EWCE 343

Geotechnical and Foundation Engineering

Footing with Eccentricity

TABLE 4-1
Bearing-capacity equations by the several authors indicated

Terzaghi (1943). See Table 4-2 for typical values and for K_{pr} values.

$$q_{ult} = cN_c s_c + \bar{q}N_q + 0.5\gamma B N_\gamma s_\gamma \qquad N_q = \frac{a^2}{a\cos^2(45 + \phi/2)}$$

$$a = e^{(0.75\pi - \phi/2)\tan\phi}$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = \frac{\tan\phi}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right)$$

For:	strip	round	square
s_c =	1.0	1.3	1.3
s_γ =	1.0	0.6	0.8

Meyerhof (1963).* See Table 4-3 for shape, depth, and inclination factors.

Vertical load: $q_{ult} = cN_c s_c d_c + \bar{q}N_q s_q d_q + 0.5\gamma B N_\gamma s_\gamma d_\gamma$

Inclined load: $q_{ult} = cN_c d_c i_c + \bar{q}N_q d_q i_q + 0.5\gamma B N_\gamma d_\gamma i_\gamma$

$$N_q = e^{\pi\tan\phi}\tan^2\left(45 + \frac{\phi}{2}\right)$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = (N_q - 1)\tan(1.4\phi)$$

Hansen (1970).* See Table 4-5 for shape, depth, and other factors.

General:† $q_{ult} = cN_c s_c d_c i_c g_c b_c + \bar{q}N_q s_q d_q i_q g_q b_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$

when $\phi = 0$

use $q_{ult} = 5.14 s_u(1 + s'_c + d'_c - i'_c - b'_c - g'_c) + \bar{q}$

N_q = same as Meyerhof above

N_c = same as Meyerhof above

$N_\gamma = 1.5(N_q - 1)\tan\phi$

Vesić (1973, 1975).* See Table 4-5 for shape, depth, and other factors. Use Hansen's equations above.

N_q = same as Meyerhof above

N_c = same as Meyerhof above

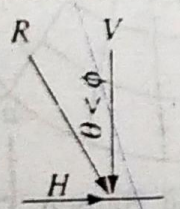
$N_\gamma = 2(N_q + 1)\tan\phi$

*These methods require a trial process to obtain design base dimensions since width B and length L are needed to compute shape, depth, and influence factors.

†See Sec. 4.6 when $i_c < 1$.

TABLE 4-3

Shape, depth, and inclination factors for the Meyerhof bearing-capacity equations of Table 4-1

Factors	Value	For
Shape:	$s_c = 1 + 0.2K_p\frac{B}{L}$	Any ϕ
	$s_q = s_\gamma = 1 + 0.1K_p\frac{B}{L}$	$\phi > 10^\circ$
	$s_q = s_\gamma = 1$	$\phi = 0$
Depth:	$d_c = 1 + 0.2\sqrt{K_p}\frac{D}{B}$	Any ϕ
	$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B}$	$\phi > 10$
	$d_q = d_\gamma = 1$	$\phi = 0$
Inclination:	$i_c = i_q = \left(1 - \frac{\theta^\circ}{90^\circ}\right)^2$	Any ϕ
	$i_\gamma = \left(1 - \frac{\theta^\circ}{\phi^\circ}\right)^2$	$\phi > 0$
	$i_\gamma = 0$ for $\theta > 0$	$\phi = 0$

Where $K_p = \tan^2(45 + \phi/2)$ as in Fig. 4-2

θ = angle of resultant R measured from vertical without a sign; if $\theta = 0$ all $i_i = 1.0$.

B, L, D = previously defined

TABLE 4-2

Bearing-capacity factors for the Terzaghi equations

Values of N_γ for ϕ of 0, 34, and 48° are original Terzaghi values and used to back-compute $K_{p\gamma}$

ϕ, deg	N_c	N_q	N_γ	$K_{p\gamma}$
0	5.7*	1.0	0.0	10.8
5	7.3	1.6	0.5	12.2
10	9.6	2.7	1.2	14.7
15	12.9	4.4	2.5	18.6
20	17.7	7.4	5.0	25.0
25	25.1	12.7	9.7	35.0
30	37.2	22.5	19.7	52.0
34	52.6	36.5	36.0	
35	57.8	41.4	42.4	82.0
40	95.7	81.3	100.4	141.0
45	172.3	173.3	297.5	298.0
48	258.3	287.9	780.1	
50	347.5	415.1	1153.2	800.0

*$N_c = 1.5\pi - 1$. [See Terzaghi (1943), p. 127.]

TABLE 4-4

Bearing-capacity factors for the Meyerhof, Hansen, and Vesić bearing-capacity equations

Note that N_c and N_q are the same for all three methods; subscripts identify author for N_γ

ϕ	N_c	N_q	$N_{\gamma(H)}$	$N_{\gamma(M)}$	$N_{\gamma(V)}$	N_q/N_c	$2\tan\phi(1-\sin\phi)^2$
0	5.14*	1.0	0.0	0.0	0.0	0.195	0.000
5	6.49	1.6	0.1	0.1	0.4	0.242	0.146
10	8.34	2.5	0.4	0.4	1.2	0.296	0.241
15	10.97	3.9	1.2	1.1	2.6	0.359	0.294
20	14.83	6.4	2.9	2.9	5.4	0.431	0.315
25	20.71	10.7	6.8	6.8	10.9	0.514	0.311
26	22.25	11.8	7.9	8.0	12.5	0.533	0.308
28	25.79	14.7	10.9	11.2	16.7	0.570	0.299
30	30.13	18.4	15.1	15.7	22.4	0.610	0.289
32	35.47	23.2	20.8	22.0	30.2	0.653	0.276
34	42.14	29.4	28.7	31.1	41.0	0.698	0.262
36	50.55	37.7	40.0	44.4	56.2	0.746	0.247
38	61.31	48.9	56.1	64.0	77.9	0.797	0.231
40	75.25	64.1	79.4	93.6	109.3	0.852	0.214
45	133.73	134.7	200.5	262.3	271.3	1.007	0.172
50	266.50	318.5	567.4	871.7	761.3	1.195	0.131

* $= \pi + 2$ as limit when $\phi \rightarrow 0°$.

Slight differences in above table can be obtained using program BEARING.EXE on diskette depending on computer used and whether or not it has floating point.

TABLE 4-5*a*

Shape and depth factors for use in either the Hansen (1970) or Vesić (1973, 1975*b*) bearing-capacity equations of Table 4-1. Use s'_c, d'_c when $\phi = 0$ only for Hansen equations. Subscripts H, V for Hansen, Vesić, respectively.

Shape factors	Depth factors
$s'_{c(H)} = 0.2\dfrac{B'}{L'}$ $(\phi = 0°)$	$d'_c = 0.4k$ $(\phi = 0°)$
$s_{c(H)} = 1.0 + \dfrac{N_q}{N_c} \cdot \dfrac{B'}{L'}$	$d_c = 1.0 + 0.4k$
$s_{c(V)} = 1.0 + \dfrac{N_q}{N_c} \cdot \dfrac{B}{L}$	$k = D/B$ for $D/B \le 1$
$s_c = 1.0$ for strip	$k = \tan^{-1}(D/B)$ for $D/B > 1$
	k in radians
$s_{q(H)} = 1.0 + \dfrac{B'}{L'}\sin\phi$	$d_q = 1 + 2\tan\phi(1 - \sin\phi)^2 k$
$s_{q(V)} = 1.0 + \dfrac{B}{L}\tan\phi$	k defined above
for all ϕ	
$s_{\gamma(H)} = 1.0 - 0.4\dfrac{B'}{L'} \ge 0.6$	$d_\gamma = 1.00$ for all ϕ
$s_{\gamma(V)} = 1.0 - 0.4\dfrac{B}{L} \ge 0.6$	

Notes:

1. Note use of "effective" base dimensions B', L' by Hansen but not by Vesić.
2. The values above are consistent with either a vertical load or a vertical load accompanied by a horizontal load H_B.
3. With a vertical load and a load H_L (and either $H_B = 0$ or $H_B \neq 0$) you may have to compute two sets of shape s_i and d_i as $s_{i,B}$, $s_{i,L}$ and $d_{i,B}$, $d_{i,L}$. For i, L subscripts of Eq. (4-2), presented in Sec. 4-6, use ratio L'/B' or D/L'.

TABLE 4-5*b*

Table of inclination, ground, and base factors for the Hansen (1970) equations. See Table 4-5*c* for equivalent Vesić equations.

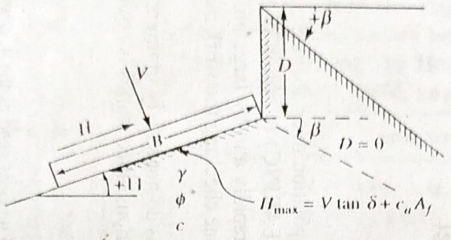
Inclination factors	Ground factors (base on slope)
$i_c' = 0.5 - \sqrt{1 - \frac{H_i}{A_f C_a}}$	$g_c' = \frac{\beta^\circ}{147^\circ}$
$i_c = i_q - \frac{1 - i_q}{N_q - 1}$	$g_c = 1.0 - \frac{\beta^\circ}{147^\circ}$
$i_q = \left[1 - \frac{0.5H_i}{V + A_f C_a \cot\phi}\right]^{\alpha_1}$ $2 \le \alpha_1 \le 5$	$g_q = g_\gamma = (1 - 0.5\tan\beta)^5$
Base factors (tilted base)	
$i_\gamma = \left[1 - \frac{0.7H_i}{V + A_f C_a \cot\phi}\right]^{\alpha_2}$	$b_c' = \frac{\eta^\circ}{147^\circ}$ $(\phi = 0)$
$i_\gamma = \left[1 - \frac{(0.7 - \eta^\circ/450^\circ)H_i}{V + A_f C_a \cot\phi}\right]^{\alpha_2}$ $2 \le \alpha_2 \le 5$	$b_c = 1 - \frac{\eta^\circ}{147^\circ}$ $(\phi > 0)$ $b_q = \exp(-2\eta\tan\phi)$ $b_\gamma = \exp(-2.7\eta\tan\phi)$ η in radians

Notes:

1. Use H_i as either H_B or H_L, or both if $H_L > 0$.
2. Hansen (1970) did not give an i_c for $\phi > 0$. The value above is from Hansen (1961) and also used by Vesić.
3. Variable C_a = base adhesion, on the order of 0.6 to 1.0 × base cohesion.
4. Refer to sketch for identification of angles η and β, footing depth D, location of H_i (parallel and at top of base slab; usually also produces eccentricity). Especially note V = force *normal* to base and is not the resultant R from combining V and H_i.

Notes: $\beta + \eta$ 90° (Both β and η have signs (+) shown.)

β ϕ



For: $L/B \le 2$ use ϕ_{tr}

$L/B > 2$ use $\phi_{ps} = 1.5\,\phi_{tr} - 17°$

$\phi_{tr} \le 34°$ use $\phi_{tr} = \phi_{ps}$

δ = friction angle between base and soil ($.5\phi \le \delta \le \phi$)

$A_f = B'L'$ (effective area)

c_a = base adhesion (0.6 to 1.0c)

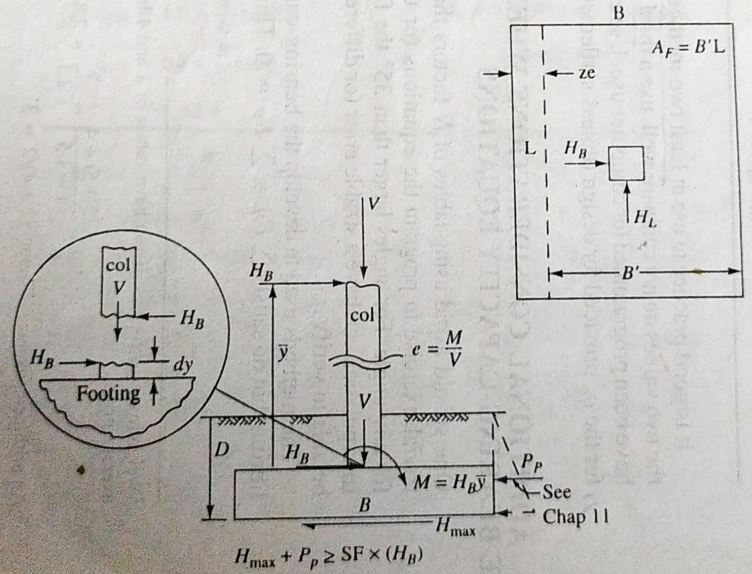


TABLE 4-5*c*

Table of inclination, ground, and base factors for the Vesić (1973, 1975*b*) bearing-capacity equations. See notes below and refer to sketch for identification of terms.

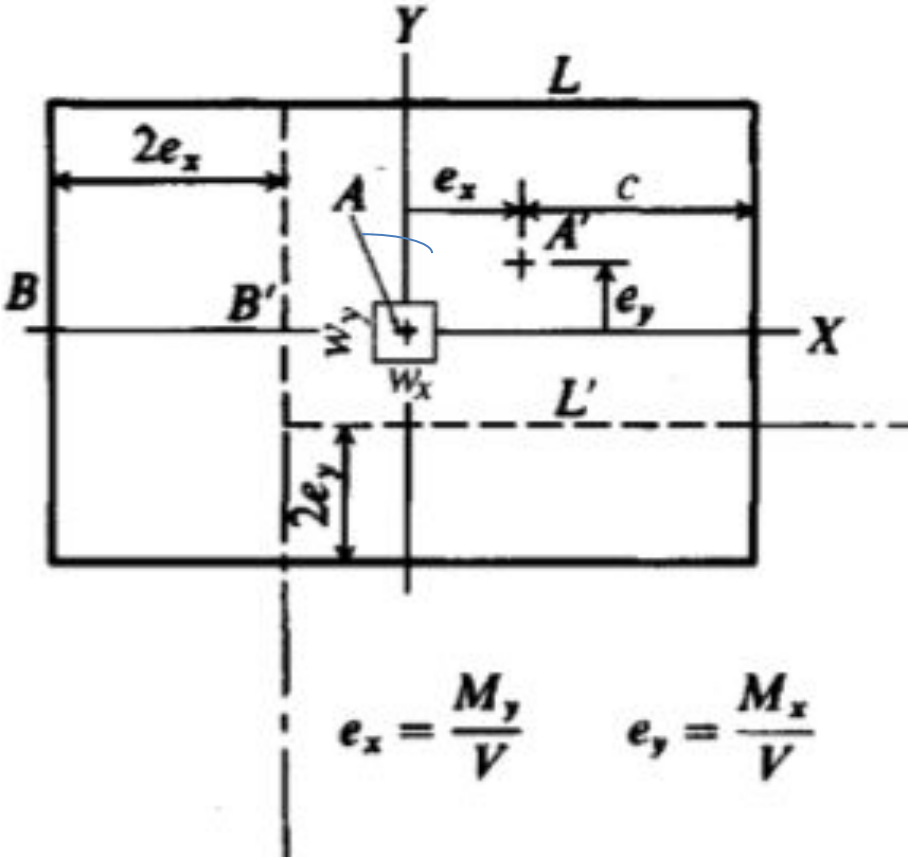
Inclination factors	Ground factors (base on slope)
$i'_c = 1 - \dfrac{mH_i}{A_f c_a N_c}$ $(\phi = 0)$	$g'_c = \dfrac{\beta}{5.14}$ β in radians
$i_c = i_q - \dfrac{1 - i_q}{N_q - 1}$ $(\phi > 0)$	$g_c = i_q - \dfrac{1 - i_q}{5.14 \tan \phi}$ $\phi > 0$
i_q, and m defined below	i_q defined with i_c
$i_q = \left[1.0 - \dfrac{H_i}{V + A_f c_a \cot \phi}\right]^m$	$g_q = g_\gamma = (1.0 - \tan \beta)^2$
	Base factors (tilted base)
$i_\gamma = \left[1.0 - \dfrac{H_i}{V + A_f c_a \cot \phi}\right]^{m+1}$	$b'_c = g'_c$ $(\phi = 0)$
	$b_c = 1 - \dfrac{2\beta}{5.14 \tan \phi}$
$m = m_B = \dfrac{2 + B/L}{1 + B/L}$	$b_q = b_\gamma = (1.0 - \eta \tan \phi)^2$
$m = m_L = \dfrac{2 + L/B}{1 + L/B}$	

Notes:

1. When $\phi = 0$ (and $\beta \neq 0$) use $N_\gamma = -2 \sin(\pm\beta)$ in N_γ term.
2. Compute $m = m_B$ when $H_i = H_B$ (H parallel to B) and $m = m_L$ when $H_i = H_L$ (H parallel to L). If you have both H_B and H_L use $m = \sqrt{m_B^2 + m_L^2}$. Note use of B and L, not B', L'.
3. Refer to Table sketch and Tables 4-5*a*,*b* for term identification.
4. Terms N_c, N_q, and N_γ are identified in Table 4-1.
5. Vesić always uses the bearing-capacity equation given in Table 4-1 (uses B' in the N_γ term even when $H_i = H_L$).
6. H_i term ≤ 1.0 for computing i_q, i_γ (always).

Footing with Eccentricity:

- A footing may be **eccentrically loaded** from a **concentric column with an axial load and moments** about one or both axes as in following figure.



- Eccentricity may result also from a column that **is initially not centrally located** or **becomes off-center when a part of the footing is cut away** during remodeling and/or installing new equipment.
- Obviously the footing cannot be cut if an analysis indicates the recomputed soil pressure might result in a bearing failure.

Footings with Eccentricity

- **Footing dimensions** as:

$\mathbf{L' = L - 2e_x}$ $\mathbf{B' = B - 2e_y}$

Effective footing area, $\boldsymbol{A_f = B'L'}$,

Center of pressure is the center of area $B'L'$ at point A'

$2e_x + L' = L$ $e_x + c = L/2$

Putting the value of e_x, we get,

$\mathbf{c = L/2 - (L/2 - L'/2) = L'/2}$

If there is no eccentricity about either axis, use the actual footing dimension for that B' or L'.

- **For design**, the minimum dimensions (to satisfy ACI 318-) of a rectangular footing with a central column of dimensions w_x x w_y are required to be

  $B_{min} = 4e_y + w_y$ $\quad$ $B' = B - 2e_y$

  $L_{min} = 4\,e_x + w_x$ $\quad$ $L' = L - 2\,e_x$

- **Final dimensions** may be larger than B_{min} or L_{min} based on obtaining the required allowable bearing capacity.

Ultimate bearing capacity for footings with eccentricity, using either the Meyerhof or Hansen/Vesic equations, is found in either of two ways:

Method 1. Use either the **Hansen or Vesic bearing-capacity equation** with the following adjustments:

- **Use B' in the $\gamma B N_\gamma$ term.**
- **Use B' and L' in computing the shape factors.**
- **Use actual B and L for all depth factors.**

The computed ultimate bearing capacity q_{ult} is then reduced to an allowable value q_a with an appropriate safety factor SF as

$$q_a = \frac{q_{ult}}{SF} \qquad \text{(as } P_a = q_a \text{xB'L')}$$

Method 2: Use the Meyerhof general bearing-capacity equation and a reduction factor R_e used as

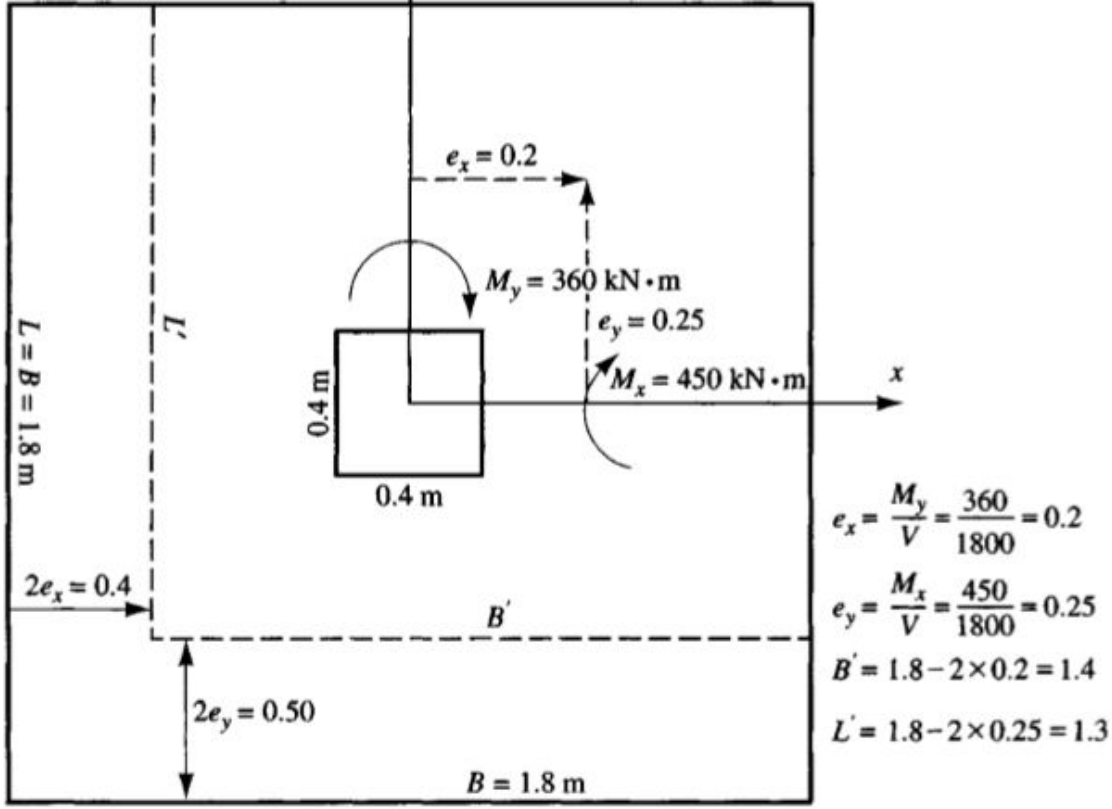
$$q_{ult.design} = q_{ult.computed} \times R_e$$

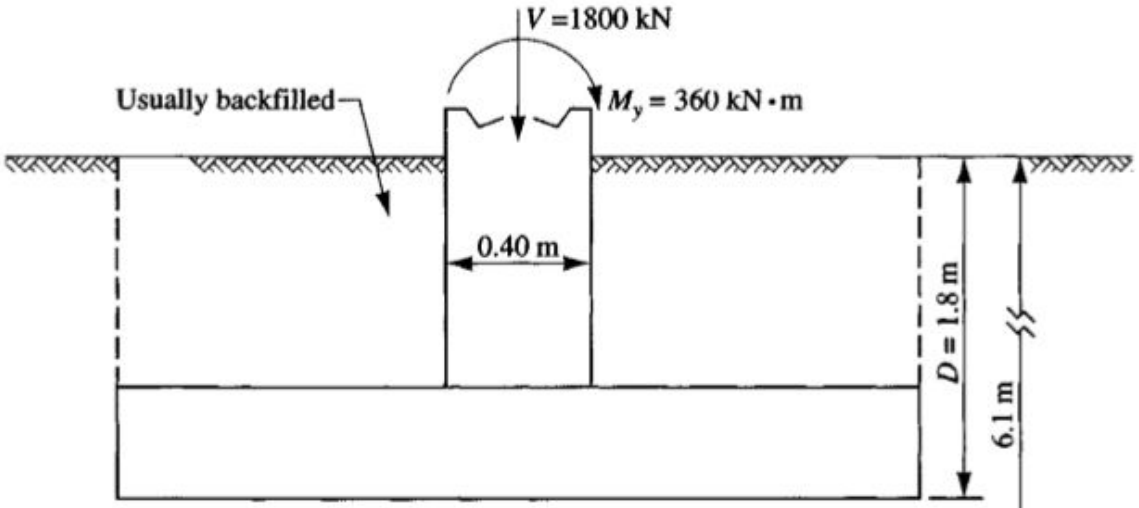
- Used only with the Meyerhof equation to compute the bearing capacity.

$$R_e = 1 - 2e/B \text{ (cohesive soil)}$$

$$R_e = 1 - \sqrt{(e/B)} \text{ (cohesionless soil and for } 0 < e/B < 0.3)$$

- **If e/B = 0.5**, the point A' falls at the edge of the base and an unstable foundation results.
- In practice the e/B ratio is seldom greater than 0.2 and is usually limited **to e ≤ B/6**.





$\phi_{tr} = 36°$ $c = 20$ kPa

$\gamma' = \gamma = 18.00$ kN/m^3

Problem 4-5. A square footing is **1.8m x1.8m** with square column. It is loaded with an axial load of **1800 kN,** and M_x**=450 kN-m**, M_y**=360 kN-m.** Undrained triaxial tests (soil not saturated) give $\emptyset = 36^0$ **and c=20 kPa**. The footing depth **D=1.8m;** the soil unit weight γ**=18.00kN/**m^3; the water table is at a depth of **6.1m** from the ground surface.

Required: What is the allowable soil pressure if **SF=3.0;** using **Hansen** bearing capacity equation with B', L'; **Meyerhof** equation and reduction factor R_e**?**

Solution:

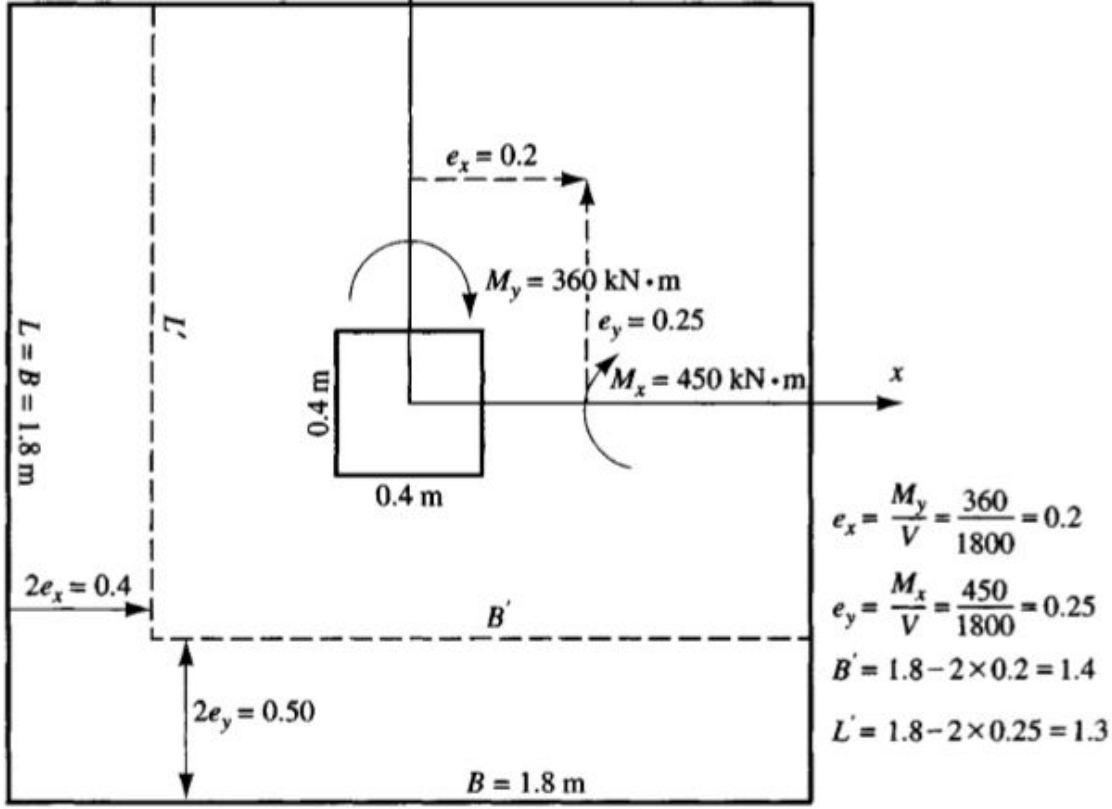
(Table 4-5)

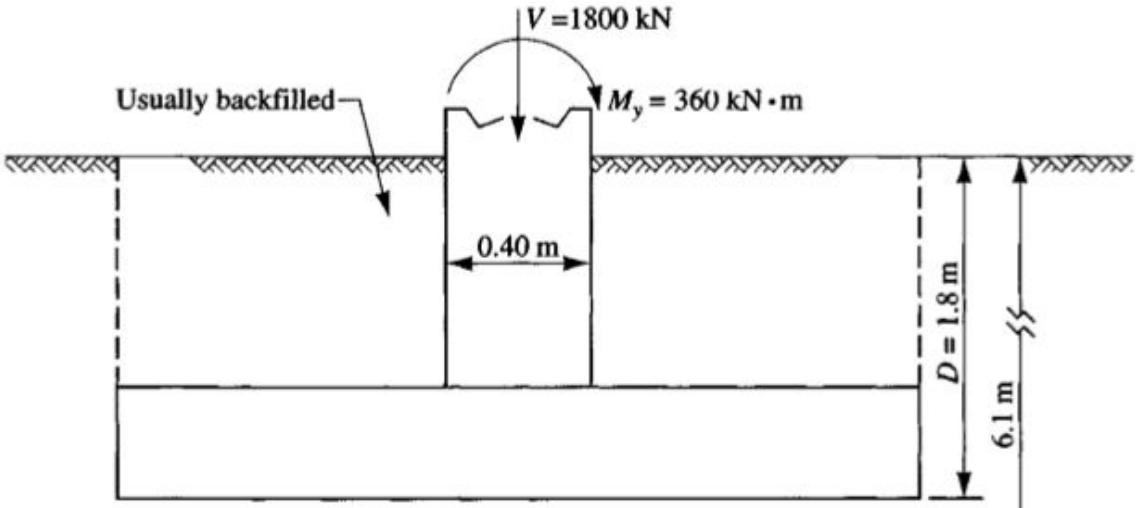
e_y**= 450/1800=0.25** e_x**= 360/1800=0.20**

Both values of **e are <B/6= 1.8/6=0.30**.

Also, B_{min}**= 4(0.25)+0.4= 1.4 <1.8m** given

L_{min}**=4(0.20)+0.4= 1.2 <1.8m** given





$\phi_{tr} = 36°$ $c = 20$ kPa

$\gamma' = \gamma = 18.00$ kN/m^3

Problem 4-5 (cont..)

Now Find,

$$B' = \text{B-}\ 2e_y = 1.8 - 2(0.25) = 1.3\text{m}\ (B' < L')$$

$$L' = \text{L-}\ 2e_x = 1.8 - 2(0.20) = 1.4\text{m}\ (L' > B')$$

By Hansen Equation, From Table 4-4 at $\emptyset = 36^0$ and rounding the integers, we obtain,

N_c=51 N_q=38 N_γ=40

N_q/N_c=0.746 $2\tan\emptyset\ (1 - sin\emptyset)^2 = 0.247$

Compute D/B= 1.8/1.8= 1.0

Now Compute: Table 4-5 & $\emptyset = 36^0$

$$s_c = 1 + (N_q/N_c)(B'/L') = 1 + (0.746)\ (1.3/1.4) = 1.69$$

Problem 4-5 (cont..)

$d_c = 1 + 0.4D/B = 1 + 0.4(1.8/1.8) = 1.40$

$s_q = 1 + (B'/L') \sin \emptyset = 1 + (1.3/1.4) \sin 36^0 = 1.55$

$d_q = 1 + 2\tan \emptyset \, (1 - sin\emptyset)^2 D/B = 1 + 0.247 \, (1.0) = 1.25$

$s_\gamma = 1 - 0.4 \, (B'/L') = 1 - 0.4(1.3/1.4) = 0.62 > 0.60$ (ok)

$d_\gamma = 1.0$

Hansen equation in Table **4-1:**

$$q_{ult} = c\, N_c s_c d_c i_c g_c b_c + q\, N_q s_q d_q i_q g_q b_q + 0.5\, \gamma B' N_\gamma s_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$$

All $i_i = g_i = b_i = 1.0$ (not 0.0)

$$q_{ult} = c\, N_c s_c d_c + q\, N_q s_q d_q + 0.5\, \gamma B' N_\gamma s_\gamma d_\gamma$$

Inserting values computed with terms of value 1.0 not shown (except d_γ) and using B'=1.3, we obtain,

Problem 4-5 (cont..)

$$q_{ult} = 20\,(51)(1.69)(1.4) + 1.8\,(18.0)(38)(1.55)(1.25) + 0.5\,(18.0)(1.3)(40)(0.62)(1.0)$$

=2413+2385+290=5088 kPa.

For SF=3.0, the allowable bearing pressure $q_a$is,

q_a= 5088/3.0=1696 ≈1700kPa.

The actual soil pressure is,

$$q_{act} = \frac{1800}{B'L'} = \frac{1800}{1.3x1.4} = 989 \text{ kPa}$$

Note:

- Both q_a and q_{act} are very large & large respectively.
- Here, settlement may be the limiting factor.
- **Some recommend max $q_a$ia around 500 kPa.**
- Small footing with large column loads are not very confidence-inspiring during construction.
- Not the location to be excessively conservative.

Problem 4-5 (cont..)

By Meyerhof Method with R_e: Method uses actual dimensions **B x L**.

$$K_p = tan^2(45 + \emptyset/2) = tan^2(45 + \mathbf{36}/2) = 3.85$$

$$\sqrt{K_p} = 1.96$$

From table 4-3:

N_c**=51** N_q**=38 (same as Hansen)** N_γ**=44.4 ≈44**

Also,

$$s_c = 1 + 0.2K_p(B/L) = 1 + 0.2\ (3.85)(1.8/1.8) = 1.77$$

$$s_q = s_\gamma = 1 + 0.1K_p(B/L) = 1 + 0.1\ (3.85)(1.8/1.8) = 1.39$$

$$d_c = 1 + 0.2\sqrt{K_p}(D/B) = 1 + 0.2\ (1.96)(1.8/1.8) = 1.39$$

$$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}(D/B) = 1 + 0.1\ (1.96)(1.8/1.8) = 1.20$$

Problem 4-5 (cont..)

Meyerhof equation for vertical load Table 4-1

$$q_{ult} = c\,N_c s_c d_c + q\,N_q s_q d_q + 0.5\,\gamma B N_\gamma s_\gamma d_\gamma$$

Inserting the values,

q_{ult}= 20(51)(1.77)(1.39) + 1.8(18.0)(38)(1.39)(1.20) +0.5(18.0)(1.8)(44)((1.39)(1.20)

=2510+2054+1189= 5752kPa

Two reduction factors since two way eccentricity. Use equation for cohesion less soils since c=20kPa only.

$R_{eB} = 1 - \sqrt{(e_y/B)} = 1 - \sqrt{(0.25/1.8)} = 1-0.37=0.63$

$R_{eL} = 1 - \sqrt{(e_x/L)} = 1 - \sqrt{(0.20/1.8)} = 0.67$

Now, $R_e = R_{eB} X R_{eL}$=0.63x0.67

q_{ult}=5752x(0.63x0.67)= 2428 kPa

$q_a = q_{ult}$/SF= 2428/3.0=809 ≈810 kPa

Actual soil pressure q_{act}= 1800/BL= 1800/(1.8x1.8)=555 kPa

Problem 4-5 (cont..)

Note:

- **Hansen** gives high q_a with satisfactory base and Meyerhof also give satisfactory base with very different q_a.
- As a geotechnical engineers what you can do?
- **Probably limit the soil pressure to q_a=500 kPa which is less either of these twos.**
- Now compute a base as follows:

$$B'L' = 1800/500 = 3.6\ m^2$$

Original ratio of $B'/L' = 1.3/1.4 = 0.929$

$$0.929 \times L'^2 = 3.6$$

$$L' = 1.97\text{m}$$

$B'/1.97 = 0.929$ $\qquad$ $B' = 1.83$

From these,

B= B'+ $2e_y$=1.83+2x0.25=**2.33m** and **L= L'+ $2e_x$=1.97+2x0.20=2.37**

Suggest a square footing= 2.40m X 2.40m

Thanks