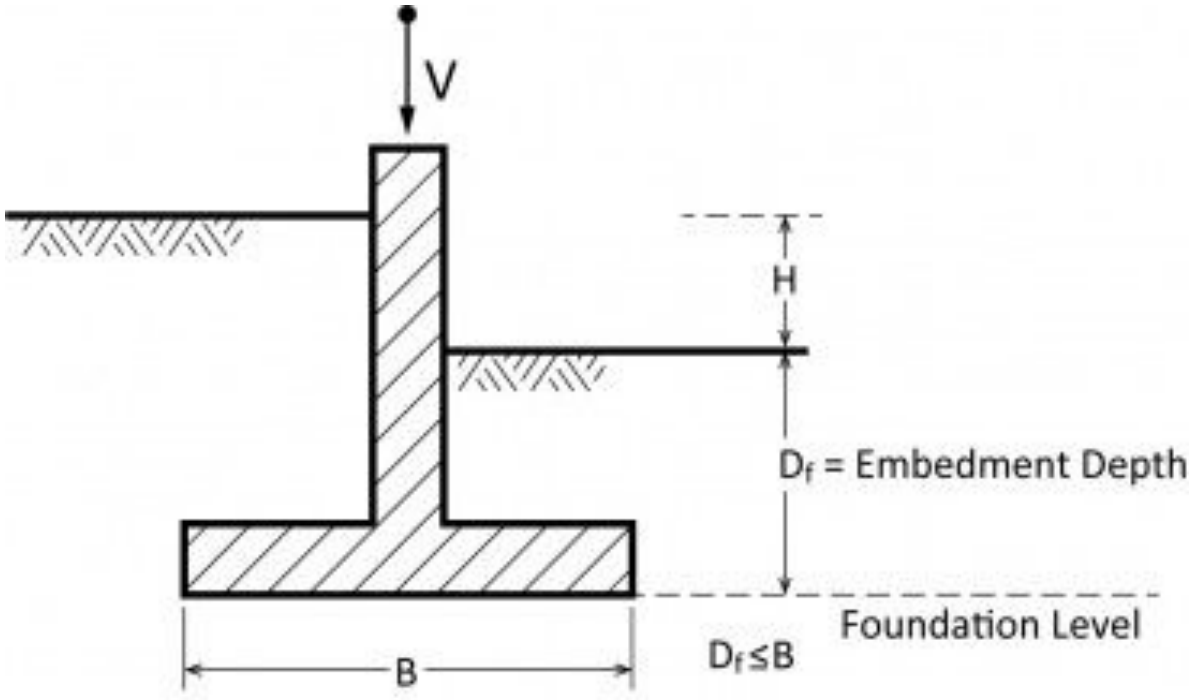
EWCE 343

Geotechnical and Foundation Engineering

Bearing Capacity Equation

Terzaghi Bearing-Capacity Equation:

- **One of the early set** of bearing-capacity equation proposed by Terzaghi in 1943.
- Terzaghi used **shape factors**.
- Terzhagi's bearing capacity equations were intended for **'shallow' foundations** where $D \leq B$.



Terzaghi (1934) (P-220,

TABLE 4-1

Bearing-capacity equations by the several authors indicated

Terzaghi (1943). See Table 4-2 for typical values and for $K_{p\gamma}$ values.

$$q_{ult} = cN_c s_c + \bar{q}N_q + 0.5\gamma BN_\gamma s_\gamma$$

$$N_q = \frac{a^2}{a\cos^2(45 + \phi/2)}$$

$$a = e^{(0.75\pi - \phi/2)\tan\phi}$$

$$N_c = (N_q - 1)\cot\phi$$

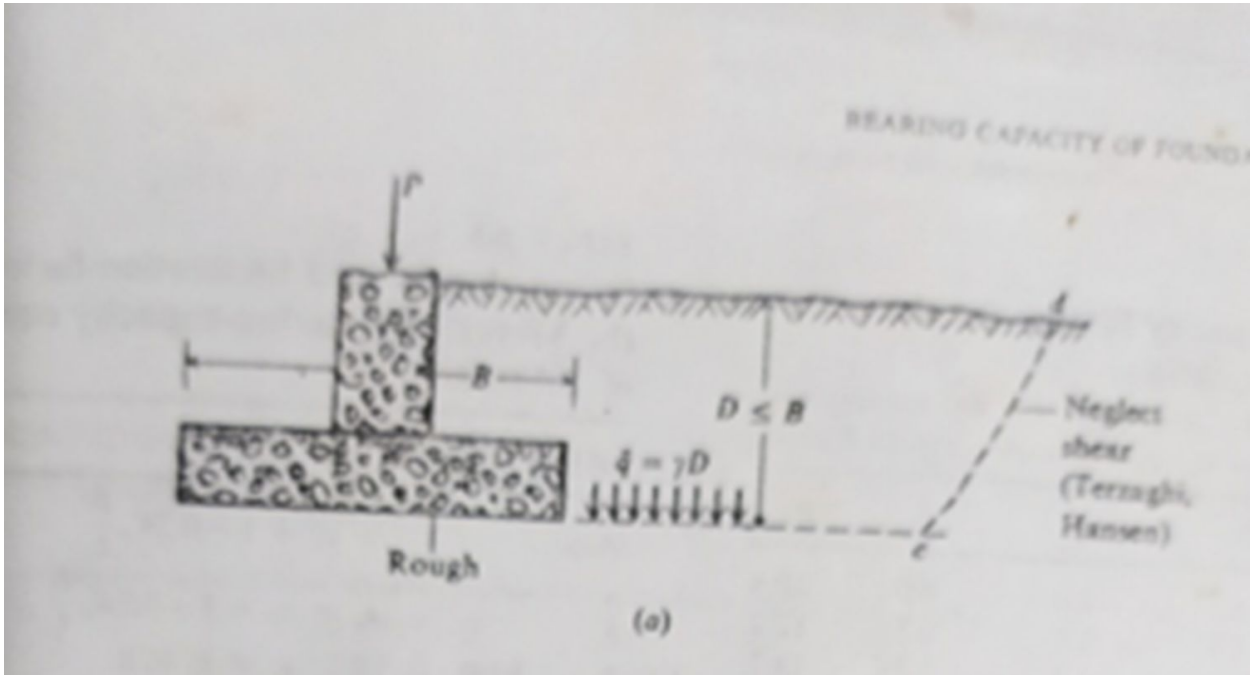
$$N_\gamma = \frac{\tan\phi}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right)$$

For:	strip	round	square
s_c =	1.0	1.3	1.3
s_γ =	1.0	0.6	0.8

Bearing Capacity Equation

Terzaghi Bearing-Capacity Equation:

- Figure-4.3a, P-221, shallow foundation with **rough base** defined.
- **Neglect the shear along cd, where D≤ B.**
- Table 4-2, is a short **table of N factor for rapid** use by the reader.



P-222

TABLE 4-2

Bearing-capacity factors for the Terzaghi equations

Values of N_γ for ϕ of 0, 34, and 48° are original Terzaghi values and used to back-compute $K_{p\gamma}$

ϕ, deg	N_c	N_q	N_γ	$K_{p\gamma}$
0	5.7*	1.0	0.0	10.8
5	7.3	1.6	0.5	12.2
10	9.6	2.7	1.2	14.7
15	12.9	4.4	2.5	18.6
20	17.7	7.4	5.0	25.0
25	25.1	12.7	9.7	35.0
30	37.2	22.5	19.7	52.0
34	52.6	36.5	36.0	
35	57.8	41.4	42.4	82.0
40	95.7	81.3	100.4	141.0
45	172.3	173.3	297.5	298.0
48	258.3	287.9	780.1	
50	347.5	415.1	1153.2	800.0

*$N_c = 1.5\pi + 1$. [See Terzaghi (1943), p. 127.]

Meyerhof's Bearing-Capacity Equation:

- Meyerhof proposed a bearing capacity equation similar to that of Terzaghi but included **shape factor s_c, s_q & s_γ and depth factor d_c, d_q & d_γ for vertical load.**

- Also included **inclination factors i_c, i_q & i_γ**

- Up to a depth of **D≈B,** Meyerhof **q_{ult} is not greatly different** from that of Terzaghi value.

- Difference becomes more pronounced at **larger D/B ratios.**

Meyerhof (1963)

Meyerhof (1963).* See Table 4-3 for shape, depth, and inclination factors.

Vertical load: $q_{ult} = cN_c s_c d_c + \bar{q} N_q s_q d_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma$

Inclined load: $q_{ult} = cN_c d_c i_c + \bar{q} N_q d_q i_q + 0.5\gamma B' N_\gamma d_\gamma i_\gamma$

$$N_q = e^{\pi \tan\phi} \tan^2\left(45 + \frac{\phi}{2}\right)$$

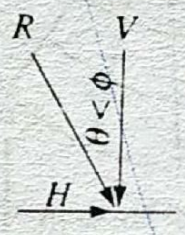
$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = (N_q - 1)\tan(1.4\phi)$$

Meyerhof

TABLE 4-3

Shape, depth, and inclination factors for the Meyerhof bearing-capacity equations of Table 4-1

Factors	Value	For
Shape:	$s_c = 1 + 0.2K_p\frac{B}{L}$	Any ϕ
	$s_q = s_\gamma = 1 + 0.1K_p\frac{B}{L}$	$\phi > 10°$
	$s_q = s_\gamma = 1$	$\phi = 0$
Depth:	$d_c = 1 + 0.2\sqrt{K_p}\frac{D}{B}$	Any ϕ
	$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B}$	$\phi > 10$
	$d_q = d_\gamma = 1$	$\phi = 0$
Inclination:	$i_c = i_q = \left(1 - \frac{\theta°}{90°}\right)^2$	Any ϕ
	$i_\gamma = \left(1 - \frac{\theta°}{\phi°}\right)^2$	$\phi > 0$
	$i_\gamma = 0$ for $\theta > 0$	$\phi = 0$

Where $K_p = \tan^2(45 + \phi/2)$ as in Fig. 4-2

θ = angle of resultant R measured from vertical without a sign; if $\theta = 0$ all $i_i = 1.0$.

Meyerhof Bearing Capacity Equation:

- **Table 4-3** for shape, depth and inclination factors.

- **Table 4-4** is for Bearing-capacity **N factors** for the Meyerhof, Hansen and Vesic bearing capacity equation.

TABLE 4-4

Bearing-capacity factors for the Meyerhof, Hansen, and Vesić bearing-capacity equations

Note that N_c and N_q are the same for all three methods; subscripts identify author for N_γ

ϕ	N_c	N_q	$N_{\gamma(H)}$	$N_{\gamma(M)}$	$N_{\gamma(V)}$	N_q/N_c	$2\tan\phi(1-\sin\phi)^2$
0	5.14*	1.0	0.0	0.0	0.0	0.195	0.000
5	6.49	1.6	0.1	0.1	0.4	0.242	0.146
10	8.34	2.5	0.4	0.4	1.2	0.296	0.241
15	10.97	3.9	1.2	1.1	2.6	0.359	0.294
20	14.83	6.4	2.9	2.9	5.4	0.431	0.315
25	20.71	10.7	6.8	6.8	10.9	0.514	0.311
26	22.25	11.8	7.9	8.0	12.5	0.533	0.308
28	25.79	14.7	10.9	11.2	16.7	0.570	0.299
30	30.13	18.4	15.1	15.7	22.4	0.610	0.289
32	35.47	23.2	20.8	22.0	30.2	0.653	0.276
34	42.14	29.4	28.7	31.1	41.0	0.698	0.262
36	50.55	37.7	40.0	44.4	56.2	0.746	0.247
38	61.31	48.9	56.1	64.0	77.9	0.797	0.231
40	75.25	64.1	79.4	93.6	109.3	0.852	0.214
45	133.73	134.7	200.5	262.3	271.3	1.007	0.172
50	266.50	318.5	567.4	871.7	761.3	1.195	0.131

* $= \pi + 2$ as limit when $\phi \to 0°$.

Slight differences in above table can be obtained using program BEARING.EXE on diskette depending on computer used and whether or not it has floating point.

Hansen's Bearing-Capacity Equation:

- Hansen proposed the general bearing capacity equation in 1970.
- Seen to **be further extension** of Meyerhof's equation.
- Hansen's **shape, depth and other factors** making up the general bearing-capacity equation.
- Meyerhof's; the additions consists in the introduction of $\boldsymbol{b_i}$ **that accounts for the possible inclination of the footing to the horizontal** and a factor $\boldsymbol{g_i}$ for inclined soil surface
- For **footing on a slope**, both the Hansen and Vesic′, g_i factors can be used to reduce the bearing capacity using N factors.
- Hansen's equation implicitly allows **any D/B ratios** and thus can be used for both shallow and deep foundation.

Hansen (1970)

Hansen (1970).* See Table 4-5 for shape, depth, and other factors.

General:† $q_{ult} = cN_c s_c d_c i_c g_c b_c + \bar{q} N_q s_q d_q i_q g_q b_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$

when $\phi = 0$

use $q_{ult} = 5.14 s_u (1 + s'_c + d'_c - i'_c - b'_c - g'_c) + \bar{q}$

N_q = same as Meyerhof above

N_c = same as Meyerhof above

$N_\gamma = 1.5(N_q - 1)\tan\phi$

Hansen and Vesic'

TABLE 4-5*a*

Shape and depth factors for use in either the Hansen (1970) or Vesić (1973, 1975*b*) bearing-capacity equations of Table 4-1. Use s'_c, d'_c when $\phi = 0$ only for Hansen equations. Subscripts H, V for Hansen, Vesić, respectively.

Shape factors	Depth factors
$s'_{c(H)} = 0.2\dfrac{B'}{L'}$ $(\phi = 0°)$ $s_{c(H)} = 1.0 + \dfrac{N_q}{N_c} \cdot \dfrac{B'}{L'}$ $s_{c(V)} = 1.0 + \dfrac{N_q}{N_c} \cdot \dfrac{B}{L}$ $s_c = 1.0$ for strip	$d'_c = 0.4k$ $(\phi = 0°)$ $d_c = 1.0 + 0.4k$ $k = D/B$ for $D/B \le 1$ $k = \tan^{-1}(D/B)$ for $D/B > 1$ k in radians
$s_{q(H)} = 1.0 + \dfrac{B'}{L'}\sin\phi$ $s_{q(V)} = 1.0 + \dfrac{B}{L}\tan\phi$ for all ϕ	$d_q = 1 + 2\tan\phi(1 - \sin\phi)^2 k$ k defined above
$s_{\gamma(H)} = 1.0 - 0.4\dfrac{B'}{L'} \ge 0.6$ $s_{\gamma(V)} = 1.0 - 0.4\dfrac{B}{L} \ge 0.6$	$d_\gamma = 1.00$ for all ϕ

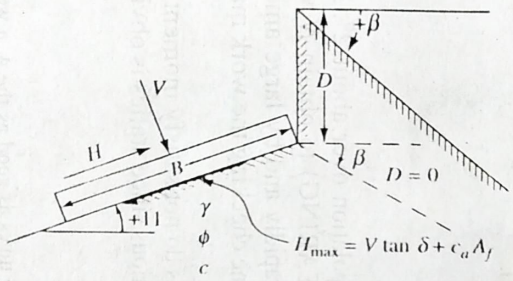
Notes:

1. Note use of "effective" base dimensions B', L' by Hansen but not by Vesić.
2. The values above are consistent with either a vertical load or a vertical load accompanied by a horizontal load H_B.
3. With a vertical load and a load H_L (and either $H_B = 0$ or $H_B = 0$) you may have to compute two sets of shape s_i and d_i as $s_{i,B}$, $s_{i,L}$ and $d_{i,B}$, $d_{i,L}$. For i, L subscripts of Eq. (4-2), presented in Sec. 4-6, use ratio L'/B' or D/L'.

Hansen

Notes: $\beta + \eta \quad 90^\circ$ (Both β and η have signs (+) shown.)

$\beta \quad \phi$



For: $L/B \le 2$ use ϕ_{tr}

$L/B > 2$ use $\phi_{ps} = 1.5\,\phi_{tr} - 17^\circ$

$\phi_{tr} \le 34^\circ$ use $\phi_{tr} = \phi_{ps}$

δ = friction angle between base and soil ($.5\phi \le \delta \le \phi$)

$A_f = B'L'$ (effective area)

c_a = base adhesion (0.6 to 1.0c)

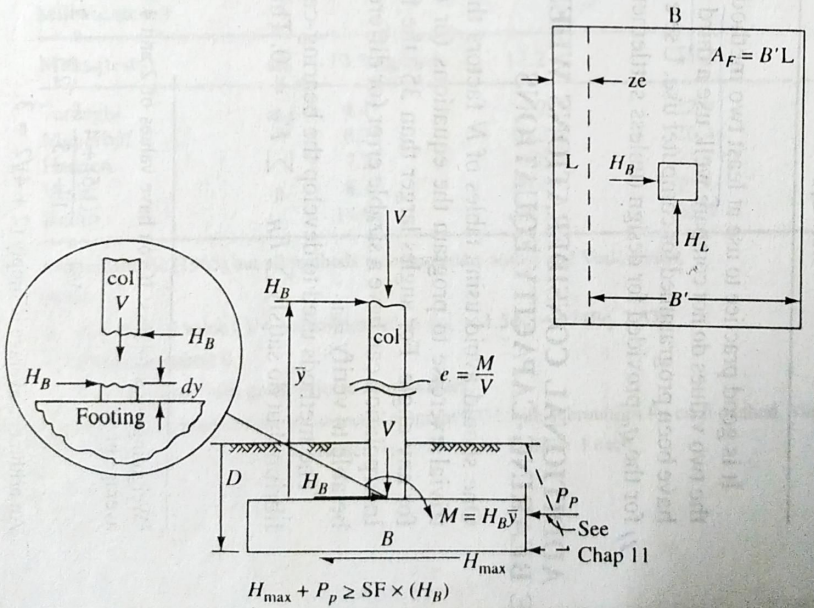


TABLE 4-5*b*

Table of inclination, ground, and base factors for the Hansen (1970) equations. See Table 4-5*c* for equivalent Vesić equations.

Inclination factors	Ground factors (base on slope)
$i'_c = 0.5 - \sqrt{1 - \frac{H_i}{A_f C_a}}$	$g'_c = \frac{\beta^\circ}{147^\circ}$
$i_c = i_q - \frac{1 - i_q}{N_q - 1}$	$g_c = 1.0 - \frac{\beta^\circ}{147^\circ}$
$i_q = \left[1 - \frac{0.5H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_1}$ $2 \le \alpha_1 \le 5$	$g_q = g_\gamma = (1 - 0.5\tan\beta)^5$
$i_\gamma = \left[1 - \frac{0.7H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_2}$	**Base factors (tilted base)**
$i_\gamma = \left[1 - \frac{(0.7 - \eta^\circ/450^\circ)H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_2}$ $2 \le \alpha_2 \le 5$	$b'_c = \frac{\eta^\circ}{147^\circ} \quad (\phi = 0)$ $b_c = 1 - \frac{\eta^\circ}{147^\circ} \quad (\phi > 0)$ $b_q = \exp(-2\eta\tan\phi)$ $b_\gamma = \exp(-2.7\eta\tan\phi)$ η in radians

Notes:

1. Use H_i as either H_B or H_L, or both if $H_L > 0$.
2. Hansen (1970) did not give an i_c for $\phi > 0$. The value above is from Hansen (1961) and also used by Vesić.
3. Variable c_a = base adhesion, on the order of 0.6 to 1.0 × base cohesion.
4. Refer to sketch for identification of angles η and β, footing depth D, location of H_i (parallel and at top of base slab; usually also produces eccentricity). Especially note V = force *normal* to base and is not the resultant R from combining V and H_i.

Vesic’

TABLE 4-5c

Table of inclination, ground, and base factors for the Vesić (1973, 1975*b*) bearing-capacity equations. See notes below and refer to sketch for identification of terms.

Inclination factors	Ground factors (base on slope)
$i_c' = 1 - \dfrac{mH_i}{A_f c_a N_c}$ $(\phi = 0)$	$g_c' = \dfrac{\beta}{5.14}$ β in radians
$i_c = i_q - \dfrac{1 - i_q}{N_q - 1}$ $(\phi > 0)$	$g_c = i_q - \dfrac{1 - i_q}{5.14 \tan \phi}$ $\phi > 0$
i_q, and m defined below	i_q defined with i_c
$i_q = \left[1.0 - \dfrac{H_i}{V + A_f c_a \cot \phi}\right]^m$	$g_q = g_\gamma = (1.0 - \tan \beta)^2$
$i_\gamma = \left[1.0 - \dfrac{H_i}{V + A_f c_a \cot \phi}\right]^{m+1}$	**Base factors (tilted base)**
$m = m_B = \dfrac{2 + B/L}{1 + B/L}$	$b_c' = g_c'$ $(\phi = 0)$
$m = m_L = \dfrac{2 + L/B}{1 + L/B}$	$b_c = 1 - \dfrac{2\beta}{5.14 \tan \phi}$
	$b_q = b_\gamma = (1.0 - \eta \tan \phi)^2$

Notes:

1. When $\phi = 0$ (and $\beta \neq 0$) use $N_\gamma = -2 \sin(\pm\beta)$ in N_γ term.
2. Compute $m = m_B$ when $H_i = H_B$ (H parallel to B) and $m = m_L$ when $H_i = H_L$ (H parallel to L). If you have both H_B and H_L use $m = \sqrt{m_B^2 + m_L^2}$. Note use of B and L, not B', L'.
3. Refer to Table sketch and Tables 4-5*a*,*b* for term identification.
4. Terms N_c, N_q, and N_γ are identified in Table 4-1.
5. Vesić always uses the bearing-capacity equation given in Table 4-1 (uses B' in the N_γ term even when $H_i = H_L$).
6. H_i term ≤ 1.0 for computing i_q, i_γ (always).

Vesic′'s Bearing-capacity equation:

- Vesic′ equation is **essentially same** as the Hansen with some change.
- N_c and N_q terms are those of Hansen but **N_γ is slightly different.**
- There are also **differences in the i_i, b_i and g_i** terms.
- Vesic′'s Bearing-capacity equation is somewhat **easier to use than Hansen's** because Hansen uses the i terms in computing shape factors s_i.

Vesic' (1973, 1975)

Vesić (1973, 1975).* See Table 4-5 for shape, depth, and other factors. Use Hansen's equations above.

$$N_q = \text{same as Meyerhof above}$$

$$N_c = \text{same as Meyerhof above}$$

$$N_\gamma = 2(N_q + 1)\tan\phi$$

*These methods require a trial process to obtain design base dimensions since width B and length L are needed to compute shape, depth, and influence factors.

Which Equations are to be Used?

- Summary of **eight load tests of footing carried** out beside the model tests and the computed q_{ult}.

- Comparing the computed q_{ult} to the measured values indicates **none of the several theories/methods has a significant advantage over any other.**

Which Equation is to be Use?

Terzaghi Equation

- Terzaghi equations, being **the first proposed**, have been **very widely used.**
- Because of their **greater ease of use**, does not need to compute all the extra shape, depth, and inclination factors, they are **still used more than expected.**
- **Not suitable** for footings carrying a horizontal shear /or a moment or for a tilted base.

TABLE 4-1

Bearing-capacity equations by the several authors indicated

Terzaghi (1943). See Table 4-2 for typical values and for $K_{p\gamma}$ values.

$$q_{ult} = cN_c s_c + \bar{q}N_q + 0.5\gamma B N_\gamma s_\gamma$$

$$N_q = \frac{a^2}{a\cos^2(45 + \phi/2)}$$

$$a = e^{(0.75\pi - \phi/2)\tan\phi}$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = \frac{\tan\phi}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right)$$

For:	strip	round	square
s_c =	1.0	1.3	1.3
s_γ =	1.0	0.6	0.8

Meyerhof Equation :

- Meyerhof proposed a bearing capacity equation **similar to that of Terzaghi** but he included **shape factors, depth factors and also included inclination factors** for cases where the footing load inclined from the vertical.

- **Up to a depth of D ≈ *B*,** the Meyerhof q_{ult} is not greatly different from the Terzaghi.

- **Difference becomes more pronounced** at larger depth.

Meyerhof (1963)

Meyerhof (1963).* See Table 4-3 for shape, depth, and inclination factors.

Vertical load: $q_{ult} = cN_c s_c d_c + \bar{q} N_q s_q d_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma$

Inclined load: $q_{ult} = cN_c d_c i_c + \bar{q} N_q d_q i_q + 0.5\gamma B' N_\gamma d_\gamma i_\gamma$

$$N_q = e^{\pi \tan\phi} \tan^2\left(45 + \frac{\phi}{2}\right)$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = (N_q - 1)\tan(1.4\phi)$$

Hansen Equation:

- Both the Meyerhof and Hansen methods are **widely used.**
- Hansen equation implicitly **allows any D/B** and thus can be used for both shallow (Footing) and deep (piles, drilled caisson) bases.
- Hansen used:

For $\frac{D}{B} \leq 1$:

$$d_c = 1 + 0.4\,\frac{D}{B} \qquad d_q = 1 + 2\tan\varphi\,(1+\sin\varphi)^2\,\frac{D}{B}$$

For $\frac{D}{B} > 1$:

$$d_c = 1 + 0.4\tan^{-1}\frac{D}{B} \qquad d_q = 1 + 2\tan\varphi\,(1+\sin\varphi)^2\,\tan^{-1}\frac{D}{B}$$

- Expressions give a discontinuity at $\frac{D}{B} = 1$. **For φ=0**

D/B=	0	1	1.5	2.0	5.0	10	20	100
d'_c =	0	0.40	0.42	0.44	0.55	0.59	0.61	0.62

Hansen (1970)

Hansen (1970).* See Table 4-5 for shape, depth, and other factors.

General:† $q_{ult} = cN_c s_c d_c i_c g_c b_c + \bar{q} N_q s_q d_q i_q g_q b_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$

when $\phi = 0$

use $q_{ult} = 5.14 s_u (1 + s'_c + d'_c - i'_c - b'_c - g'_c) + \bar{q}$

N_q = same as Meyerhof above

N_c = same as Meyerhof above

$N_\gamma = 1.5(N_q - 1)\tan\phi$

Vesic′ Equation

- There is **very little difference** between the Hansen and Vesic′ methods.
- Vesic procedure is essentially same as the method of Hansen with the following differences:

  -N_c and N_q terms are those as Hansen but **N_γ is slightly different** (as shown in Table 4-4)

  -Are also **differences in i_i, b_i and g_i** as in Table 4-5
- Vesic equation is somewhat easier to use.

Vesic' (1973, 1975)

Vesić (1973, 1975).* See Table 4-5 for shape, depth, and other factors. Use Hansen's equations above.

$$N_q = \text{same as Meyerhof above}$$

$$N_c = \text{same as Meyerhof above}$$

$$N_\gamma = 2(N_q + 1)\tan\phi$$

*These methods require a trial process to obtain design base dimensions since width B and length L are needed to compute shape, depth, and influence factors.

Suggested Equations to be used:

Teraghi Equation:

- Terzaghi equations, being the first proposed, have been **very widely used.**
- Because of their **greater ease of use**, does not need to compute all the extra shape, depth, and inclination factors, they are **still used more than expected.**
- **Not suitable** for footings carrying a horizontal shear /or a moment or for a tilted base.

Meyerhof and Hansen Equation:

- Both the Meyerhof and Hansen methods are **widely used/**

Vesic′Equation:

- Method has not been much used.
- There is **very little difference** between the Hansen and Vesic′methods.

One may Suggest the Following Equations to Use

Use	Best For
Terzaghi	
	• **Any situation** that applies. • Depending on user preference or familiarity with particular methods.

How to use the Bearing Capacity Equation?

- Good practice to use **at least two methods** and **compare** the computed values of q_{ult}.

- If the two values **do not compare well**, **use a third method,** a trivial exercise where equations have been **programmed** for computer use.

- Use either an **arithmetic or weighted average value** for the q_a, provided for design **unless settlement is controlling**.

$$\text{Weighted value} = \frac{2(1.5)+4}{1.5+1} = \frac{7}{2.5} = 2.8$$

$$\text{Average value} = \frac{2+4}{1+1} = 3$$

Problem 1: Compute the **allowable bearing pressure** using Terzaghi equation for the footing and soil parameters shown in Figure 1-1. Use **safety factor** 3 to obtain q_a. Compare this with the value obtained from the **general equation of bearing capacity of c-φ soil**. The soil data obtained from a series of undrained (U) tri-axial tests.

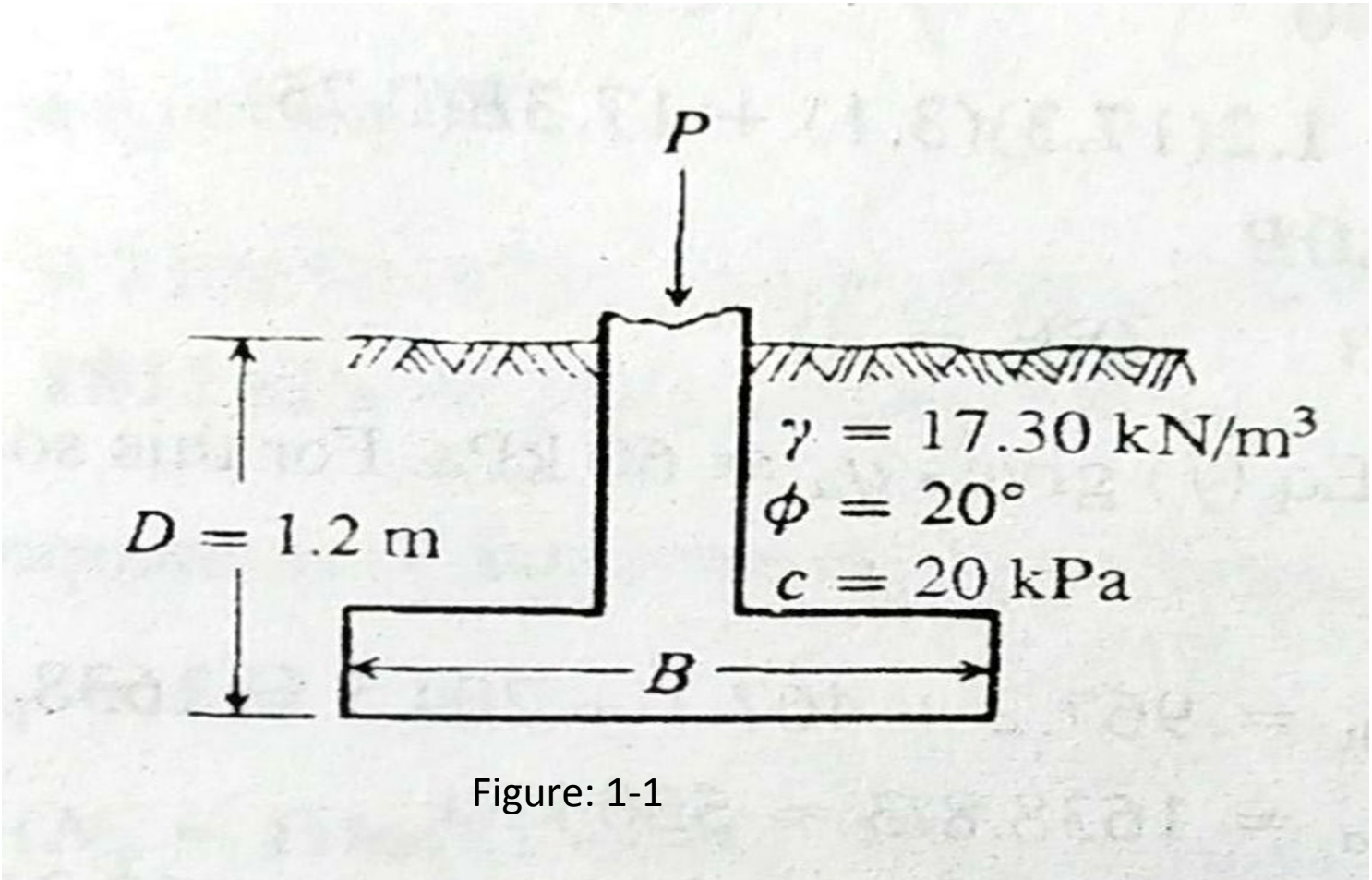


Figure: 1-1

Solution:

B is not known, but **D is known**.

Geotechnical consultant would have to **recommend the value of B**.

From **Table 4-2 (P -222** or S-6), we obtain,

For $\emptyset = 20^o$

$N_c = 17.7$ $N_q = 7.4$ $N_\gamma = 5.0$

$s_c = 1.3$ $s_\gamma = 0.8$

(from Table 4-1, 4-2, P-220 & 222, **square footing** or S-6/39)

Terzaghi eqn,

$$q_{ult} = c\, N_c s_c + q\, N_q + 0.5\, \gamma B\, N_\gamma s_\gamma$$

$$= c\, N_c s_c + \gamma D\, N_q + 0.5\, \gamma B\, N_\gamma s_\gamma$$

$$= 20(17.7)(1.3) + (17.3)(1.2)(7.4) + 0.5(17.3)(B)(5)(0.8)$$

$$= \mathbf{(613.8 + 34.6B)}$$

A SF =3.0 is commonly used **when c > 0**

Allowable pressure is, $q_a = \frac{q_{ult}}{3}$

P-222

TABLE 4-2

Bearing-capacity factors for the Terzaghi equations

Values of N_γ for ϕ of 0, 34, and 48° are original Terzaghi values and used to back-compute $K_{p\gamma}$

ϕ, deg	N_c	N_q	N_γ	$K_{p\gamma}$
0	5.7*	1.0	0.0	10.8
5	7.3	1.6	0.5	12.2
10	9.6	2.7	1.2	14.7
15	12.9	4.4	2.5	18.6
20	17.7	7.4	5.0	25.0
25	25.1	12.7	9.7	35.0
30	37.2	22.5	19.7	52.0
34	52.6	36.5	36.0	
35	57.8	41.4	42.4	82.0
40	95.7	81.3	100.4	141.0
45	172.3	173.3	297.5	298.0
48	258.3	287.9	780.1	
50	347.5	415.1	1153.2	800.0

*$N_c = 1.5\pi + 1$. [See Terzaghi (1943), p. 127.]

Terzaghi (1934) (P-220,

TABLE 4-1

Bearing-capacity equations by the several authors indicated

Terzaghi (1943). See Table 4-2 for typical values and for $K_{p\gamma}$ values.

$$q_{ult} = cN_c s_c + \bar{q}N_q + 0.5\gamma BN_\gamma s_\gamma$$

$$N_q = \frac{a^2}{a\cos^2(45 + \phi/2)}$$

$$a = e^{(0.75\pi - \phi/2)\tan\phi}$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = \frac{\tan\phi}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right)$$

For:	strip	round	square
s_c =	1.0	1.3	1.3
s_γ =	1.0	0.6	0.8

- $q_a = \frac{q_{ult}}{3}$

$= \frac{613.8+34.6B}{3} =$ **(205 + 11.5B)** kPa

Since B is likely to range from 1.5m to 3.0 m; r_γ=0.95

q_a= 205 + 11.5 **(1.5)** =220 kPa (rounding)

q_a= 205 + 11.5 **(3.0)** **(0.95)** =240 kPa

Hence, **recommend $q_a \cong$ 200 – 220 kPa**

Reduction factor:

- Soil **stress state is indeterminate** at the **design stress level q_a**.
- It is only **at failure (q_{ult})** that state **stress is determined**.
- Bearing capacity equation to be **conservative** most of the time because of the common practice is **to use conservative estimates for soil parameters.**
- Additionally, **after estimating a conservative q_{ult},** this is **further reduced** to the allowable soil pressure q_a, using a Safety factor
- This means the probability is very high that q_a is "safe".

Question: "Bearing capacity equation of soil for foundation is to be conservation"". Explain.

Terzaghi's Reduction (q_a):

- When Terzaghi (1943) developed his bearing capacity equations, he considered a **general shear failure in a dense soil** and a **local shear failure for a loose soil.**

- For **local shear f***ailure*, he proposed **reducing** c and $\emptyset$ *as*:

  $c'' = \mathbf{0.67c}$

  $\emptyset'' = \boldsymbol{tan^{-1}(0.67 tan\emptyset)}$

- Terzaghi (and others) consider **both smooth** and **rough base** contact with the soil.

Vesic and De Beer Reduction:

- There are some evidence, from using small **footing up to about 1m for B**, that $\mathbf{BN_\gamma}$ term does not increase the bearing capacity without bound.

- For **very large values of B**, both Vesic (1969) and De Beer (1965) suggest that the limiting value of q_{ult}, **approaches that of a deep foundation.**

- They suggest the following reduction factor,

$$r_\gamma = 1 - 0.25 \log\left(\frac{B}{k}\right) \quad \text{where, } B \geq 2.0m \ (6.0\ ft)$$

Where **k= 2.0** for SI **and k= 6.0** for fps.

Vesic and De Beer Reduction:

- Equation gives the following results:

B = 2	2.5	3	3.5	4	5	10	20	100 m
r_γ = 1.0	0.97	0.95	0.93	0.92	0.90	0.82	0.75	0.57

- One can use this reduction factor with any of the bearing capacity methods to give $\mathbf{0.5\gamma B N_\gamma s_\gamma d_\gamma r_\gamma}$.

- This equation is particularly applicable for **large bases,** where $\boldsymbol{BN_\gamma}$ term is predominating.

Using basic equation,

$$q_{ult} = c\left[\frac{2K_p}{Cos\,\varphi} + \sqrt{K_p}\right] + q\,\frac{\sqrt{K_p\,xK_p}}{Cos\,\varphi} + \frac{\gamma B}{4}\left[\frac{K_p^{\,2}}{Cos\,\varphi} - \sqrt{K_p}\right] \text{-----(1)}$$

$$q_{ult} = c\,N_c + q\,N_q + \gamma B\,N_\gamma$$

Here, $\cos\emptyset = \cos 20° = 0.940$

$$\tan\left(45 + \frac{\emptyset}{2}\right) = \sqrt{K_p} = \tan 55° = 1.428 = 1.43$$

$$tan^2\left(45 + \frac{\emptyset}{2}\right) = K_p = tan^2 55° = 2.04$$

- $N_c = \left[\frac{2K_p}{Cos\,\varphi} + \sqrt{K_p}\right) = \frac{2(2.040)}{0.940} + 1.428 = 5.8$

$N_q = \frac{\sqrt{K_p}\, xK_p}{Cos\,\varphi} = \frac{1.428\ (2.04)}{0.940} = 3.1$

$N_\gamma = \frac{1}{4}\left(\frac{K_p{}^2}{Cos\,\varphi} - \sqrt{K_p}\right) = 0.25\left(\frac{(2.04)^2}{0.940} - 1.428\right) = 0.75$

Hence, $q_{ult} = c\,N_c + q\,N_q + \gamma B\,N_\gamma$

$= 20\ (5.8) + 1.2(17.3)(3.1) + (17.3)\ (B)(0.75)$

$= 180 + 13.0B$

Here, SF= 3, allowable bearing Capacity, $q_a = 60 + 4.3B$

- q_a= 60+ 4.3(1.5) = 67 kPa

  q_a= 60+ 4.3(3.0) = 73 kPa

For a reasonable size footing, Eqn (1), gives $q_a \cong 60 - 70\ kPa$

For this soils, with ∅= 30° , B= 1.5m, and SF=3

Terzaghi equation, q_{ult}= c $N_c s_c$+ q N_q + 0.5 γB $N_\gamma s_\gamma$

N_c= 37.2, N_q =22.5, N_γ = 19.7

s_c= 1.3 s_γ = 0.8 (from table -2, square footing)

q_{ult}=20(37.2)(1.3)+1.2 (17.3)(22.5)+ 0.5(17.3)(1.5)(19.7(0.8)

= 967.2+ 467.1+ 204.5= 1638.8

$q_a = \frac{1638.8}{3}$ = **546.26 ≈550 kPa**

From Eqn (1), q_{ult}= c N_c+ q N_q + γB N_γ

Here, cosØ= cos 30°= 0.87

$$\tan\left(45+\frac{\emptyset}{2}\right) = \sqrt{K_p} = \tan 60° = 1.73$$

$$tan^2\left(45+\frac{\emptyset}{2}\right) = K_p = tan^2 60° = 3.0$$

- $N_c = \left[\frac{2K_p}{Cos\,\varphi} + \sqrt{K_p}\right) = \frac{2(3.0)}{0.87} + 1.73 = 8.626$

$$N_q = \frac{\sqrt{K_p}\, xK_p}{Cos\,\varphi} = \frac{1.73\,(3.0)}{0.87} = 5.965$$

$$N_\gamma = \frac{1}{4}\left(\frac{K_p{}^2}{Cos\,\varphi} - \sqrt{K_p}\right) = 0.25\left(\frac{(3.0)^2}{0.87} - 1.73\right) = 2.15$$

Now Eqn from (1),

$$q_{ult} = c\,N_c + q\,N_q + \gamma B\,N_\gamma$$

$$= 20(8.626) + 1.2\,(17.3)\,(5.965) + 17.3(B)(2.15)$$

$$= 299 + 38B = 299 + 38\,(1.5) = 330\ \text{kPa}$$

$$q_a = \frac{330}{3} = 110\ \text{kPa}$$

Hence, Eqn (1) is so conservative as to be nearly useless.

Problem-2: A footing **load test** produced the following results:

D=0.5m, B=0.5m L=2.0m

$\gamma' = 9.31\ \text{kN}/m^3$ $\emptyset_{triaxial}$=42.7° c=0

P_{ult}=1863 kN (measured)

$$q_{ult} = \frac{P_{ult}}{BxL} = \frac{1683}{0.5x2.0} = 1863\ \text{kPa (measured)}$$

Determine the ultimate bearing capacity by both **Hansen** and **Meyerhof** equations.

Solution: Using Hansen Equation:

- General equation of Hansen

$$q_{ult} = cN_c s_c d_c i_c g_c b_c + \bar{q} N_q s_q d_q i_q g_q b_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$$

- Since c=0, any factors with subscript c do not need to computing.
- All i_i g_i and b_i factors are 1.00
- With all this, Hansen Equation simplifies to;

$$q_{ult} = \gamma' D\, N_q s_q d_q + 0.5\, \gamma' B\, N_\gamma s_\gamma d_\gamma \text{ ---------(1)}$$

For L/B $\leq$ 2, use $\emptyset_{tr}$

For L/B >2, use $\emptyset_{ps}$ =**1.5 $\emptyset_{tr}$ - 17°**

$\emptyset_{tr} \leq 34°$ ***use*** $\emptyset_{tr} = \emptyset_{ps}$

(P-227, S-17)

Here, $\frac{L}{B} = \frac{2.0}{0.5}$ =4>2, **$\emptyset_{ps}$ = 1.5 $\emptyset_{tr}$ - 17°**

= 1.5x 42.7° - 17°

= 46.75° $\approx$ 47° (use)

From Table 4-4 (P- 223, S-12), for ∅=47°; by 1° increment (or Interpolation), we get, **N_q=208,** **N_γ(H)=274**

From Table 4-5a, (P-226, S-16)

$s_{q(H)}$ **= 1.0 +** $\frac{B'}{L'}$ **cos∅** = 1.0 + $\frac{0.5/2}{2.0/2}$ cos47°= 1.1705≈1.18

d_q **= 1.0 + 2tan ∅(1-sin∅)²k**

where **k=D/B for D/B≤1**

k=0.5/0.5= 1

k=tan^{-1}(D/B), for D/B> 1

d_q= 1.0 + 2tan ∅(1-sin∅)²k= 1.15

From Table 4-5a, (P-226, S-16)

$s_{\gamma(H)} = 1.0 - \frac{B'}{L'} \geq 0.6$ or, $s_{\gamma(H)} = 0.93$

$d_\gamma = 1.0$ for all Ø

$q_{ult} = \gamma' D N_q s_q d_q + 0.5 \gamma' B N_\gamma s_\gamma d_\gamma$

$= 9.31(0.5)(208)(1.18)(1.155) + 0.5(9.31)(0.5)(274)(0.9)(1)$

$= 1906.99 \approx 1907$ kPa

$q_{ult} = \frac{P_{ult}}{BxL} = \frac{1863}{0.5x2} = 1863$ kPa

Difference is (1907-1863)= 44 kPa

Meyerhof Equation: From Table 4-1(P-220, S-3) and Table 4-2(S-4) and $\emptyset_{ps}$=47°

From Table 4-4 , we get N_q=187,

For $\emptyset > 40°$, we use $N_\gamma = (N_q - 1)\tan(1.4\ \emptyset)$

$= 413.47 \approx 414$

From Table 4-3(S-8), we get,

$k_P = \tan^2(45 + \frac{\emptyset}{2}) = 6.44$ or, $\sqrt{k_P} = 2.538 \approx 2.54$

$s_q = s_\gamma = 1.0 + 0.1\ k_P \frac{B}{L} = 1 + 0.1(6.44)\frac{0.5}{2.0} = 1.161$

$d_q = d_\gamma = 1.0 + 0.1\ \sqrt{k_P} \frac{D}{B} = 1 + 0.1\ (2.54)\frac{0.5}{0.5} = 1.254$

Meyerhof Equation: From Table 4-1(P-220, S-9)

$$q_{ult} = cN_c s_c d_c + \gamma' D\, N_q s_q d_q + 0.5\, \gamma' B\; N_\gamma s_\gamma d_\gamma$$

Ignoring the c subscript term,

$$q_{ult} = \gamma' D\, N_q s_q d_q + 0.5\, \gamma' B\; N_\gamma s_\gamma d_\gamma \text{ ---------- (1)}$$

Table 4-4(P-223, S-12) and $\emptyset_{ps}$=47°

From Table 4-4 , we get N_q=187,

For $\emptyset > 40°$, we use $N_\gamma = (N_q - 1)\tan(1.4\,\emptyset)$

$= 413.47 \approx 414$

Ignoring any c subscript terms, Meyerhof equation, we get

$$q_{ult} = \gamma' D\, N_q s_q d_q + 0.5\, \gamma' B\, N_\gamma s_\gamma d_\gamma$$

$=9.31(0.5)(187)(1.161)(1.254)+0.5(9.31)(414)(1.161)(1.254)$

$=1262+1369=$ **2659 kPa vs 1863 kPa measured**.

Reference Books:

Foundation Analysis and Design, Fifth edition

By

Joseph E. Bowels

Chapter 4: Bearing Capacity of Foundation

JOSEPH E. BOWLES

FOUNDATION ANALYSIS AND DESIGN

FIFTH EDITION

McGRAW-HILL INTERNATIONAL EDITIONS
Civil Engineering Series

Assignment: (graded)

2x design problems given in Class Notes.

By 2400 hrs on 21 October 20