EWCE 343

Geotechnical and Foundation Engineering

Footing with Inclined Load

Footing with inclined Load:

- Inclined loads are produced when footing is loaded with **both vertical load and horizontal components of loading.**
- Such loads are common for horizontal **wind load** and gravity load on footings.
- Footings with inclined loadings are in **rolling equipment** (steel mils) and industrial foundations.
- Subjected to horizontal loads from materials going through the equipment or materials **push/pull applied** to foundations.
- Load inclination results in a BC reductions.

Inclination factors are given in Table 4-3 and Table 4-5b & 4-5c.

TABLE 4-1
Bearing-capacity equations by the several authors indicated

Terzaghi (1943). See Table 4-2 for typical values and for K_{pr} values.

$$q_{ult} = cN_c s_c + \bar{q}N_q + 0.5\gamma BN_\gamma s_\gamma \qquad N_q = \frac{a^2}{a\cos^2(45 + \phi/2)}$$

$$a = e^{(0.75\pi - \phi/2)\tan\phi}$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = \frac{\tan\phi}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right)$$

For:	strip	round	square
s_c =	1.0	1.3	1.3
s_γ =	1.0	0.6	0.8

Meyerhof (1963).* See Table 4-3 for shape, depth, and inclination factors.

Vertical load: $q_{ult} = cN_c s_c d_c + \bar{q}N_q s_q d_q + 0.5\gamma BN_\gamma s_\gamma d_\gamma$

Inclined load: $q_{ult} = cN_c d_c i_c + \bar{q}N_q d_q i_q + 0.5\gamma BN_\gamma d_\gamma i_\gamma$

$$N_q = e^{\pi\tan\phi}\tan^2\left(45 + \frac{\phi}{2}\right)$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = (N_q - 1)\tan(1.4\phi)$$

Hansen (1970).* See Table 4-5 for shape, depth, and other factors.

General:† $q_{ult} = cN_c s_c d_c i_c g_c b_c + \bar{q}N_q s_q d_q i_q g_q b_q + 0.5\gamma BN_\gamma s_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$

when $\phi = 0$

use $q_{ult} = 5.14 s_u(1 + s'_c + d'_c - i'_c - b'_c - g'_c) + \bar{q}$

N_q = same as Meyerhof above

N_c = same as Meyerhof above

$N_\gamma = 1.5(N_q - 1)\tan\phi$

Vesić (1973, 1975).* See Table 4-5 for shape, depth, and other factors. Use Hansen's equations above.

N_q = same as Meyerhof above

N_c = same as Meyerhof above

$N_\gamma = 2(N_q + 1)\tan\phi$

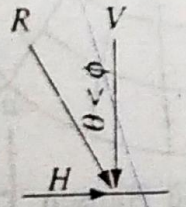
---

*These methods require a trial process to obtain design base dimensions since width B and length L are needed to compute shape, depth, and influence factors.

†See Sec. 4.6 when $i_c < 1$.

TABLE 4-3

Shape, depth, and inclination factors for the Meyerhof bearing-capacity equations of Table 4-1

Factors	Value	For
Shape:	$s_c = 1 + 0.2K_p\frac{B}{L}$	Any ϕ
	$s_q = s_\gamma = 1 + 0.1K_p\frac{B}{L}$	$\phi > 10^\circ$
	$s_q = s_\gamma = 1$	$\phi = 0$
Depth:	$d_c = 1 + 0.2\sqrt{K_p}\frac{D}{B}$	Any ϕ
	$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B}$	$\phi > 10$
	$d_q = d_\gamma = 1$	$\phi = 0$
Inclination:	$i_c = i_q = \left(1 - \frac{\theta^\circ}{90^\circ}\right)^2$	Any ϕ
	$i_\gamma = \left(1 - \frac{\theta^\circ}{\phi^\circ}\right)^2$	$\phi > 0$
	$i_\gamma = 0$ for $\theta > 0$	$\phi = 0$

Where $K_p = \tan^2(45 + \phi/2)$ as in Fig. 4-2

θ = angle of resultant R measured from vertical without a sign; if $\theta = 0$ all $i_i = 1.0$.

B, L, D = previously defined

TABLE 4-5*b*

Table of inclination, ground, and base factors for the Hansen (1970) equations. See Table 4-5*c* for equivalent Vesić equations.

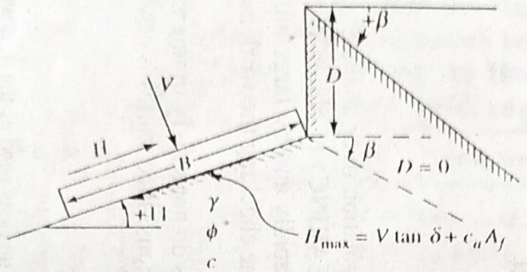
Inclination factors	Ground factors (base on slope)
$i_c' = 0.5 - \sqrt{1 - \frac{H_i}{A_f C_a}}$	$g_c' = \frac{\beta^\circ}{147^\circ}$
$i_c = i_q - \frac{1 - i_q}{N_q - 1}$	$g_c = 1.0 - \frac{\beta^\circ}{147^\circ}$
$i_q = \left[1 - \frac{0.5H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_1}$ $2 \le \alpha_1 \le 5$	$g_q = g_\gamma = (1 - 0.5\tan\beta)^5$
Base factors (tilted base)	
$i_\gamma = \left[1 - \frac{0.7H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_2}$	$b_c' = \frac{\eta^\circ}{147^\circ}$ $(\phi = 0)$
$i_\gamma = \left[1 - \frac{(0.7 - \eta^\circ/450^\circ)H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_2}$ $2 \le \alpha_2 \le 5$	$b_c = 1 - \frac{\eta^\circ}{147^\circ}$ $(\phi > 0)$ $b_q = \exp(-2\eta\tan\phi)$ $b_\gamma = \exp(-2.7\eta\tan\phi)$ η in radians

Notes:

1. Use H_i as either H_B or H_L, or both if $H_L > 0$.
2. Hansen (1970) did not give an i_c for $\phi > 0$. The value above is from Hansen (1961) and also used by Vesić.
3. Variable c_a = base adhesion, on the order of 0.6 to 1.0 × base cohesion.
4. Refer to sketch for identification of angles η and β, footing depth D, location of H_i (parallel and at top of base slab; usually also produces eccentricity). Especially note V = force *normal* to base and is not the resultant R from combining V and H_i.

Notes: $\beta + \eta \quad 90^\circ$ (Both β and η have signs (+) shown.)

$\beta \quad \phi$



For: $L/B \le 2$ use ϕ_{tr}

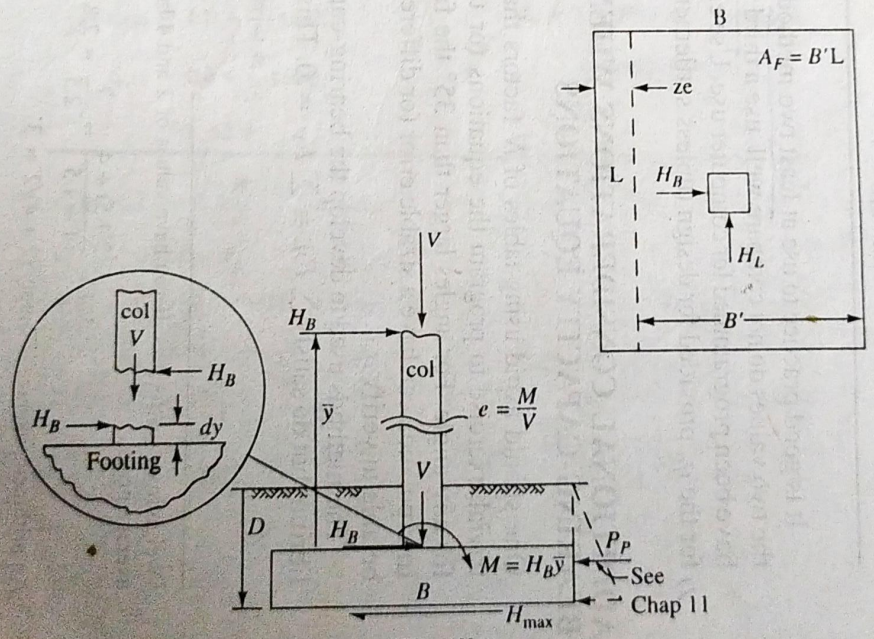
$L/B > 2$ use $\phi_{ps} = 1.5\,\phi_{tr} - 17^\circ$

$\phi_{tr} \le 34^\circ$ use $\phi_{tr} = \phi_{ps}$

δ = friction angle between base and soil ($.5\phi \le \delta \le \phi$)

$A_f = B'L'$ (effective area)

c_a = base adhesion (0.6 to 1.0c)



$H_{max} + P_p \ge SF \times (H_B)$

TABLE 4-5*c*

Table of inclination, ground, and base factors for the Vesić (1973, 1975*b*) bearing-capacity equations. See notes below and refer to sketch for identification of terms.

Inclination factors	Ground factors (base on slope)
$i'_c = 1 - \dfrac{mH_i}{A_f c_a N_c}$ $(\phi = 0)$	$g'_c = \dfrac{\beta}{5.14}$ β in radians
$i_c = i_q - \dfrac{1 - i_q}{N_q - 1}$ $(\phi > 0)$	$g_c = i_q - \dfrac{1 - i_q}{5.14 \tan \phi}$ $\phi > 0$
i_q, and m defined below	i_q defined with i_c
$i_q = \left[1.0 - \dfrac{H_i}{V + A_f c_a \cot \phi}\right]^m$	$g_q = g_\gamma = (1.0 - \tan \beta)^2$
	Base factors (tilted base)
$i_\gamma = \left[1.0 - \dfrac{H_i}{V + A_f c_a \cot \phi}\right]^{m+1}$	$b'_c = g'_c$ $(\phi = 0)$
$m = m_B = \dfrac{2 + B/L}{1 + B/L}$	$b_c = 1 - \dfrac{2\beta}{5.14 \tan \phi}$
$m = m_L = \dfrac{2 + L/B}{1 + L/B}$	$b_q = b_\gamma = (1.0 - \eta \tan \phi)^2$

Notes:

1. When $\phi = 0$ (and $\beta \neq 0$) use $N_\gamma = -2 \sin(\pm\beta)$ in N_γ term.
2. Compute $m = m_B$ when $H_i = H_B$ (H parallel to B) and $m = m_L$ when $H_i = H_L$ (H parallel to L). If you have both H_B and H_L use $m = \sqrt{m_B^2 + m_L^2}$. Note use of B and L, not B', L'.
3. Refer to Table sketch and Tables 4-5*a*,*b* for term identification.
4. Terms N_c, N_q, and N_γ are identified in Table 4-1.
5. Vesić always uses the bearing-capacity equation given in Table 4-1 (uses B' in the N_γ term even when $H_i = H_L$).
6. H_i term ≤ 1.0 for computing i_q, i_γ (always).

TABLE 4-4

Bearing-capacity factors for the Meyerhof, Hansen, and Vesić bearing-capacity equations

Note that N_c and N_q are the same for all three methods; subscripts identify author for N_γ

ϕ	N_c	N_q	$N_{\gamma(H)}$	$N_{\gamma(M)}$	$N_{\gamma(V)}$	N_q/N_c	$2\tan\phi(1-\sin\phi)^2$
0	5.14*	1.0	0.0	0.0	0.0	0.195	0.000
5	6.49	1.6	0.1	0.1	0.4	0.242	0.146
10	8.34	2.5	0.4	0.4	1.2	0.296	0.241
15	10.97	3.9	1.2	1.1	2.6	0.359	0.294
20	14.83	6.4	2.9	2.9	5.4	0.431	0.315
25	20.71	10.7	6.8	6.8	10.9	0.514	0.311
26	22.25	11.8	7.9	8.0	12.5	0.533	0.308
28	25.79	14.7	10.9	11.2	16.7	0.570	0.299
30	30.13	18.4	15.1	15.7	22.4	0.610	0.289
32	35.47	23.2	20.8	22.0	30.2	0.653	0.276
34	42.14	29.4	28.7	31.1	41.0	0.698	0.262
36	50.55	37.7	40.0	44.4	56.2	0.746	0.247
38	61.31	48.9	56.1	64.0	77.9	0.797	0.231
40	75.25	64.1	79.4	93.6	109.3	0.852	0.214
45	133.73	134.7	200.5	262.3	271.3	1.007	0.172
50	266.50	318.5	567.4	871.7	761.3	1.195	0.131

* $= \pi + 2$ as limit when $\phi \rightarrow 0°$.

Slight differences in above table can be obtained using program BEARING.EXE on diskette depending on computer used and whether or not it has floating point.

Footing with inclined Load

Table 4-3: Self explanatory of Meyerhof's factors.

Table 4-5b: Hansen factors. $2 \leq \alpha_1 \leq 5$ for i_c, i_q, i_γ

Table 4-5c: Vesic Equation factors

$$m = m_B = \frac{2 + \frac{B}{L}}{1 + \frac{B}{L}} \qquad m = m_A = \frac{2 + \frac{L}{B}}{1 + \frac{L}{B}}$$

Using inclination factors:

- General case of inclined loading:

  H=H_B parallel to B dimension. **If H_B=0**, $i_{cB} = i_{qB} = i_{\gamma B}$=1.0

  H=H_L parallel to L dimension. **If H_L=0,** $i_{cL} = i_{qL} = i_{\gamma L}$=1.0

Footing with inclined Load:

- **H_i values are used to compute inclination factors** for **Hansen Equation.**

Compute i_i using Table **4-5b**

Compute shape factors for Hansen equation

$$s'_{cB} = 0.2\,\frac{B_{icB}}{L} \qquad s'_{cL} = 0.2\,\frac{L_{icB}}{B} \text{ for } \emptyset=0$$

$$s_{cB} = 1+\frac{N_q}{N_c}\,\frac{B'_{ic_B}}{L'} \qquad s_{cL} = 1+\frac{N_q}{N_c}\,\frac{L'_{ic_L}}{B'}$$

$$s_{qB} = 1+\sin\varphi\,\frac{B'_{iq_B}}{L'} \qquad s_{qL} = 1+\sin\varphi\,\frac{L'_{iq_L}}{B'}$$

$$s_{\gamma B} = 1-0.4\,\frac{B'_{i\gamma_B}}{L'_{i\gamma_L}} \qquad s_{\gamma L} = 1-0.4\,\frac{L'_{i\gamma_L}}{B'_{i\gamma_B}} \qquad \boldsymbol{s_{\gamma i} \geq 0.6}$$

Footing with inclined Load:

Hansen bearing capacity equation

$$q_{ult} = cN_c s_{c.B} d_{c.B} i_{c.B} + qN_q s_{q.B} d_{q.B} i_{q.B} + 0.5\,\gamma B' N_\gamma s_{\gamma.B} d_{\gamma.B} i_{\gamma.B}$$

Or

$$q_{ult} = cN_c s_{c.L} d_{c.L} i_{c.L} + qN_q s_{q.L} d_{q.L} i_{q.L} + 0.5\,\gamma B' N_\gamma s_{\gamma.L} d_{\gamma.L} i_{\gamma.L}$$

Use smaller of q_{ult}.

Footing with inclined Load:

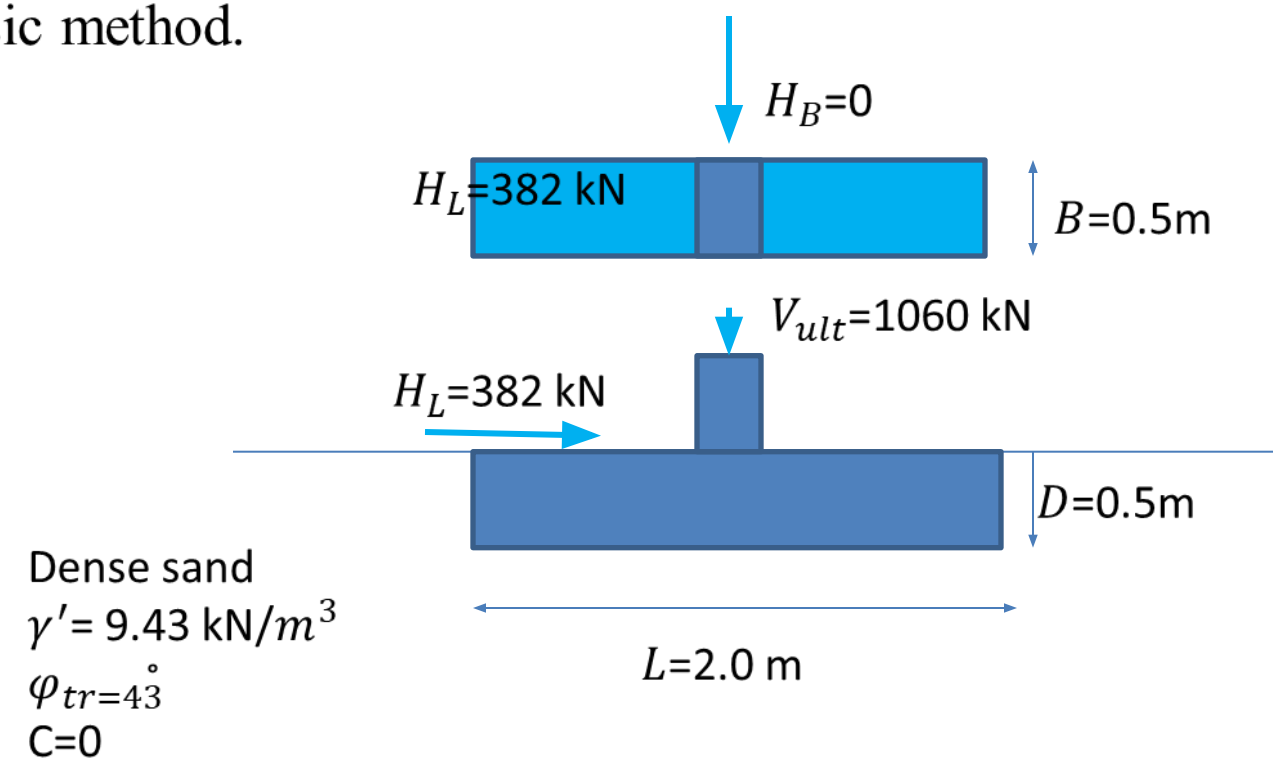
Ex. 4-6. Data shown in the sketch of load test:

$H_{L.ult}$= 382 kN

V_{ult}=1060 kN

Required:

- Find out the q_{ult} by Hansen method
- Find out the q_{ult} by vesic method.



Footing with inclined Load:

Solution:

φ_{tr}=43°

Check: $\frac{L}{B}=\frac{2}{0.5}$ =4 >2 use φ_{ps}

φ_{ps}=1.1 φ_{tr}= 1.1x43= 47.3 °

φ_{ps}=1.5 φ_{tr}-17= 1.5x43 -17= 47.5 °

use $\varphi = 47$ °

Hansen method

From Table 4.4, for $\varphi = 47$ ° N_c= 187 N_q= 208 N_γ= 347

N_q/N_c=1.10 $2\tan\varphi\,(1-sin\varphi)^2$=0.155

Since c=0, and b_i=g_i=1.0

q_{ult}= q$N_q s_q d_q i_q$ + 0.5 $\gamma B' N_\gamma s_\gamma d_\gamma i_\gamma$

Footing with inclined Load:

Table 4-5a: depth factor

dq= 1+2tan $\varphi(1-sin\varphi)^2$k **For D/B≤ 1, *use* k=D/B=0.5/0.5=1.0**

dq_B=1+0.155x1=1.155

dq_L=1+2tan $\varphi(1-sin\varphi)^2$k **D/L= 0.5/2.0=0.25 use k=0.25**

= 1+ 0.155x0.25=1.04

$d\gamma_L$=$d\gamma_B$= 1.0

Table 4-5c: Inclination factor

$$i_q = \left(1 - \frac{0.5H_i}{V + A_f c_a \cot\emptyset}\right)^{\sigma_1}$$

$2 \leq \sigma_1 \geq 5$

i_{qB}=1.0

V=vertical load

c_a=base cohesion

H_i= horizontal load either H_B or H_L

Footing with inclined Load:

$$i_q = \left(1 - \frac{0.5H_i}{V + A_f c_a \cot\emptyset}\right)^{\sigma_1}$$

$$i_{qL} = \left(1 - \frac{0.5x382}{V+0}\right)^{2.5} = 0.608$$

$$i_\gamma = \left(1 - \frac{0.7H_i}{V + A_f c_a \cot\emptyset}\right)^{\sigma_2} \qquad 2 \leq \sigma_2 \geq 5 \qquad \sigma_2 = 3.5$$

$$i_{\gamma B} = 1.0$$

$$i_{\gamma L} = \left(1 - \frac{0.7x382}{V+0}\right)^{3.5} = 0.361$$

Shape factor Table 4.5a

$$s_{q(H)} = 1 + \sin\emptyset \frac{B}{L}$$

$$s_{qB} = 1 + \sin\emptyset\left(\frac{Bi_{\gamma B}}{L}\right) = \mathbf{1 + \sin 47\left(\frac{0.5\ x\ 1}{2}\right) = 1.18}$$

$$s_{qL} = 1 + \sin\emptyset\left(\frac{Li_{\gamma L}}{B}\right) = \mathbf{1 + \sin 47\left(\frac{2\ x\ 0.608}{0.5}\right) = 2.78}$$

Footing with inclined Load:

Shape factor Table 4.5a

$$s_{q(H)} = 1 + \sin\emptyset \frac{B}{L}$$

$$s_{qB} = 1 + \sin\emptyset\left(\frac{Bi_{\gamma B}}{L}\right) = \mathbf{1 + \sin 47\ \left(\frac{0.5\ x\ 1}{2}\right) = 1.18}$$

$$s_{qL} = 1 + \sin\emptyset\left(\frac{Li_{\gamma L}}{B}\right) = \mathbf{1 + \sin 47\ \left(\frac{2\ x\ 0.608}{0.5}\right) = 2.78}$$

$$s_{\gamma(H)} = 1 - 0.4\frac{B}{L} \qquad \geq \mathbf{0.6}$$

$$s_{\gamma B} = 1 - 0.4\frac{Bi_{\gamma B}}{Li_{\gamma L}} = \mathbf{1 - 0.4\ \frac{0.5x1.0}{2x0.361} = 0.723 > 0.6}\quad \mathbf{ok}$$

$$s_{\gamma L} = 1 - 0.4\frac{Li_{\gamma L}}{Bi_{\gamma B}} = \mathbf{1 - 0.4\ \frac{2x0.361}{0.5x1} = 0.422 < 0.6} \qquad \mathbf{use\ 0.6}$$

Footing with inclined Load:

Along B,

$$q_{ult} = q\,(=\gamma D)N_q s_{q.B} d_{q.B} i_{q.B} + 0.5\,\gamma B' N_\gamma s_{\gamma.B} d_{\gamma.B} i_{\gamma.B}$$

$$=9.43(0.5)(187)(1.18)(1.16)(1)+9.43(0.5)(300)(0.73)(1)(1)$$

$$=1206.9+ 311.3= 1718.2\approx 1700 \text{ kpa}$$

Along L,

$$q_{ult} = q\,(=\gamma D)N_q s_{q.L} d_{q.L} i_{q.L} + 0.5\,\gamma L' N_\gamma s_{\gamma.L} d_{\gamma.L} i_{\gamma.L}$$

$$=9.43(0.5)(187)(2.78)(1.04)(.608)+9.43(0.5)((2)(300)(0.6)(1)(0.361)$$

$$=1549.9+612.76=2162.7 \approx 2150 \text{ kPa}$$

Footing with inclined Load:

- Smaller Value, q_{ult}=1700 kPa

- From load test, $q_{ult}=\frac{V}{BxL}=\frac{1060}{0.2x2.0}$ =1060 kPa

Footing with inclined Load (Vesic'):

Solution:

Vesic' Equation

φ_{tr}=43°

Check: $\frac{L}{B}=\frac{2}{0.5}=4>2$ use φ_{ps}

φ_{ps}=1.1 φ_{tr}= 1.1x43= 47.3 °

φ_{ps}=1.5 φ_{tr}-17= 1.5x43 -17= 47.5 °

use $\varphi = 47$ °

Vesic' method

From **Table 4.4, for $\varphi = 47$ °** N_q= 187 N_γ= 404

$2\tan\varphi\,(1-sin\varphi)^2$=0.155

Since c=0, and b_i=g_i=1.0

$q_{ult}= qN_q s_q d_q i_q + 0.5\,\gamma B' N_\gamma s_\gamma d_\gamma i_\gamma$

Footing with inclined Load:

Shape factors and depth factors: **Table 4-5a**

$$s_{q(v)} = 1 + \frac{B}{L}\tan\emptyset = 1 + \frac{0.5}{2}\tan 47 = 1.27$$

$$s_{\gamma(v)} = 1 - 0.4\frac{B}{L} \geq 0.6$$

$$= 1 - 0.4 \times \frac{0.5}{2.0} = 0.9 > 0.6 \text{ ok}$$

$$d_q = 1 + 2\tan\emptyset(1 - \sin\emptyset)k \quad \text{For} \frac{D}{B} \leq 1, use \ k = D/B = 0.5/0.5 = 1.0$$

$$= 1 + 0.155(1) = 1.16$$

$$d_\gamma = 1.0 \text{ for all } \emptyset$$

Footing with inclined Load (Vesic' equation):

Inclination Factor $\mathbf{m}=\frac{2+\frac{L}{B}}{1+\frac{L}{B}}=\frac{2+\frac{2}{0.5}}{1+\frac{2}{0.5}}=\frac{2+4}{1+4}=\frac{6}{5}=1.2$

$$i_q=(1-\frac{H_L}{V+A_f c_a cot\emptyset})^m$$

$$=(1-\frac{382}{1060+0})^{1.2}=0.585$$

$$i_\gamma=(1-\frac{H_i}{V+A_f c_a cot\emptyset})^{m+1}$$

$$=(1-\frac{H_i}{V+A_f c_a cot\emptyset})^{m+1}$$

$$=(1-1060)^{1.2+1}=0.374$$

Footing with inclined Load (Vesic' equation):

Since c=0, and b_i=g_i=1.0

$$q_{ult} = qN_q s_q d_q i_q + 0.5\,\gamma B' N_\gamma s_\gamma d_\gamma i_\gamma$$

Putting values,

$$q_{ult} = qN_q s_q d_q i_q + 0.5\,\gamma B' N_\gamma s_\gamma d_\gamma i_\gamma$$

= 9.43(0.5)(1.87)(1.27)(1.16)(0.585)+0.5(9.43)(0.5)(404)(0.9)(1)(0.374)

=759.9+320.6=1080.5≈ $1080kPa$

From Load test, $q_{ult} = \frac{V}{BxL} \frac{1060}{0.5x2}$=1060 kPa

Thanks