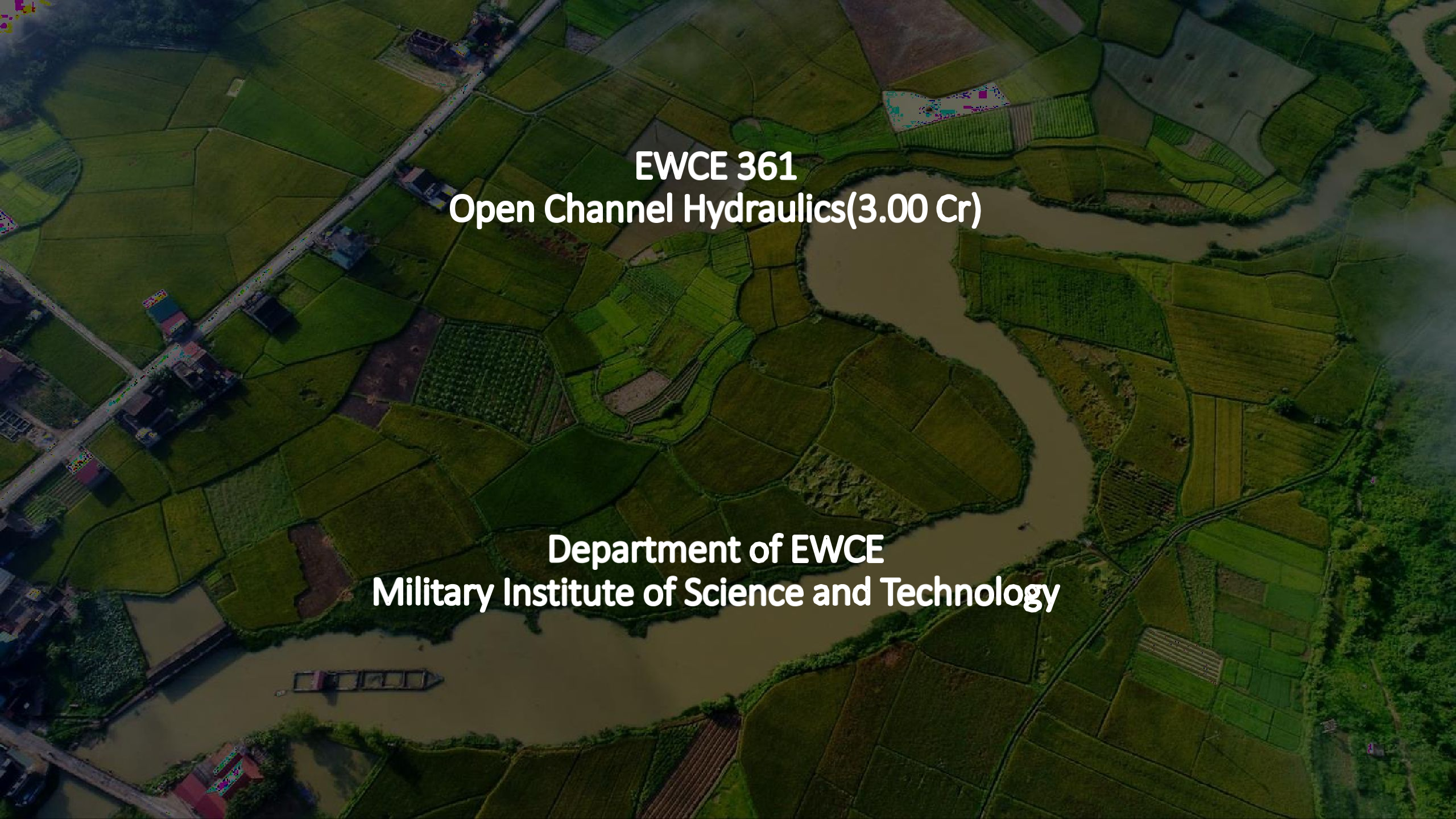
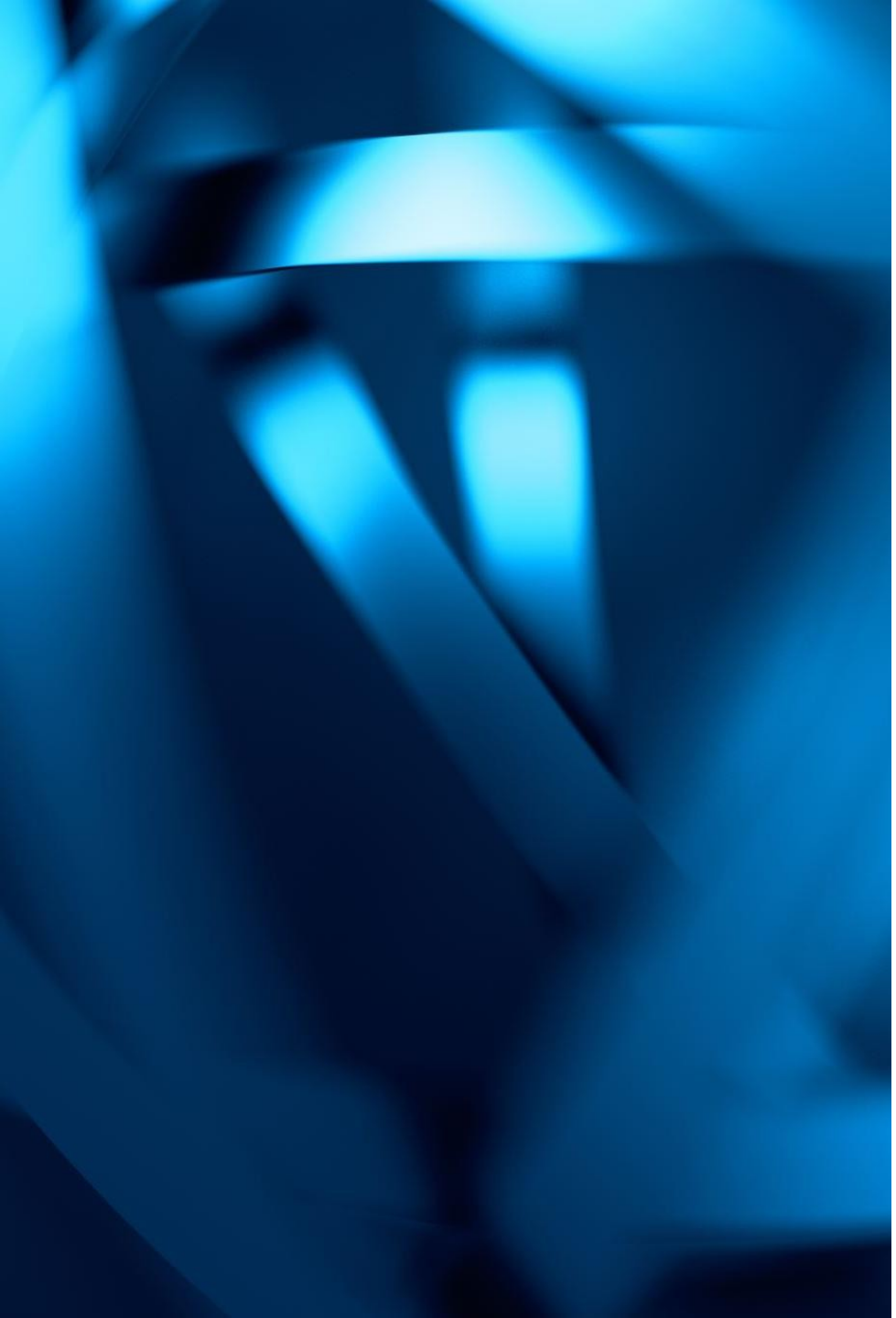


EWCE 361
Open Channel Hydraulics(3.00 Cr)

Department of EWCE
Military Institute of Science and Technology



Reference Book

Open Channel Flow by MD. ABDUL HALIM
Open Channel Hydraulics by Ven TE

Chow Flow in Open Channel by K

Subramanya

Topics

1. Basic concepts of OCF
2. Design of Channels
3. Gradually Varied Flow
4. Hydraulic Jump



Class	Topic
1	Introduction to OCF, Examples, Characteristics, Classification, Geometric Properties and Derivation, Effect of Viscosity and Gravity
2	Types of Flow, Velocity Distribution, Math Example
3	Math Example, Pressure Distribution
4	Math Example
5	Design of Channel(Theory) , best hydraulic section derivation
6	Best hydraulic section math problem
7	Shear stress ratio, Lane theory
8	Yalin Karahan Approach, lacey theory
9	Math problem, overall review of the chapter

Class	Topic
10	Introduction to Gradually Varied Flow
11	Flow profile method of drawing
12	Flow profile drawing examples and math
13	Direct step method math
14	Standard step method math
15	Introduction to hydraulic jump, types, derivations
16	Derivations, characteristics, Explanation of some figures related to this chapter
17	Math Problem
18	Math problem

Definition and Example of Open Channel Flow

1.1 INTRODUCTION

The flow of water in a conduit may be either open channel flow or pipe flow. In pipe flow the flowing water is completely enclosed by solid boundary and flow occurs under pressure. In open channel flow (Fig.1.1) the flowing water is not completely enclosed by solid boundary and flow occurs with a free surface. A free surface is subjected to atmospheric pressure. Therefore, open channel flow may be defined as the flow of water in a conduit with a free surface. Open channel flow is also known as the *free surface flow*.

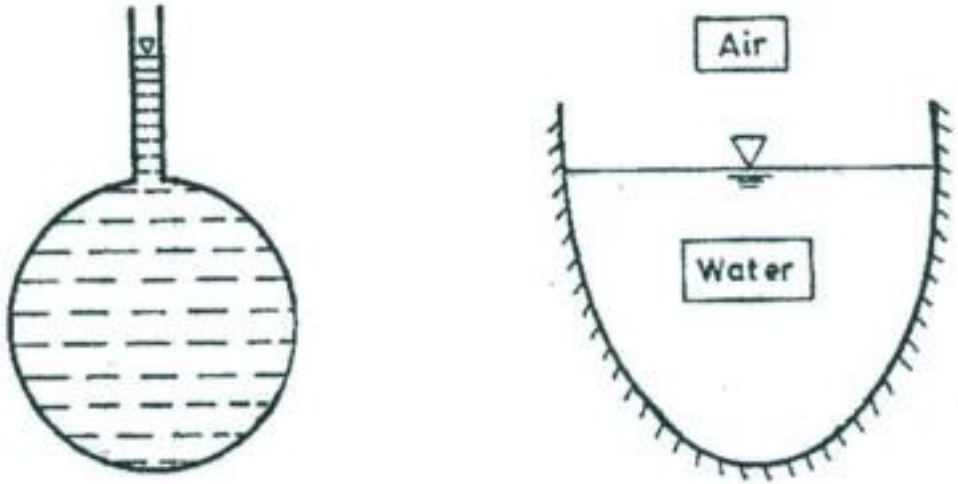


Fig. 1.1 Pipe flow and open channel flow

Flows in rivers and canals are some of the many familiar examples of open channel flow. Flow of water in a closed conduit, e.g. in an underground sewer or in a culvert, may be open channel flow if the flow occurs with a free surface (Fig.1.2). The flow of groundwater with a free surface is also an example of open channel flow.

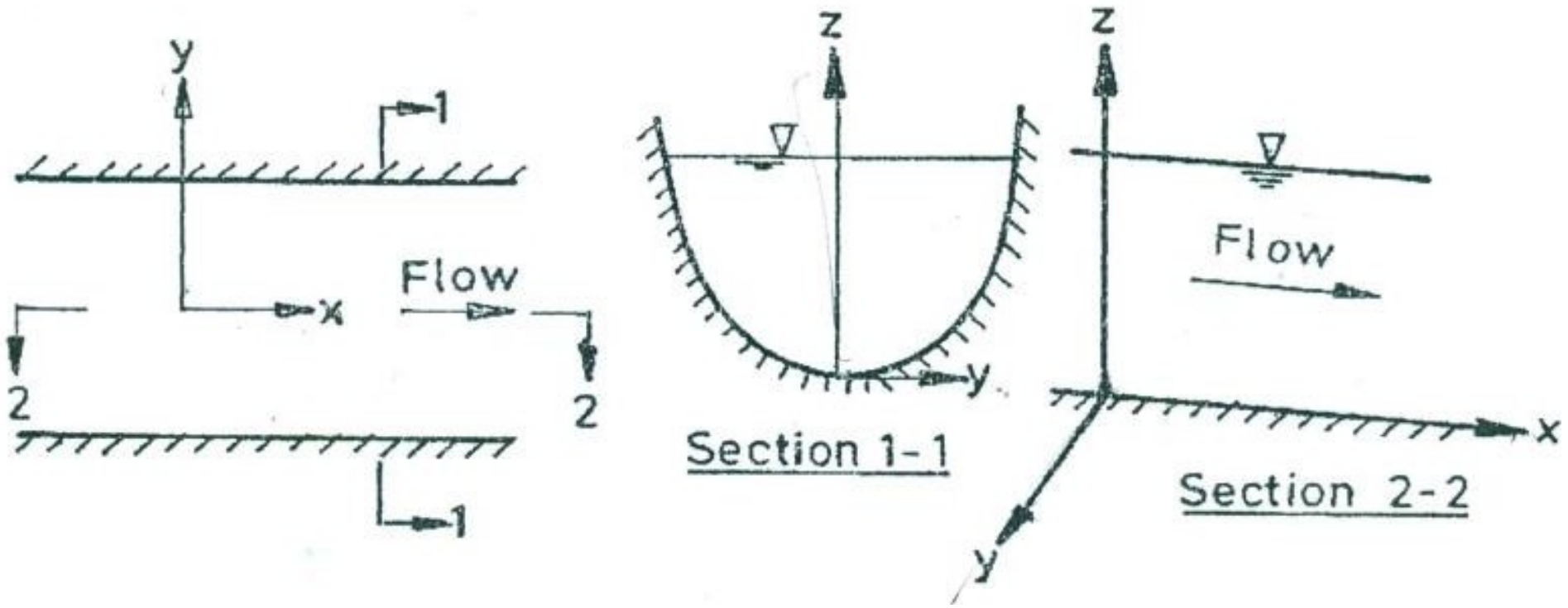


Fig.1.2 Flow in an underground sewer

Characteristics of Open Channel Flow

Open channel flow occurs under the action of gravity and at atmospheric pressure. Basically all open channels have a bottom slope and flow occurs downstream along the slope. The component of the gravity force or the weight of water along the slope acts as the driving force. Obviously, for open channel flow to occur, the total energy at an upstream section must be greater than the total energy at a downstream section.

Long Section and Cross Section in Open Channel



Classification of Open Channel

1.2 KINDS OF OPEN CHANNEL

An open channel is a conduit in which water flows with a free surface. Open channels are classified on different criteria as follows.

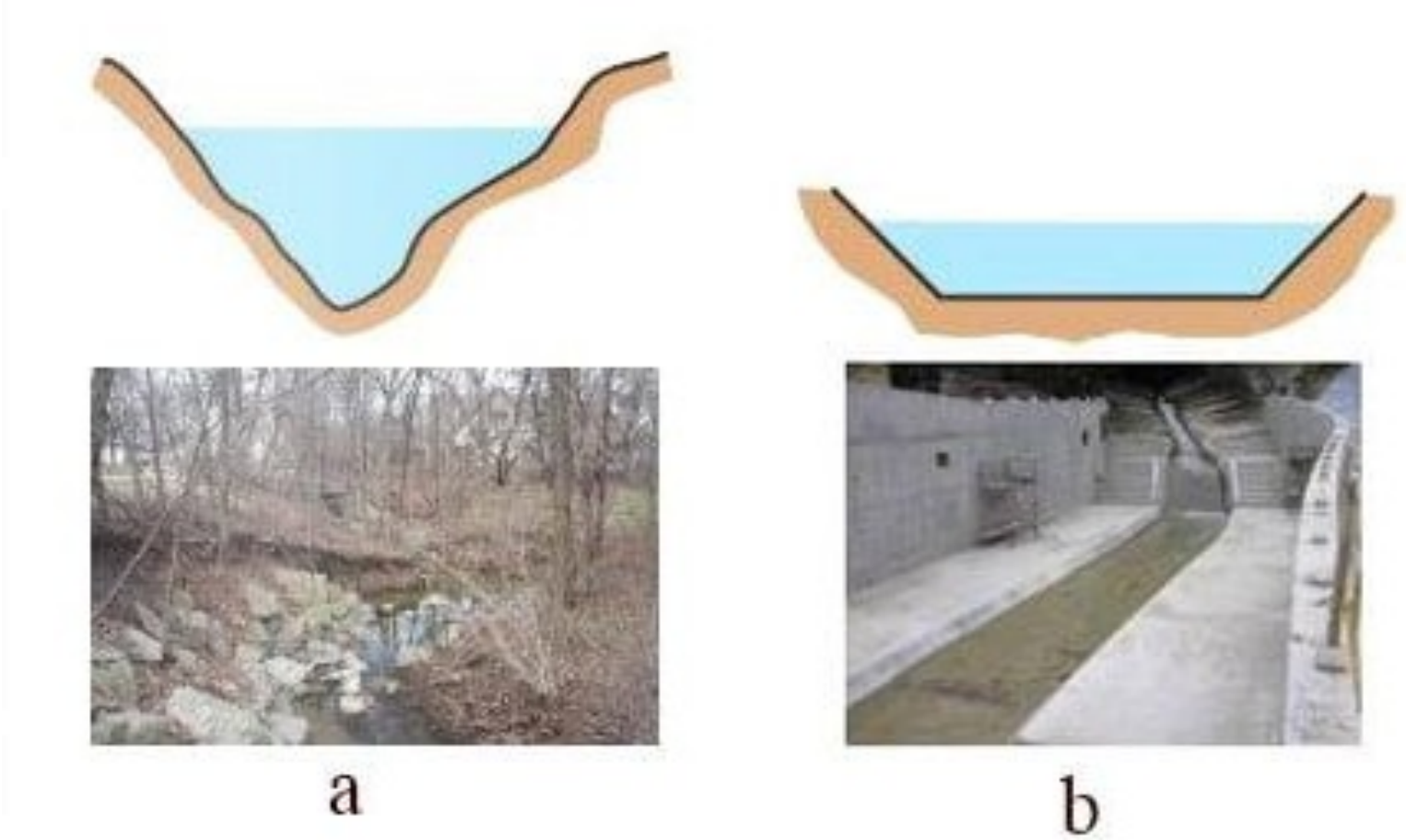
(a) Natural and Artificial Channels

Natural open channels include all channels that exist naturally on the earth, e.g. rivers and tidal estuaries. They are generally very irregular in shape.

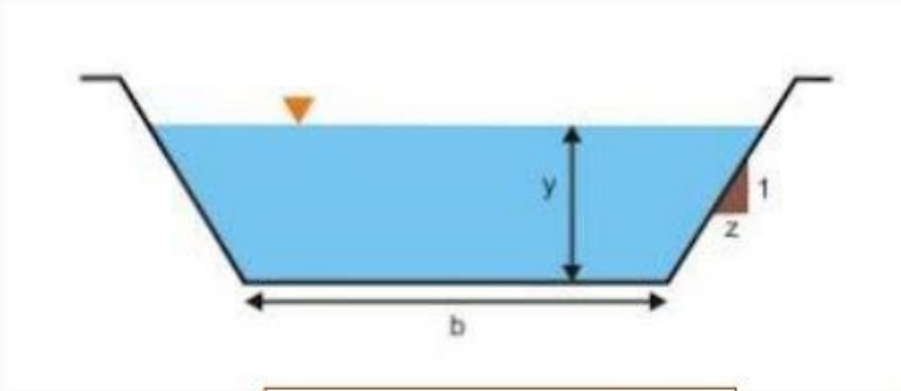
Artificial open channels are the channels developed by men, e.g. irrigation canals, laboratory flumes, spillway chutes, drops, culverts, roadside gutters etc. They are usually designed with regular geometric shapes.

(b) Prismatic and Non-prismatic Channels

A channel with unvarying cross-section and constant bottom slope is called a prismatic channel; otherwise it is non-prismatic. The artificial channels are usually prismatic and the natural channels are generally non-prismatic.



(a) Natural Channel and (b) Artificial Channel



Prismatic channels



Non-prismatic channels

(c) Rigid and Mobile Boundary Channels

A channel with immovable bed and sides is known as a rigid boundary channel, e.g. lined canals, sewers and non-erodible unlined canals. If the channel boundary is composed of loose sedimentary particles moving under the action of flowing water, the channel is called a mobile boundary channel. An alluvial channel is a mobile boundary channel transporting the same type of material as that comprising the channel perimeter.

(d) Small and Large Slope Channels

An open channel having a bottom slope greater than 1 in 10 is called a channel of large slope; otherwise it is a channel of small slope (Chow, 1959). The slopes of ordinary channels, natural or artificial, are far less than 1 in 10. However, some artificial channels like drops and chutes have slopes far more than 1 in 10.

Note: Alluvial channels are water channels made up of loose sediments. The bank of these channel are subjected to erosion,

Chute is an open channel like structure which is constructed on steep slope of the gully face with a suitable inlet and outlet. By chute, water flows mainly from upstream to downstream.



Rigid Boundary Channel



Mobile Boundary Channel



Large slope channels



Small slope channels

Some Examples of Open Channel

- Canal
- Flume
- Chute
- Drop
- Culvert
- Open-Flow Tunnel

Some Examples of Open Channel

- CULVERT is a covered channel flowing partly full, which is installed to drain water through highway and railroad embankments.







- CANAL is usually a long and mild-sloped channel built in the ground.



- FLUME is a channel usually supported on or above the surface of the ground to carry water across a depression.



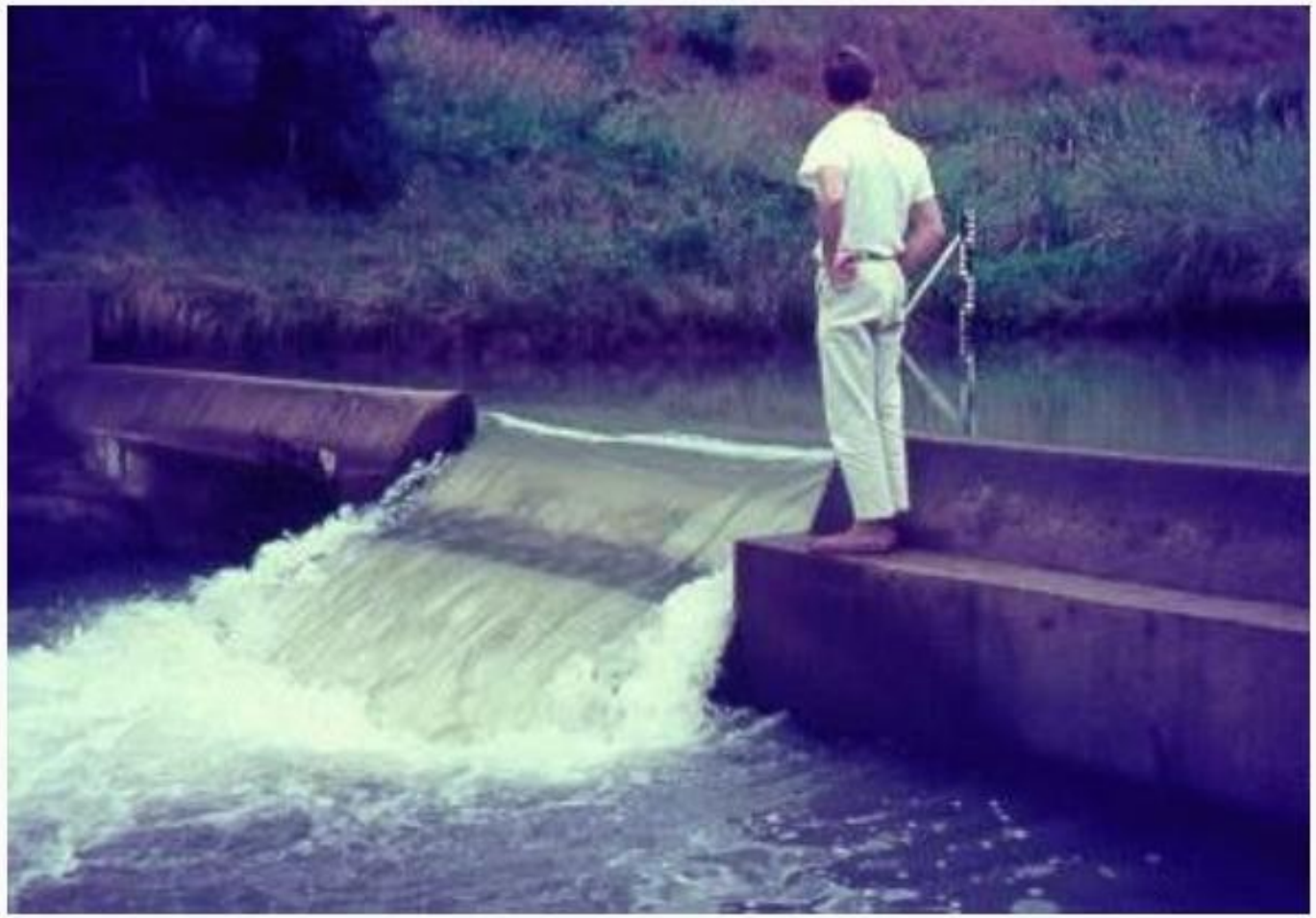


- CHUTE is a channel having steep slopes. A chute spillway is a common and basic design that transfers excess water from behind the dam down a smooth decline into the river below.





- DROP is similar to a chute, but the change in elevation is affected in a short distance.



- OPEN-FLOW TUNNEL is a comparatively long covered channel used to carry water through a hill or any obstruction on the ground.

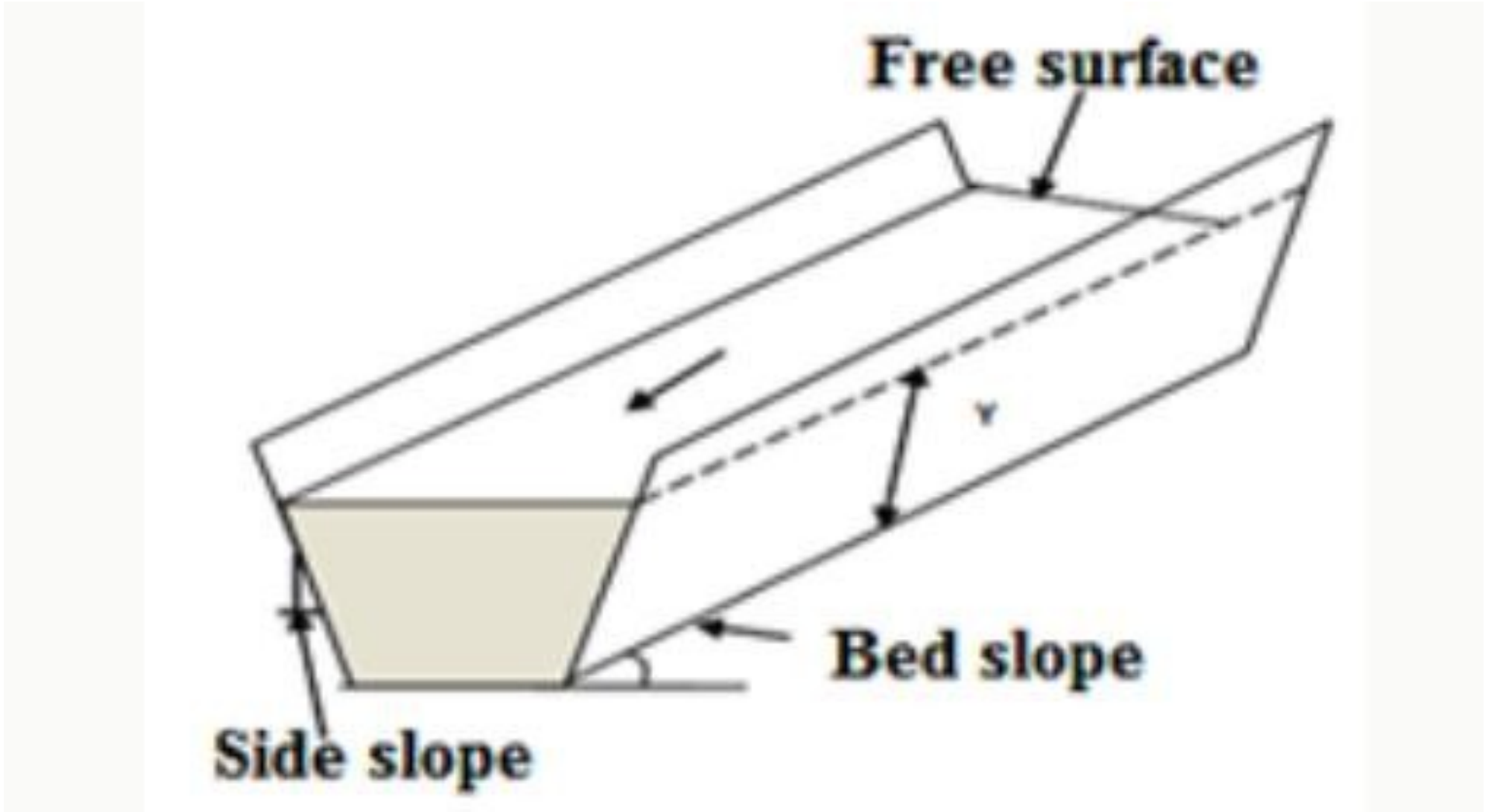




1.3 CHANNEL GEOMETRY AND SECTION ELEMENTS

(a) Prismatic Channels

The rectangle, trapezoid, triangle, parabola and circle are the most commonly used shapes of prismatic or regular or uniform channels. Table 1.1 furnishes the formulas for the cross-sectional area A, the wetted perimeter P and the top width B for these channel shapes.


Free surface
y
Bed slope
Side slope

The properties of a channel section which are determined by the geometric shape of the channel and the depth of flow are known as the *geometric elements of a channel section*. They are defined below.

Depth of flow h and depth of flow section d: The depth of flow is the vertical distance from the lowest point of a channel section to the water surface. The depth of flow section is the depth of flow normal to the direction of flow. The relation between h and d is

$$d = h\cos\theta \tag{1.1}$$

where θ is the angle made by the channel bottom with the horizontal.

Stage: The stage is the elevation of the water surface relative to a horizontal datum and may be positive or negative.

Flow area A: The flow area is the cross-sectional area of the flow normal to the direction of flow.

Wetted perimeter P: The wetted perimeter is the length of the interface between water and channel boundary.

Top width B: The top width is the width of a channel section at the water surface.

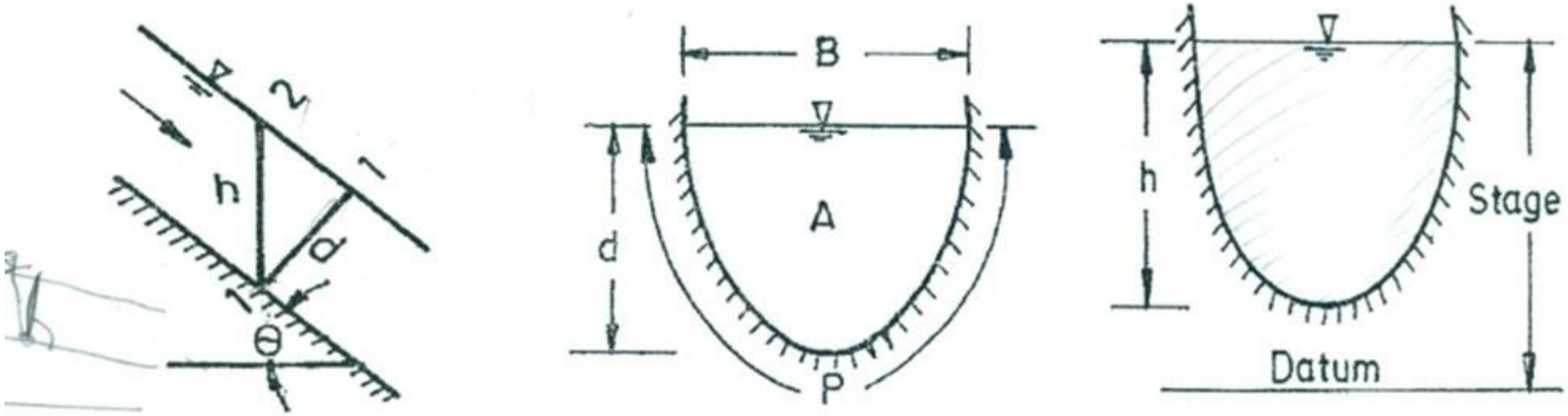
Hydraulic radius R: The hydraulic radius is the ratio of the flow area to the wetted perimeter, i. e.

$$R = A/P \tag{1.2}$$

Hydraulic depth D: The hydraulic depth is the ratio of the flow area to the top width, i.e.

$$D = A/B \tag{1.3}$$

The cross-section of a channel taken normal to the direction of flow is called a *channel section* (Fig. 1.4). A *vertical channel section* is the vertical section passing through the bottom or lowest point of a channel section.



Channel section and vertical channel section

Channel section (section 1-1)

Vertical channel section (section 1-2)

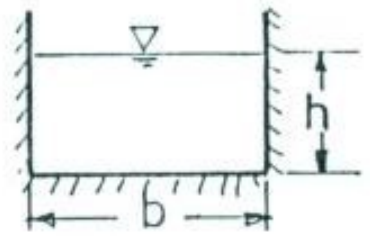
Fig.1.4 Channel section and vertical channel section

Formula for Geometric Elements of a Channel

A=cross sectional area of flow b=bottom width of channel B=top width of channel P=wetted perimeter s=side slope of channel

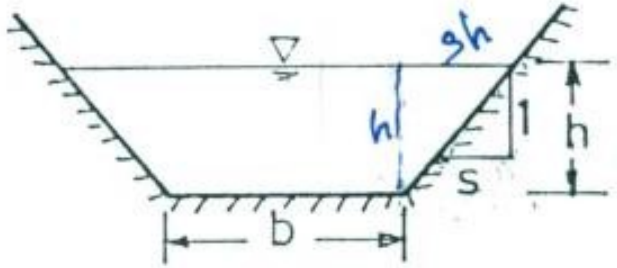
1. Rectangle

$$A = bh$$
$$P = b + 2h$$
$$B = b$$

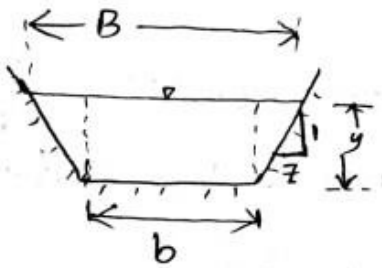


2. Trapezoid

$$A = (b + sh)h$$
$$P = b + 2\sqrt{1 + s^2}\,h$$
$$B = b + 2sh$$



Formula Derivation for Trapezoidal Channel



side slope = z:1
Depth of flow, y

$B = b + yz + yz$

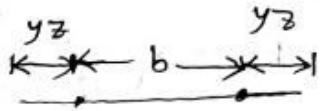
$= b + 2yz$

Area of trapezoid $= \frac{1}{2}\left(b + (b+2yz)\right) \times y$

$= \frac{1}{2}(2b + 2yz) \times y$

$= \frac{1}{2} \times 2(b+yz)y$

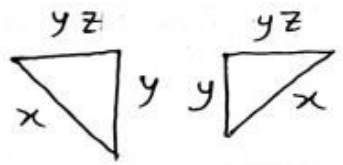
$= (b+yz)y$



Top width, $B = b + 2yz$

Perimeter, $P = b + y\sqrt{1+z^2} + y\sqrt{1+z^2}$

$= b + 2y\sqrt{1+z^2}$



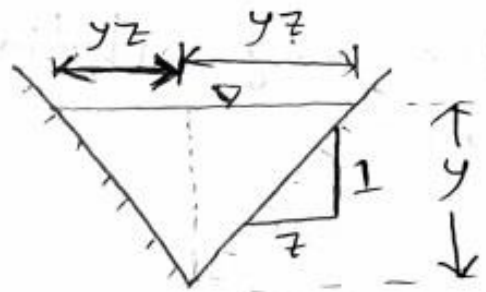
$x^2 = \sqrt{y^2 + (yz)^2}$

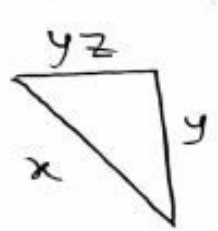
$= \sqrt{y^2 + y^2z^2}$

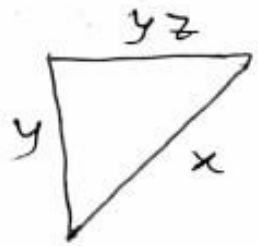
$= \sqrt{y^2(1+z^2)}$

$= y\sqrt{1+z^2}$

Formula Derivation for Triangular Channel







Area of triangle, $A = \frac{1}{2} \times 2yz \times y$

$= y^2 z$

Perimeter, $P = y\sqrt{1+z^2} + y\sqrt{1+z^2}$

$= 2y\sqrt{1+z^2}$

Top width, $B = yz + yz = 2yz$

$x = \sqrt{y^2 + (yz)^2}$

$= \sqrt{y^2 + y^2 z^2}$

$= \sqrt{y^2(1+z^2)}$

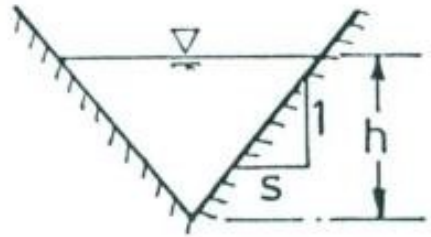
$= y\sqrt{1+z^2}$

3. Triangle

$$A = sh^2$$

$$P = 2\sqrt{1+s^2}\,h$$

$$B = 2sh$$



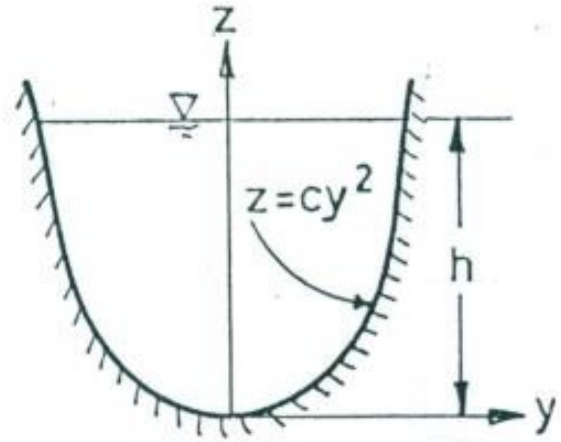
4. Parabola (perimeter equation $z = cy^2$)

$$A = \frac{2}{3}Bh = \frac{4h^{3/2}}{3\sqrt{c}}$$

$$P = \frac{B}{2}\left[\sqrt{1+\left(\frac{4h}{B}\right)^2} + \frac{B}{4h}\ln\left(\frac{4h}{B} + \sqrt{1+\left(\frac{4h}{B}\right)^2}\right)\right]$$

$$\approx B + \frac{8h^2}{3B}\,(when\ \ 0 < 4h/B \le 1)$$

$$B = \frac{3A}{2h} = \frac{2h^{1/2}}{\sqrt{c}}$$



5. Circle

$$h = d_0[1 - \cos(\omega/2)]/2$$

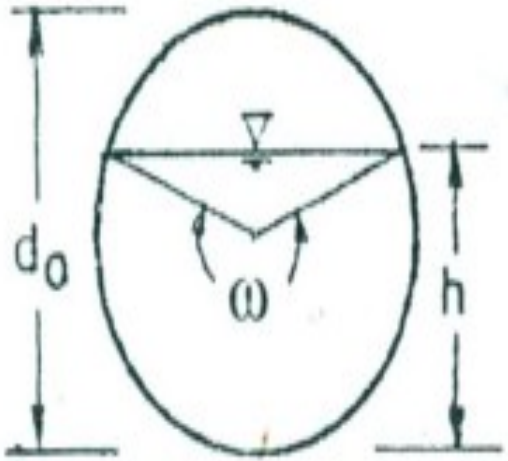
$$\omega = 2\cos^{-1}(1 - 2h/d_0)$$

$$A = (\omega - \sin\omega)d_0^{\,2}/8$$

$$P = \omega d_0/2$$

$$B = d_0 \sin(\omega/2)$$

radian



1.5 EFFECTS OF VISCOSITY AND GRAVITY

(a) Effect of Viscosity

The effect of the viscous forces relative to the inertial forces on open channel flow is expressed by the *Reynolds number*, which may be written as

$$\mathrm{Re} = \frac{Inertial\ \ forces}{Viscous\ \ forces} = \frac{Mass * acceleration}{Viscous\ \ shear\ \ stress * area} = \frac{\rho L^3\left(L/T^2\right)}{\tau . A}$$

$$= \frac{\rho L^3\left(L/T^2\right)}{\mu(du/dz)L^2} = \frac{\rho L^3\left(L/T^2\right)}{\mu(L/T/L)L^2} = \frac{\rho L^2}{\mu T} = \frac{(L/T)L}{\mu/\rho} = \frac{UL}{\nu} \qquad (1.9)$$

where U is a characteristic or representative velocity, taken as the mean velocity of flow, L is a characteristic length and ν is the kinematic viscosity of water. For water at 20^0C, $\nu = 10^{-6}$ m^2/s. This value of ν is normally used.

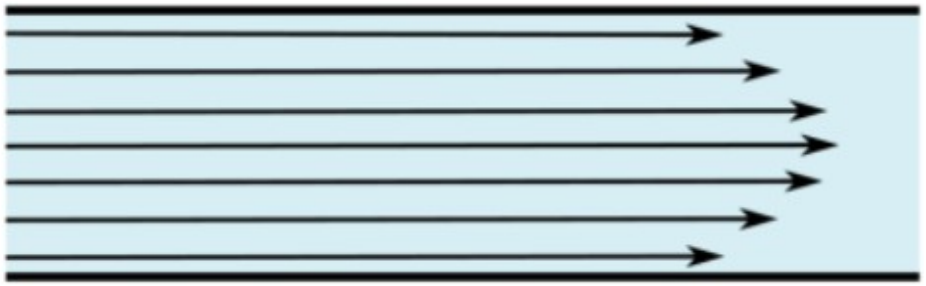
In open channel flow, the characteristic length commonly used is the hydraulic radius R (= A/P), so that

$$\mathrm{Re} = \frac{UR}{\nu} \qquad (1.10)$$

Then, when

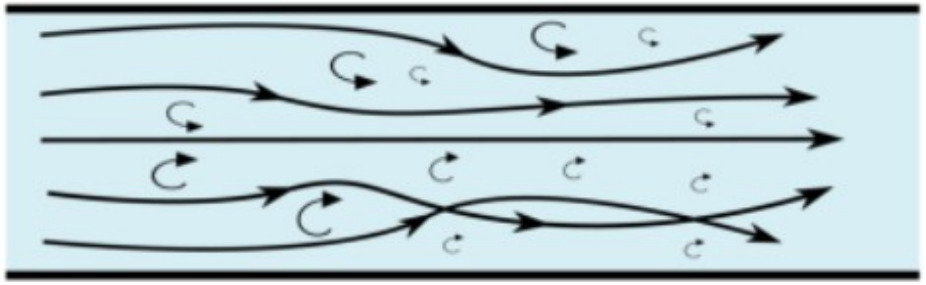
i) Re < 500, the flow is *laminar*,
ii) Re > 12,500, the flow is *turbulent*, and
iii) 500 ≤ Re ≤ 12,500, the flow is *transitional*.

laminar flow



Laminar flow is the movement of fluid particles along well-defined paths or streamlines, where all the streamlines are straight and parallel.

turbulent flow



Turbulent flow is defined as the flow in which the fluid particles move in a zigzag way.

(b) Effect of Gravity

The effect of the gravity forces relative to the inertial forces on open channel flow is determined by the *Froude number*, which may be written as

Froude Number

$$F_r = \sqrt{\frac{\text{Inertia force}}{\text{Gravitational force}}}$$

Inertia force = Mass x inertial acceleration

= Density x vol^m x inertial acceleration

= Density x vol^m x $\frac{\text{velocity}}{\text{time}}$

= Density x $\frac{vol^m}{\text{time}}$ x velocity

$= \rho \times Q \times V$

$= \rho \times AV \times V = \rho AV^2$

D=

Gravitational force = mass × gravitational acceleration

$= m \times g$

$= \rho V \times g$ $\quad$ volume = (length)^3

$= \rho \times L^3 \times g$

$= \rho \times L^2 \times L \times g$

$= \rho A L g$

$\therefore F_R = \sqrt{\frac{\rho A V^2}{\rho A L g}} = \sqrt{\frac{V^2}{Lg}} = \frac{V}{\sqrt{Lg}} = \frac{V}{\sqrt{gD}}$

In open channel flow, the characteristic length commonly used is the hydraulic depth, $D = \frac{A}{B}$

D=

Combined Effect of Viscosity and Gravity: State of Flow

A combined effect of viscosity and gravity may produce any of the following four states of flow in an open channel based on the numerical values of the Froude and the Reynolds numbers:

i) Subcritical laminar $Fr < 1, Re < 500$
ii) Supercritical laminar $Fr > 1, Re < 500$
iii) Subcritical turbulent $Fr < 1, Re > 12{,}500$
iv) Supercritical turbulent $Fr > 1, Re > 12{,}500$

The first two flow states, subcritical laminar and supercritical laminar, are very rare. The flow in most rivers and canals is subcritical turbulent and that at the feet of drops and spillways is supercritical turbulent.

Wide Channel

When the width of a rectangular channel is very large compared to the depth, i.e. b >> h (generally if b ≥ 10h), the sides of the channel have practically no influence on the velocity distribution in the central region. Such a channel is known as a wide channel (Chow, 1959). Many rivers in alluvial plains are treated as a wide channel. For a wide channel

$$R = \frac{A}{P} = \frac{bh}{b+2h} \approx \frac{bh}{b} \approx h \qquad (1.4)$$

and the discharge is usually expressed per unit width of the channel which is designated by q (= Q/b, where Q is the total discharge and b is the bottom width) and the units of q are $m^3/s/m$ or simply m^2/s.

Example 1.1

A trapezoidal channel has a bottom width of 6 m and side slopes of 2:1. Compute the discharge and determine the state of flow in this channel if the depth of flow is 1.5 m and the mean velocity of flow is 2.30 m/s. If elementary waves are created in this channel, determine the speed of the wave fronts upstream and downstream.

Solution Trapezoidal channel, b = 6 m, s = 2, h = 1.5 m, U = 2.30 m/s

$$A = (b + sh)h = (6 + 2 \times 1.5) \times 1.5 = 13.5\ m^2$$

$$P = b + 2\sqrt{1 + s^2}\,h = 6 + 2\sqrt{1 + 2^2} \times 1.5 = 12.71\ m$$

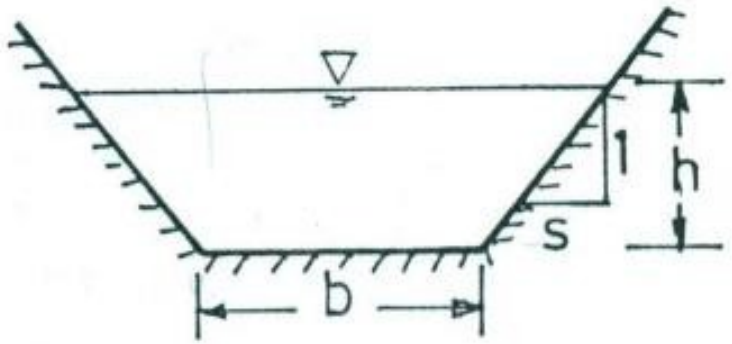
$$B = b + 2sh = 6 + 2 \times 2 \times 1.5 = 12\ m$$

$$R = A/P = 13.5/12.71 = 1.06\ m$$

$$D = A/B = 13.5/12 = 1.13\ m$$

$$\therefore Q = AU = 13.5 \times 2.30 = 31.05\ m^3/s$$

$$\mathrm{Re} = \frac{UR}{\nu} = \frac{2.30 \times 1.06}{10^{-6}} = 2.44 \times 10^6 > 12{,}500$$



$$Fr = \frac{U}{\sqrt{gD}} = \frac{2.30}{\sqrt{9.81 \times 1.13}} = 0.69 < 1$$

Hence, the flow is subcritical turbulent.

Now, $c = \sqrt{gD} = \sqrt{9.81 \times 1.13} = 3.33\ m/s$

$\therefore$ Speed of the wave fronts upstream = c – U = 3.33 – 2.30 = 1.03 m/s

and, speed of the wave fronts downstream = c + U = 3.33 + 2.30 = 5.63 m/s

Example 1.2

A circular channel 2.75 m in diameter carries a discharge of 6.55 m^3/s at a depth of 1.1 m. Determine the state of flow.

Solution Circular channel, $d_0 = 2.75$ m, $Q = 6.55$ m^3/s, $h = 1.1$ m

$$\omega = 2\cos^{-1}(1-2h/d_0) = 2\cos^{-1}(1-2\times1.1/2.75) = 2.74\ rad$$

$$A = (\omega - \sin\omega)d_o^{\,2}/8 = (2.74 - Sin2.74)2.75^2/8 = 2.22\ m^2$$

$$P = \omega d_o/2 = 2.74\times2.75/2 = 3.77\ m$$

$$B = d_o \sin(\omega/2) = 2.75\times\sin(2.74/2) = 2.69\ m$$

$$R = A/P = 2.22/3.77 = 0.59\ m$$

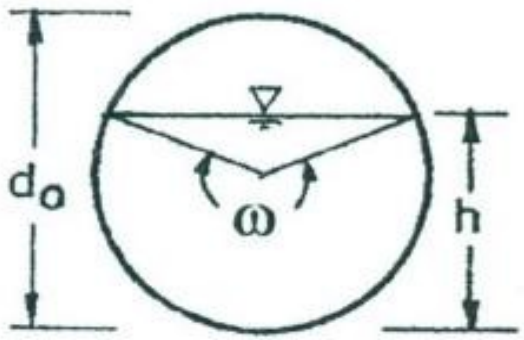
$$D = A/B = 2.22/2.69 = 0.82\ m$$

$$U = Q/A = 6.55/2.22 = 2.95\ m/s$$

$$\text{Re} = \frac{UR}{\nu} = \frac{2.95\times0.59}{10^{-6}} = 1.74\times10^6 > 12{,}500$$

$$Fr = \frac{U}{\sqrt{gD}} = \frac{2.95}{\sqrt{9.81\times0.82}} = 1.04 > 1$$

Hence, the flow is supercritical turbulent.



Dynamic Viscosity

Unit:

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{\text{shear stress}}{\frac{\text{velocity}}{\text{displacement}}} = \frac{\frac{N}{m^2}}{\frac{m/s}{m}}$$

$$= \frac{N/m^2}{\left(\frac{m}{s} \times \frac{1}{m}\right)} = \frac{N.s}{m^2}$$

$$= N s m^{-2}$$

C.G.S. unit $\mu = \frac{\text{dyne.sec}}{cm^2}$ (1 poise)

$$1 \text{ poise} = \frac{\text{dyne.sec}}{cm^2} = \frac{10^{-5} N.sec}{\left(\frac{1}{100} m\right)^2} = \frac{10^{-5}}{\frac{1}{10^4}} N.s/m^2 \quad [1 \text{ dyne} = 10^{-5} N]$$

$$= 10^{-5} . 10^4 \ N.S/m^2$$

$$= 10^{-1} \ N.S/m^2$$

$$\therefore 1 \text{ poise} = \frac{1}{10} N.S/m^2$$

Kinematic Viscosity

→ kinematic viscosity is the ratio of dynamic viscosity of the fluid to its density.

Mathematically, $\nu = \frac{\mu}{\rho}$

unit: kinematic viscosity, $\nu = \frac{\mu}{\rho} = \frac{N.S/m^2}{kg/m^3} = \frac{\left[kg.\frac{m}{s^2}\right].S/m^2}{kg/m^3}$

$= kg.\frac{m}{s^2}.\frac{s}{m^2} \times \frac{m^3}{kg}$

$= m^2/sec$ (SI unit)

CGS unit: cm^2/sec (1 stoke)

$cm^2/sec = \left(\frac{1}{100}\right)^2 m^2/sec = 10^{-4} m^2/sec$

1 centistoke $= \frac{1}{100}$ stoke

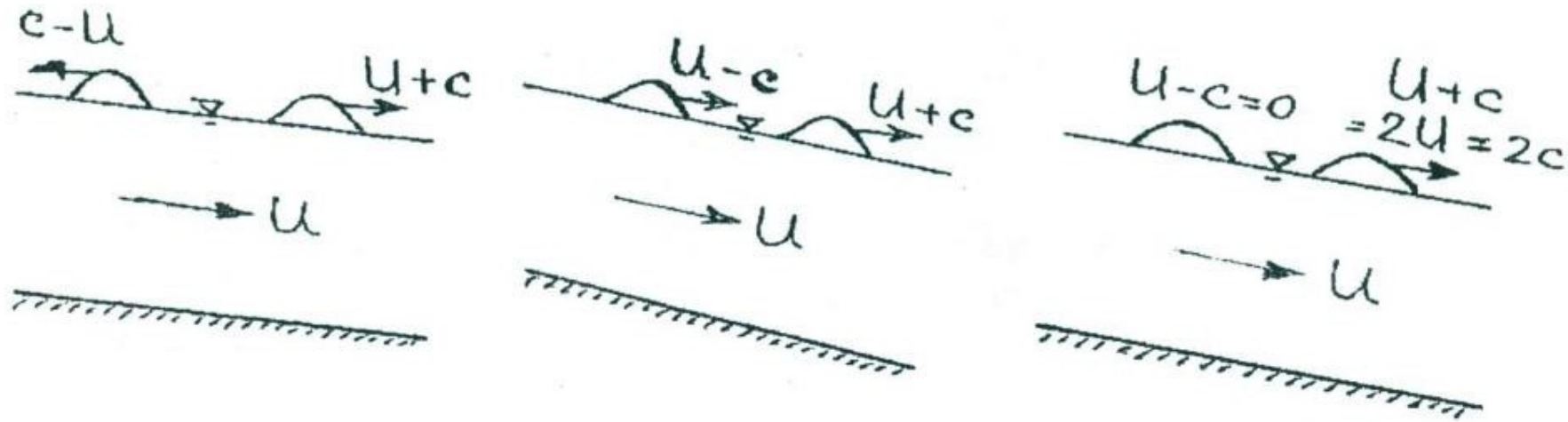
Celerity

Equation (1.12) states that at the critical state of flow (Fr = 1), the flow velocity is equal to $\sqrt{gD}$. It can be shown that $\sqrt{gD}$ is the velocity (or celerity) of an elementary or small-amplitude wave on the surface of still water (see Appendix 1.1). The Froude number can then be defined as the ratio between the flow velocity U and the wave celerity c, i.e.

$$Fr = \frac{U}{c} \tag{1.13}$$

Obviously, then

i) for subcritical flow, Fr < 1 and U < c,
ii) for critical flow, Fr = 1 and U = c, and
iii) for supercritical flow, Fr > 1 and U > c.



Fr < 1, U < c
(a) Subcritical flow

Fr > 1, U > c
(b) Supercritical flow

Fr = 1, U = c
(c) Critical flow

Fig. 1.7 Propagation of wave in subcritical, supercritical and critical flows

The fact that a small disturbance such as an elementary wave can propagate upstream against the flow in subcritical flow, but not in supercritical flow, has important practical significance. In subcritical flow, conditions upstream are affected by downstream conditions. Therefore, subcritical flow is controlled from downstream. In supercritical flow, conditions upstream are not affected by downstream conditions, and hence, supercritical flow is controlled from upstream.

Q: Does the highest velocity always occur in the surface of water for an open channel flow? Explain

1.6 VELOCITY DISTRIBUTION

Velocity Distribution in a Channel Section

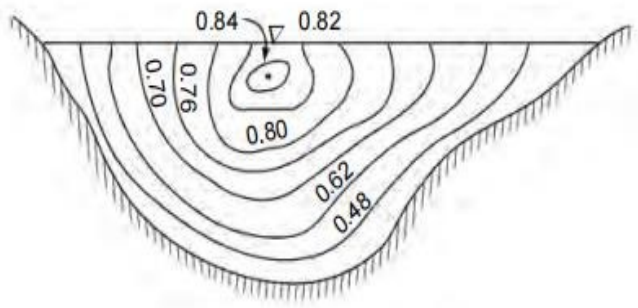
Owing to the presence of the free surface and the friction over the channel bed and banks, the velocities are not uniformly distributed in an open channel flow section. The velocity is zero at the solid boundary and gradually increases with distance from the boundary (Fig.1.8a,b). The velocity distribution in a channel section also depends on the channel geometry, the roughness of the channel, the presence of bends etc. The measured maximum velocity usually occurs below the free surface at a distance of 0.05 to 0.25 of the depth and is about 10% to 30% higher than the cross-sectional mean velocity.

In turbulent flow, the variation of velocity along a vertical can be approximated by a *logarithmic* or *power law*. In this case, the average of the velocities at 0.2 and 0.8 of the depth or the velocity at 0.6 of the depth below the water surface (Fig.1.8c) is approximately equal to the mean velocity in the vertical.

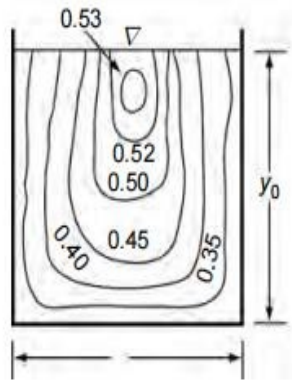
Measurement of Velocity

The velocity of flow in an open channel can be measured with a current meter. It is the standard practice of the U.S. Geological Survey to determine the average velocity in a vertical $\overline{U}$ by measuring the velocities at 0.2 and 0.8 of the depth below the free surface when the depth is more than 0.61 m (2 ft), or at 0.6 of the depth when the depth is less than 0.61 m, i.e.

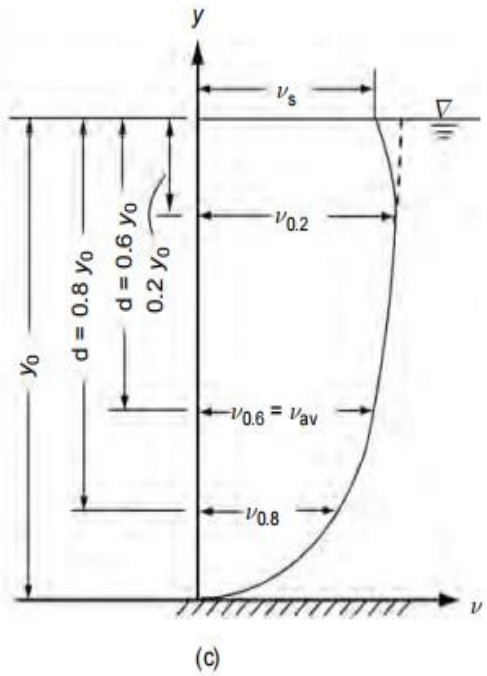
$$\overline{U} \approx \frac{u_{0.2} + u_{0.8}}{2} \approx u_{0.6} \qquad (1.14)$$



(a)



(b)



(c)

Measurement of Velocity

The velocity of flow in an open channel can be measured with a current meter. It is the standard practice of the U.S. Geological Survey to determine the average velocity in a vertical $\bar{U}$ by measuring the velocities at 0.2 and 0.8 of the depth below the free surface when the depth is more than 0.61 m (2 ft), or at 0.6 of the depth when the depth is less than 0.61 m, i.e.

$$\bar{U} \approx \frac{u_{0.2} + u_{0.8}}{2} \approx u_{0.6} \qquad (1.14)$$

1.7 VELOCITY DISTRIBUTION COEFFICIENTS

The flow in a straight prismatic channel is in fact three-dimensional, i.e. the flow properties like the velocity and the pressure vary in the longitudinal, lateral and vertical directions. However, the variations of the flow parameters in the lateral and vertical directions are usually small compared to those in the longitudinal direction. Consequently, a majority of open channel flow problems are analyzed by considering that the flow is one-dimensional, i.e. we consider the cross-sectional mean values of the flow parameters that vary from section to section only.

In the one-dimensional method of flow analysis, the discharge and the mean velocity of flow in a channel section are computed as follows. Let u be the velocity over an elementary area ΔA of the channel cross-section (Fig. 1.9). Then, the discharge past a channel section is given by

$$Q = \int_O^A u dA \qquad (1.18)$$

and the mean velocity of flow U for the entire cross-section is given by

$$U = \frac{Q}{A} = \frac{1}{A}\int_O^A u dA \qquad (1.19)$$

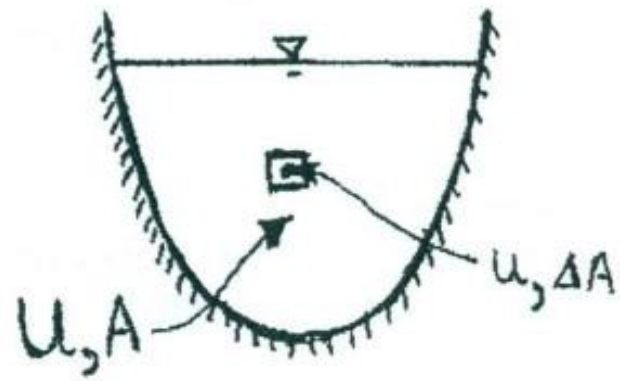


Fig.1.9 Definition sketch

In the one-dimensional method of flow analysis, owing to non-uniform velocity distribution in a channel section, the kinetic energy and the momentum of flow computed from the cross-sectional mean velocity are generally less than their actual values. To get the actual kinetic energy of flow, the kinetic energy based on the mean velocity is multiplied by the coefficient α, known as the *kinetic energy coefficient* or the *Coriolis coefficient*. Similarly, to get the actual momentum, the momentum based on the mean velocity is multiplied by the coefficient β, known as the *momentum coefficient* or the *Boussinesq coefficient*. The energy and momentum coefficients α and β together are known as the *velocity distribution coefficients*.