Owing to non-uniform velocity distribution in a channel section, the kinetic energy and the momentum of flow computed from the cross-sectional mean velocity are generally less than their actual values. To get the actual kinetic energy of flow, the kinetic energy based on the mean velocity is multiplied by the coefficient α, known as the *kinetic energy coefficient.* Similarly, to get the actual momentum, the momentum based on the mean velocity is multiplied by the coefficient β, known as the *momentum coefficient.* The energy and momentum coefficients α and β together are known as the *velocity distribution coefficients.*

The kinetic energy of flow passing ΔA (Fig.1.9) per unit time is equal to

$$\frac{1}{2}\times\rho u\Delta A\times u^2=\frac{\rho}{2}u^3\Delta A$$

where ρ is the mass density of water. Therefore, the total kinetic energy of flow passing the channel section is equal to

$$\frac{\rho}{2}\int_o^A u^3 dA$$

where A is the total area of the cross-section. The total kinetic energy based on the mean velocity U and corrected for the non-uniform distribution of velocity is

$$\alpha\frac{\rho}{2}U^3A$$

Equating the above two quantities and rearranging

$$\alpha=\frac{\int_o^A u^3dA}{U^3A}\approx\frac{\sum u^3\Delta A}{U^3A} \qquad (1.20)$$

The momentum of flow passing ΔA (Fig.1.9) per unit time is $\rho u\Delta A \times u = \rho u^2\Delta A$. Therefore, the total momentum of flow passing the channel section per unit time is equal to

$$\rho\int_o^A u^2 dA$$

The total momentum based on the mean velocity U and corrected for the non-uniform distribution of velocity is $\beta\rho U^2A$ so that

$$\beta=\frac{\int_o^A u^2dA}{U^2A}\approx\frac{\sum u^2\Delta A}{U^2A} \qquad (1.21)$$

The energy and momentum coefficients are always positive and never less than unity. For uniform velocity distribution in the channel section, $\alpha=\beta=1$. In all other cases, $\alpha>\beta>1$. The values of α and β are higher for laminar flow than for turbulent flow, since the effect of turbulence is to make the distribution of velocity more uniform over the channel section and reduce α and β.

Experimental results (Watts et al., 1967) suggest that, when the channel is straight and prismatic and the flow is turbulent and uniform or gradually varied, the two velocity distribution coefficients do not normally exceed 1.10 and 1.04, respectively, and one can assume $\alpha=\beta=1$. However, in channels of complex cross-section, upstream from weirs, in the vicinity of obstructions, near pronounced irregularities in alignment or when the flow is concentrated in one part of the section, values of α and β may even be greater than 2 and 1.35, respectively.

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Although the numerical values of α and β vary over a wide range depending on the velocity distribution, the ratio (α-1)/(β-1) tends to vary only slightly, in the range 2.8 to 3.0 (Henderson, 1966).

Example 1.4

In a wide channel the velocity varies along a vertical as u = 1 + 3z/h, where h is the depth of flow and u is the velocity at a distance z from the channel bottom. (i) Compute the discharge per unit width, (ii) determine the state of flow, and (iii) compute the velocity distribution coefficients α and β and the ratio (α-1)/(β-1), if h = 5 m.

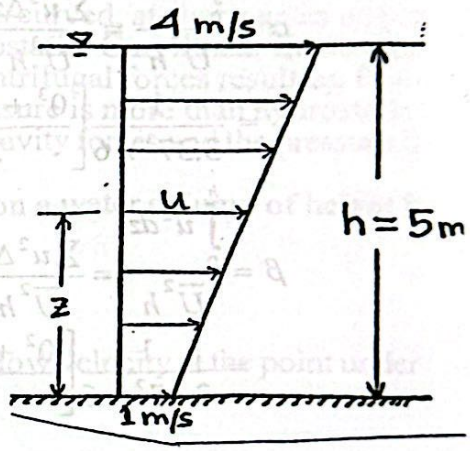
Solution

For a wide channel we can consider unit width of the channel and replace the area A in Eq.(1.19) by the flow depth h and dA by dz. Then, the cross-sectional mean velocity U becomes the depth-averaged velocity $\overline{U}$. Therefore,

$$\overline{U}=\frac{1}{h}\int_0^h u\,dz=\frac{1}{h}\int_o^h\left(1+\frac{3z}{h}\right)dz=\frac{1}{h}\left[h+\frac{3h^2}{2h}\right]=1+\frac{3}{2}=2.50\ m/s$$

Alternatively, since the velocity varies linearly from 1 m/s at the channel bottom (z/h = 0) to 4 m/s at the water surface (z/h = 1), the depth-averaged velocity is obtained as

$$\overline{U}=\frac{1+4}{2}=2.50\ m/s$$



(i) Then, the discharge per unit width is

$$q=\overline{U}h=2.50\times5=12.50\ \ m^2/s$$

(ii) The Reynolds and Froude numbers Re and Fr are then

$$\text{Re}=\frac{\overline{U}R}{\nu}=\frac{\overline{U}h}{\nu}=\frac{2.50\times5}{10^{-6}}=12.5\times10^6>12{,}500$$

(for a wide channel, R ≈ h)

$$Fr=\frac{\overline{U}}{\sqrt{gD}}=\frac{\overline{U}}{\sqrt{gh}}=\frac{2.50}{\sqrt{9.81\times5}}=0.36<1\quad(\because D=h)$$

Hence, the flow is subcritical turbulent.

(iii) The kinetic energy and momentum coefficients α and β are then

$$\alpha=\frac{1}{\overline{U}^3h}\int_0^h u^3dz=\frac{1}{\overline{U}^3h}\int_0^h\left(1+\frac{3z}{h}\right)^3dz$$

$$=\frac{1}{\overline{U}^3h}\int_0^h\left(1+\frac{9z}{h}+\frac{27z^2}{h^2}+\frac{27z^3}{h^3}\right)dz$$

$$=\frac{1}{\overline{U}^3h}(h+4.5h+9h+6.75h)=\frac{21.25}{\overline{U}^3}=\frac{21.25}{2.5^3}=1.36$$

$$\beta=\frac{1}{\overline{U}^2h}\int_0^h u^2dz=\frac{1}{\overline{U}^2h}\int_0^h\left(1+\frac{3z}{h}\right)^2dz$$

$$=\frac{1}{\overline{U}^2h}\int_0^h\left(1+\frac{6z}{h}+\frac{9z^2}{h^2}\right)dz=\frac{1}{\overline{U}^2h}(h+3h+3h)=\frac{7}{\overline{U}^2}=\frac{7}{2.5^2}=1.12$$

$$\therefore(\alpha-1)/(\beta-1)=0.36/0.12=3$$

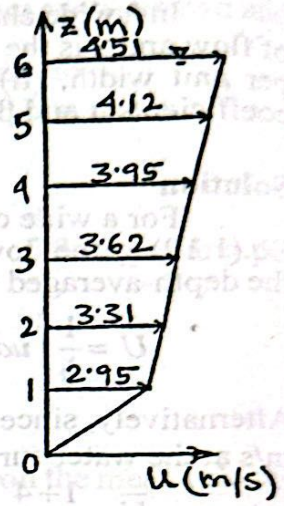
Note that when u is expressed as a function of z/h, the numerical values of $\overline{U}$, α and β become independent of the depth of flow.

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Example 1.5

Using the trapezoidal rule of numerical integration, compute the discharge per unit width, the mean velocity and the numerical values of α and β for the following velocity measurements (u is the velocity at a distance z from the channel bottom) along a vertical in a wide channel, when the total depth is 6 m.

z (m)	0	1	2	3	4	5	6
u (m/s)	0	2.95	3.31	3.62	3.95	4.12	4.51



Solution $h = 6m \qquad \Delta z = 1\ m$

$$q = \int_0^h u\,dz = \sum u \Delta z = \Delta z \sum u$$

$$= 1 \times \left[\frac{0 + 4.51}{2} + 2.95 + 3.31 + 3.62 + 3.95 + 4.12 \right] = 20.21\ m^2/s$$

$$\overline{U} = \frac{q}{h} = \frac{20.21}{6} = 3.57\ m/s$$

$$\alpha = \frac{\int_0^h u^3\,dz}{\overline{U}^3 h} = \frac{\sum u^3 \Delta z}{\overline{U}^3 h} = \frac{\Delta z}{\overline{U}^3 h} \sum u^3$$

$$= \frac{1}{3.37^3 \times 6} \left[\frac{0^3 + 4.51^3}{2} + 2.95^3 + 3.31^3 + 3.62^3 + 3.95^3 + 4.12^3 \right] = 1.25$$

$$\beta = \frac{\int_0^h u^2\,dz}{\overline{U}^2 h} = \frac{\sum u^2 \Delta z}{\overline{U}^2 h} = \frac{\Delta z}{\overline{U}^2 h} \sum u^2$$

$$= \frac{1}{3.37^2 \times 6} \left[\frac{0^2 + 4.51^2}{2} + 2.95^2 + 3.31^2 + 3.62^2 + 3.95^2 + 4.12^2 \right] = 1.11$$

1.8 PRESSURE DISTRIBUTION

Hydrostatic Pressure Distribution

Let us consider a vertical column of water of height h and cross-sectional area ΔA (Fig. 1.10). Let p be the intensity of pressure or unit pressure (force/unit area) at the bottom of the water column. Then,

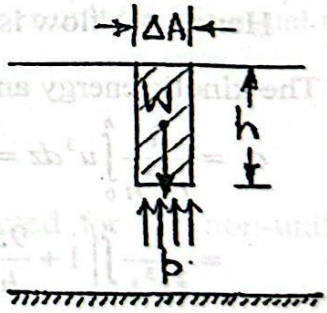


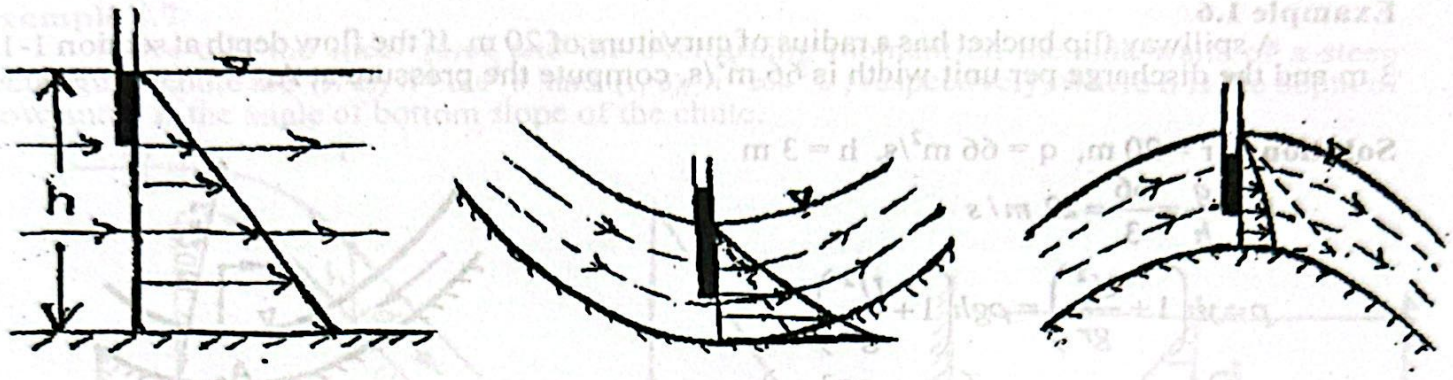
Fig.1.10 Definition sketch

$$p = \frac{Force}{Area} = \frac{Weight\ of\ water\ column}{Area} = \frac{W}{\Delta A} = \frac{\gamma h \Delta A}{\Delta A} = \gamma h = \rho g h \qquad (1.22)$$

Equation (1.22) indicates that the pressure at any point is directly proportional to the depth of the point below the free surface. This is known as the *hydrostatic distribution of pressure* and h is the *hydrostatic pressure head.*

The hydrostatic distribution of pressure is valid for parallel flow in a horizontal or small slope channel. When the curvature of the streamlines is small, the flow is known as *parallel flow* (Fig.1.11a). Uniform flow is practically parallel flow. Gradually varied flow may also be regarded as parallel flow since the curvature of the streamlines is small and negligible. Hence, the hydrostatic law of pressure distribution holds exactly for uniform flow and approximately for gradually varied flow in a horizontal or small slope channel.

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$p = \gamma h$ (a) Parallel flow

$p > \gamma h$ (b) Concave flow

$p < \gamma h$ (c) Convex flow

Fig.1.11 Pressure distribution in parallel, concave and convex flows

Pressure Distribution in Curvilinear Flow

When the curvature of the streamlines is considerable, the flow is known as *curvilinear flow*. Such situations may occur when the bottom of the channel is curved, at sluice gates and at free overfalls. In such cases, the pressure distribution is not hydrostatic. Curvilinear flows may either be *concave* or *convex*. In concave flow (Fig.1.11b), the centrifugal forces resulting from streamline curvature combine with the gravity forces and the pressure is more than hydrostatic. In convex flow (Fig.1.11c), the centrifugal forces act against the gravity forces and the pressure is less than hydrostatic.

Let us consider the forces acting in the vertical direction on a water column of height h and cross-sectional area ΔA (Fig.1.12). Then,

Mass of the water column = $\rho h \Delta A$

If r is the radius of curvature of the streamline and u is the flow velocity at the point under consideration, then

$$\text{Centrifugal acceleration} = \frac{u^2}{r}$$

and

$$\text{Centrifugal force} = \rho h \Delta A \times \frac{u^2}{r}$$

The intensity of pressure as a result of the centrifugal force is

$$p_c = \frac{Force}{Area} = \frac{\rho h \Delta A}{\Delta A} \times \frac{u^2}{r} = \rho h \frac{u^2}{r} \qquad (1.23)$$

and the pressure head

$$h_c = \frac{p_c}{\gamma} = \frac{\rho h}{\rho g} \frac{u^2}{r} = \frac{h u^2}{g r} \qquad (1.24)$$

For practical purposes, the velocity u is replaced by the cross-sectional mean velocity U.

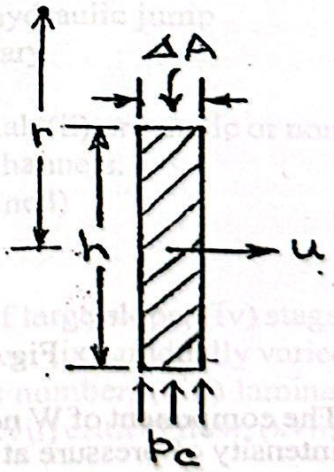


Fig.1.12 Definition sketch

The total pressure acting at the bottom of the water column is the algebraic sum of the hydrostatic pressure and the pressure due to the centrifugal action, i.e.

$$\text{Total pressure head} = h \pm h_c = h \pm \frac{hU^2}{gr} = h\left(1 \pm \frac{U^2}{gr}\right) \qquad (1.25)$$

and

$$\text{Total pressure} = \gamma(h \pm h_c) = \gamma h\left(1 \pm \frac{U^2}{gr}\right) \qquad (1.26)$$

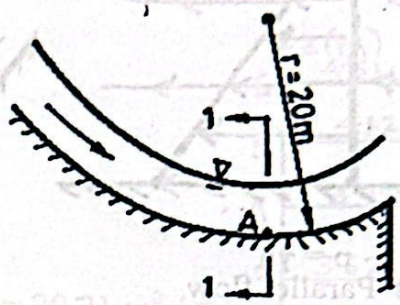
where the plus and minus signs are used with concave and convex flows, respectively.

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Example 1.6

A spillway flip bucket has a radius of curvature of 20 m. If the flow depth at section 1-1 is 3 m and the discharge per unit width is 66 m^2/s, compute the pressure at A.

Solution r = 20 m, q = 66 m^2/s, h = 3 m



$$U=\frac{q}{h}=\frac{66}{3}=22\ m/s$$

$$p=\gamma h\left(1+\frac{U^2}{gr}\right)=\rho gh\left(1+\frac{U^2}{gr}\right)$$

$$=1000\times 9.81\times 3\times\left(1+\frac{22^2}{9.81\times 20}\right)$$

$$=102030\ N/m^2$$

Effect of Slope on Pressure Distribution

Consider a water column of height h and cross-sectional area ΔA (Fig.1.13). The pressure at B in this case balances the component of the weight of the water column AB normal to the bed. Now, weight of the water column

$$W=\gamma d\Delta A$$

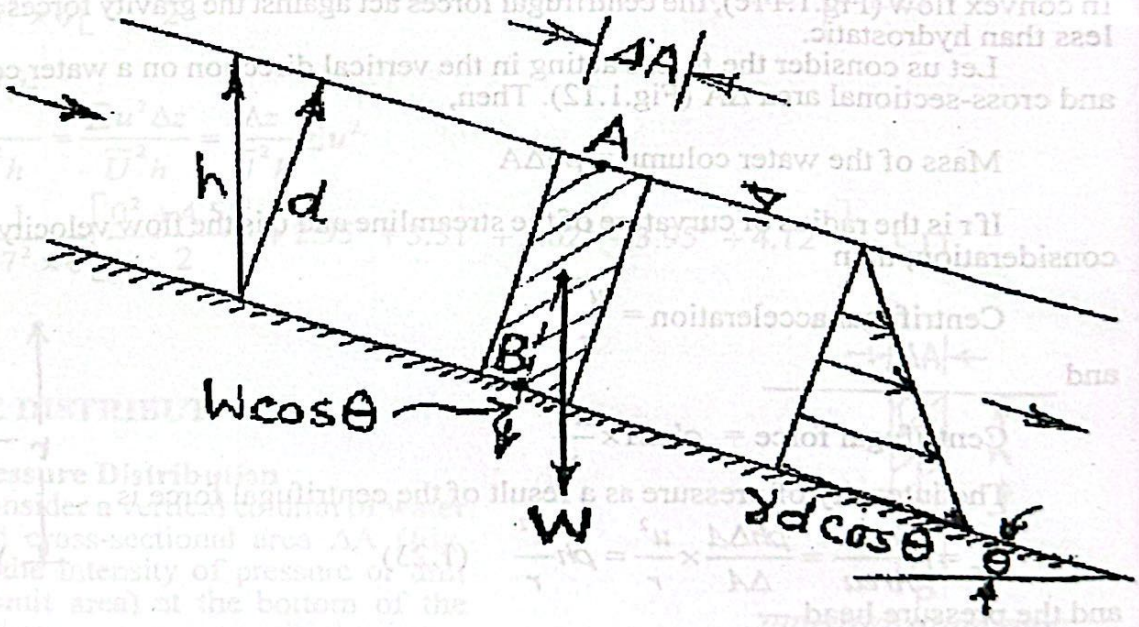


Fig.1.13 Pressure distribution in a channel of large slope

The component of W normal to the channel bed, i.e. along AB = $W\cos\theta = \gamma d\Delta A\cos\theta$. Hence, the intensity of pressure at the bottom of the water column

$$p=\frac{Force}{Area}=\frac{W\cos\theta}{\Delta A}=\frac{\gamma d\Delta A\cos\theta}{\Delta A}=\gamma d\cos\theta=\gamma h\cos^2\theta\ \ (\because d=h\cos\theta)\quad (1.27)$$

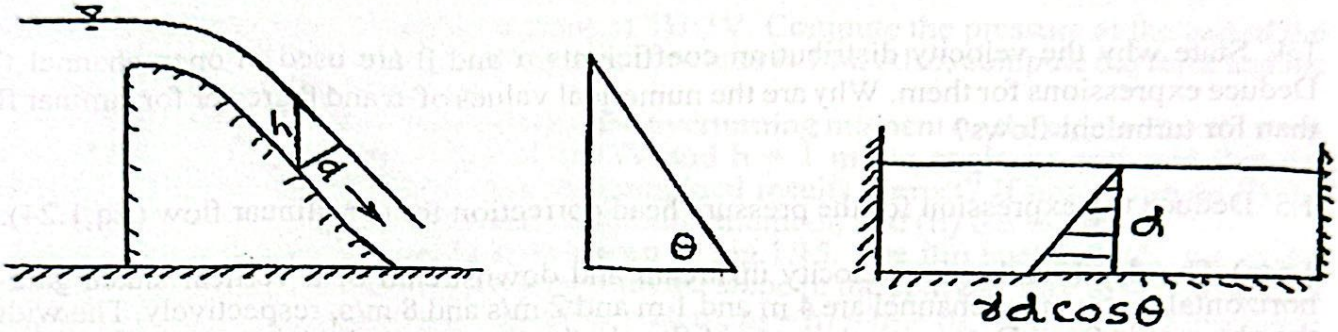
$$\therefore\ \text{Pressure head, } \frac{p}{\gamma}=d\cos\theta=h\cos^2\theta \quad (1.28)$$

Thus, for channels of large slope, the pressure head at any vertical depth is equal to the depth of flow h multiplied by a correction factor $\cos^2\theta$. For a small slope channel, $\theta \le 6°$, the use of Eq. (1.22) instead of Eq. (1.28) involves an error less than about 1% and can safely be ignored. So, the effect of slope on pressure distribution is considered for a channel of large slope only.

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Example 1.7

Prove that the shear force and the overturning moment on the side walls of a steep rectangular chute are $(1/2)\gamma h^2 \cos^3\theta$ and $(1/6)\gamma h^3 \cos^4\theta$, respectively, where h is the depth of flow and θ is the angle of bottom slope of the chute.



Solution Pressure at the bed of the chute, $p = \gamma d \cos\theta$

Shear force on the side walls, $F = \frac{1}{2} \times p \times d = \frac{1}{2} \times \gamma d \cos\theta \times d$ *(per unit length)*

$$= \frac{1}{2}\gamma d^2 \cos\theta = \frac{1}{2}\gamma h^2 \cos^3\theta \quad (\because d = h\cos\theta)$$

Overturning moment (about point O) $= F \times arm = F \times d/3$

$$= \frac{1}{2}\gamma d^2 \cos\theta \times \frac{d}{3} = \frac{1}{6}\gamma d^3 \cos\theta = \frac{1}{6}\gamma h^3 \cos^4\theta$$

PROBLEMS AND EXERCISES

1.1(a) State whether the following open channel flows are (i) steady or unsteady, (ii) subcritical or supercritical, (iii) gradually varied, rapidly varied or spatially varied, and (iv) laminar or turbulent:

1. Flood flow in a river
2. Flow upstream of a hydraulic jump
3. Overland flow
4. Tidal flow in an estuary
5. Flow downstream of a sluice gate.

(b) State whether the following open channels are (i) natural or artificial, (ii) prismatic or non-prismatic, (iii) rigid or mobile boundary, and (iv) small or large slope channels.

1. Alluvial river
2. Irrigation canal (unlined)
3. Laboratory flume
4. Spillway chute

1.2 Define (i) open channel flow, (ii) prismatic channel, (iii) channel of large slope, (iv) stage, (v) wide channel, (vi) steady flow, (vii) unsteady flow, (viii) uniform flow, (ix) gradually varied flow, (x) rapidly varied flow, (xi) spatially varied flow, (xii) Reynolds number, (xiii) laminar flow, (xiv) turbulent flow, (xv) Froude number, (xvi) subcritical flow, (xvii) critical flow, (xviii) supercritical flow, (xix) parallel flow, and (xx) curvilinear flow.

1.3(a) Write three differences between pipe flow and open channel flow.

(b) State why the solution of an open channel flow problem becomes difficult when the depth of flow is unknown.

(c) State why uniform flow can be steady only.

(d) How can you physically identify whether the flow in an open channel is laminar or turbulent?

(e) How can you physically identify whether the flow in an open channel is subcritical, critical or supercritical?

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(f) Explain why the Froude number is more important than the Reynolds number to determine the state of open channel flow.

(g) Explain why an elementary wave can move upstream in subcritical flow, but not in supercritical flow. What is the practical significance of this phenomenon?

1.4 State why the velocity distribution coefficients α and β are used in open channel flow. Deduce expressions for them. Why are the numerical values of α and β greater for laminar flows than for turbulent flows?

1.5 Deduce the expression for the pressure head correction for curvilinear flow (Eq.1.24).

1.6(a) The depth and mean velocity upstream and downstream of a vertical sluice gate in a horizontal rectangular channel are 4 m and 1 m and 2 m/s and 8 m/s, respectively. The width of the channel is 6 m. Determine the state of flow both upstream and downstream of the gate.

(b) Consider the following data for the Padma (Ganges) river at the Baruria station in Faridpur on the 2nd July, 1989: A = 33,500 m^2, Q = 56,200 m^3/s and B = 3820 m. Compute the state of flow. Assume that the river is wide.

1.7 Water flows in an open channel at a depth of 1 m and a mean velocity of 3 m/s. Compute the discharge and determine the state of flow if the channel is

i) wide
ii) rectangular with b = 6 m
iii) trapezoidal with b = 6 m and s = 2
iv) triangular with s = 2
v) parabolic with B = 4 m
vi) circular whose diameter is 2.5 m.

If elementary waves are created in these channels, determine the speeds of the wave fronts upstream and/or downstream.

1.8(a) The average depth of water in a wide river connected to sea is 5 m. Determine the time taken by a tidal wave to travel from the river mouth to 30 km upstream (i) when there is no flow in the river, and (ii) when the average flow velocity in the river is 1 m/s.

(b) Waves of small amplitude are created at the center of a circular-shaped pond of radius 50 m. The waves are found to reach the edge of the pond in 10 s. Estimate the approximate volume of water in the pond assuming that the depth of water in the pond is same everywhere.

(c) Two islands A and B situated in an ocean are 1000 km apart. An earthquake in island A creates a single tidal wave (tsunami) which propagates across the ocean to island B. If the average water depth in the ocean is 1 km, estimate the time of arrival of the tsunami in island B.

1.9(a) In a wide river the velocity varies along a vertical as $u = 1 + 2z/h$, where h is the total depth and u is the velocity at a distance z from the channel bottom. The river is 5 m deep. (i) Compute the discharge per unit width, (ii) determine the state of flow, and (iii) compute the numerical values of the velocity distribution coefficients α and β and the ratio $(\alpha-1)/(\beta-1)$.

(b) Solve the above problem if $u = 1+2(z/h)^{1/2}$

1.10(a) For laminar flow the velocity distribution along a vertical can be approximated by

$$u = u_o \frac{\pi z}{2h} \sin$$

where u is the velocity at a distance z from the channel bottom, h is the depth of flow and u_o is the velocity at the free surface. Compute the velocity distribution coefficients α and β and the ratio $(\alpha-1)/(\beta-1)$.

(b) For turbulent flow the velocity distribution along a vertical can be approximated by $u \propto z^n$, when n = 1/7 (Prandtl's one-seventh power law). Determine the velocity distribution coefficients α and β and the ratio $(\alpha-1)/(\beta-1)$ in terms of n and for n = 1/7. Compare the numerical values of α and β with those obtained for laminar flow in Problem 1.10(a).

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1.11(a) Solve Example 1.5 using the Simpson's rules of numerical integration.

(b) Figure 1.14 shows the velocity distribution downstream of a sluice gate under submerged condition. Using the trapezoidal rule of numerical integration, compute the discharge per unit width, the mean velocity of flow and the numerical values of α and β.

1.12(a) A steep rectangular chute has a slope of 1H:3V. Compute the pressure at the bed of the chute if the vertical depth of water flowing over the chute is 1 m. Also, compute the force and the overturning moment on its side walls.

(b) While computing the shear force and the overturning moment on the side walls of a steep spillway chute having an inclination of 1H:3V and h = 1 m, an engineer assumed that the pressure distribution is hydrostatic. Are the computed results correct? If not, compute (i) the correct values of the shear force and the overturning moment, and (ii) the % error.

(c) A high-head overflow spillway is shown in Fig.1.15. The flip bucket at the toe of the spillway acts to change the direction of flow from the slope of the spillway face to the horizontal and to discharge the flow into the air. If $r_1 = r_2 = 20$ m, $h_1 = h_2 = h_3 = h_4 = 1$ m and the discharge over the spillway is 6.5 $m^3/s/m$, determine the intensities of pressure at points 1, 2, 3 and 4.

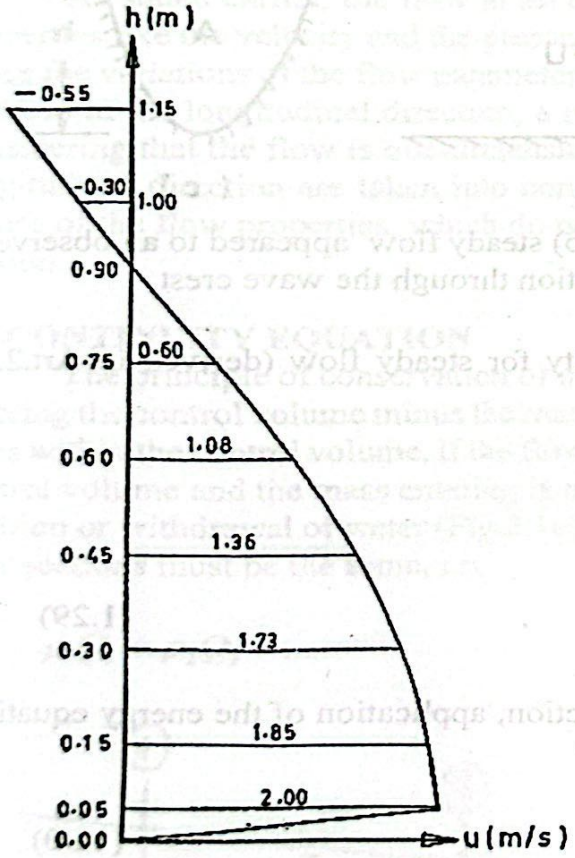


Fig. 1.14

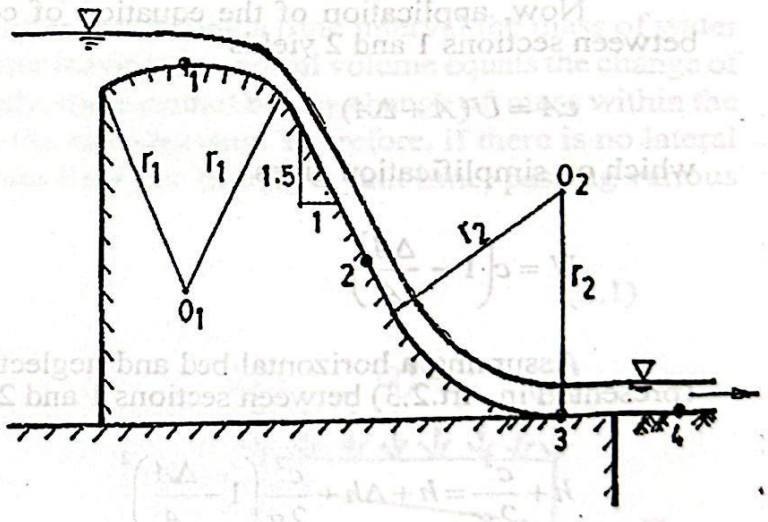


Fig. 1.15

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