EWCE 343

Geotechnical and Foundation Engineering

Bearing Capacity

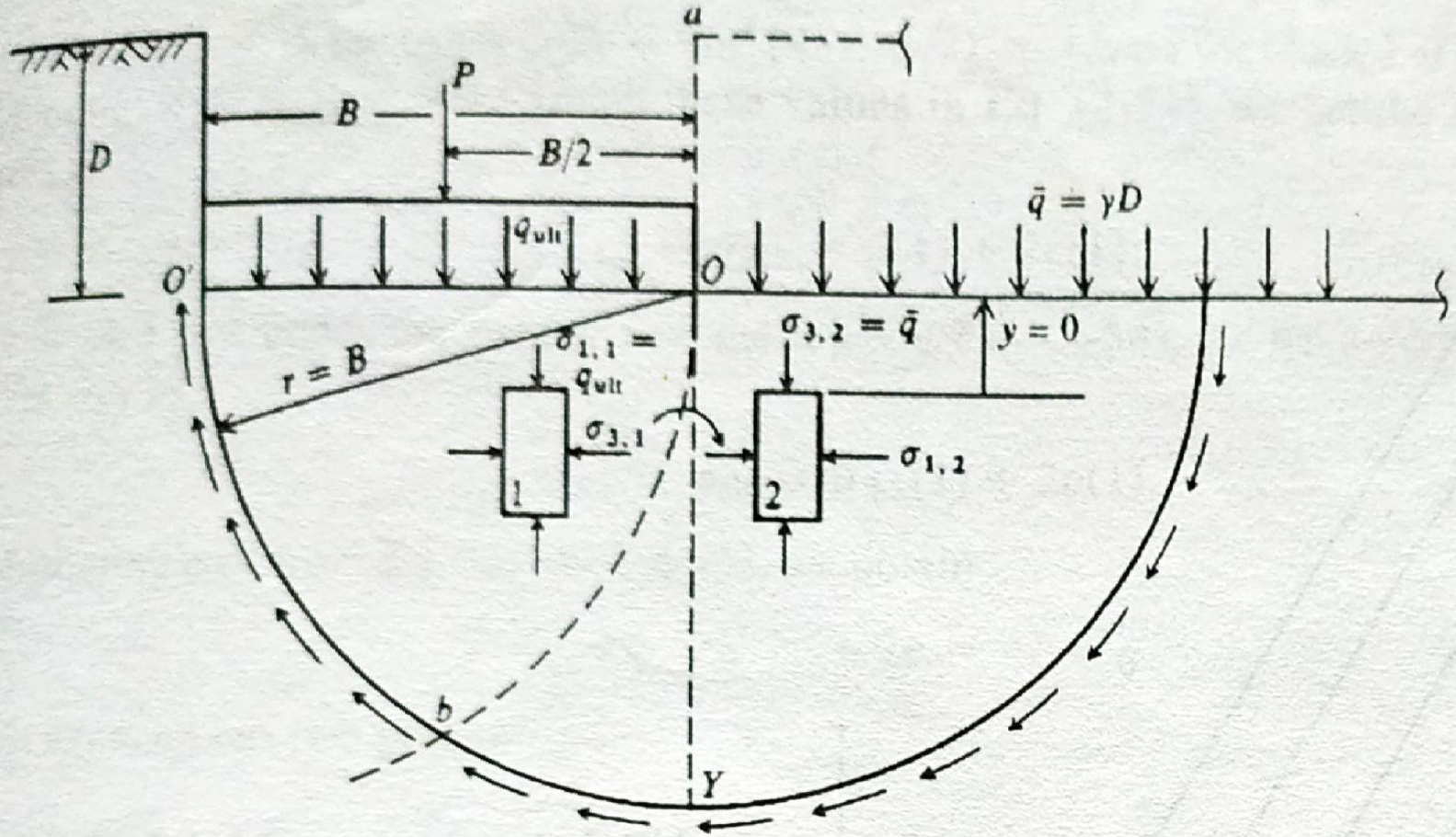


Figure-1: Footing on φ =0 Soil

Note: $\bar{q} = p_o' = \gamma' D$, but use $\bar{q}$. since this is the accepted symbol for bearing capacity computations.

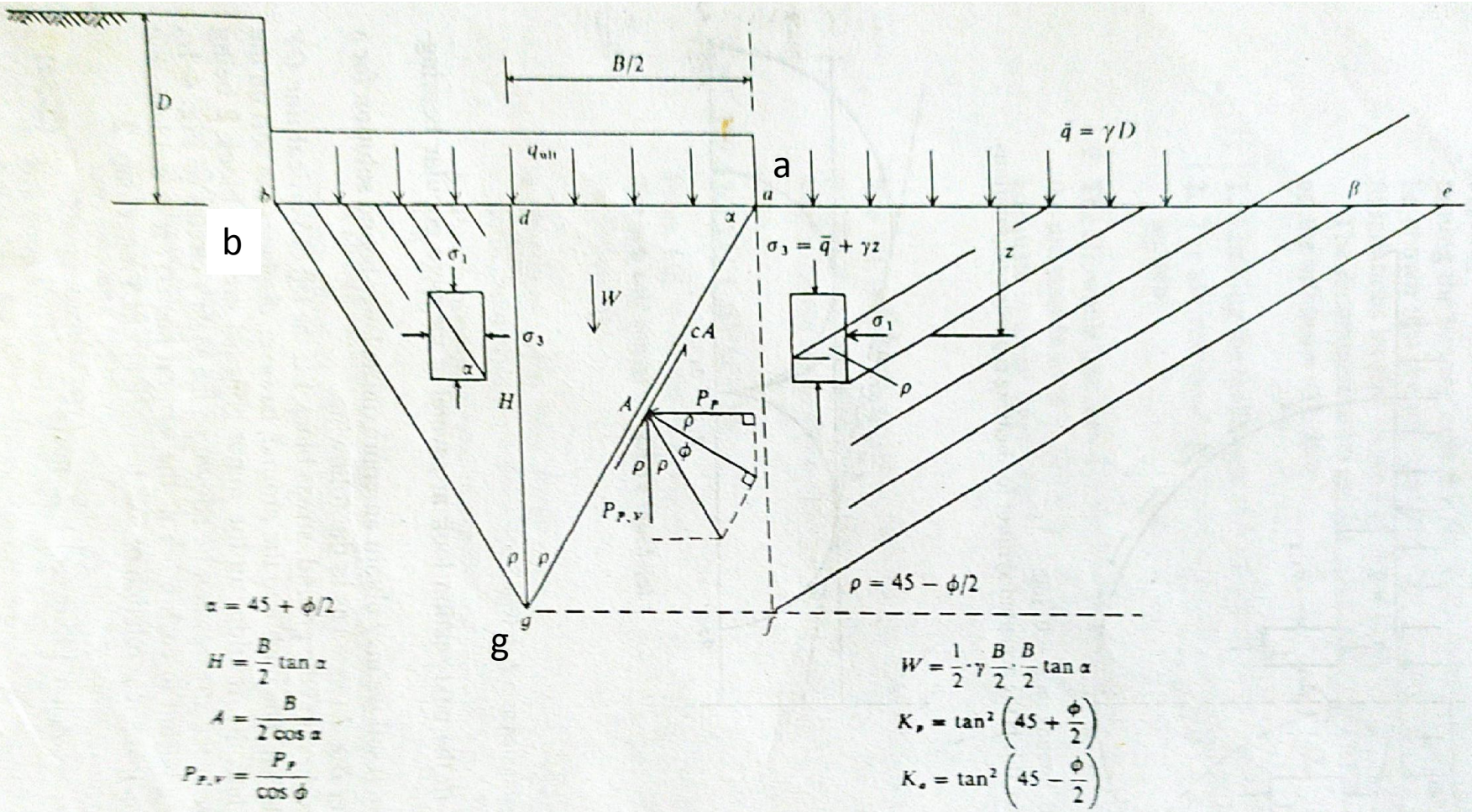


Figure-2: **Footing on ϕ-c Soil**

Bearing Capacity

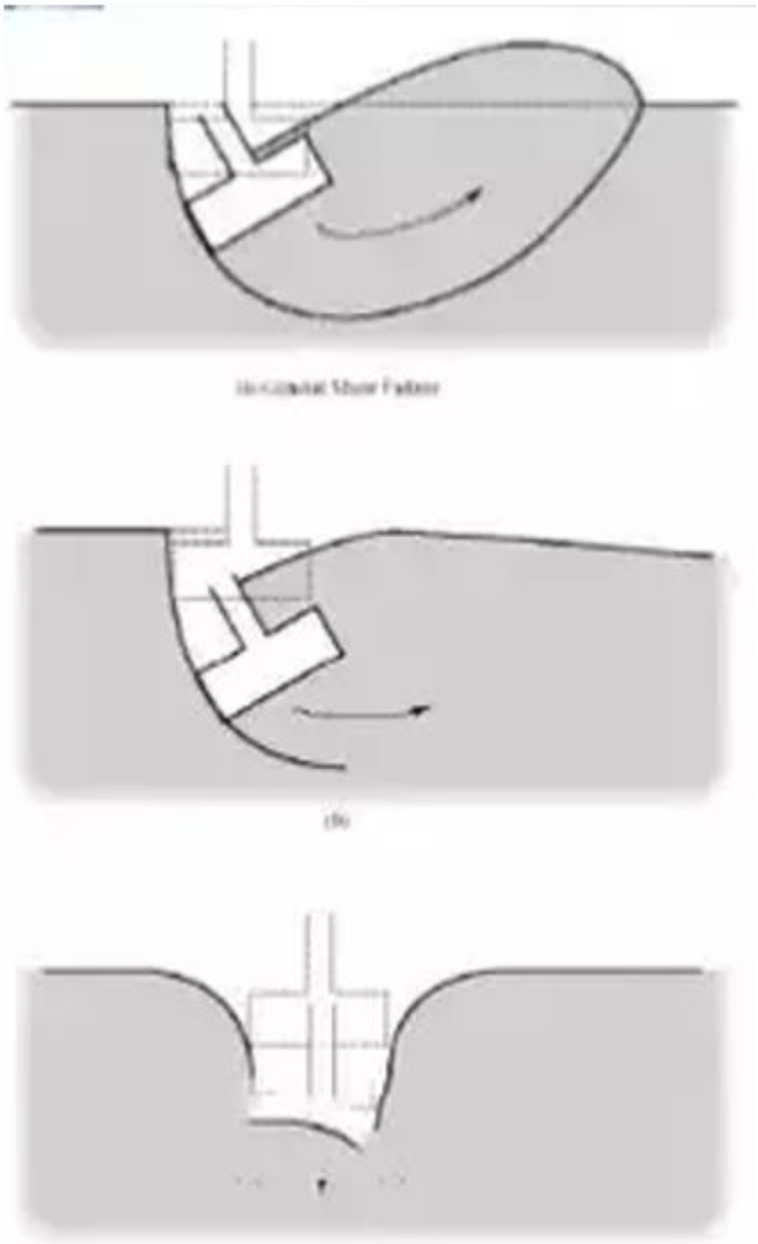
From Figure 1 and from Figure-2, it is clear that there are **two potential failure modes**, where the footing, when loaded to produce the maximum bearing pressure q_{ult}, will do one or both of he followings:

- **rotate as in Figure-1**, about some center of rotation (probably along the vertical line Oa) with **shear resistance developed** along the **perimeter of the slip zone** shown as circle.

- **Punch into the ground** as the **approximate wedge Ob$O'o$f Figure-1**.

- **Punch into the ground** as the **wedge agb of Figure-2**, or the **approximate wedge Ob$O'o$f Figure-1**.

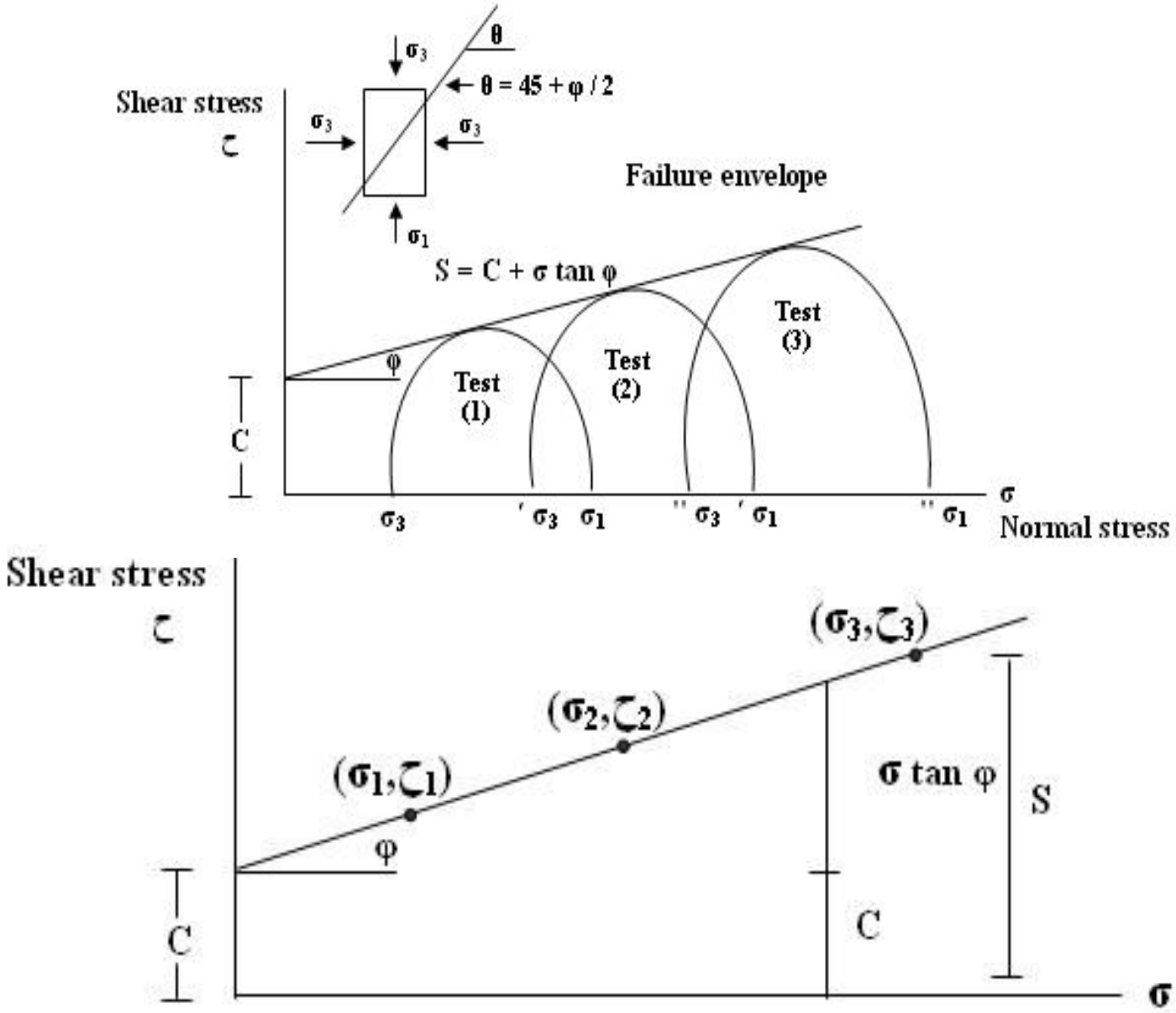
Shear Failures



- It should be apparent that **both modes of potential failure, develop the limiting soil shear strength along the slip path** according to the shear strength equation given as

$$s = c + \sigma_n \tan\varphi$$

- Although, this is the equation of the Mohr's circle **rupture envelope**, its physical meaning is illustrated in **Figure 3,** as it **applies along the slip lines.**



: *The relation between normal and shear stress*

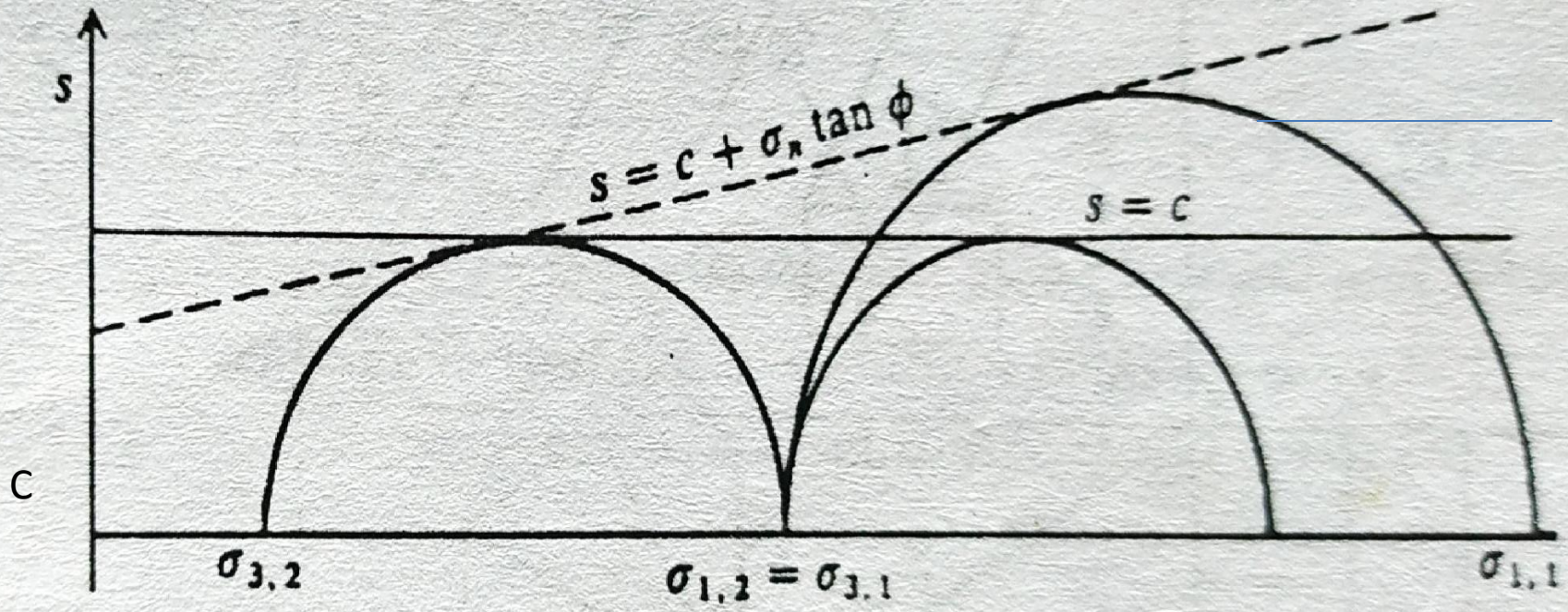


Figure-3: Mohr's Circle for φ=0 and φ-c Soil

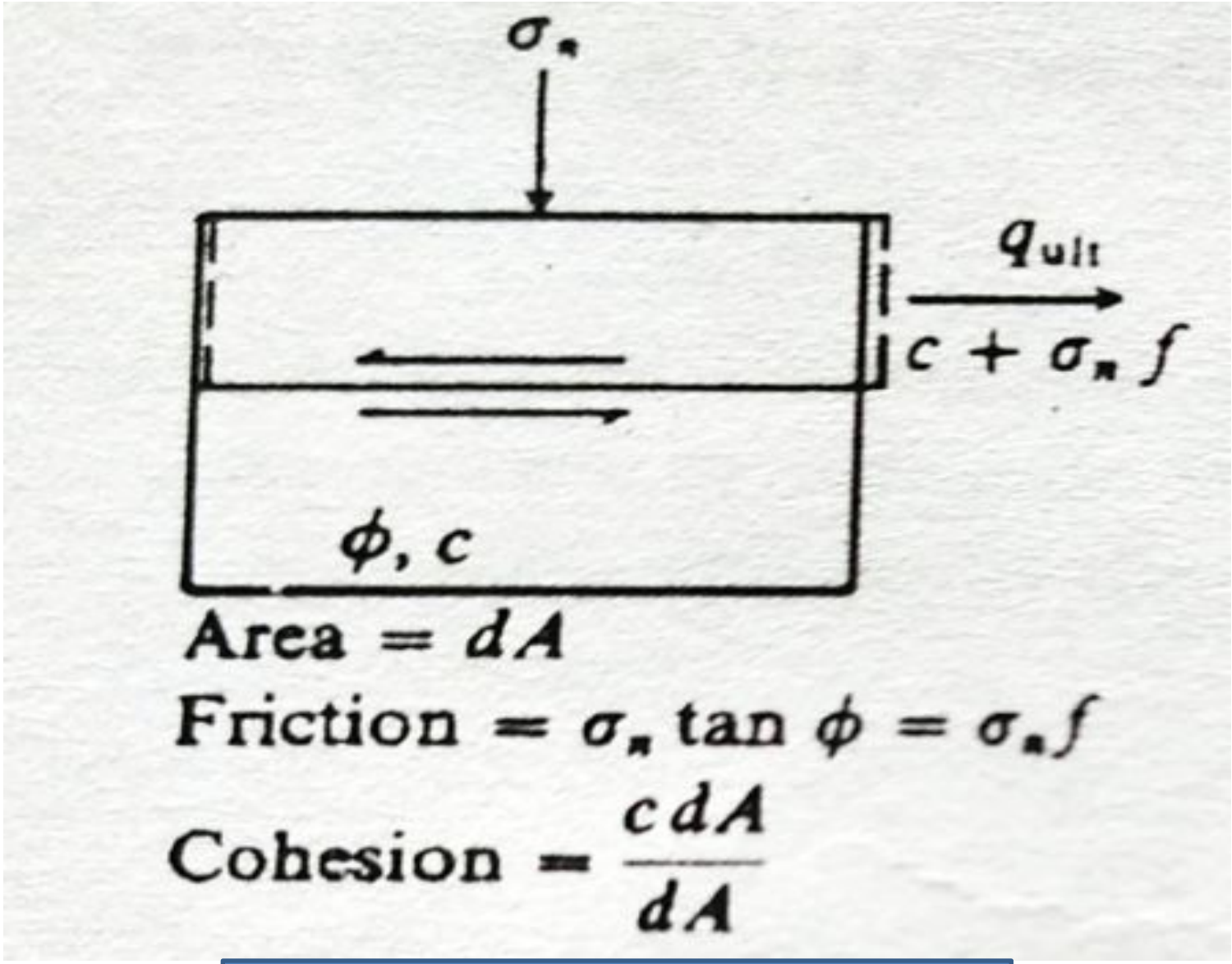


Figure- 3: Physical meaning of shear strength.

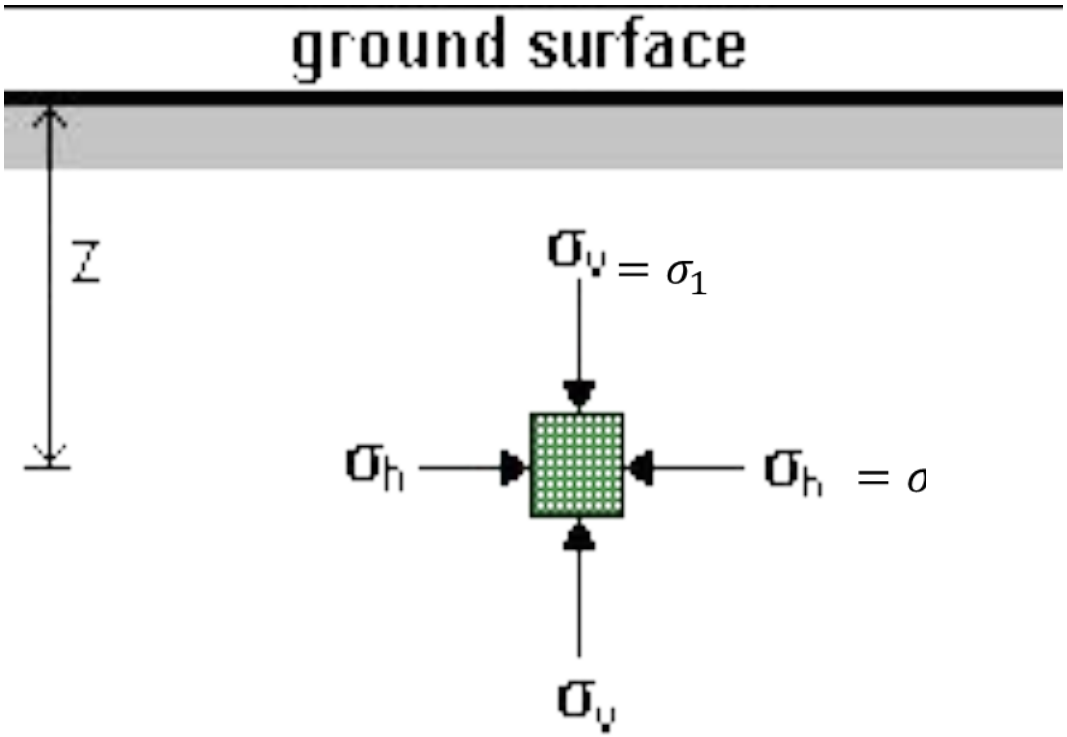
- It is common to use **total stress parameters** in this equation.

- For certain loading cases, **the effective stress parameters** may be more appropriate ie long term slowly applied loading.

- Total stress increases with depth and with unit weight:

Vertical total stress at depth z,

$\boldsymbol{\sigma_v = \gamma \, . \, z}$

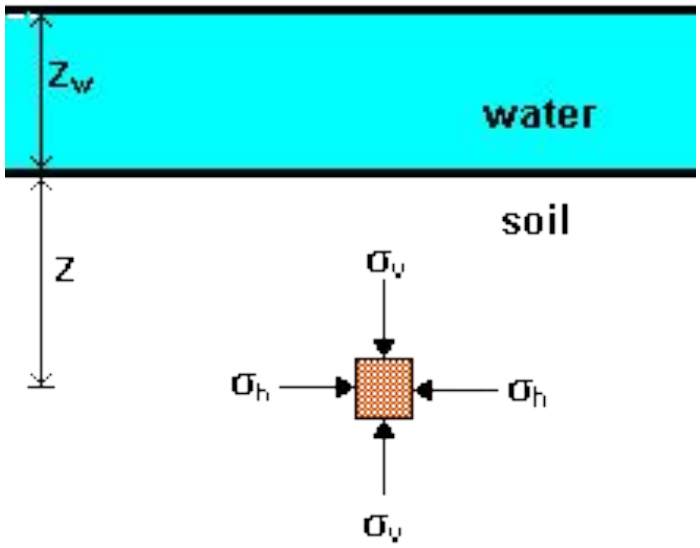


- Total stress is the sum of the weight of the soil up to the surface and the weight of water above this:

Vertical total stress at depth z, $\sigma_v = \gamma .z + \gamma_w .z_w$

where

γ_w = unit weight of water



Principle Stresses and principle Planes

- **Principle Stress:** Normal stress on a plane where shear stress is zero.

  σ_1= Maximum (major) principle stress.

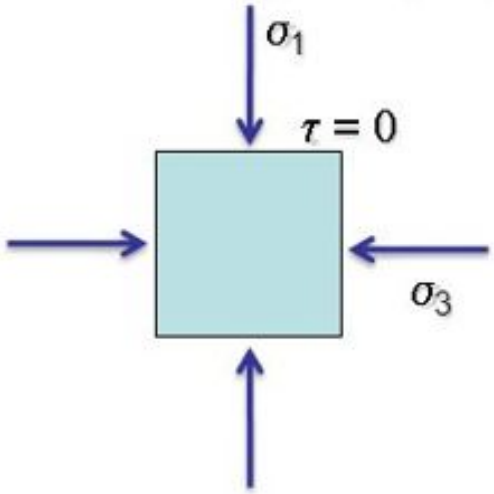
  σ_2= Intermediate principle stress (3D)

  σ_3= Minimum principle stress

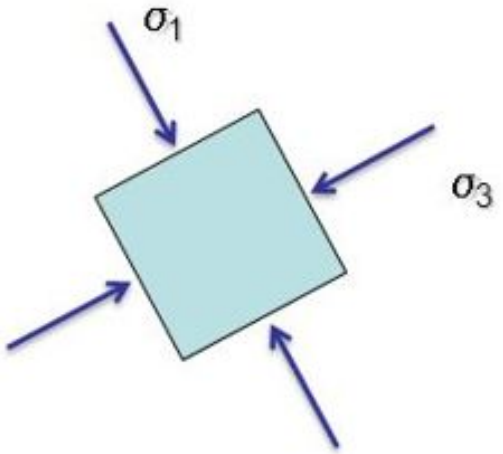
- **Principle Planes:**

  -Plane on which principle stresses act

  -Two **principle planes are perpendicular**, **but not necessarily horizontal or vertical.**

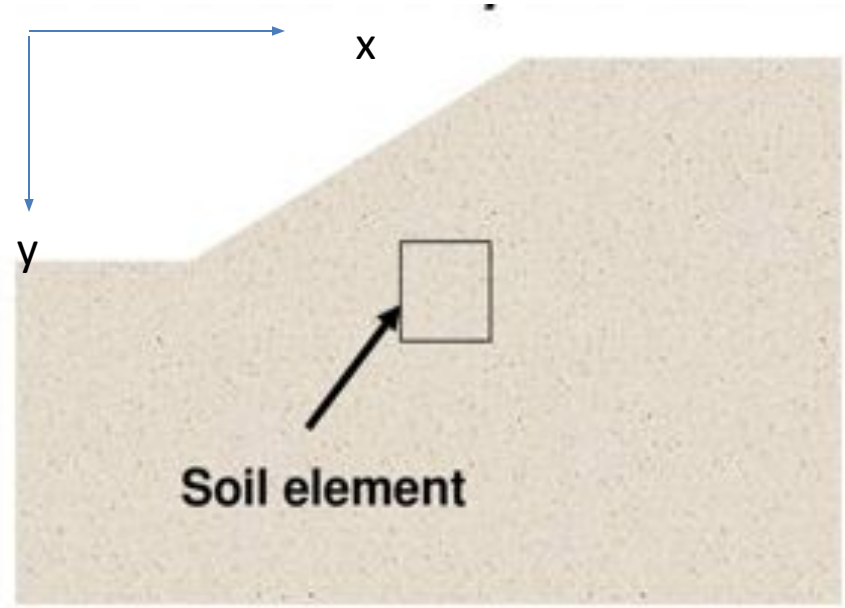


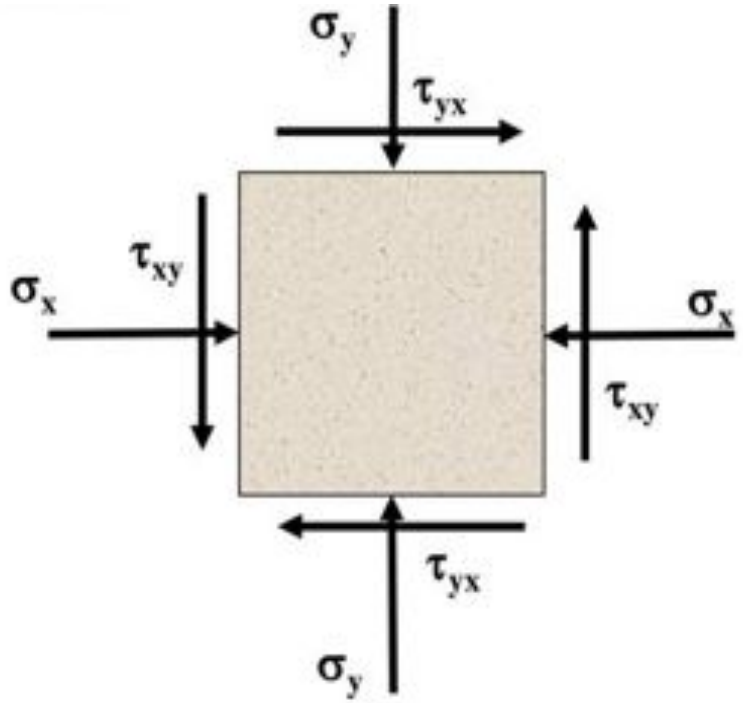
Not necessarily horizontal And vertical!



Normal Stress and shearing Stress on a plane:

- Consider only two dimensional cases.
- Soil element is in equilibrium, ie $\tau_{xy} = \tau_{yx}$



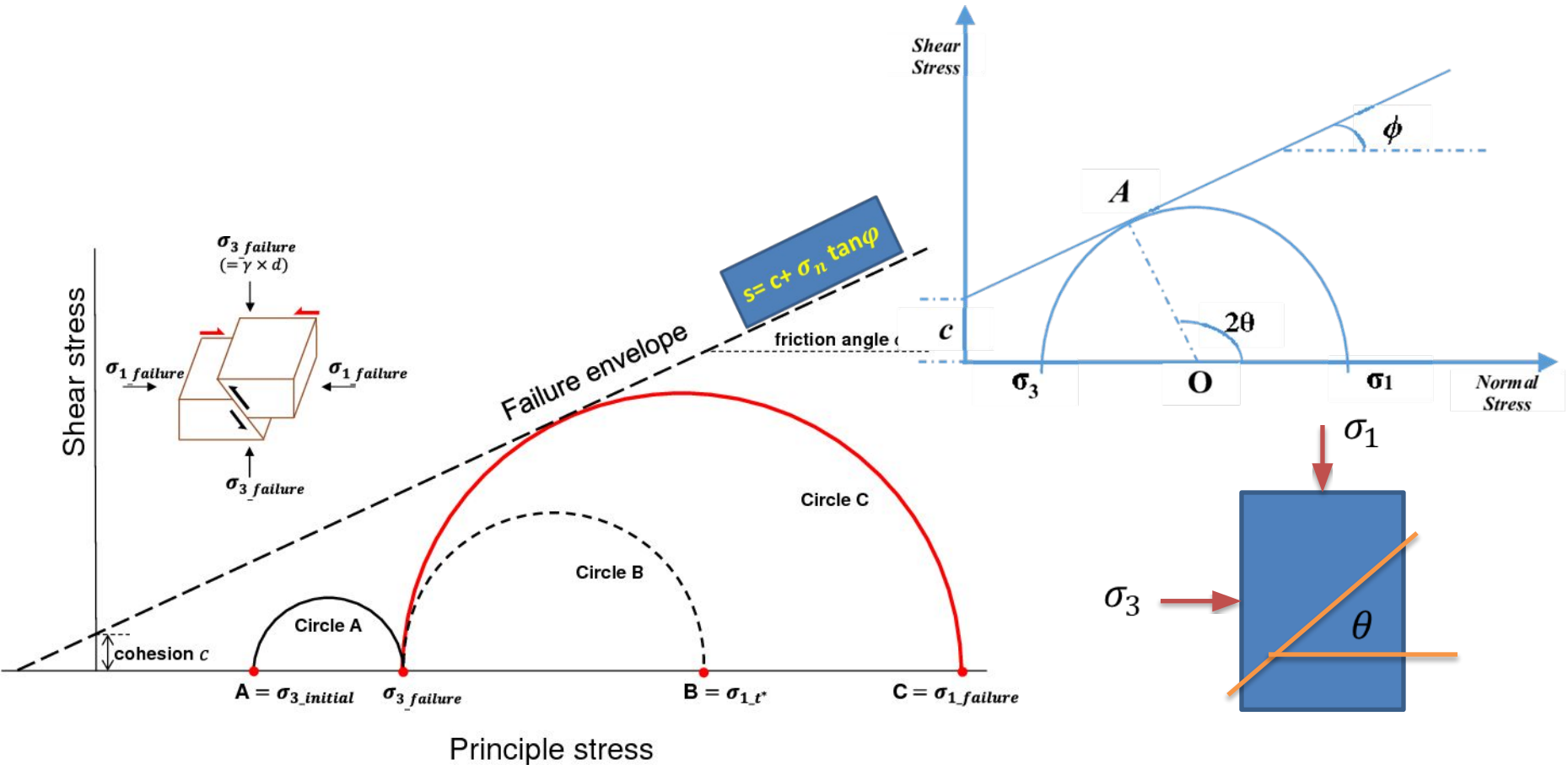


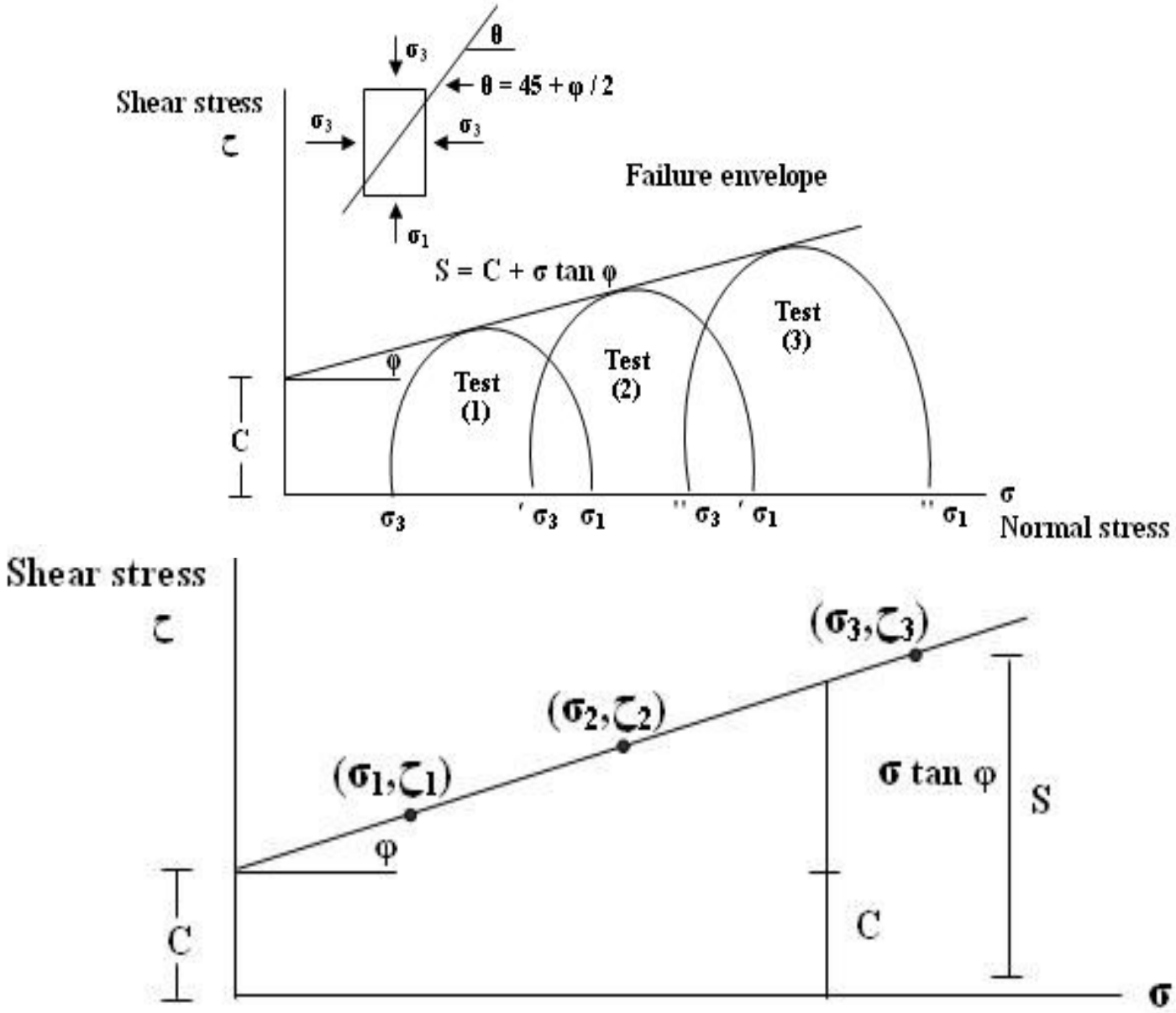
Mohr's Circle:

- **Mohr's circle**, invented by Christian Otto **Mohr**, is a two-dimensional graphical representation of general stresses at a point.



- Mohr's circle is a **graphical depiction** of two dimensional stress states.
- In soil mechanics, Mohr's circles are **used to visualize relationships between normal and shear stresses**, and to **estimate the maximum stresses,** based on **three or more soil samples** extracted from a site.





: *The relation between normal and shear stress*

Major and Minor Principal Stresses

- A **three-dimensional stress system** is applied to a soil mass. The stresses in the third direction, on the other hand, **are irrelevant**.

- *There are three planes on which the **shear stresses are zero** at any point in a stressed body.*

- Stress system is reduced to a two-dimensional model. So there are mainly **two planes** on which shear stresses are zero. *They are mainly mentioned as **principal planes**.*

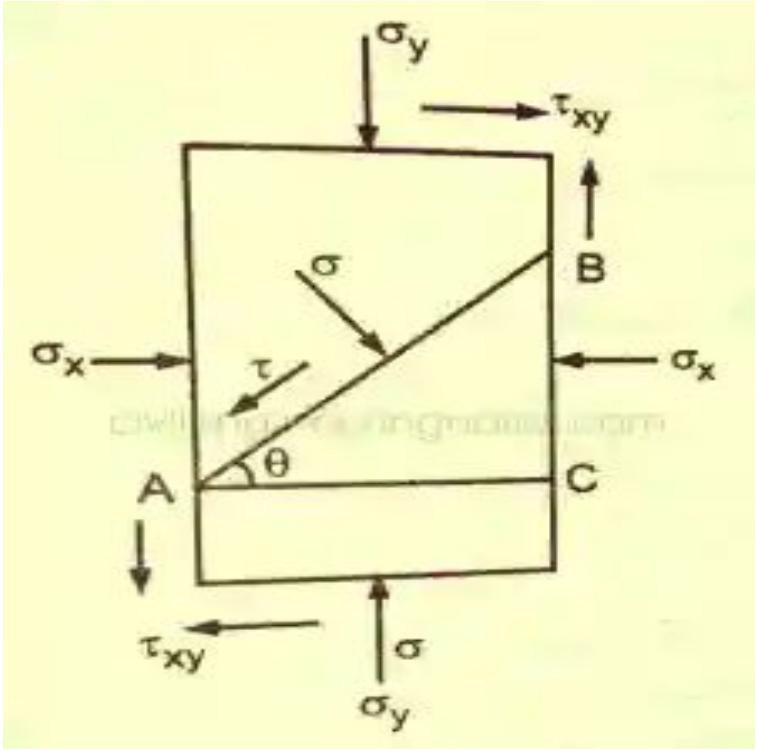
- **Compressive stresses on the principal planes** are at their highest or lowest levels.

- Plane with the **greatest compressive stress (σ_1)** is called a **major principal plane**, while the plane with the **least compressive stress** (σ_3) is called a **minor principal plane**.

- Third principle plane is subjected to the stress which is intermediate between (σ_1) and **(σ_3) which is of not much relevance.**

- Only the major principal is stress (σ_1) and the minor principal stress (σ_3) are important.

c



Unit weight of Soil, γ

- $\boldsymbol{\gamma = \frac{W}{V}}$ kN/m^3
- Dry unit weight, $\boldsymbol{\gamma_d} = \frac{weight\ of\ soil\ solids}{Total\ volume} = \frac{W_s}{V_t}$
- Bulk unit weight, $\boldsymbol{\gamma_b} = \frac{Total\ weight}{Total\ volume} = \frac{W_t}{V_t}$
- Unit weight of soil solids, $\boldsymbol{\gamma_s} = \frac{weight\ of\ soil\ solids}{Volume\ of\ soil\ solids} = \frac{W_s}{V_s}$

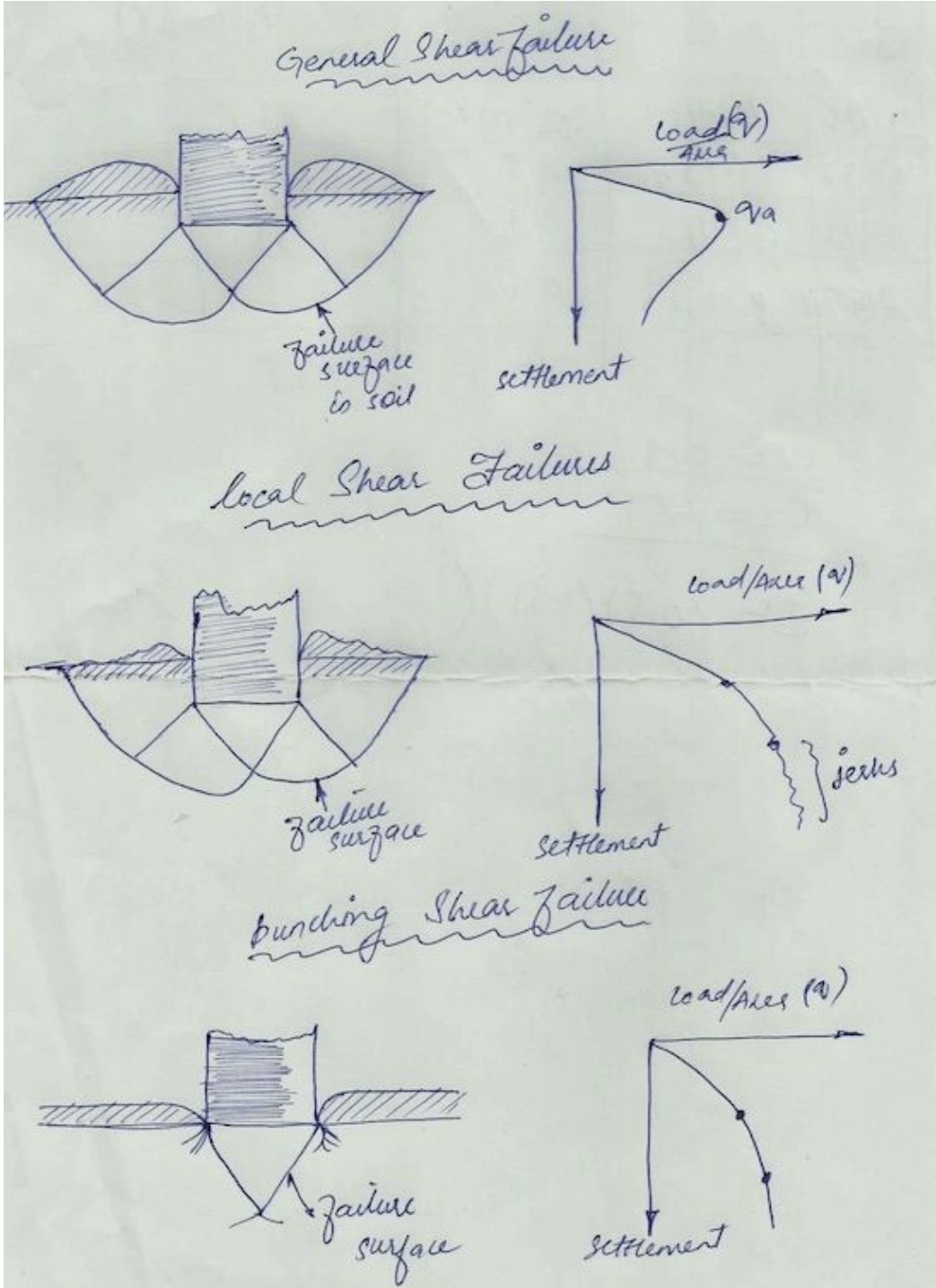
Bearing Capacity

Question: Describe the potential failure modes, where the footing, when loaded to produce maximum bearing pressure.

- Problem of **how to obtain a reliable value of maximum bearing pressure**, q_{ult}.

- Examine briefly some approximation **for** q_{ult}, to illustrate the complexity of the problem.

- q_{ult} = Maximum vertical pressure that can be applied to the ground surface, at which **point of shear failure mechanism develops** in the supporting soil.



$\varphi = 0$ Soil

Foundation Pushes in-to the ground:

- **Figure 1**, assume a **width of a strip footing B.**
- When the **foundation pushes in to the ground**, **stress block 1** to the left of vertical line OY has **principle stresses** as shown.
- **Push of foundation into the ground**, however, **displaces the soil on the right side** of the line OY laterally.
- Resulting in the **major principle stress on block 2 being horizontal** as shown.

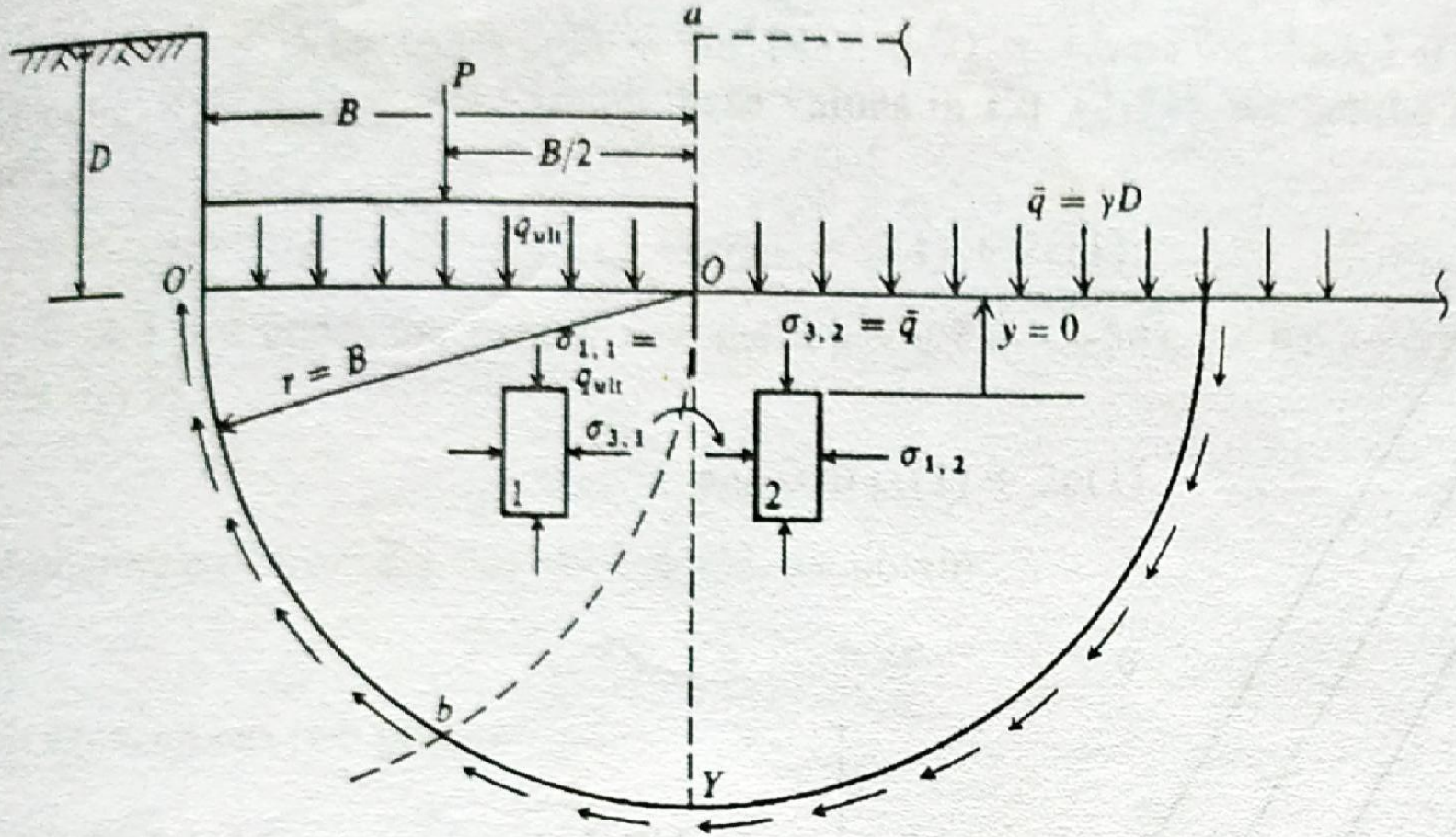


Figure-1: Footing on φ =0 Soil

Note: $\bar{q} = p_o' = \gamma' D$, but use $\bar{q}$. since this is the accepted symbol for bearing capacity computations.

- These **block-stresses can be shown** on the Mohr's Circle in **Figure 3**.

- When the two blocks are adjacent to each other at the vertical line OY, it is **evident that** $\sigma_{3,1} = \sigma_{1,2}$, but with the **principle stress rotation of 90⁰** between blocks.

- It is known that the **principal stresses** are related to the shear parameters of the backfill material by **the Bell's equation as follows :**

$\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha$ **where** $\alpha = 45 + \frac{\varphi}{2}$

$$\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\varphi}{2}\right) + 2c \tan\left(45 + \frac{\varphi}{2}\right) \text{ --------- (1)}$$

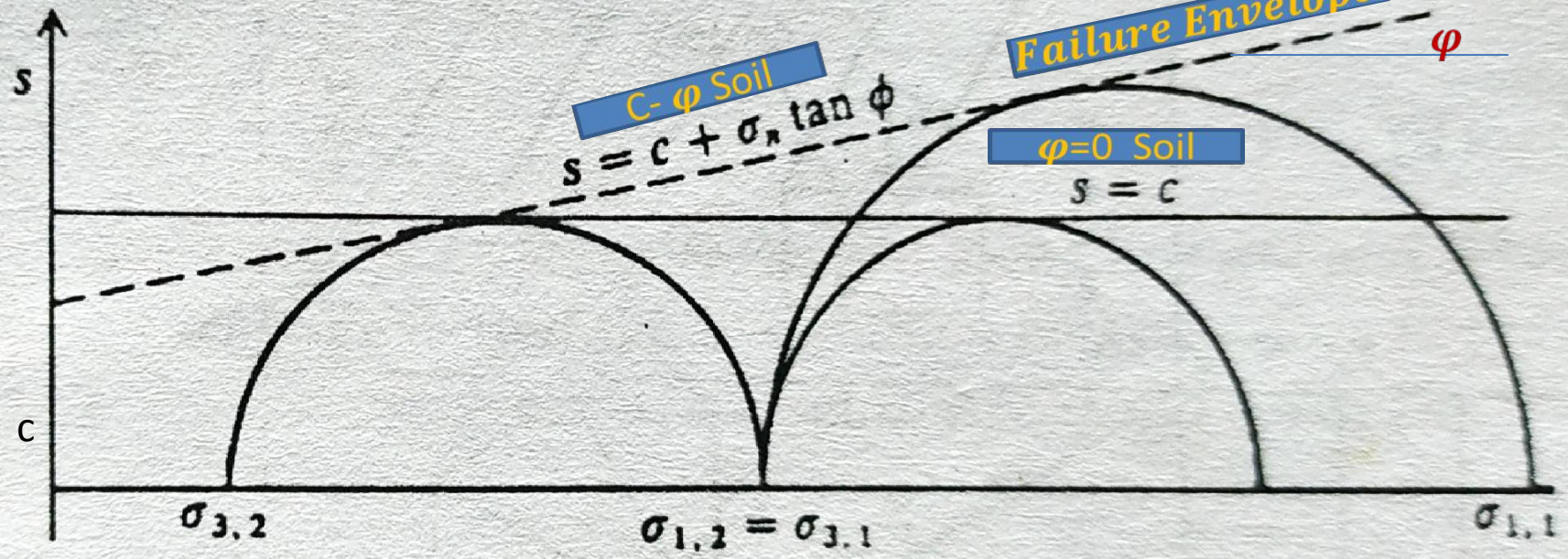
- We have φ=0, $\tan(45+\frac{\varphi}{2}) = tan^2(45+\frac{\varphi}{2}) = 1$

$$\sigma_1 = \sigma_3 (1) + 2c (1)$$

- For **block 2** at point O (corner of footing), $\sigma_{3.2}$= q = γD, which may be either **an effective** or a **total stress**.

$$\sigma_{1.2} = \sigma_{3.1} = \sigma_{3.2} (1) + 2c (1)$$

$$\sigma_{1.2} = \sigma_{3.1} = q (1) + 2c (1) \text{ ---------- (a)}$$



(c) Mohr's circle for (a) and for a ϕ–c soil.

Figure-3: Mohr's Circle for φ=0 and φ-c Soil

- **For Block 1**, just under the footing and again from the equation (a), we get the major principle stress $\sigma_{1,1}$ as

$$\sigma_{1.1} = q_{ult} = \sigma_{3.1}(1) + 2c\,(1) \quad \text{---------- (b)}$$

- Substituting, $\sigma_{3,1}$ from equation (a) in equation (b), we get,

$$q_{ult} = \bar{q} + 2c + 2c$$

$$= \mathbf{4c + \bar{q}} \quad \textbf{----------\ (c)}$$

- This equation has **two possibilities:**

  -Use for bearing capacity

  -Use for critical excavation depth in clay.

Bearing Capacity:

- For bearing capacity and with the **footing located on the ground surface** so that **aO → 0, $\bar{q} = 0$;** we get the ultimate bearing pressure,

$$q_{ult} = 4c$$

Critical excavation depth:

For critical excavation depth,

- Where **a wall** is used to **support the soil along line *aO*** and it is necessary to estimate the depth of excavation D.
- So that the **overburden pressure** does not squeeze the soil from the toe of the wall in to the excavation; we set $q_{ult}=0$, we obtain

$$q_{ult}=0=4c+q = 4c + \gamma D$$

- Critical depth of excavation D_c, (safety factor=1) is then,

$$D_c = \frac{4c}{\gamma}$$

Footing Rotation about point O

Take rotation about point O.

- Rotation could be about point O'.

- If the **soils in the two rotation zones are exactly equal in strength**, the wedge ObO' would punch in to the ground.
- It is **most improbable** due to worm holes, roots and other inclusion, that **rotation will not develop first**; which is substantiated by **observed field failures**.
- **Summing moment about point O**, we have

$$q_{ult} \text{xB} \frac{B}{2} - c\pi \text{ B x B} - \bar{q} \text{ x } \frac{B x B}{2} = 0$$

$$q_{ult} = 2c\pi + q$$

$$\mathbf{q_{ult} = 6.28c}, \text{ when } q=0$$

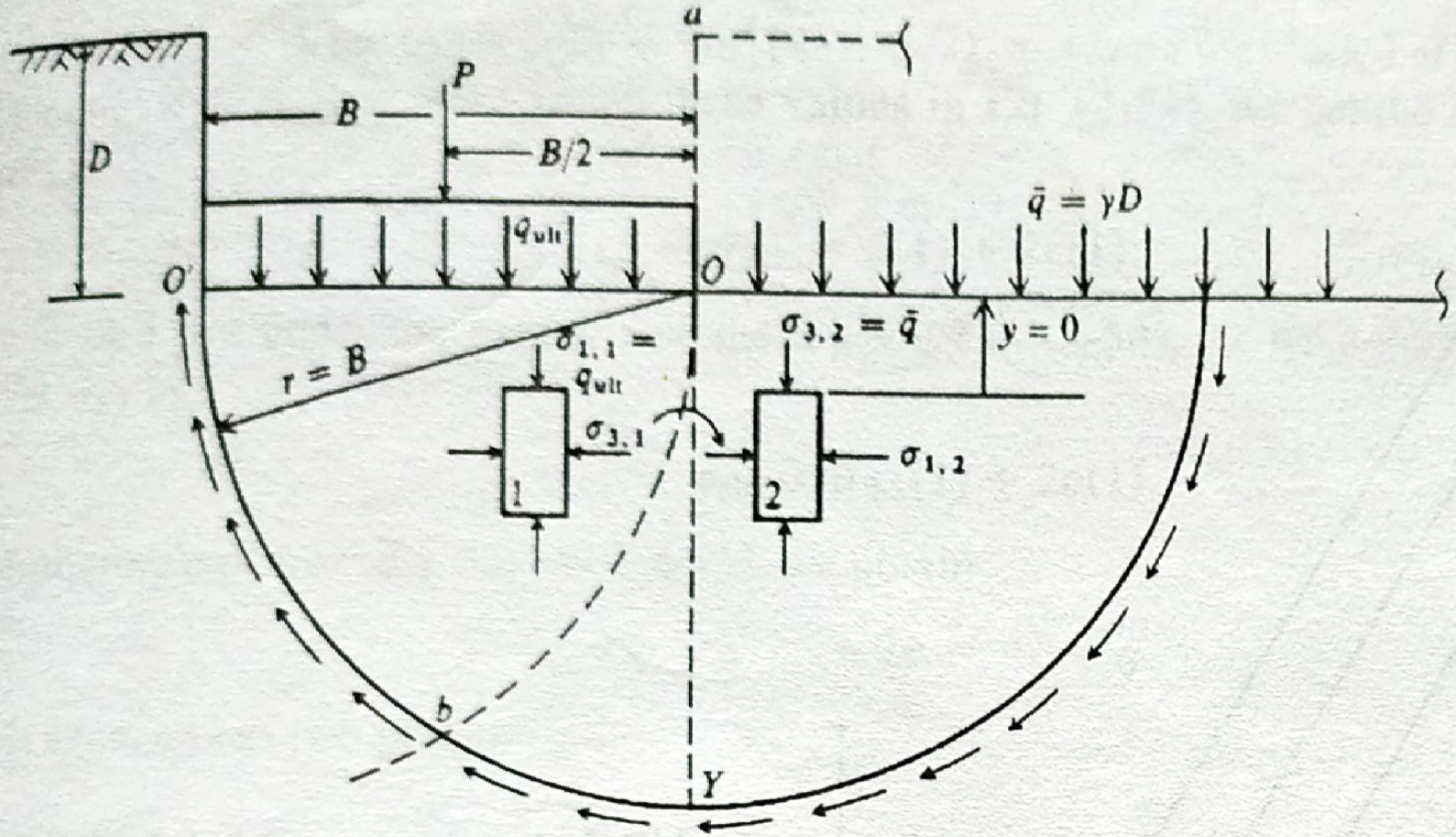


Figure-1: Footing on φ =0 Soil

Note: $\bar{q} = p_o' = \gamma' D$, but use $\bar{q}$. since this is the accepted symbol for bearing capacity computations.

Assignment:

Question: Derive the basic equation of ultimate bearing pressure in φ=0 soil due to footing pushes in to the ground as well as footing rotation.

$$q_{ult} = 4c$$

$$q_{ult} = 6.28c$$

Question: Derive the basic equation of ultimate bearing pressure in $c - \varphi$ soil due to footing pushes and rotation in to the ground as well footing rotation.

$$q_{ult} = c\,N_c + q\,N_q + \gamma B\,N_\gamma$$

Thank you