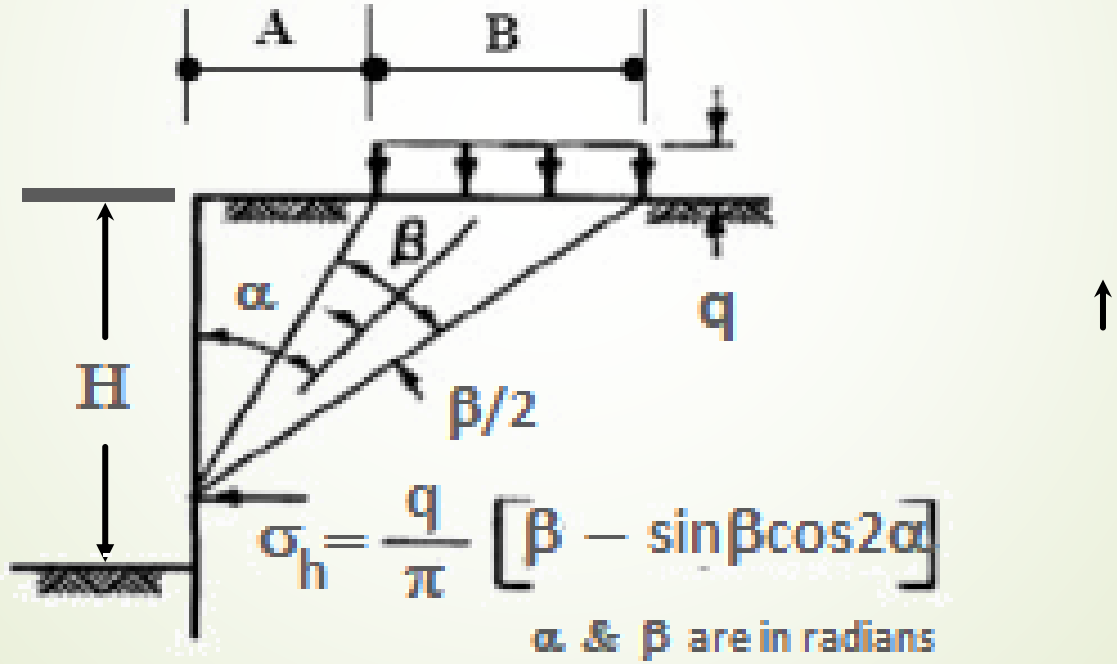


Fig. 13 Lateral pressure due to strip vertical load.

4.1.9 Active (Passive or Rest) Pressure: Cohesionless Soil: Effect of Surcharge

(5) Ramp Load Surcharge

Referring to Fig. 14, the lateral pressure due to vertical ramp load at any depth z is given by:

$$\sigma_h = \frac{q}{\pi c}\left[c\beta + x\alpha + 2z\log_e \frac{R_1}{R_2}\right] \qquad (24)$$

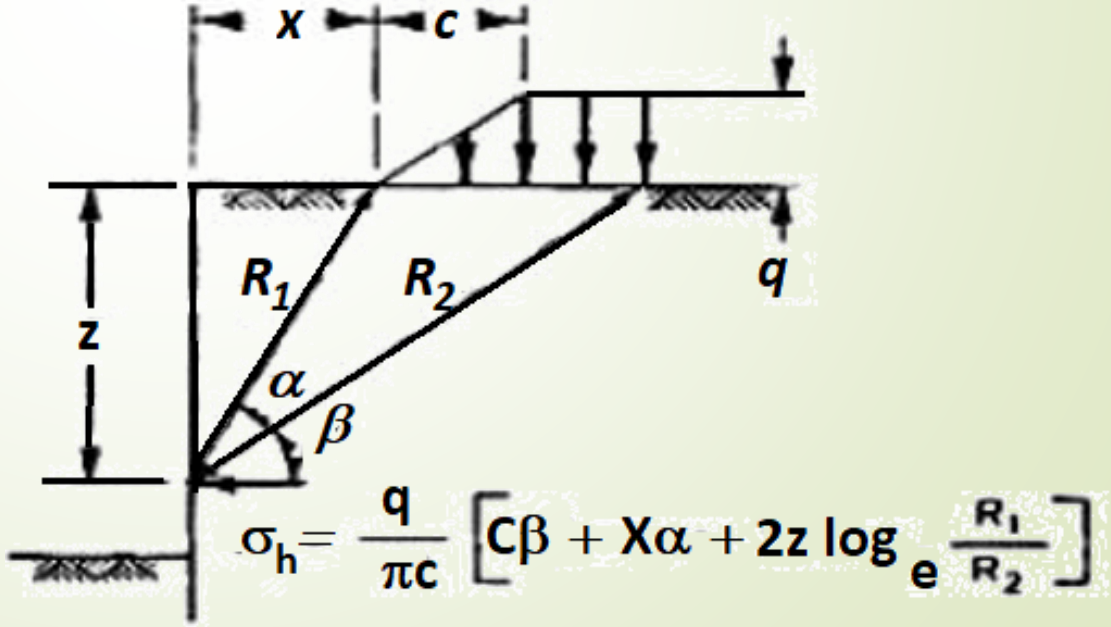


Fig. 14 Lateral pressure due to vertical ramp load.

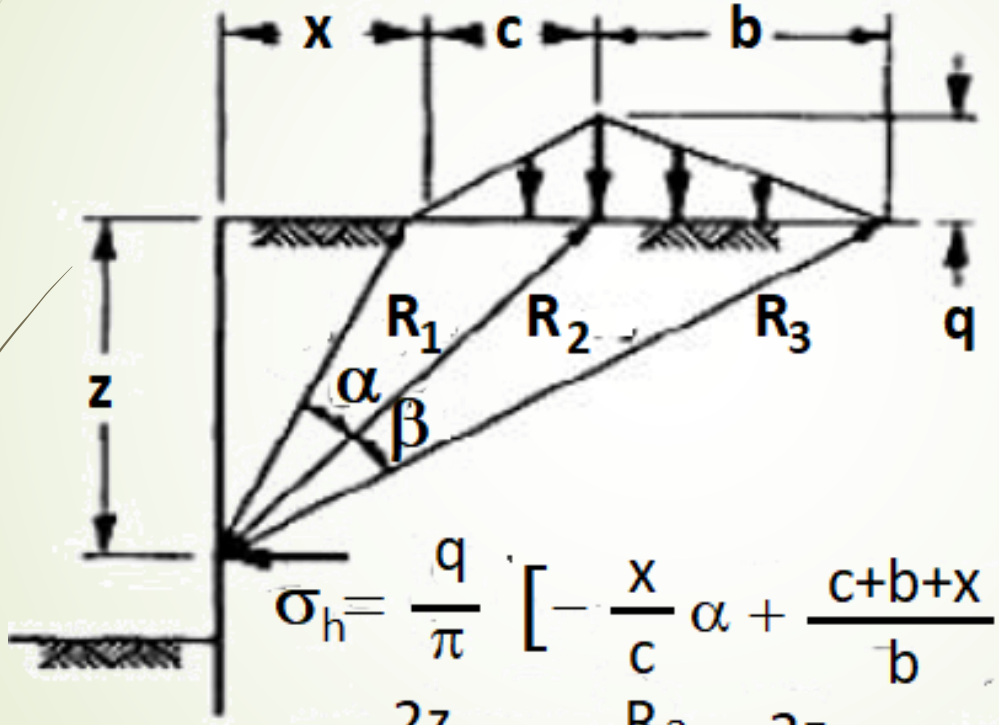
4.1.9 Active (Passive or Rest) Pressure: Cohesionless Soil: Effect of Surcharge

(6) Triangular Load Surcharge

A triangular load varies perpendicular to the wall as illustrated in Fig. 15. The load is assumed to vary parallel to the wall. The lateral pressure due at any depth z is given by:

$$\sigma_h = \frac{q}{\pi}\left[-\frac{x}{c}\alpha + \frac{c+b+x}{b}\beta + \frac{2z}{c}\log_e\frac{R_2}{R_1} + \frac{2z}{b}log_e\frac{R_2}{R_3}\right] \quad (24)$$

4.1.9 Active (Passive or Rest) Pressure: Cohesionless Soil: Effect of Surcharge



$$\sigma_h = \frac{q}{\pi}\left[-\frac{x}{c}\alpha + \frac{c+b+x}{b}\beta + \frac{2z}{c}\log_e\frac{R_2}{R_1} + \frac{2z}{b}\log_e\frac{R_2}{R_3}\right]$$

Fig. 15 Lateral pressure due to vertical triangular load.

4.2 Coulomb's Theory of Lateral Earth Pressure

In finding lateral earth pressure, Coulomb considered that in both active and passive conditions, failure of backfill soil would occur in a wedge. He considered the basic principles of mechanics that is the equilibrium of the soil wedge on the verge of its failure. Coulomb's theory of earth pressure is based on the following assumptions:

(1) Soil mass is homogenous and isotropic.

(2) The failure wedge is a rigid body.

(3) Both the rupture and the pressure surfaces are plane.

(4) There may be friction between wall and soil.

(5) Failure is two dimensional.

4.2 Coulomb's Theory of Lateral Earth Pressure

The analysis of earth pressure is based on the assumptions that a failure plane develops in a soil mass, and along this failure surface the normal and shear stresses are related by shear strength expression, $\tau = c + \sigma tan\varphi$. This makes the problem statically determinate.

Free body diagram of a wedge of uniform soil bounded by the soil surface, retaining wall, and a failure plane is shown in Fig. 16. Equilibrium analysis of the forces as indicated in the Fig. 16 allows the active force P_a and passive force P_p to be expressed in terms of the geometry and shear strength. The following terminologies are used.

4.2 Coulomb's Theory of Lateral Earth Pressure

$\gamma =$ unit weight of soil.

$\varphi =$ angle of internal friction of soil

$c =$ cohesion of soil

$\delta =$ angle of wall friction

$\alpha =$ angle of back wall with horizontal

$\theta =$ angle of failure plane with horizontal

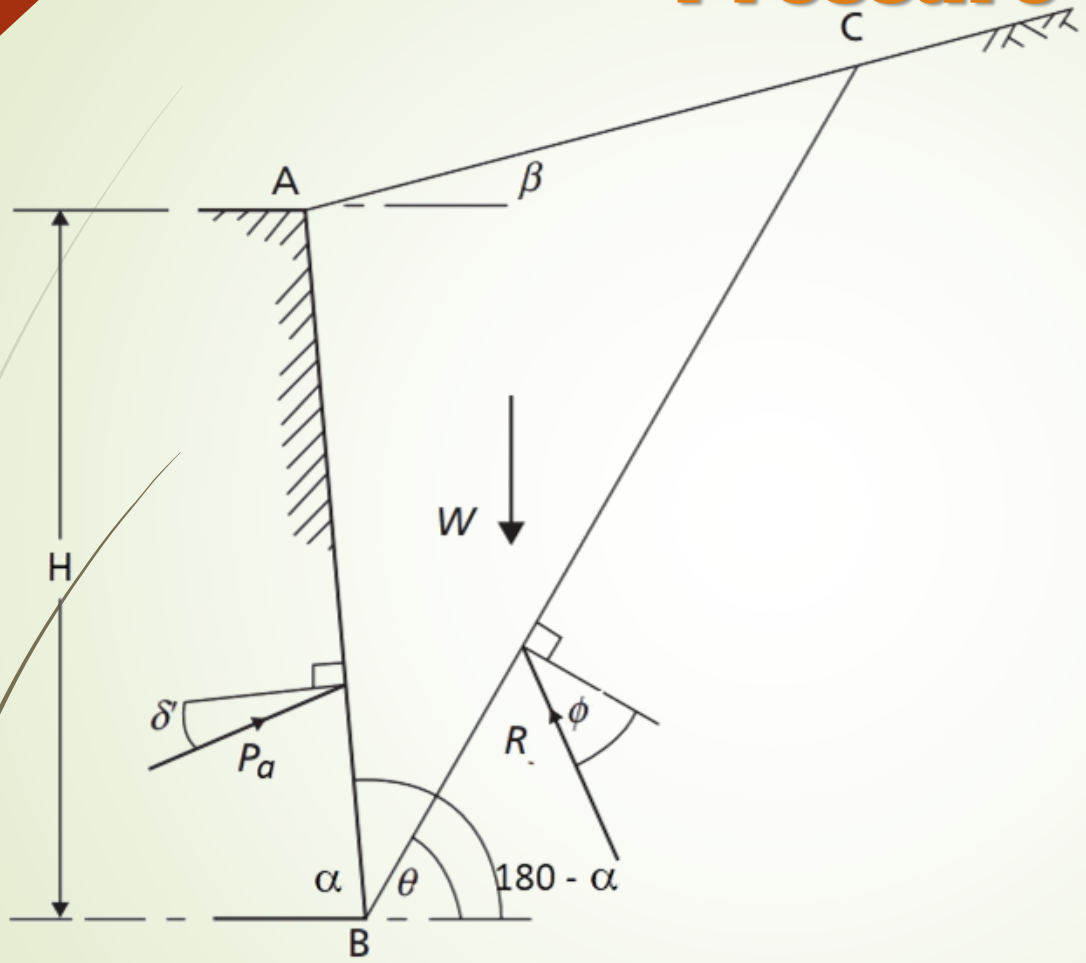
$z =$ depth below the ground surface

$\beta =$ slope of retained soil surface

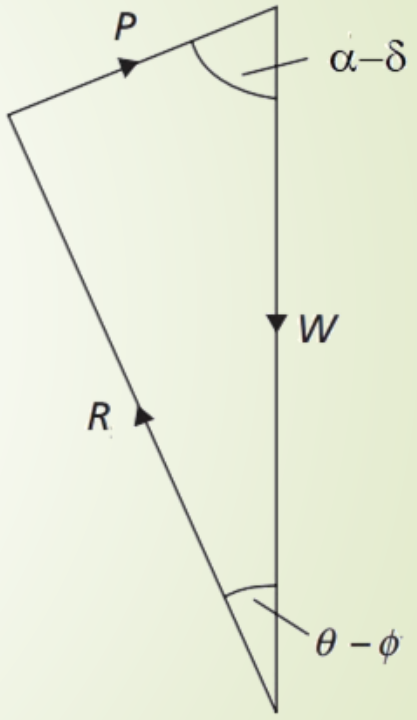
4.2 Coulomb's Theory of Lateral Earth Pressure

Figure 16 shows the forces acting on the soil wedge between a wall surface AB, inclined at angle α to the horizontal, and a trial failure plane BC, at angle θ to the horizontal. The soil surface AC is inclined at angle β to the horizontal. The shear strength parameter c is taken as zero, the apparent friction parameter of the soil is φ. For the failure condition, the soil wedge is in equilibrium under its own weight (W), the reaction to the force (P_a) between the soil and the wall, and the resultant reaction (R) on the failure plane. Because the soil wedge tends to move down the plane BC at failure, the reaction P_a acts at angle δ below the normal to the wall.

4.2 Coulomb's Theory of Lateral Earth Pressure



(a) Wedge geometry



(b) Force polygon

Fig. 16 Soil wedge and force polygon for Coulomb earth pressure theory.

4.2 Coulomb's Theory of Lateral Earth Pressure

If the wall were to settle more than the backfill, the reaction P_a would act at angle δ above the normal. At failure, when the shear strength of the soil has been fully mobilized, the direction of R is at an angle φ below the normal to the failure plane. The directions of all three forces, and the magnitude of W, are known, and therefore a triangle of forces (Fig. 16b) can be drawn and the magnitude of P_a determined for the trial in question.

4.2 Coulomb's Theory of Lateral Earth Pressure

A number of trial failure planes would have to be selected to obtain the maximum value of P_a, which would be the total active thrust on the wall. However, using the sine rule, P_a can be expressed in terms of W and the angles in the triangle of forces. Then the maximum value of P_a, corresponding to a particular value of θ, is given by $\frac{\partial P_a}{\partial \theta} = 0$. This leads to the following solutions for P_a in dry cohesionless soil:

$$P_a = \frac{1}{2} K_a \gamma H^2 \qquad \text{(25a)}$$

and, $$K_a = \frac{sin^2(\alpha+\varphi)}{sin^2\alpha \, sin^2(\alpha-\delta)\left[1-\sqrt{\frac{\sin(\varphi+\delta)\,\sin(\varphi-\beta)}{sin(\alpha-\delta)\,sin(\alpha+\beta)}}\right]^2} \qquad \text{(25b)}$$

4.2 Coulomb's Theory of Lateral Earth Pressure

Similarly, for passive condition the following solutions for passive force P_p in dry cohesionless soil can be obtained:

$$P_p = \frac{1}{2} K_p \gamma H^2 \qquad (26a)$$

and, $$K_p = \frac{\sin^2(\alpha - \varphi)}{\sin^2\alpha \, \sin^2(\alpha+\delta)\left[1+\sqrt{\frac{\sin(\varphi+\delta)\,\sin(\varphi+\beta)}{\sin(\alpha+\delta)\,\sin(\alpha+\beta)}}\right]^2} \qquad (26b)$$

4.3 Graphical Solution of Coulomb's Earth Pressure Theory

There are various graphical methods of solution of Coulomb's theory for determining lateral earth pressure. The prominent methods are due to Culmann, Poncelet and Rebhan.

4.3.1 Culmann's Method

This graphical procedure was devised by Karl Culmann over a century ago (1875). It is used to determine the magnitude and location of the resultant earth pressure, both active and passive, on retaining walls. This method is applicable with acceptable accuracy to cases where the backfill surface is level or sloped, regular or irregular, and where the backfill material is uniform or stratified.

4.3 Graphical Solution of Coulomb's Earth Pressure Theory

Also, it considers such variables as wall friction, cohesionless soils, and with some procedural modifications, cohesive soils and surcharge loads, both concentrated and uniformly distributed. It does, however, require that the angle of internal friction of the soil be a constant for the total backfill. The procedure presented here, as such, is limited to cohesionless soil.

Active Pressure

The Culmann's procedure for determining active pressure is illustrated in Fig. 17, and stepwise description follows.

4.3 Graphical Solution of Coulomb's Earth Pressure Theory

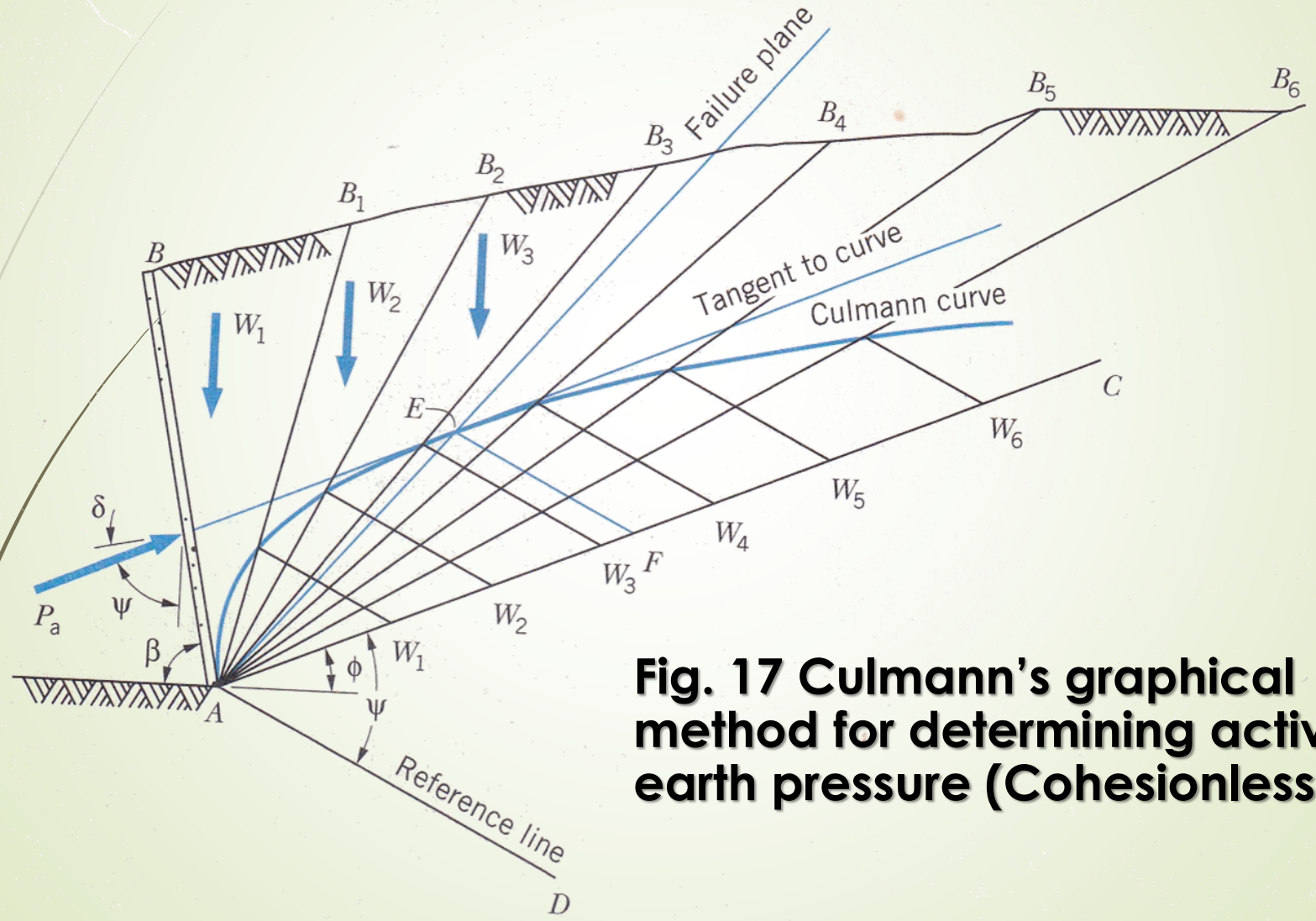


Fig. 17 Culmann's graphical method for determining active earth pressure (Cohesionless soil).

4.3 Graphical Solution of Coulomb's Earth Pressure Theory

(1) A convenient scale is chosen to show representative configuration of retaining wall and backfill. This includes height and slope of the retaining wall, surface configuration of the backfill, location and magnitude of surcharge loads, and so on.

(2) From point A, the line AC is drawn such that it makes an angle φ with the horizontal (above horizontal).

(3) A line AD is drawn at an angle of ψ from the line AC in the direction of backfill. From Fig. 17, the angle to be the angle between the vertical and the resultant active pressure.

4.3 Graphical Solution of Coulomb's Earth Pressure Theory

(4) The rays (lines) of assumed failure surfaces AB_1, AB_2, AB_3 and so on are drawn.

(5) The weight of each wedge is determined, accounting for variations in densities if the backfill is a layered system, for variable moisture content etc.

(6) A convenient scale of weight is chosen and the weights of each wedge are plotted along line AC. For example, the distance from A to W_1 along the line AC represents weight W_1; similarly the distance from W_1 to W_2 along line AC equals W_2 and so on.

4.3 Graphical Solution of Coulomb's Earth Pressure Theory

(7) From each of the points located on line AC, lines parallel to line AD (reference line)are drawn to intersect the corresponding assumed failure surfaces; that is, the line from W_1 will intersect line AB_1, the line one from W_2 will intersect line AB_2, and so on.

(8) These points of intersections are connected with a smooth line, known as Culmann's curve.

(9) A tangent parallel to line AC is drawn on Culmann's curve. In Fig. 17, point E represents such a tangent point. More than one tangent is possible if the Culmann's curve is irregular.

4.3 Graphical Solution of Coulomb's Earth Pressure Theory

(10) From point of tangency, line EF is drawn parallel to line AD. The magnitude of EF, based on the weight scale represents the active pressure P_a. If several tangents to the curve are possible, the largest of such values becomes the value of P_a.

Surcharge loads and their respective effects on the location of the resultant could also be accounted for .

Passive Pressure

Figure 18 illustrates the procedure for determining the passive resistance via Culmann's method. The approach is similar to that for the active pressure, with some differences.

4.3 Graphical Solution of Coulomb's Earth Pressure Theory

a) Line AC makes an angle of below rather than above the horizontal line.

b) The reference line AD makes an angle of ψ with line AC as a manner indicated in Fig. 18.

For the assumed sliding wedges, the weights W_1, W_2 and so on are plotted along line AC. From these points, lines are drawn parallel to the reference line to intersect the corresponding rays. Culmann's line represents a smooth curve connecting such points of intersection. A tangent to the Culmann's curve parallel to the line AC is drawn., with the resultant earth pressure being the scaled value of line EF.

4.3 Graphical Solution of Coulomb's Earth Pressure Theory

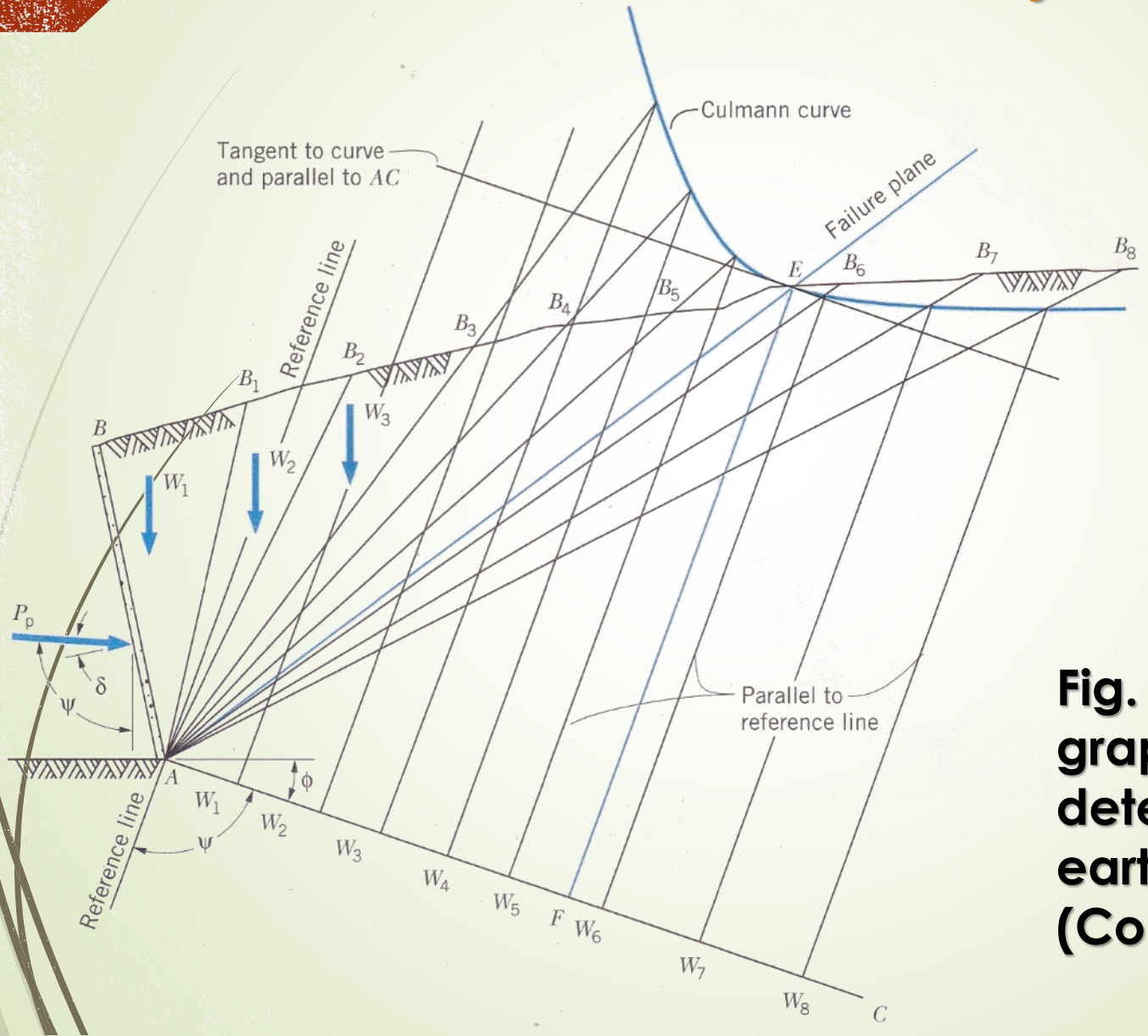


Fig. 18. Culmann's graphical method for determining passive earth pressure (Cohesionless soil).