EWCE 361 — SA

Open Channel Hydraulics

flow w/ atm pressure

free surface

Prismatic → regular cross-section (rect, circular, tra...) ← most artificial channels

Non-prismatic → irregular shape ← natural

trapezoidal – best hydraulic section

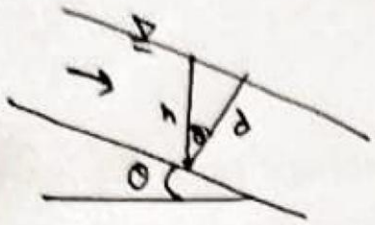
least concrete, max vol

Rigid, mobile boundary channel

Small, large " "

Depth of flow, h.

depth of flow section, d

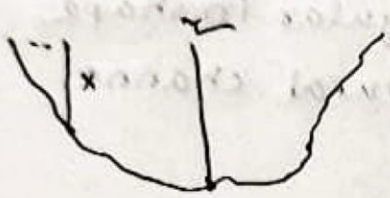


section depth normal to the dir. of flow

$d = h\cos\theta$

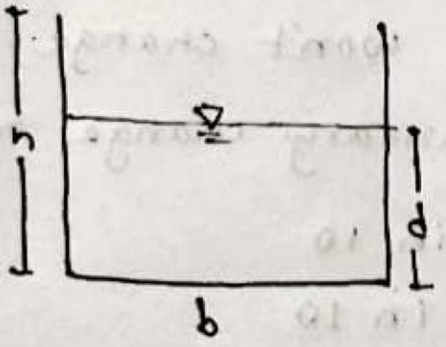
slope na thakle h = d hoito

consider lowest point



Stage - datum lvl., can be neg. w.r.t. datum

Flow area: A



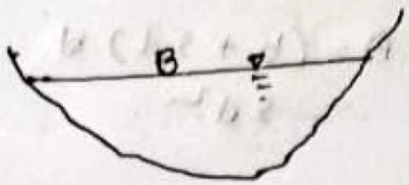
$A = bd$

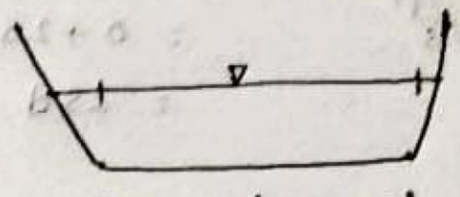
Wetted perimeter, P

(channel er shatha jeta lege ashe

$P = b + 2 \times d$

Top width, B

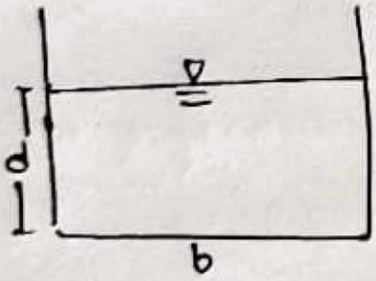




hydraulic radius, $R = \frac{A}{P}$

depth, $D = \frac{A}{B}$

A = hori. belonging area (A) x dist.



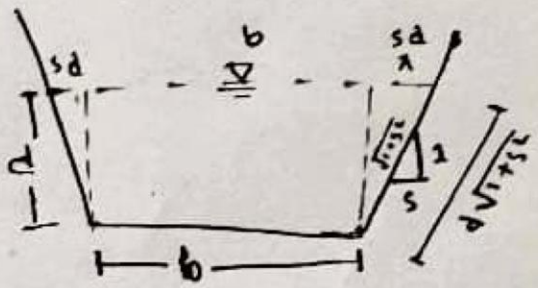
$A = bd$

$B = b$

$P = b + 2 \times d$

$R = \frac{bd}{b+2d}$

$D = \frac{bd}{b} = d$



$\frac{x}{s} = \frac{d}{1}$

$x = sd$

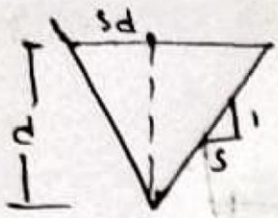
$A = \frac{b + b + 2sd}{2} \times d = \frac{2(b+sd)}{2} \times d = (b+sd)d$

$B = b + 2 \times sd$

$P = b + 2d\sqrt{1+s^2}$

$R = \frac{(b+sd)d}{b + 2d\sqrt{1+s^2}}$

$D =$



$B = b + 2sd$ $\quad P = 2d\sqrt{1+s^2}$

$= 0 + 2sd$ $\quad A = (b + sd)d$

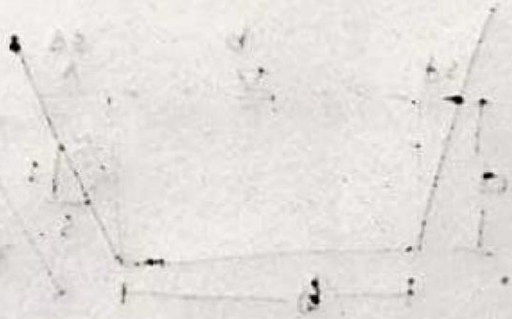
$= 2sd$ $\quad = sd^2$

trapezoidal er moloi just b nai, so, $b = 0$.

—

parabolic — shuttro diye dibe

circle —





SA/L03 30/11

Effects of viscosity & gravity

Viscosity — inertial / viscous

$$Re = \frac{VR}{\nu}$$

$R = \frac{\text{area}}{\text{perimeter}}$

500 — 12500 transitional (inclusive)

Gravity

Froude Number — $\sqrt{\frac{\text{inertia}}{\text{gravitational}}}$

$$Fr = \frac{V}{\sqrt{gD}}$$

$D = \frac{A}{B}$ — $\frac{\text{area}}{\text{top width}}$

Combined eff. state of flow

wide channel

$b >> h$

$m^3/s/m$ (per meter width)

---

side slope → j : j

side : ~~ht~~

Ex. 1.1



Celerity

$C \rightarrow g/d$

Ex. 1.2

Velocity distribution



Velocity distribution

Kinetic energy of flow passing ΔA per unit time

$$\frac{1}{2} m u^2$$

$$\frac{1}{2} \rho \Delta V \times u^2$$

$$\frac{1}{2} \rho \Delta A L u^2$$

$$\frac{1}{2} \rho \Delta A \frac{1}{t} u^2$$

$$\frac{1}{2} \rho \Delta A u \times u^2$$

$$= \frac{\rho}{2} u^3 \Delta A$$

$$\frac{\rho}{2} \int_0^A u^3 dA$$

$$\alpha \frac{\rho}{2} v^3 A = \frac{\rho}{2} \int_0^A u^3 dA$$

$$\Rightarrow \alpha = \cdots$$

velocity distribution coeff.

$$V = \frac{Q}{A} = \frac{1}{A}\int_0^A u\,dA$$

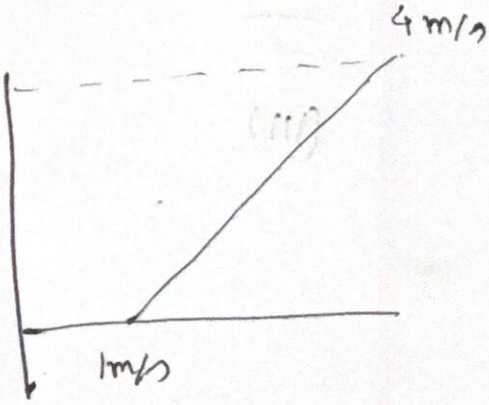
α, β

$$\beta = \frac{\int_0^A u^2\,dA}{V^2 A} \simeq \frac{\Sigma u^2 \Delta A}{V^2 A}$$

$\alpha > \beta$ usually

Ex 1.4

$$u = 1 + \frac{3z}{h}$$



velocity profile

$Q = AV$

$\Rightarrow \frac{Q}{b} = \frac{A}{b} V$

$\Rightarrow q = hV$

Sol: Alt 1: $V = \frac{1}{A}\int_0^A u\,dA = \frac{1}{h}\int_0^h u\,dz = \frac{1}{h}\int_0^h \left(1 + \frac{3z}{h}\right) dz$

$= \ldots = 2.5$ m/s

Alt 2: $\bar{V} = \frac{1+4}{2} = 2.5$ m/s

(i) $q = \bar{v}h = 2.5 \times 5 = 12.5 \ m^3/s/m$

—

Wide channel,

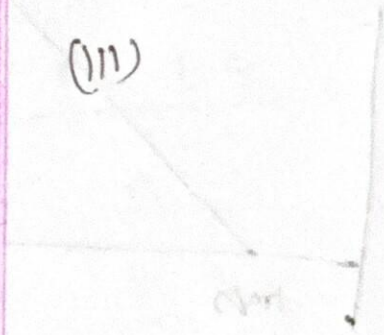
$R = D = h$

—

(ii)

—

(iii)



Ex. 1.5

$$\frac{0+2.95}{2} + \frac{2.95+3.31}{2} + \frac{3.31+3.62}{2} + \cdots$$

$$\frac{1}{2}\left[\frac{0}{2} + 2.95 + 3.31 + 3.62 \cdots\right]$$



SA/L06 08/12

Pressure distribution

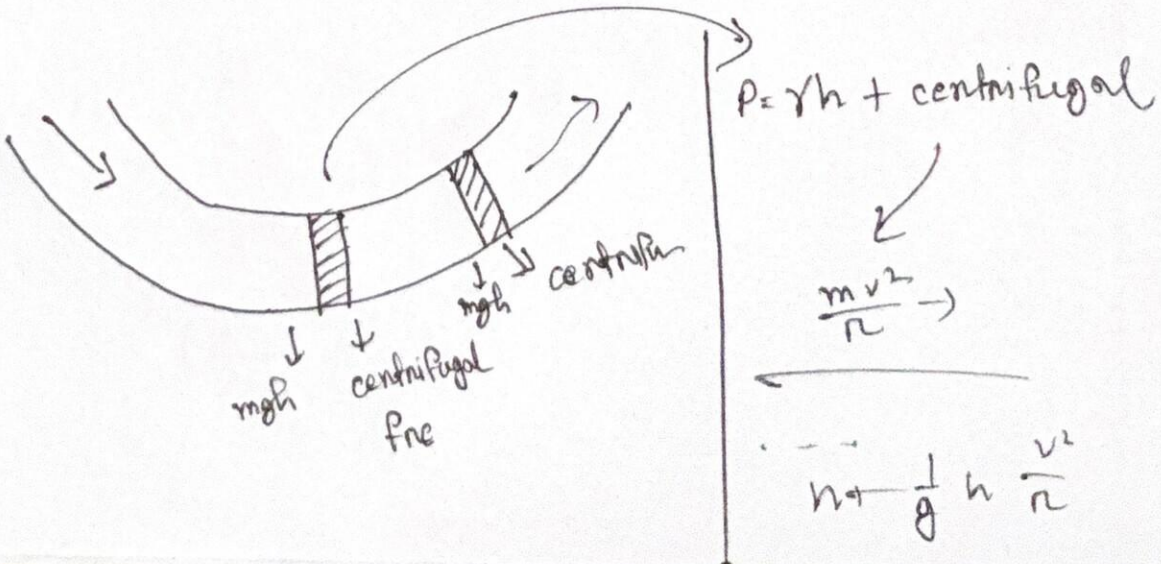
Hydrostatic pressure distribution

$$W = mg$$
$$= \rho V \times g$$
$$= \rho g \times V$$
$$= \gamma \times h \Delta A$$
$$= \dots$$

$$p = \gamma h$$

$$\frac{p}{\gamma} = h$$

Pressure distribution on concave flow



Ex 1.6

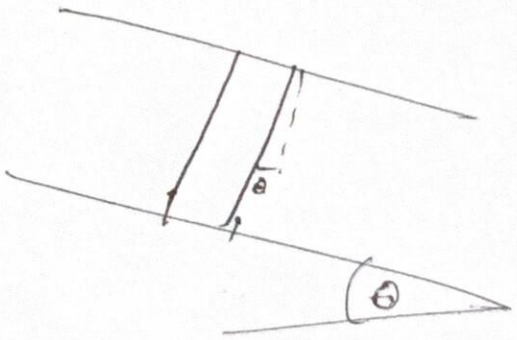
do if inclined

$Q = AV$

$q = hv$

$\Rightarrow V = \frac{q}{h} = 22 \text{ m/s}$

$P = \gamma h \left(1 + \frac{v^2}{g r_c}\right)$



$P = \gamma d \cos\theta$

$= \gamma h \cos\theta \times \cos\theta$

$= \gamma h \cos^2\theta$

Governing equations for steady one-dimensional flow

continuity eqn, energy eqn, momentum eqn

Continuity eqn: $P_1 Q_1 = P_2 Q_2 = \cdots$

$Q = AV$

$Q_1 = Q_2 = \cdots$

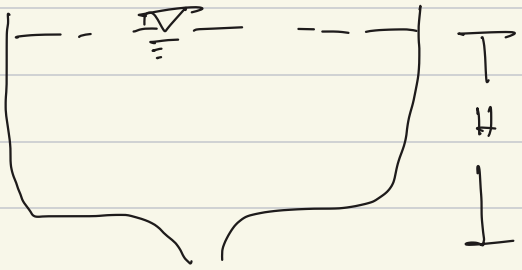
Energy eqn: $H = z + \frac{p}{\gamma} + \frac{u^2}{2g}$

In context of OCF: $H = z_b + h + \alpha \frac{u^2}{2g}$

Momentum eqn: $\rho Q(\beta_2 v_2 - \beta_1 v_1) = F_{p1} - F_{p2} + W \sin\theta - F_f$

$$\frac{mv}{t} = \frac{\rho(\text{Vol}) \times v}{t} = \rho Q V_1 \times \beta_1 = \rho Q V_2 \times \beta_2$$

Law of Torricelli



$v = \sqrt{2gH}$

Example 2.2

$V = \frac{Q}{A}$ → per m width

$V = \frac{q}{h} \rightarrow \frac{qb}{bh}$

$= \frac{q}{h}$

$V_1 = ---$

$V_2 = ---$

$---$

$h_f = ---$

} Bernoulli

Apply momentum eqn:

$$\rho q (V_2 - V_1) = \tfrac{1}{2}\gamma h_1^2 - \tfrac{1}{2}\gamma h_2^2 - F$$

$\Rightarrow F = ---$

$= 5.92$ kN

CT- 1 Chapter 1 Theory, math, ...
not derivation

Derivation won't be given from first chapter

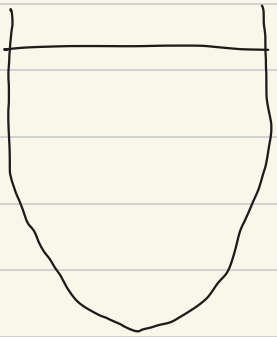
Specific energy and critical flow

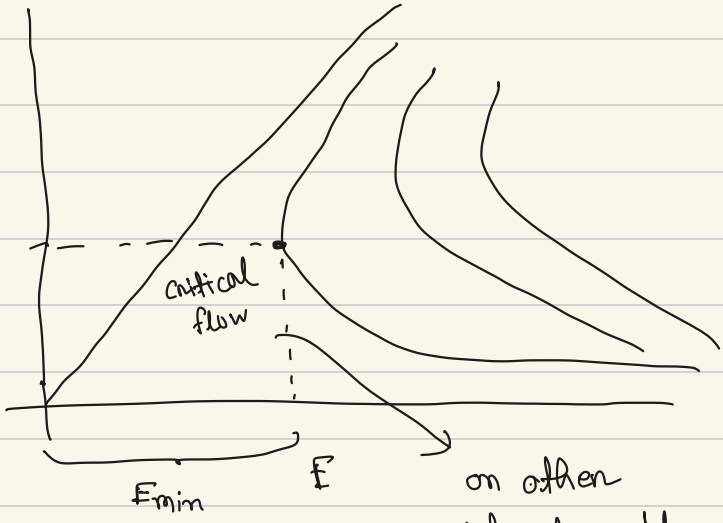
↳ w/ respect to channel bottom (datum = 0)

$$E = h + \frac{v^2}{2g}$$

$$= h + \frac{Q^2}{2gA^2}$$

Specific energy curve:



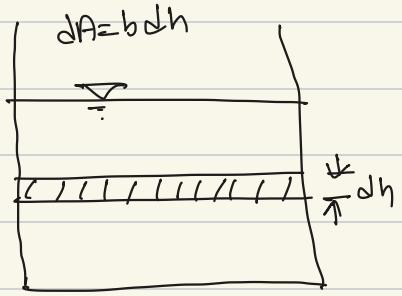


on other points, two state possible → super sub

at $h = 0$, $\frac{v^2}{2g} = \infty$

Rectangular channel, holding Q = const

Prove $F_n = 1$ at critical depths



$\Psi = \tan^{-1} \sec^2 \theta$

Discharge depth curve

$Q^2 = 2gA^2(E-h)$ | $E = h + \frac{Q^2}{2gA} \Rightarrow \dots \Rightarrow Q^2 = \dots$

Prove: For a given discharge, at critical flow ...

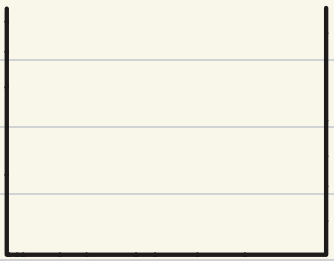
Critical flow

$$Fr = \frac{v}{\sqrt{gD}} = 1$$

$$\Rightarrow \frac{v^2}{gD} = 1$$

$$\Rightarrow \frac{v_c^2}{g} = D_c$$

$$\Rightarrow \frac{v^2}{2g} = \frac{D_c}{2} = \frac{h_c}{2}$$



$D = h$

$E_c = \dots = 1.5\, h_c$

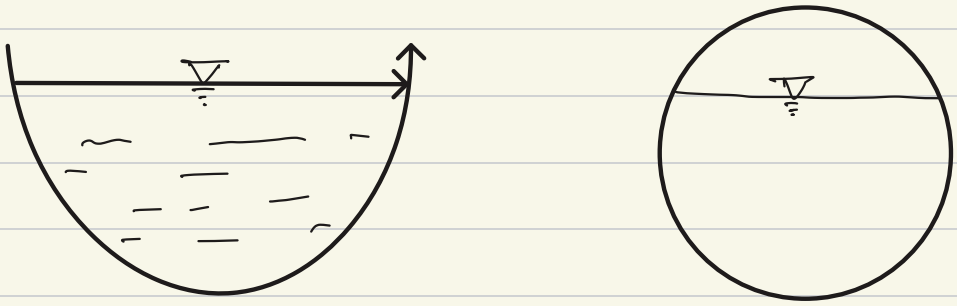
only valid for rectangular

Rough / CT-1 17/12

Open channel flow (OCF)

Flow of water in a conduit with a free surface.

E.g., flow in river/canals.



Examples of OCF

Types / classification

a) Natural & artificial

b) Prismitic — non-varying cross-section & constant bottom slope; usually artificial.

Non-prismitic — not above

c) Rigid — immovable bed, i.e; non-erodible; concrete

Mobile boundary channel — loose sedimentary particles

d) Large slope — bottom slope > 1:10

Small slope — ... ← almost all natural

Geometric elements of channel flow

Properties that are defined by geometric shape

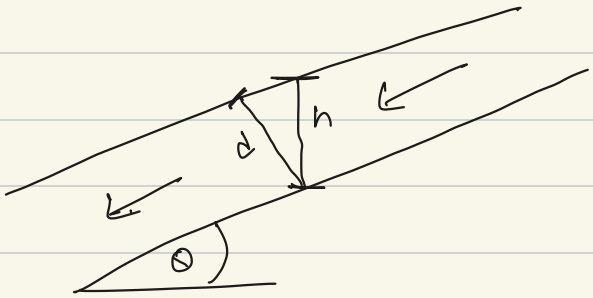
Depth of flow (h)

vertical dist. from lowest point to water surface

Depth of flow section (d)

depth of flow normal to flow direction

$d = h \cos\theta$



Stage — elevation of water surface w.r.t a hor. datum.

Flow area — Area normal to flow direction

Wetted perimeter — P

Top width — B

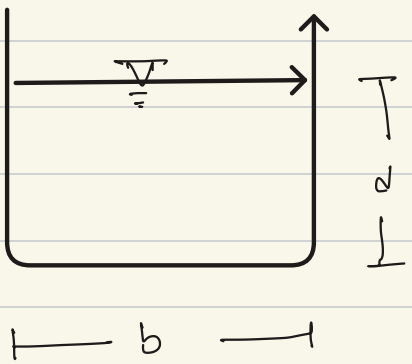
Hydraulic radius — $R = A/P$

Hydraulic depth — $D = A/B$

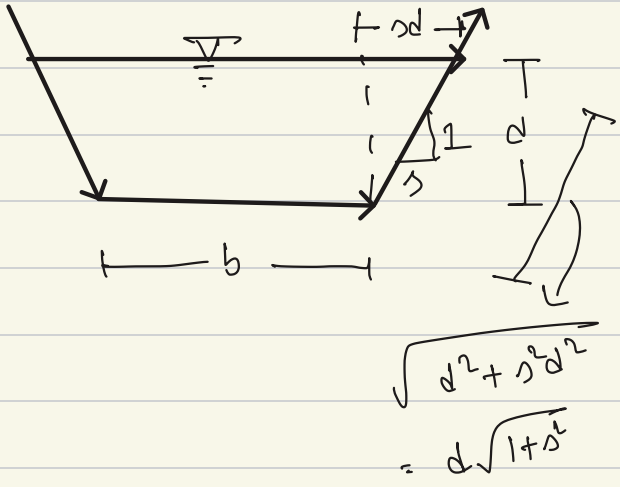
$A = bd$

$P = 2d + b$

$B = b$



$$A = bd + 2\frac{1}{2}sd^2$$
$$= d(b+sd)$$
$$B = b + 2sd$$
$$p = b + 2d\sqrt{1+s^2}$$



Effects of viscosity & gravity

Reynolds Number — effect of viscosity
ratio between inertial force & viscous force

$$Re = \frac{UL}{\nu} = \frac{UR}{\nu} \quad ;$$

U = mean velocity
ν = kinematic viscosity
$= 10^{-6}\ m^2/s$ at 20°C (water)
$R = \frac{A}{P}$

$Re < 500 \longrightarrow$ laminar
$Re > 12500 \longrightarrow$ turbulent
else $\longrightarrow$ transitional

Froude Number — effect of gravity

$$Fr = \frac{U}{\sqrt{gL}} = \frac{U}{\sqrt{gD}} \quad ; \qquad D = \frac{A}{B}$$

$Fr < 1 \longrightarrow$ subcritical; $Fr > 1 \longrightarrow$ supercritical
$Fr = 1 \longrightarrow$ critical

Wide channel

$b \gg h \longrightarrow R \simeq h$

$Q/b \rightarrow q \ (m^3/s/m)$; $b \rightarrow$ bottom width

$R \simeq h$

Celerity

Speed of wave on still water surface

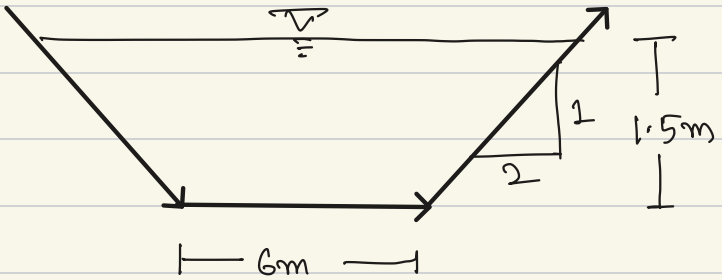
$$c = \sqrt{gD}$$

Example 1.1

$$Re = \frac{UR}{\nu}$$

$$= \frac{2.3 \times 1.06}{10^{-6}}$$

$$= 2.4 \times 10^6 \longrightarrow \text{turbulent}$$



$$Fr = \frac{U}{\sqrt{gD}}$$

$$= \frac{2.3}{\sqrt{9.81 \times 1.125}}$$

$$= 0.69 \longrightarrow \text{subcritical}$$

$\therefore$ Subcritical turbulent

$V = 2.3$ m/s

$A = 6 \times 1.5 + 2\frac{1}{2}(2 \times 1.5)1.5 \ m^2$

$= 13.5 \ m^2$

$P = 6 + 2\sqrt{(1.5 \times 2)^2 + 1.5^2} = 12.71$ m

$R = A/P = 1.06$ m

$B = 6 + 2 \times 2 \times 1.5 = 12$ m

$D = A/B = 1.125$ m

$$C = \sqrt{gD} = \sqrt{9.81 \times 1.125} = 3.32 \ ms^{-1}$$

$\therefore$ Speed of wavefront,

upstream $= 3.32 - 2.3 = 1.02 \ ms^{-1}$

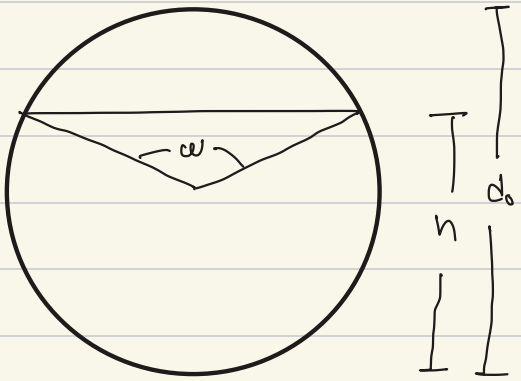
downstream $= 3.32 + 2.3 = 5.62 \ ms^{-1}$

$$w = 2\cos^{-1}\left(1 - \frac{2h}{d_0}\right) \quad (\text{rad})$$

$$A = \frac{(w - \sin w)\, d_0^2}{8}$$

$$P = d_0 \frac{w}{2}$$

$$B = d_0 \sin\left(\frac{w}{2}\right)$$



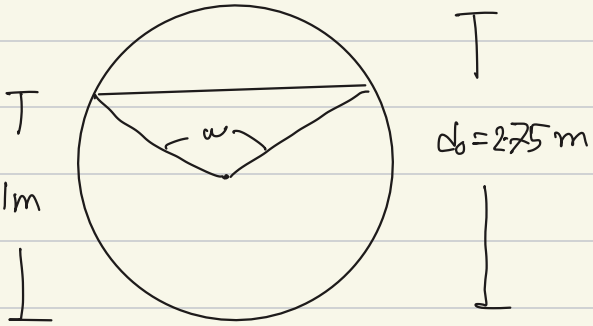
$$w = 2\cos^{-1}\left(1 - \frac{2h}{d_0}\right)$$

$$= 2\cos^{-1}\left(1 - \frac{2 \times 1.1}{2.75}\right) = 2.74^{c}$$

$$A = w - \sin(w)\, d_0^2 \times \frac{1}{8}$$

$$= \left[2.74 - \sin(2.74)\right] \times 2.75^2 \times \frac{1}{8}\ m^2$$

$$= 2.22\ m^2$$



$$P = d_0 \frac{w}{2} = 2.75 \times \frac{2.74}{2} = 3.77\ m$$

$B = \ldots = 2.69\ m$

$D = \ldots$

$Re = \ldots$

$Fr = \ldots$

$Q = 6.55\ m^3/s$

$$U = \frac{6.55}{2.22} = 2.95\ m/s$$

Velocity distribution

$$\bar{U} = \frac{u_{0.2} + u_{0.8}}{2} \quad ; \quad d > 0.61\ m$$

$$= u_{0.6} \quad ; \quad d < 0.61\ m$$

$$U = \frac{Q}{A} = \frac{1}{A}\int_0^A u\, dA$$

Coefficients

$\alpha \rightarrow$ kinetic energy coeff.

$\beta \rightarrow$ momentum coeff.

$$\alpha = \frac{\int_0^A u^3 dA}{U^3 A}$$

$$\beta = \frac{\int_0^A u^2 dA}{U^2 A}$$

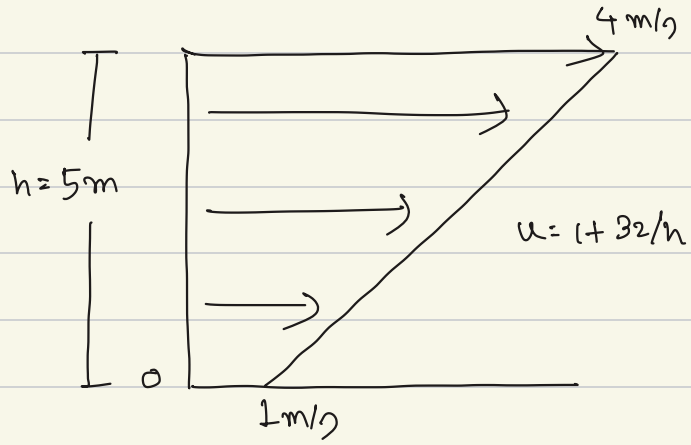
always +ve,

" ≥ 1

$\alpha > \beta > 1$

$$\frac{\alpha - 1}{\beta - 1} \rightarrow 2.8 - 3.0$$

Ex. 1.4



(I) $Q = AV$

$q = \frac{A}{b} V$

$= hV = h\bar{U}$

$\bar{U} = \int_0^5 (1 + \frac{3z}{5})\, dz \times \frac{1}{5}$

$= 2.5 \text{ m/s}$

$\therefore q = 5 \times 2.5 = 12.5 \text{ m}^3/\text{s/m}$

wide channel: $R \simeq h$

$= 5 \text{ m}$

(II) $Re = \frac{\bar{U} R}{\nu}$

$= \frac{2.5 \times 5}{10^{-6}} = 12.5 \times 10^6 \longrightarrow$ turbulent

$Fr = \frac{\bar{U}}{\sqrt{gD}} = \frac{\bar{U}}{\sqrt{gh}}$

$= \frac{2.5}{\sqrt{9.81 \times 5}}$

$= 0.36 \longrightarrow$ subcritical

(III) $\alpha = \frac{\int_0^A u^3 dA}{\bar{U}^3 A} = \frac{1}{\bar{U}^3 h} \int_0^h u^3 dz$

$= \frac{1}{2.5^3 \times 5} \int_0^5 (1 + \frac{3z}{5})^3 dz$

$= 1.36$

alternatively,

$(a+b)^3 = a^3 + 3ab^2 + 3ba^2 + b^3$

$\longrightarrow \frac{1}{\cancel{h}} \times (\cdots \cancel{h} + \cdots \cancel{h} + \cdots \cancel{h})$

$\beta = \frac{\int_0^A u^2 dA}{\bar{U}^2 A} = \frac{\int_0^h u^2 dz}{\bar{U}^2 h}$

$= \frac{1}{2.5^2 \times 5} \int_0^5 (1 + \frac{3z}{5})^2 dz$

$= 1.12$

$\longrightarrow \frac{\alpha - 1}{\beta - 1} = 3$

Ex. 1.5

$Q = \int u dA$

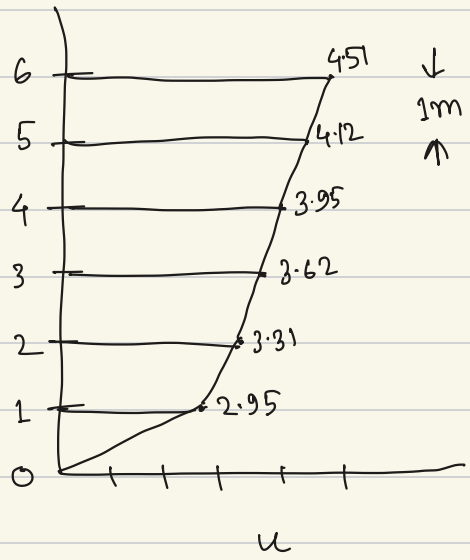
$q = \int u dh$

$= \Sigma u_i dh$

$= 1\left[\frac{0+2.95}{2} + \frac{2.95+3.31}{2} + \cdots + \frac{4.12+4.51}{2}\right] h$

$= 1\left[\frac{0+4.51}{2} + 2.95+3.31+3.62+3.95+4.12\right]$

$= 20.21 \ m^3/s/m$



$\bar{U} = \frac{q}{h} = \frac{20.21}{6} = 3.37 \ m/s$

$\alpha = \frac{\Sigma u^3 \Delta z}{\bar{U}^3 h} = \frac{1}{3.37^3 \times 6} \times 1\left[2.95^3 + 3.31^3 + 3.62^3 + 3.95^3 + 4.12^3 + \frac{4.51^3}{2}\right]$

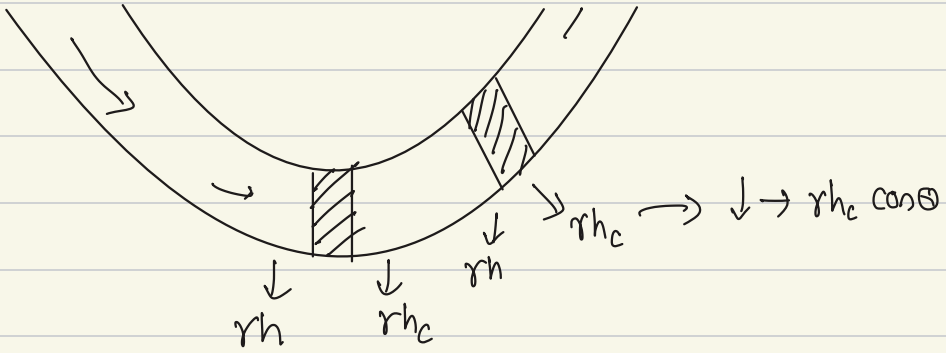
$= 1.25$

$\beta = \frac{\Sigma u^2 \Delta z}{\bar{U}^2 h} = \frac{1}{3.37^2 \times 6} \times 1\left[2.95^2 + 3.31^2 + 3.62^2 + 3.95^2 + 4.12^2 + \frac{4.51^2}{2}\right)$

$= 1.11$

Pressure distribution

$p = \rho g h = \gamma h$ ⟵ hydrostatic distribution



Concave flow

$$h_c = \frac{hu^2}{gR}$$

Total pressure, $p = \gamma h + \gamma \frac{hu^2}{gR}$

$= \gamma h \left(1 + \frac{u^2}{gR}\right)$

$\frac{mv^2}{R}$

↓

$\frac{mv^2}{RA} \rightarrow \frac{\rho V v^2}{RA}$

$p_c \rightarrow \rho h \frac{v^2}{R}$

$h_c \rightarrow \frac{p_c}{\gamma = \rho g}$

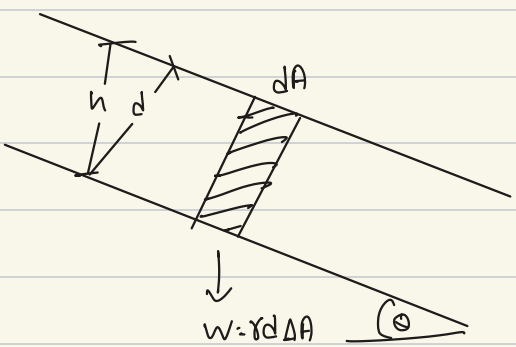
$\rightarrow \frac{hu^2}{gR}$

Effect of slope

$$p = \frac{\gamma d \Delta A \cos\theta}{\Delta A}$$

$= \gamma (h\cos\theta) \cos\theta$

$= \gamma h \cos^2\theta$



$d = h\cos\theta$

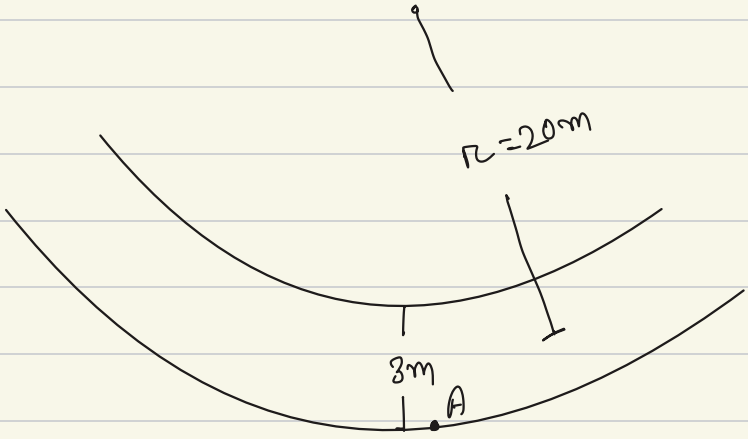
Ex. 1.6

$$P_A = \gamma h + \gamma h_c$$

$$= \gamma h + \gamma \frac{hu^2}{rg}$$

$$= 9.81 \times 1000 \times 3 + 9.81 \times 1000 \times \frac{3 \times 22^2}{20 \times 9.81}$$

$$= 102.03 \text{ KN/m}$$



$q = 66 \text{ m}^3/\text{s/m}$

$$U = \frac{q}{h} = \frac{66}{3} = 22 \text{ m/s}$$

Section factor for critical flow computation

$$Z = A\sqrt{D} \rightarrow Z_c = A_c\sqrt{D_c}$$

$$\rightarrow Z_c = \frac{Q}{\sqrt{g/\alpha}}$$

$$F_R = \frac{\alpha V}{\sqrt{gD}} \Rightarrow F_D^2 = \alpha \frac{V^2}{gD}$$

$$= \frac{\alpha Q^2}{gDA^2}$$

$$= \dots$$

$$\Rightarrow F_D^2 = \frac{Z_c^2}{Z^2}$$

$\frac{V^2}{2g} =$

Build relation btwn F_R & section fact

$Z^2 = A^2 D \rightarrow Z^2 = C_1 h^m$; $m \rightarrow$ exponent fun critical flow

$$\Rightarrow 2\ln Z = \ln C_1 + m \ln h$$

$$\Rightarrow \dots \quad \frac{d\ln Z}{dh} = \frac{m}{2h}$$

$$Z = A\sqrt{D} \rightarrow \ln Z = \ln A + \frac{1}{2}\ln D$$

$$= A\sqrt{A/B}$$

$$\Rightarrow \ln Z = \ln A + \frac{1}{2}[\ln A - \ln B]$$

$$\Rightarrow \ln Z = \frac{3}{2}\ln A - \frac{1}{2}\ln B$$

$$\Rightarrow \frac{d\ln Z}{dh} = \frac{3}{2}\frac{\ln A}{dh}\frac{1}{A} - \frac{1}{2}\frac{\ln B}{dh} \times \frac{1}{B}$$

$$= \dots \frac{1}{2A}\left[3B - D\frac{dB}{dh}\right]$$

3.13 $\rightarrow Z = A\sqrt{D}$

$D = A/B$

value of m for different channel sections

$$m = \frac{h}{A}\left[3B - D\frac{dB}{dh}\right]$$ ← use this eqn not necessary to memo other eqn.

Trapezium → b, h, s from given

Ex. 3.2

$$\frac{dB}{dh} \to \frac{d(b+2sh)}{dh} = 0 + 2s$$

Computation of critical depth (h_c)

memo

$$h_c = \sqrt[3]{\frac{\alpha Q^2}{gb^2}} \longrightarrow \text{rectangular}$$

↓ wide

$$h_c = \sqrt[3]{\frac{\alpha q^2}{g}}$$

Triangular channel,

$$h_c = \sqrt[5]{\frac{2\alpha Q^2}{gs^2}}$$

Parabolic, $z = cy^2$ ← convert to this format

$$h_c = \sqrt[4]{\frac{27\alpha c Q^2}{32g}}$$

$$E_c = \frac{3}{2}h_c$$

$$\Rightarrow h_c + \frac{V_c^2}{2g} = \frac{3}{2}h_c$$

$$\Rightarrow h_c + \frac{Q^2}{2gA^2} = \frac{3}{2}h_c$$

$$\Rightarrow h_c = \dots$$

Ex. 3.3

(iv) $y^2 = 42$ $\quad [z = cy^2]$

$\Rightarrow z = \frac{1}{4} y^2$

$h_c = \dots$

$\dots$

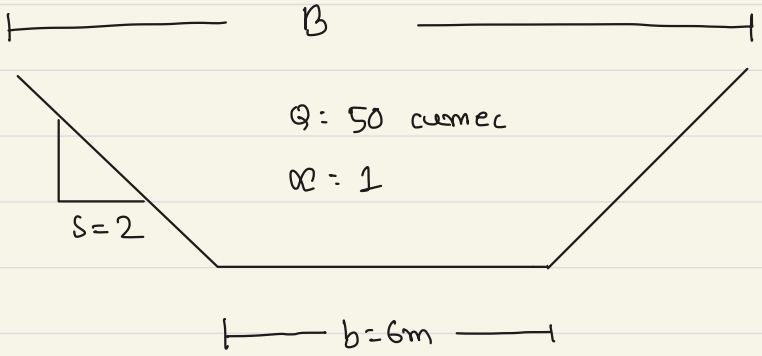
question will be in same unit as likely → S.I unit

Trapezium → 2 var ;

trial & error method used

% for trapezium channel — trial & error

Ex 34



$$z_c = \frac{Q}{\sqrt{g\alpha}} \rightarrow 15.$$

z to be $\approx z_c$

$$z = A\sqrt{D} \; ; \quad D = \frac{A}{B}$$

Assume, h = 1

A	B	D	z	Rmk
—	—	—		

$$v_c = \frac{Q}{A_c} = \ldots$$

Circle math

Numerical method

$\rightarrow f(h) = 1 - \frac{\alpha Q^2 B}{g A^3}$; $[1 - Fr^2]$

Ex. 3.6

$A = (6+2h)h$, $B = 6+4h$, $\alpha Q^2/g = 19.98$

$$f(h) = 1 - \frac{\alpha Q^2 B}{g A^3} = 1 - \frac{19.98(6+4h)}{[(6+2h)h]^3}$$

h_{min}	h_{max}	h	$f(h)$	Root lies btwn
-3	4	0.5	-2.73	05 — 4
05	4	2.25	0.977	0.5 — 2.25
0.5	2.25	1.375	0.87	0.5 — 1.375
0.5	1.375	0.938	0.52	0.5 — 0.938
0.5	0.938	0.719	-0.16	0.719 — 0.938
0.719	0.938			

Newton - Raphson

$$f(h) = A^3 - \frac{\alpha Q^2 B}{g}$$

Ex. 3.7

- - -

Table connection : first col → h

Critical Flow & Control

Change in channel → obstacle, slope
← control

Chng in downstream to control u/s → downstream control
" " u/s " " d/s → upstream control
produces critical flow at chng

after control point, u/s → sub / super critical ?
d/s → " " ?

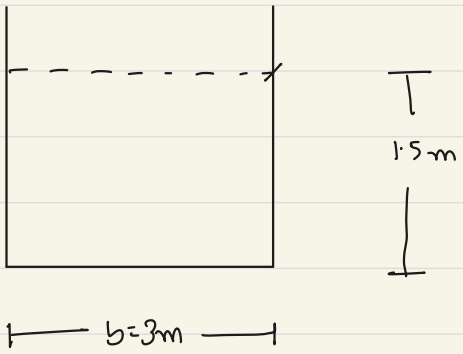
Transition Problem

Ex. 3.8

a) $A = 1.5 \times 3 = 4.5\ m$

$Q = AV = 4.5\ m^3/s$

$h_c = \sqrt[3]{\frac{Q^2}{gb^2}} = \sqrt[3]{\frac{45^2}{9.81 \times 3^2}} = 0.61\ m$



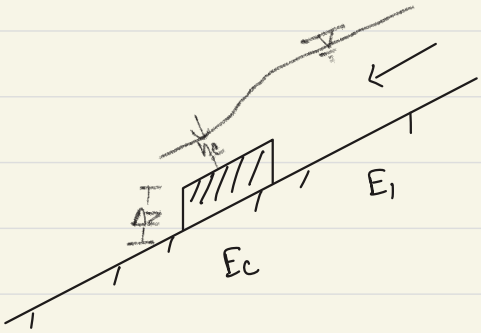
u/s → subcritical

$E_1 = h_1 + \frac{v^2}{2g} = 1.55\ m$

$E_1 = E_c + \Delta z$

$E_c = 1.5\, h_c = \cdots = 0.92\ m$

$\therefore \Delta z = 0.63\ m$



b) $E_2 = E_1 - \Delta z$

$= 1.55 - 0.45 = 1.1\ m$

$\Rightarrow h_2 + \frac{v^2}{2g} = 1.1$ $\qquad v^2 = \frac{Q^2}{A^2}$

$\Rightarrow h_2 + \frac{Q^2}{2g \times b^2 \times h_2^2} = 1.1$ $\qquad = \frac{Q^2}{b^2 h_2^2}$

$\Rightarrow h_2^3 + \frac{Q^2}{2gb^2} = 1.1\, h_2^2$

$\Rightarrow h_2^3 - 1.1\, h_2^2 + \frac{Q^2}{2gb^2} = 0$

h_2 → two real value

$h_2 = 0.93,\ 0.42$ m

$V_2 = \text{---},\ \text{---}$

F_r : sub, super critical

[usually upper value, but check]

$V = \frac{Q}{A}$

$E_{min} \rightarrow E_c$ at critical

$E_c < E_2, E_1$

$E_2 < E_c \rightarrow$

✓✓ E_c, E_2 check →

alternatively, check w/ h_c

For mid

until transition problem

incl. derivations

Ex. 3.8 b. (ii)

$E_1 = \Delta z + E_2$

$\Rightarrow E_2 = 1.55 - 0.8 = 0.75\ m < E_c$

$E_{min} \rightarrow E_c$ at critical

$E_c < E_2, E_1$

$\Rightarrow h_2 + \frac{Q^2}{2g b^2 h_2^2} = 0.75$

$\Rightarrow h_2^3 - 0.75 h_2^2 + \frac{Q^2}{2g b^2} = 0 \rightarrow h_2$ = one neg, two imaginary

$E_2 = E_c = 0.92m$

$E_1 = \Delta z + E_c = 0.8 + 0.92\ m$

$\Rightarrow E_1 = 1.72\ m$

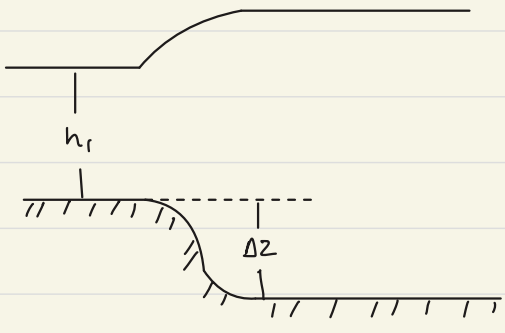
$\Rightarrow h_1' + \frac{Q^2}{2g b^2 h_1'^2} = 1.72$

$\Rightarrow h_1'^3 - 1.72 h_1'^2 + \frac{Q^2}{2g b^2} = 0 \Rightarrow h_1'^3 - 1.72 h_1'^2 + \frac{4.5^2}{2 \times 9.81 \times 3^2} = 0$

$\Rightarrow h_1' = 1.68m,\ 0.28\ m$

$h_1' = 1.68\ m$

(iii)

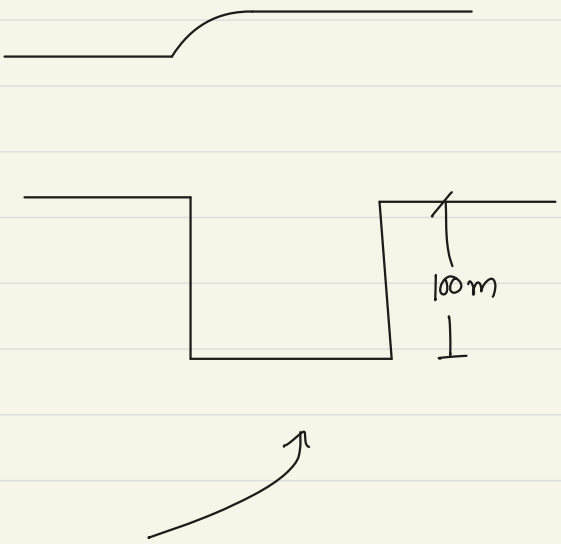


Sol

$$E_2 = 1.55 + 100 = 101.55 \text{ m}$$

$$\Rightarrow h_2 + \frac{Q^2}{2gb^2 \times h_2^2} = 101.55 \text{ m}$$

$h_2 \rightarrow$ 101.5 m, 0.3 m



width matters

narrow width → rise

wide → acts as slope

Rough/ midterm

31/12

Chap 2

Continuity eq^n : $\rho_1 Q_1 = \rho_2 Q_2 \rightarrow Q_1 = Q_2 \Rightarrow U_1 A_1 = U_2 A_2$

Energy eq^n : $H = z + \frac{p}{\gamma} + \frac{u^2}{2g}$

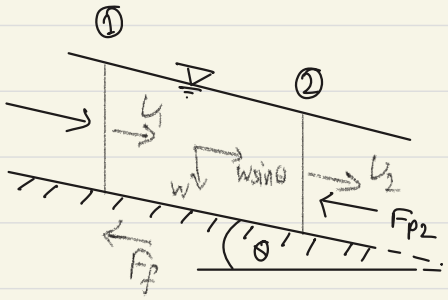
for OCF : $H = z_b + h + \alpha \frac{U^2}{2g}$; $z + \frac{p}{\gamma}$ → piezometric head = el above datum

z_b : ht of channel bed above datum

h = depth of flow

Momentum eq^n :

$$\rho Q (\beta_2 U_2 - \beta_1 U_1) = F_{p1} - F_{p2} + W \sin\theta - F_f$$

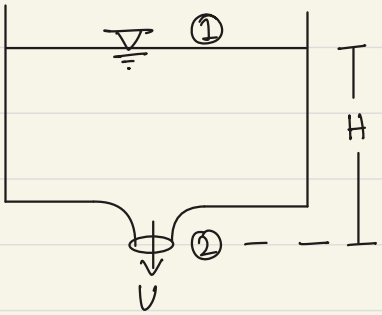


Ex 2.1 Law of Torricelli :

$$H_1 + 0 + \frac{U_1^2}{2g} = 0 + 0 + \frac{U_2^2}{2g}$$

$$\Rightarrow H_1 = \frac{U_2^2}{2g}$$

$$\Rightarrow U = \sqrt{2gH} \quad [U_1 << U_2, H_1 = H]$$

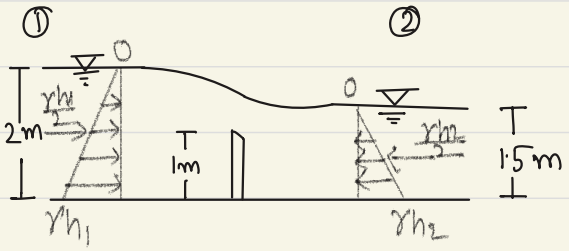


Ex. 2.2

Energy loss:

$$2+0+\frac{2^2}{2\times 9.81} = 1.5+0+\frac{2.67^2}{2\times 9.81}+h_f$$

$$\Rightarrow h_f = 0.34\ m$$



Force on wein plate:

$q = 4\ m^3/s/m$

$\therefore U_1 = \frac{q}{h_1} = \frac{4}{2} = 2\ m/s$

$U_2 = \frac{q}{h_2} = \frac{4}{1.5} = 2.67\ m/s$

$$\rho q(U_2-U_1) = \Sigma F = \frac{\gamma h_1}{2}\times(h_1\times 1) - \frac{\gamma h_2}{2}\times(h_2\times 1) - F$$

$$\Rightarrow 1000\times 4(2.67-2) = \frac{9810\times 2}{2}\times 2 - \frac{9810\times 1.5}{2}\times 1.5 - F$$

$$\Rightarrow F = 5904\ N$$

$$= 5.9\ kN$$

Channel w/ change in width

Ex 3.9 a)

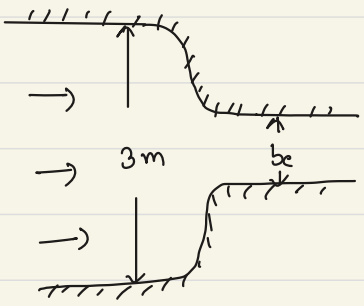
Alt $h_c = 2/3\ E_c = \dots$

$h_c = 3\sqrt{\frac{Q^2}{gb^2}} \rightarrow b = \dots$

$v = 1\ m/s;\ d = 1.5\ m;\ b = 3m$

$Q = AV = 4.5\ m^3/s$

$E_1 = \dots = 1.55\ m = E_c$



$h_c = 2/3\ E_c = \dots = 1.03\ m$

$v_c = \sqrt{g D_c} = 3.19\ m/s$

$Q = A_c h_c = b_c h_c v_c \Rightarrow b_c = \dots = 1.37m$

Drop in water = ---

b) (i) $b_2 = 2m$ ($> b_c \rightarrow$ supercritical, unchanged)

---

b) (ii) $b_2 = 1m$

b) (iii)

Combination of Transitions

both width & ht change

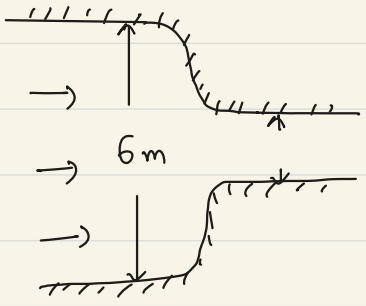
$\Delta z \longleftrightarrow b$

Ex 3.10

$$E_1 = 2 + \frac{2^2}{9.81} = 2.2$$

$$E_1 = E_c = 2.2 = \tfrac{2}{3} h_c$$

$$\Rightarrow h_c = \quad m$$



$$\text{Again, } h_c = \sqrt[3]{\frac{Q^2}{g b^2}} = \sqrt[3]{\frac{24}{9.81 \times b_c}}$$

$$\Rightarrow b_c = \quad m$$

L13 19/01

Review — chap 3

Energy curve

Prove $E_c = 1.5 h_c \longrightarrow$ $D = h$ rect

$$\frac{V}{\sqrt{gD}} = 1 \Rightarrow V^2 = gD \Rightarrow \frac{v^2}{g} = D \Rightarrow \frac{v^2}{2g} = \frac{D}{2}$$

$$E_c = h_c + \frac{v_c^2}{2g} \leftarrow \frac{h_c}{2}$$

Critical depth calc

Trial & error

Numerical

L14

21/01

L15

26/01

L16

26/01

Flow measurement

when (i) $b_c \geq b$ on, (ii) $\Delta z_c \leq \Delta z \rightarrow h_c$



$\rightarrow h_c = f(Q) \rightarrow \sqrt[3]{Q^2/gb^2}$

on, $Q = f(h_c)$

$Q = \dots$

$= \left(\frac{2}{3}\right)^{1.5} b\sqrt{g}\, h_1^{1.5} = 1.705\, b h_1^{1.5}$

neglect $\leftarrow$

$\rightarrow 1.705\, b \left(h_1 + \frac{v_1^2}{2g}\right)^{1.5}$

E.O. Chap 3

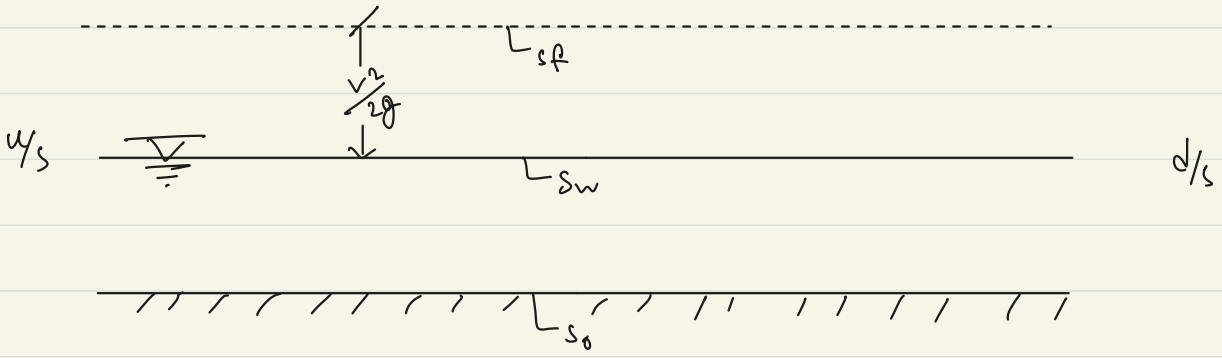
Chap 4 $\rightarrow$ Uniform flow

Steady flow $\rightarrow \frac{dQ}{dt} = 0$

Unsteady $\rightarrow \frac{dQ}{dt} \geq 0$

Uniform flow $\rightarrow \frac{dQ}{dx} = 0$

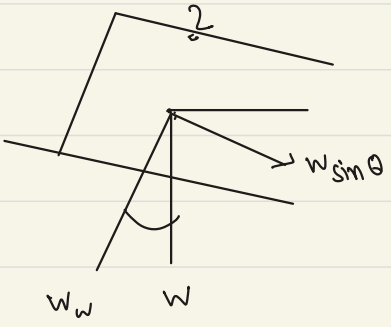
Can unsteady flow be uniform



$\tan\theta = S_0 = S_w = S_f$ [uniform]

Establishment of uniform flow

$$W \sin\theta = F_f$$



Velocity distribution

Shear stress & friction velocity

$$F_f = \tau_0 PL$$

$$W \sin\theta = \tau_0 PL$$

$$\gamma AL S_0 = \tau_0 PL$$

$$\therefore \tau_0 = \gamma \frac{A}{P} S_0$$

$$\Rightarrow \gamma R S_0 = \rho g R S_0$$

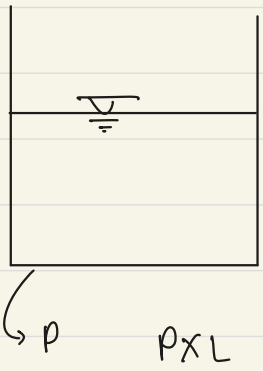
$\sin\theta \approx \tan\theta$

$\tan\theta = S_0$

$S_0 = S_f$

$W \sin\theta = \gamma AL \sin\theta$

$= \dots$



Rough / midterm

Chap 2 Governing equations for one-dimensional flow

1) Continuity eqn: $\rho_1 Q_1 = \rho_2 Q_2 \Rightarrow Q_1 = Q_2 \Rightarrow U_1 A_1 = U_2 A_2$

2) Energy eqn: $H = z + \frac{p}{\gamma} + \frac{v^2}{2g}$

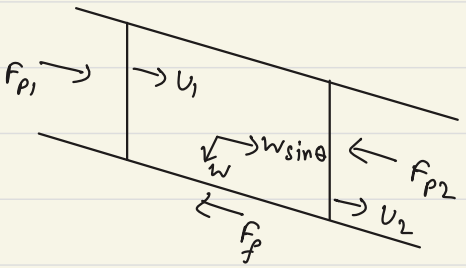
for OCF, $H = z_b + h + \alpha \frac{U^2}{2g}$

$z_b \rightarrow$ ht of channel bd above datum

$h \rightarrow$ depth of flow

3) Momentum eqn:

$$\rho Q(\beta_1 U_1 - \beta_2 U_2) = F_{p1} - F_{p2} + W\sin\theta - F_f$$

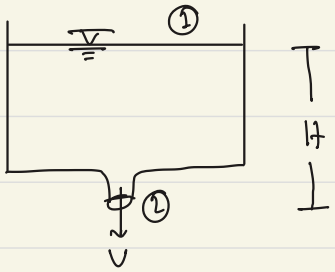


Ex. 2.1

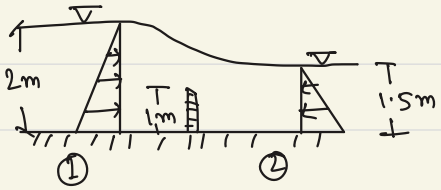
$$H + 0 + \frac{U_1^2}{2g} = 0 + 0 + \frac{U_2^2}{2g}$$

$$\Rightarrow H = \frac{U_2^2}{2g} \qquad [U_1 \ll U_2]$$

$$\Rightarrow U = \sqrt{2gH}$$



Ex. 2.2



Energy eqn:

$$2 + 0 + \frac{2^2}{2\times 9.81} = 1.5 + 0 + \frac{2.67^2}{2\times 9.81} + h_f$$

$$\Rightarrow h_f = 0.34\ m$$

$q = 4\ m^2/s$

$\therefore U_1 = \frac{q}{h_1} = \frac{4}{2} = 2m$

$U_2 = 2.67\ m$

Momentum eqn:

$$\rho Q(U_2 - U_1) = \tfrac{1}{2}\gamma h_1 \times h_1 - \tfrac{1}{2}\gamma h_2 \times h_2 - F$$

$$\Rightarrow 1000\times 4(2.67 - 2) = \tfrac{1}{2}\,9810\,[2^2 - 1.5^2] - F$$

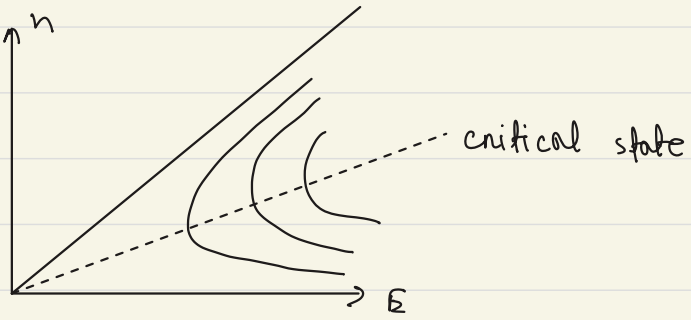
$$\Rightarrow F = 5904\ N$$

Chap 3

Specific energy: energy measured w/ respect to channel bottom. $\rightarrow z_b = 0$

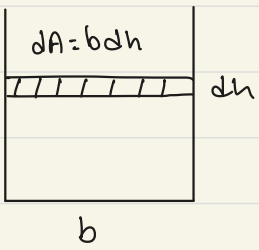
$$E = h + \frac{v^2}{2g}$$
$$= h + \frac{Q^2}{2gA^2}$$

Specific energy curve:



Proof at critical state, $F_R = 1$:

$$\frac{dE}{dh} = 1 + \frac{Q^2}{2g}(-2)A^{-3}\frac{dA}{dh}$$
$$= 1 - \frac{2v^2}{2gA}\frac{dA}{dh}$$
$$= 1 - \frac{v^2 b}{gA}$$
$$= 1 - \frac{v^2}{g\,A/b} = 1 - \frac{v^2}{gD}$$
$$= 1 - F_R^2$$

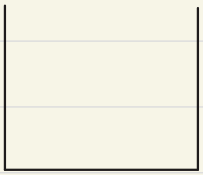


$\Rightarrow \frac{dA}{dh} = b$

At E = min $\rightarrow$ critical state, $\frac{dE}{dh} = 0 \Rightarrow 1 - F_R^2 = 0$

$\Rightarrow F_R = 1$

Proof: $E_c = 1.5 h_c$ for rect.



$$F_R = \frac{v_c}{\sqrt{g D_c}} = 1$$

$$\Rightarrow \frac{v_c^2}{g} = D_c$$

$$\Rightarrow \frac{v_c^2}{2g} = \frac{D_c}{2} = \frac{h_c}{2}$$

$$\therefore E_c = h_c + \frac{v_c^2}{2g} = h_c + \frac{h_c}{2} = 1.5 h_c$$

Relation between F_R & section factor:

$$Z = A\sqrt{D}$$

$$Z_c = A_c \sqrt{D_c}$$

$$F_R = \frac{v}{\sqrt{gD}}$$

$$\Rightarrow F_R^2 = \alpha \frac{v^2}{gD}$$

$$= \frac{\alpha Q^2}{gDA^2}$$

$$= \frac{Q^2}{g/\alpha} \Big/ DA^2$$

$$\Rightarrow F_R^2 = \frac{Z_c^2}{Z^2}$$

$$\alpha \frac{v_c^2}{2g} = \frac{D_c}{2}$$

$$\Rightarrow \alpha \frac{Q^2}{2g A_c^2} = \frac{D_c}{2}$$

$$\Rightarrow A_c^2 D_c = \alpha \frac{Q^2}{g}$$

$$\Rightarrow Z_c = \frac{Q}{\sqrt{g/\alpha}}$$

Hydraulic exponent for critical flow computation

$$Z^2 = C_1 h^M \quad ; \quad M = \frac{h}{A}\left(3B - D\frac{dB}{dh}\right)$$

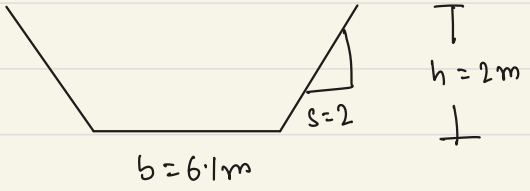
Ex. 3.2

$$M = \frac{h}{A}\left(3B - D\frac{dB}{dh}\right)$$

$$= \frac{2}{20.2}\left(3\times 14.1 - 1.43\frac{d}{dh}[6.1+4h]\right)$$

$$= \frac{2}{20.2}\left(3\times 14.1 - 1.43[0+4]\right)$$

$$\therefore 3.62$$



$$A = bh + sh^2 = 20.2\,m$$

$$B = b + 2\times sh = 14.1\,m$$

$$D = A/B = 1.43\,m$$

$$B = 6.1 + 2\times 2h$$

Computation of critical depth

Analytical methods:

wide channel $\rightarrow$ $h_c = \sqrt[3]{\frac{\alpha q^2}{g}}$

Rect. " $\rightarrow$ $h_c = \sqrt[3]{\frac{\alpha Q^2}{g B^2}}$

Triangular " $\rightarrow$ $h_c = \sqrt[5]{\frac{2\alpha Q^2}{g s^2}}$

Parabolic " $\rightarrow$ $h_c = \sqrt[4]{\frac{27\alpha c Q^2}{32 g}}$ $\quad [Z = cy^2]$

Ex. 3.3

(i) $h_c = \sqrt[3]{\frac{\alpha q^2}{g}} = \sqrt[3]{\frac{1.12 \times 3^2}{9.81}} = 1.01 \text{ m}$

$v_c = \frac{q}{h_c} = \frac{3}{1.01} = 2.97 \text{ m/s}$

(ii) Rect, $b = 6\text{m}$; $Q = 20 \text{ m}^3/\text{s}$

$h_c = \sqrt[3]{\frac{\alpha Q^2}{g B^2}} = \sqrt[3]{\frac{1.12 \times 20^2}{9.81 \times 6^2}} = 1.083 \text{ m}$

$v_c = \frac{Q}{b h_c} = \frac{20}{6 \times 1.083} = 3.078 \text{ m/s}$

(iii) Tri; $s = 2$; $Q = 10 \text{ m}^3/\text{s}$

$h_c = \sqrt[5]{\frac{2\alpha Q^2}{g s^2}} = \sqrt[5]{\frac{2 \times 1.12 \times 10^2}{9.82 \times 2^2}} = 1.416 \text{ m}$

$v_c = \frac{Q}{2 \times \frac{1}{2} s h_c^2} = \frac{10}{2 \times 1.416^2} = 2.49 \text{ m/s}$

(iv) Para; $Q = 20 \text{ m}^3/\text{s}$

$y^2 = 4z$

$\Rightarrow z = \frac{1}{4} y^2$; $c = 1/4$

$h_c = \sqrt[4]{\frac{27 \alpha c Q^2}{32 g}} = \sqrt[4]{\frac{27 \times 1.12 \times 1/4 \times 20^2}{32 \times 9.81}} = 1.76 \text{ m}$

$v_c = \frac{Q}{A_c} =$

Ex 3.4 Trapezoidal

$b = 6m$ $\quad\quad$ $\alpha = 1$

$s = 2m$

$Q = 50\ m^3/s$

$$z_c = \frac{Q}{\sqrt{g/\alpha}} = \frac{50}{\sqrt{9.81/1}} = 15.96$$

$$z = A\sqrt{D}$$

$A \leftarrow bh + sh^2$ $\quad$ $B \leftarrow b + 2sh$ $\quad$ $D \leftarrow A/B$

h	A	B	D	$z = A\sqrt{D}$
1				
2				
1.6				

$$v_c = \frac{Q}{A} = \dots$$

Ex 3.5

$d_0 = 2m$; $\quad Q = 4\ m^3/s$; $\quad \alpha = 1.10$

$$z_c = \frac{Q}{\sqrt{g/\alpha}} = 1.339 \quad ; \quad z = A\sqrt{D}$$

$$A = \frac{d_0^2}{8}(w - \sin w)$$

$$B = d_0 \sin\frac{w}{2}$$

$$D = A/B$$

w	A	B	D	$z = A\sqrt{D}$
1	0.079	0.959	0.0826	0.023
3				1.21
3.1				1.34

$w = 3.1$

$$h_c = \frac{d_0}{2}\left(1 - \cos\frac{w}{2}\right) = 0.98\ m$$

Numerical $\quad 1 - F_n^2 = 0 \Rightarrow 1 - \left(\frac{z_c}{z}\right)^2 = 1 - \left(\frac{Q}{\sqrt{g}k}\right)^2 \times \left(\frac{1}{A\sqrt{D}}\right)^2$

$$= 1 - \frac{\alpha Q^2}{g} \times \left(\frac{1}{A\sqrt{A/B}}\right)^2$$

$$= 1 - \frac{\alpha Q^2}{g} \times \frac{B}{A^3}$$

$$= 1 - \frac{\alpha Q^2 B}{g A^3}$$

Ex. 3.6/ Trapezium; $b = 6m$; $s = 2m$; $Q = 14\ m^3/s$; $\alpha = 1$

$$f(h) = 1 - \frac{\alpha Q^2 B}{A^3 g} = 1 - \frac{196(6+4h)}{(6+2h)^3 h^3 \times 9.81}$$

$$= 1 - \frac{19.98(6+4h)}{[(6+2h)h]^3}$$

$B = 6 + 2 \times 2h$

$= 6 + 4h$

$A = (b + sh)h$

$= (6 + 2h)h$

h_{min}	h_{max}	h_{av}	$f(h_{av})$
0	10	5	0.999
0	5	2.5	0.985
0	2.5	1.25	0.82
0	1.25	0.625	−0.825
0.625	1.25	0 938	0.52
0.625	0.938	0.782	0.12
0.625	0.782	0.704	−0.24
0.704	0.782	0.743	−0.042
0.743	0.782	0.763	0.043

Ex 3.7/ Trapezium; $b = 6m$; $s = 2m$; $Q = 14\ m^3/s$; $\alpha = 1$

$$1 - F_r^2 = 0 \Rightarrow 1 - \left(\frac{z_c}{2}\right)^2 = 1 - \frac{\alpha Q^2 B}{g A^3} = 0$$

$$\Rightarrow A^3 - \frac{\alpha Q^2 B}{g} = 0$$

$$\therefore f(h) = A^3 - \frac{\alpha Q^2 B}{g}$$

$$f'(h) = 3A^2 B - \frac{\alpha Q^2}{g}\frac{dB}{dh}$$

$A = bh + sh^2$ $\quad = 6h + 2h^2$

$B = b + 2sh$ $\quad = 6 + 4h$

$\frac{dB}{dh} = \frac{d}{dh}[b + 2sh]$ $\quad = 4$

$$\therefore f(h) = [6h + 2h^2]^3 - \frac{1 \times 196 \times [6 + 4h]}{9.81}$$

$$= [6h + 2h^2]^3 - 19.98[6 + 4h]$$

$$f'(h) = 3[6h + 2h^2]^2[6 + 4h] - 19.98 \times 4 = 24[3h + h^2]^2[3 + 2h] - 79.9$$

f	$f(h)$	$f'(h)$	$\frac{f(h)}{f'(h)}$	h
2	312.2	1840.08	0.170	0 830
0.830	70.78	1050.27	0.067	0.763
0.763	8.49	815.53	0.01	0.762
0.762	7.68	812.32	0.009	0.762

L17 16/02

Revise — uniform flow

---

velocity distribution

Shear stress, τ_0 ; $F_f = \tau_0 \times PL$

...

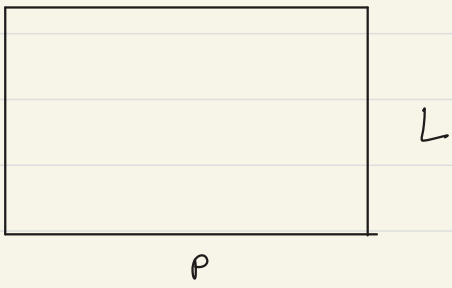
$\tau_0 = \rho g R S_0$

...

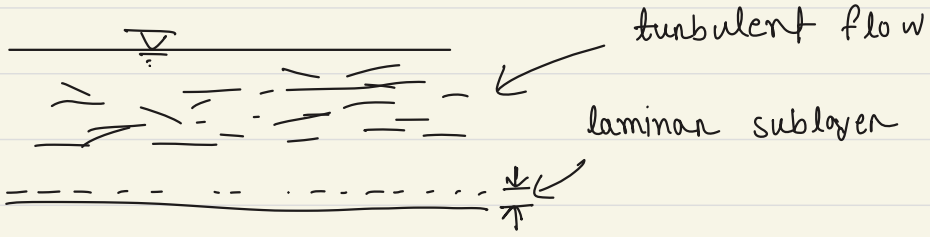
Drag velocity, $u^* = \sqrt{\tau_0/\rho}$

$= \sqrt{gRS_0}$





Laminar or viscous sublayer



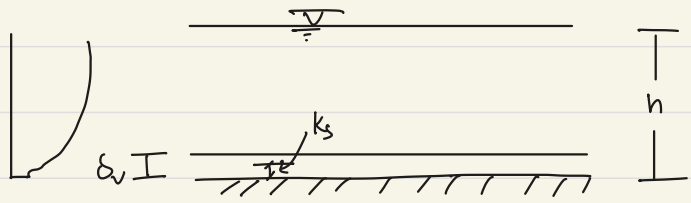
$$\delta_v = \frac{11.6\nu}{u^*} \leftarrow \sqrt{gRS_0}$$

Smooth & rough boundaries

$\frac{u^* k_s}{\nu} \leq 5 \rightarrow$ hydraulically smooth

$$\delta_v = 11.6 \frac{\nu}{u^*}$$

$$k_s = 5 \frac{\nu}{u^*}$$



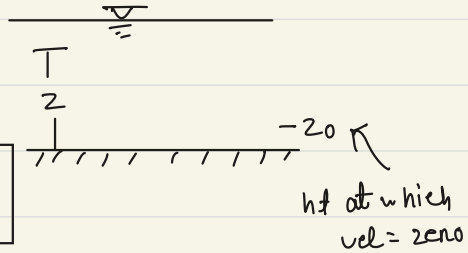
$\frac{u^* k_s}{\nu} \geq 70 \rightarrow$ rough

$$k_s = 70 \frac{\nu}{u^*}$$

in-between → transition regime

Velocity distribution along a vertical

$$\frac{u_z}{u^*} = \frac{1}{\kappa} \ln \frac{z}{z_0} \; ; \; \kappa = 0.4$$



$z_0 =$

$0.11 \frac{\nu}{u^*}$	$0.033 k_s$	$0.11 \frac{\nu}{u^*} + 0.033 k_s$
Smooth	Rough	Transition

Depth averaged velocity → 0.2h & 0.8h ; 0.6h

Cross-sectional mean velocity

$$\frac{V}{u^*} = \begin{array}{|c|c|c|} \hline 5.75 \log \frac{3.64 u^* R}{\nu} & 5.75 \log \frac{12.2R}{K_s} & 5.75 \log \frac{12.2R}{K_s + 3.35 \nu/u^*} \\ \hline \text{Smooth} & \text{Rough} & \text{Transition} \\ \hline \end{array}$$

Diff ; which to use

Depth averaged vs cross-sectional mean vel

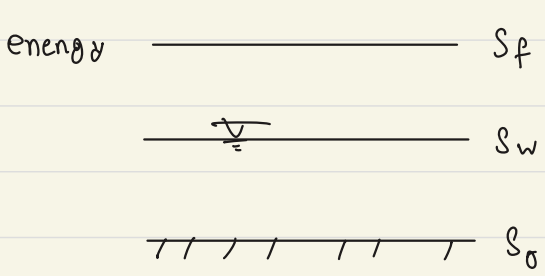
Ex. 4.1 $b = 6m$ $k_s = 2\ mm$

$d = 0.5m$

$S_0 = 0.0025$

$R = \frac{A}{P} = \frac{6 \times 0.5}{0.5 + 2(6)} =$

Uniform flow formulas



$S_f = S_w = S_o$ on uniform flow

Chezy formula

$$\sin\theta \simeq \tan\theta = S_o$$
$$= S_f$$

$$W\sin\theta = F_f$$

$$F_f = KPLU^2$$

$$Q = C\sqrt{R}$$

L20

02/03

Chezy's formula

30–80 in BD's given

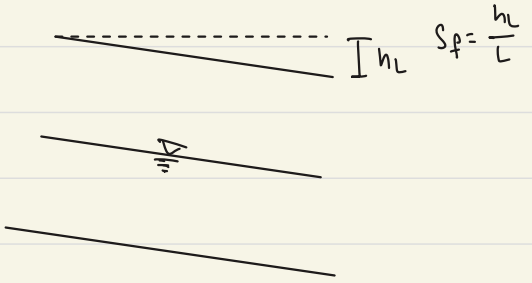
$V = C[\cdots]$

Darcy-Weisbach formula

$h_f = f \frac{L}{d_0} \frac{V^2}{2g}$ → pipe flow

$d_0 = 4R$; $S_f = \frac{h_f}{L}$

$V = \sqrt{\frac{8g}{f}} R^{1/2} S_f^{1/2}$ [works on any units]



Manning's formula

$V = \frac{1}{n} R^{2/3} S_f^{1/2}$ → metric

$= \frac{1.486}{n} R^{2/3} S_f^{1/2}$ → fps

Relationship btwn Chezy's C, Darcy's f, Manning's n

Estimation of C:

Estimation of n:

CT

Before today's class

no derivations

Rough / CT-3 08/03

Uniform flow

Characteristics :

1. Discharge, velocity, depth are same along its length
2. Energy line, water level, and channel beds are parallel

Requirements :

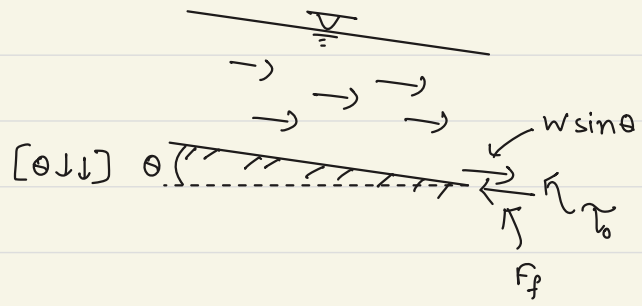
$$W \sin\theta = F_f$$

i) Must have a downward slope ($\theta > 0$)
ii) Must not have adverse slope where direction of gravity and friction are opposite of flow direction
iii) Must have friction ($F_f > 0$) ← ideal flow non-uniform

Velocity distribution :

Shear stress,

$$W \sin\theta = F_f$$
$$\Rightarrow \gamma A L (\sin\theta = \tan\theta = S_0) = \tau_0 L P$$
$$\Rightarrow \gamma \frac{A}{P} S_0 = \tau_0$$
$$\Rightarrow \tau_0 = \gamma R S_0$$
$$\Rightarrow \tau_0 = \rho g R S_0$$



Drag velocity, $u^* = \sqrt{\tau_0/\rho}$

$$\Rightarrow u^* = \sqrt{g R S_0}$$

Laminar sublayer,

Thickness, $\delta_v = 11.6 \frac{\nu}{u^*}$

Boundaries, $\frac{k_s u^*}{\nu} \rightarrow$

≤ 5	≥ 70	btwn 5 & 70
Smooth	Rough	Transition

Velocity distribution along a vertical,

$$\frac{u_z}{u^*} = \frac{1}{K} \ln \frac{z}{z_0} \; ; \quad K = 0.4$$

$z_0 =$

$0.11 \frac{\nu}{u^*}$	$0.033 k_s$	$0.11 \frac{\nu}{u^*} + 0.033 k_s$
Smooth	Rough	Transition

Cross-sectional mean velocity,

$\frac{U}{u^*} =$

$5.75 \log \left(\frac{3.64 R}{\nu / u^*} \right)$	$5.75 \log \frac{12.2R}{k_s}$	$5.75 \log \frac{12.2R}{k_s + 3.35 \frac{\nu}{u^*}}$
Smooth	Rough	Transition

Formulas :

General, $\tau_0 = \gamma R S_0$

$u^* = \sqrt{g R S_0}$

$\delta_v = \frac{11.6 \nu}{u^*}$

velocity distribution,

Component	Smooth	Rough	Transition
Boundary	$\frac{k_s u^*}{\nu} \leq 5$	$\frac{k_s u^*}{\nu} \geq 70$	$5 < \frac{k_s u^*}{\nu} < 70$
velocity distribution	$\frac{U}{u^*} = \frac{1}{K} \log\left(\frac{z}{z_0}\right)$		
z_0	$\frac{0.11 \nu}{u^*}$	$0.033 k_s$	$0.11 \frac{\nu}{u^*} + 0.033 k_s$
Cross-sectional mean vel, $\frac{U}{u^*} =$	$5.75 \log \frac{3.64 R}{\nu / u^*}$	$5.75 \log \frac{12.2 R}{k_s}$	$5.75 \log \frac{12.2 R}{k_s + 3.35 \frac{\nu}{u^*}}$

Ex 4.1 $b = 6m = B$; $S_o = 0.025$; $K_s = 2\,mm$; $h = 0.50\,m$

mean vel = ?

Boundary condition: $\frac{K_s u^*}{\nu} = \frac{0.002 \times 0.103}{10^{-6}}$

$= 206 \geq 70 \rightarrow$ Rough

$u^* = \sqrt{gRS_o}$

$R = \frac{A}{P} = \frac{0.5 \times 6}{6 + 2 \times 0.5}$

$= 0.429\,m$

$\therefore u^* = \sqrt{9.81 \times 0.429 \times 0.025}$

$= 0.103$

Cross-sectional mean: $\frac{U}{u^*} = 5.75 \log \frac{12.2R}{K_s}$

$\Rightarrow \frac{U}{0.103} = 5.75 \log \frac{12.2 \times 0.429}{0.002}$

$\Rightarrow U = 2.02\,m/s$

Chezy's formula:

Two assumptions,

1. $W \sin\theta = F_f$
2. $F_f = KPLU^2$

Hence,

$W \sin\theta = KPLU^2$

$\Rightarrow \gamma AL (\sin\theta \simeq \tan\theta \simeq S_o = S_f) = KPLU^2$

$\Rightarrow \gamma/K \frac{A}{P} S_f = U^2$

$\Rightarrow U = C S_f^{1/2} R^{1/2}$; $C = \sqrt{\gamma/K}$

$C \rightarrow 30 \sqrt{m}/s$ to $80 \sqrt{m}/s$

$\rightarrow 50 \sqrt{m}/s$

L21 30/03

Ex. 4.2

Normal depth (P53 of another copy of book; on missing page)

Section factor ($AR^{2/3}$)

$$Q_n = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$\Rightarrow AR^{2/3} = \frac{nQ_n}{\sqrt{S}}$$

Conveyance ($K = \frac{1}{n} AR^{2/3}$) [capability to convey; ↑ → more flow]
[Convenient to express as K^2]

$Q_n = \dots$

$= KS^{1/2}$

On Chezy's → $K = C A_n R^{1/2}$

--- [See book]

Ex. 4.3

L22

30/03

P79 of PDF to P

L23

06/04

P82 of PDF

Rough / Final 22/04

Chapter 1

Open channel flow: Flow of water in a conduit w/ free surface.

Classifications:

(a) Natural, artificial

(b) Prismitic, non-prismitic

↳ non-varying cross-section & uniform slope (e.g., artificial)

(c) Rigid boundary: Immovable bed & sides

Mobile boundary: Bed w/ loose sands that move under the flow of water. E.g., alluvial channel

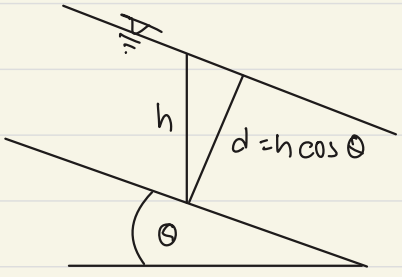
(d) Large slope, small slope

↳ slope more than 1 in 10

Geometric elements:

Depth of flow: vert. dist from the lowest point of a section to water surface → h

Depth of flow section: Depth of flow normal to flow direction.



Stage: Elevation of water surface relative to a datum.

Flow area, A: x-sectional area normal to flow dir, ← channel cross-section as opposed to vertical cross-sec

Wetted perimeter, P:

Top width, B:

Hydraulic radius, R: $R = A/P$

Hydraulic depth, D: $D = A/B$

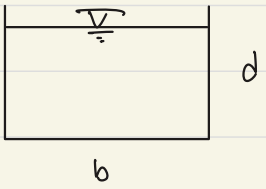
Formulas:

Rectangular:

$A = bd$

$P = b + 2d$

$B = b$



Trapezoidal:

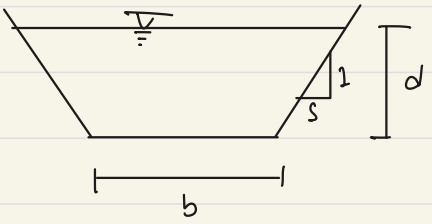
$A = bd + 2 \times \frac{1}{2} sd^2$

$= bd + sd^2$

$P = b + 2\sqrt{(sd)^2 + d^2}$

$= b + 2d\sqrt{1+s^2}$

$B = b + 2sd$



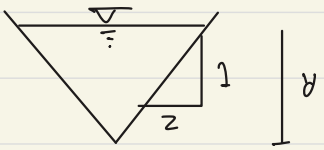
Triangular:

$A = zy^2$

$P = 2\sqrt{(zy)^2 + y^2}$

$= 2y\sqrt{1+z^2}$

$B = 2zy$



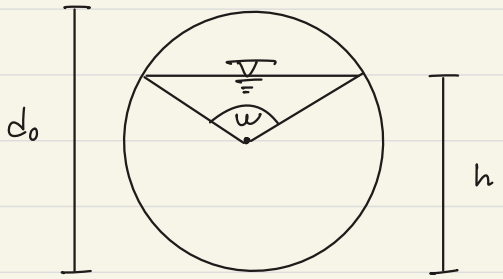
Circle: $h = d_0 \frac{1-\cos(\omega/2)}{2}$

$A = (\omega - \sin\omega)\frac{d_0^2}{8}$

$P = d_0 \frac{\omega}{2}$

$B = d_0 \sin\frac{\omega}{2}$

$\omega \rightarrow$ find from 'h' eqn



Effects of viscosity: [$\nu \rightarrow m^2/s$; 1 stoke $= 10^{-4}\ m^2/s$]

$Re = \frac{UR}{\nu}$; $\nu = 10^{-6}\ m^2/s$ (at 20°C)

→ < 500 → laminar
> 12,500 → turbulent } transitional in btwn

Effects of gravity:

$Fr = \frac{V}{\sqrt{gD}}$

→ < 1 → subcritical
> 1 → supercritical } critical in btwn

wide channel: when $b \geq 10h$

Then, $R \approx h$ [$R = \frac{A}{P} = \frac{bh}{b+2h} \simeq \frac{bh}{b} \simeq h$]

Discharge → $q = Q/b$

→ discharge per unit width (of channel btm)

→ $m^3/s/m$

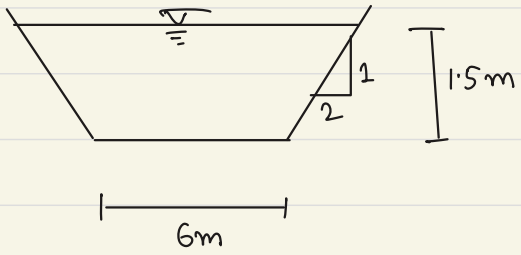
Ex 1.1 Trapezoidal

$b = 6m$

side: 2:1 (H:V)

$d = 1.5m$

$v = 2.3$ m/s



Discharge:

$$A = bh + shxh = 6 \times 1.5 + 2 \times 1.5 \times 1.5 = 13.5 \ m^2$$

$$Q = AV = 13.5 \times 2.3 = 31.05 \ m^3/s$$

State of flow:

$$P = b + 2\sqrt{(sh)^2 + h^2} = 6 + 2\sqrt{2^2 + 1} \times 1.5 = 12.71 \ m$$

$$B = b + 2sh = 6 + 2 \times 2 \times 1.5 = 12 \ m$$

$$R = A/P = 1.06$$

$$D = A/B = 1.13$$

$$Re = \frac{VR}{\nu} = \frac{2.3 \times 1.06}{10^{-6}} = 2.43 \times 10^6$$

$$Fr = \frac{V}{\sqrt{gD}} = \frac{2.3}{\sqrt{9.81 \times 1.13}} = 0.69$$

$\therefore$ Subcritical turbulent

Now, $c = \sqrt{gD} = \sqrt{9.81 \times 1.13} = 3.33$ m/s

speed of wave fronts u/s $= c - V = 3.33 - 2.3 = \cdots$

d/s $= c + V = 2.3 + 3.33 = \cdots$

Ex. 1.2 Circular

$d_0 = 2.75\ m$

$Q = 6.55\ m^3/s$

$h = 1.1\ m$

$$A = (\omega - \sin\omega)\frac{d_0^2}{8}$$

$$= (2.74 - \sin 2.74)\frac{2.75^2}{8}$$

$$= 2.22\ m^2$$

$$h = d_0\left(1 - \cos\frac{\omega}{2}\right) \times 1/2$$

$$\Rightarrow 1.1 = 2.75\left(1 - \cos\frac{\omega}{2}\right) \times 1/2$$

$$\therefore \omega = 2.74^c$$

$$P = d_0\frac{\omega}{2} = 2.75\ \frac{2.74}{2} = 3.77\ m$$

$$B = d_0 \sin\left(\frac{\omega}{2}\right) = 2.69\ m$$

$$V = Q/A = \frac{6.55}{2.22} = 2.95\ m/s$$

$$Re = \frac{VR}{\nu} = \frac{2.95 \times 2.22/3.77}{10^{-6}} = 1.74 \times 10^6$$

$$Fr = \frac{V}{\sqrt{gD}} = \frac{2.95}{\sqrt{9.81 \times \frac{2.22}{2.69}}} = 1.04$$

$\therefore$ Supercritical turbulent

Celerity — $\sqrt{gD}$ is the velocity of an elementary or small amplitude wave on the surface of still water

$$Fr = \frac{V}{\sqrt{gD}}$$

when $Fr = 1$, $V = \sqrt{gD} = C$

(Subcritical) < 1, $V < C$ [wave propagates u/s]

(Super critical) > 1, $V > C$ [" carit " "]

↳ flow controlled by u/s

flow controlled from d/s

Velocity distribution

The kinetic energy & momentum computed from cross-sectional mean vel are generally less than their actual val.

Hence, multiplied by α (Kinetic energy coeff) (coriolis coeff)

β (momentum coeff) (Boussinesq coeff)

$$\alpha = \frac{\int_0^A u^3 dA}{V^3 A}$$

$$\beta = \frac{\int_0^A u^2 dA}{V^2 A}$$

} Always positive, never < 1

$\alpha > \beta > 1$

$\alpha, \beta \uparrow$ for laminar than turbulent

$$\frac{\alpha - 1}{\beta - 1} \approx 2.8 \text{ to } 3.0$$

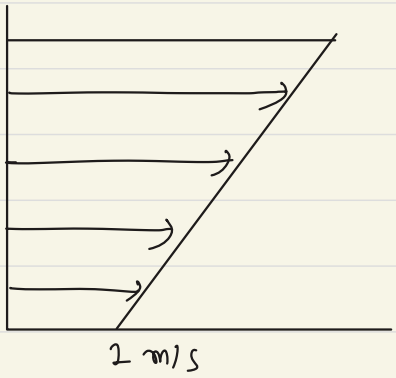
Ex. 1.4 $u = 1 + \frac{32}{h}$; $h = 5m$; wide channel

$1 + \frac{3 \times 5}{5} = 4$ m/s

$$U = \frac{\int u dA}{A}$$

$$= \frac{\int u dz}{h} = \frac{\int_0^5 (1 + \frac{32}{5}) dz}{5}$$

$$= 2.5 \text{ m/s}$$



(i) $q = hU = 5 \times 2.5 = 12.5 \text{ m}^3/\text{s/m}$

(ii) $Re = \frac{UR}{\nu} \simeq \frac{Uh}{\nu} = \frac{2.5 \times 5}{10^{-6}} = 12.5 \times 10^6$

$Fr = \frac{U}{\sqrt{gD}} = \frac{U}{\sqrt{gh}} = \frac{2.5}{\sqrt{9.81 \times 5}} = 0.36$

} subcritical turbulent

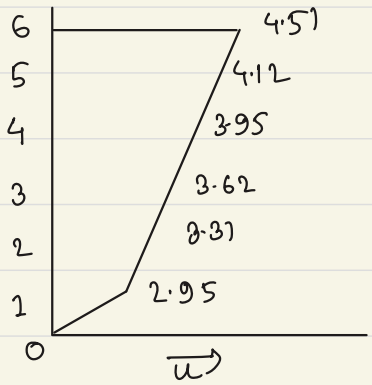
(iii) $\alpha = \frac{\int_0^A u^3 dA}{U^3 A} \simeq \frac{\int_0^h u^3 dz}{U^3 h} = \frac{\int_0^5 (1 + \frac{32}{5})^3 dz}{(2.5)^3 \times 5}$

$= 1.36$

$$\beta = \frac{\int_0^h u^2 dz}{U^2 h} = \frac{\int_0^5 (1 + \frac{3}{5}z)^2 dz}{(2.5)^2 \times 5} = 1.12$$

$$\frac{(\alpha - 1)}{(\beta - 1)} = 3$$

Ex 1.5 wide channel



$$q = \int u\,dz$$

$$= \left[\frac{0+2.95}{2} + \frac{2.95+3.31}{2} + \cdots + \frac{4.12+4.51}{2}\right] 2$$

$$= \left[\frac{0+4.51}{2} + 2.95 + 3.31 + 3.62 + 3.95 + 4.12\right]$$

$$= 20.21 \ m^3/s/m$$

$$v = \frac{q}{h} = \frac{20.21}{6} = 3.37 \ m/s$$

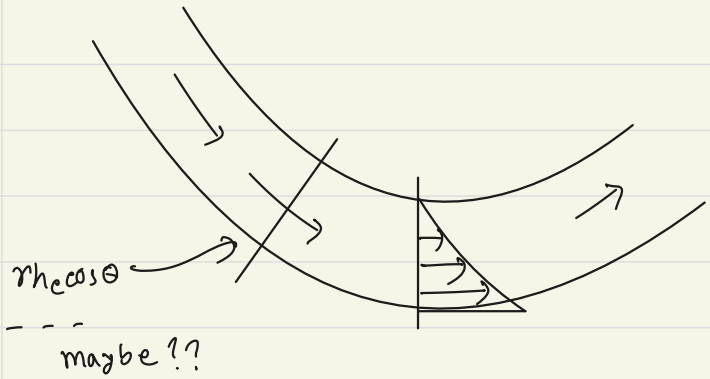
$$\alpha = \frac{\int_0^A u^3 dA}{v^3 A} = \frac{1}{v^3 h} \times \Sigma u^3 \Delta z$$

$$= \frac{1}{3.37^3 \times 6} \times \left[\frac{0+4.51^3}{2} + 2.95^3 + 3.31^3 + 3.62^3 + 3.95^3 + 4.12^3\right] \times 1$$

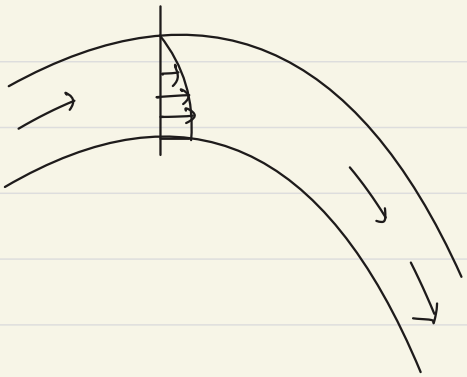
$$= 1.25$$

$$\beta = \frac{1}{v^2 h} \times \Sigma u^2 \Delta z = 1.11$$

Pressure distribution



Concave, $\rho > \gamma h$



Convex, $\rho < \gamma h$

Total pressure: $\gamma h \pm \gamma h_c$

$$h_c = \frac{h u^2}{g R}$$

$$\text{Force} = \frac{mv^2}{R}$$

$$\text{Pressure} = \frac{mv^2}{R \times A} = \frac{\rho h A v^2}{RA}$$

$$= \frac{\rho h v^2}{R}$$

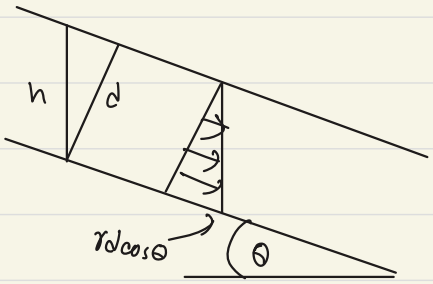
$$h_c = \frac{p}{\gamma} = \frac{\rho h v^2}{\rho g R}$$

$$= \frac{h v^2}{g R}$$

Effect of slope

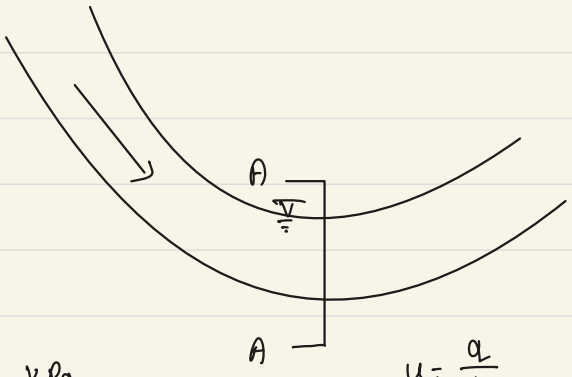
$$p = \gamma d \cos\theta = \gamma h \cos\theta \times \cos\theta$$

$$= \gamma h \cos^2\theta$$



Ex. 1.6 $r = 20m$; $h = 3m$

$q = 66\ m^2/s$



Total pressure = $\rho h + \rho h_c$

$= \rho h + \rho\left(\frac{hu^2}{rg}\right)$

$= \rho h\left(1 + \frac{u^2}{rg}\right)$

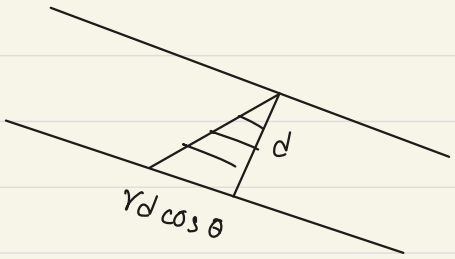
$= 9810 \times 3\left(1 + \frac{22^2}{20 \times 9.81}\right) = 102\ kPa$

$u = \frac{q}{h}$

$= \frac{66}{3}\ m/s$

$= 22\ m/s$

Ex 1.7



Shear force : $\frac{1}{2}\rho d\cos\theta \times d$

$= \frac{1}{2}\rho h^2 \cos^3\theta$

Overturning moment : $F \times d/3$

$= \frac{1}{6}\rho h^2 \cos^3\theta \times h\cos\theta$

$= \frac{1}{6}\rho h^3 \cos^4\theta$

Chapter 2

Continuity eqⁿ: $\rho_1 Q_1 = \rho_2 Q_2$

$\Rightarrow Q_1 = Q_2$

Energy eqⁿ: $H = z + \frac{p}{\gamma} + \frac{v^2}{2g} = \text{const}$

$= z + \frac{p}{\gamma} + \frac{\alpha v^2}{2g}$

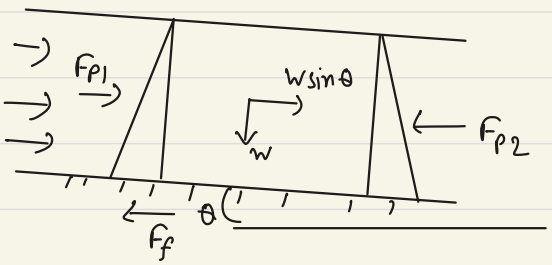
Again, $\left(z + \frac{p}{\gamma}\right)$ = piezometric head → ht of water surface from datum

$\therefore H = z_b + h + \frac{\alpha v^2}{2g}$

Momentum eqⁿ:

$\Delta P = m(\Delta v)$

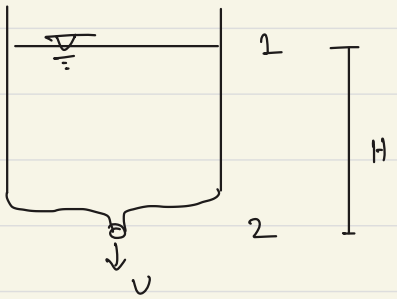
$\rightarrow \rho Q(\beta v_2 - \beta v_1) = F_{p1} - F_{p2} + w\sin\theta - f_f$



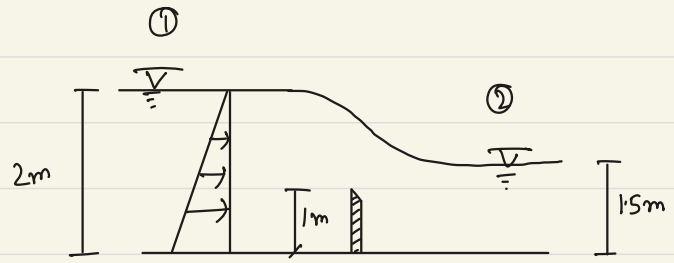
Ex. 2.1 Law of Torricelli

$0 + H + 0 = 0 + 0 + \frac{v^2}{2g}$

$\Rightarrow v = \sqrt{2gH}$



Ex. 2.2 $q = 4\ m^2/s$



Energy loss:

$$0+2+\frac{2^2}{2\times9.81} = 0+1.5+\frac{2.67^2}{2\times9.81}+h_L$$

$$\Rightarrow h_L = 0.34\ m$$

$$U_1 = \frac{4}{2} = 2\ m/s$$

$$U_2 = \frac{4}{1.5} = 2.67\ m/s$$

Force on wein plate:

$$\rho Q(\beta_2 U_2 - \beta_1 U_1) = F_{p1} - F_{p2} - F_w$$

$$\Rightarrow 1000\times 4(2.67-2) = \frac{0+\gamma h_1}{2}\times h_1 - \frac{0+\gamma h_2}{2}h_2 - F_w$$

$$\Rightarrow 2680 = \frac{9810\times 2}{2}\times 2 - \frac{9810\times 1.5}{2}\times 1.5 - F_w$$

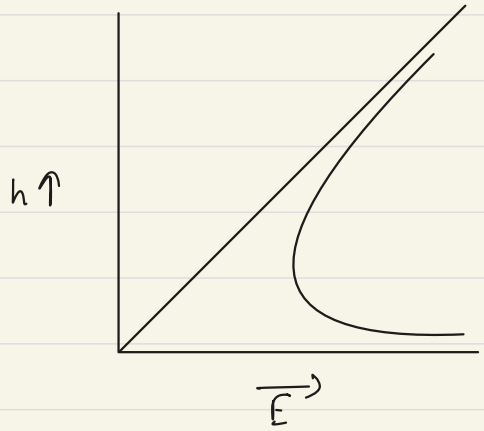
$$\Rightarrow F_w = 5.9\ kN$$

Chapter 3

Specific energy: Energy measured w/ respect to channel bottom

$$E = h + \frac{v^2}{2g} = h + \frac{Q^2}{2gA^2}$$

Specific energy curve:



Show that at min E, $F_n = 1$;

$$\frac{dE}{dh} = 1 + \frac{Q^2}{2g}(-2)A^{-3}\frac{dA}{dh}$$

$$= 1 + \frac{v^2}{2gA}(-2)\frac{dA}{dh}$$

$$= 1 - \frac{v^2}{gA}\frac{dA}{dh}$$

$$= 1 - \frac{v^2 B}{gA} \qquad dA = B\,dh$$

$$= 1 - \frac{v^2}{g\frac{A}{B}}$$

$$\Rightarrow \frac{dE}{dh} = 1 - \frac{v^2}{gD}$$

when $\frac{dE}{dh} = 0$, $1 = \frac{v^2}{gD}$

$$\Rightarrow 1 = F_n^2$$

$$\Rightarrow F_n = 1$$

Show that discharge is max at $h = h_c$ for a given specific energy

$$E = h + \frac{Q^2}{2gA^2}$$

$$\Rightarrow Q^2 = 2gA^2(E-h)$$

$$\Rightarrow 2Q\frac{dQ}{dh} = 2g\left[A^2(0-1) + (E-h)2A\frac{dA}{dh}\right]$$

$$= 2g\left(-A^2 + \frac{2AdA}{dh}(E-h)\right)$$

$$= 2g\left(-A^2 + 2ABx\frac{U^2}{2g}\right)$$

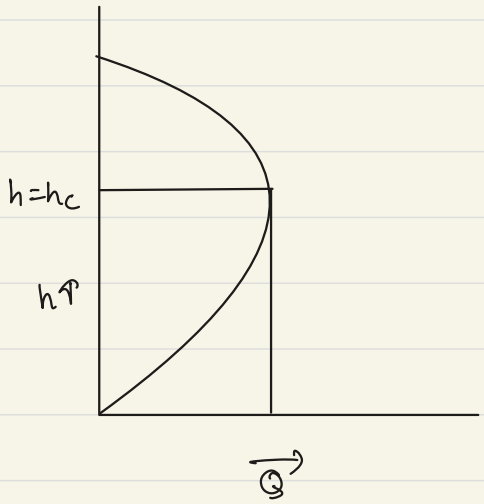
$$= -2gA^2\left(1 - \frac{BU^2}{gA}\right)$$

$$= -2gA^2\left(1 - \frac{U^2}{gD}\right)$$

$$\Rightarrow 2Q\frac{dQ}{dh} = -2gA^2(1 - Fr^2)$$

$dA = Bdh$

$D = \frac{A}{B}$



At $\frac{dQ}{dh} = 0$, $1 - Fr^2 = 0 \Rightarrow Fr =$

Show that, $E_c = 1.5\,h_c$ for rect channel

$$Fr = 1$$

$$\Rightarrow \frac{U}{\sqrt{gD}} = 1 \Rightarrow U^2 = gD$$

$$\Rightarrow \frac{U^2}{2g} = \frac{D}{2}$$

$$E_c = h_c + \frac{U_c^2}{2g} = h_c + \frac{D_c}{2} = h_c + \frac{h_c}{2}$$

$$\Rightarrow E_c = 1.5h_c$$

Section factor:

$$Z = A\sqrt{D} \quad ; \quad Z_c = A_c\sqrt{D_c}$$

$$Z_c = \frac{Q}{\sqrt{g/\alpha}} \longleftarrow \quad 1 = \alpha\frac{V_c}{\sqrt{gD_c}} \Rightarrow \alpha\frac{V_c^2}{2g} = \frac{D_c}{2}$$

$$\Rightarrow \frac{\alpha Q^2}{2gA^2} = \frac{D_c}{2} \Rightarrow A^2 D_c = \frac{\alpha Q^2}{g}$$

Relation btwn section factor & F_n

$$F_n = \alpha\frac{V}{\sqrt{gD}} \Rightarrow F_n^2 = \frac{\alpha V^2}{gD} = \frac{\alpha Q^2}{gDA^2} = \frac{\alpha Q^2/g}{A^2 D}$$

$$\Rightarrow F_n^2 = \frac{Z_c^2}{Z^2}$$

$Z^2 = C_1 h^m$; $m = \frac{h}{A}\left[3B - D\frac{dB}{dh}\right]$ → Hydraulic exponent for critical flow

$$\Rightarrow 2\ln Z = \ln C_1 + m\ln h$$

$$\Rightarrow \frac{d}{dh}\ln Z = \frac{1}{2} m \frac{1}{h} = \frac{m}{2h}$$

Again, $Z = A\sqrt{D} = A\sqrt{\frac{A}{B}}$

$$\Rightarrow \ln Z = \ln A + \frac{1}{2}(\ln A - \ln B)$$

$$= \frac{3}{2}\ln A - \frac{1}{2}\ln B$$

$$\Rightarrow \frac{d}{dh}\ln Z = \frac{3}{2}\frac{1}{A}\frac{dA}{dh} - \frac{1}{2}\frac{1}{B}\frac{dB}{dh}$$

$$= \frac{3}{2}AB - \frac{1}{2}\frac{1}{B}\frac{dB}{dh}$$

$$\Rightarrow \frac{m}{2h} = \frac{1}{2A}\left[3B - D\frac{dB}{dh}\right]$$

$$= \frac{h}{A}\left[3B - D\frac{dB}{dh}\right]$$

Ex. 3.1 Rect channel:

$$m = \frac{h}{A}\left[3B - D\frac{dB}{dh}\right]; \qquad A = Bh$$

$$= \frac{h}{Bh}\left[3B - \frac{A}{B} \times 0\right] \qquad D = \frac{A}{B}$$

$$= 3 \qquad \frac{dB}{dh} = 0$$

Ex. 3.2 Trapezoidal; $b = 6.1\,m$; $s = 2$; $h = 2m$

$$m = \frac{h}{A}\left[3B - D\frac{dB}{dh}\right]$$

$$= \frac{2}{20.2}\left[3 \times 14.1 - 1.43\,\frac{d}{dh}(6.1 + 4h)\right]$$

$$= \frac{2}{20.2}\left[42.3 - 1.43(4)\right] = 3.62$$

$A = bh + sh^2 = h(b + sh) = 20.2\,m^2$

$B = b + 2sh = 14.1\,m$

$D = A/B = 1.43$

Computation of critical depth:

Rect. channel: $y_c = \sqrt[3]{\frac{\alpha Q^2}{g B^2}}$

Wide channel: $y_c = \sqrt[3]{\frac{\alpha q^2}{g}}$

Triangular channel: $y_c = \sqrt[5]{\frac{2\alpha Q^2}{g s^2}}$

Parabolic " $y_c = \sqrt[4]{\frac{27\alpha c Q^2}{32 g}}$ $[z = cy^2]$

Ex. 3.3

(i) wide rect. channel; $q = 3\ m^2/s$

$$y_c = \sqrt[3]{\frac{\alpha q^2}{g}} = \sqrt[3]{\frac{1.12 \times 3^2}{9.81}} = 1.01\ m\ ; \quad V_c = \frac{q}{y_c} = \dots$$

(ii) Rect; $b = 6\ m$, $Q = 20\ m^3/s$

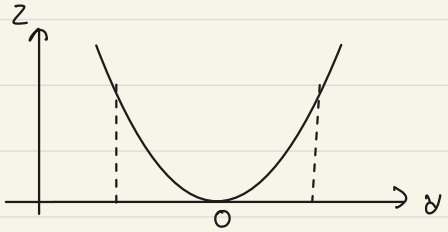
$$y_c = \sqrt[3]{\frac{\alpha Q^2}{g b^2}} = 1.08\ m\ ; \quad V_c = \frac{Q}{b \times y_c} = \dots$$

(iii) Triangular; $s = 2$, $Q = 10\ m^3/s$

$$y_c = \sqrt[5]{\frac{2\alpha Q^2}{g s^2}} = 1.42\ m\ ; \quad V_c = \frac{Q}{s y_c \times y_c} = \dots$$

(iv) Parabolic; $y^2 = 42$; $Q = 20\ m^3/s$

$\Rightarrow z = \frac{1}{4} y^2$; $c = 1/4$;



$$y_c = \sqrt[4]{\frac{27\alpha c Q^2}{32 g}} = 1.76\ m\ ; \quad A \rightarrow y = 0.25 x^2 \Rightarrow 1.76 = 0.25 x^2 \Rightarrow x = 2.65$$

$$\therefore A = 2 \times 2.65 \times 1.76 - \int_{-2.65}^{2.65} (0.25 x^2)\, dx$$

$$= 6.23\ m^2$$

Ex. 3.4 Trapezoidal; $b = 6m$; $s = 2$; $Q = 50\ m^3/s$; $\alpha = 1$

$$z_c = \frac{Q}{\sqrt{g/\alpha}} = \frac{50}{\sqrt{9.81}} = 15.96$$

$$z = A\sqrt{D}; \quad A = bh + sh^2 = h(b+sh); \quad D = \frac{A}{B}; \quad B = b + 2sh$$

h	A	B	D	z
2	20	14	1.43	23.9
1.5				14.32
1.6				16.04

Ex. 3.5 Circular channel; $d_0 = 2m$; $Q = 4\ m^3/s$; $\alpha = 1.10$

$$z_c = \frac{Q}{\sqrt{g/\alpha}} = 1.34$$

$$z = A\sqrt{D}; \quad A = (\omega - \sin\omega)\frac{d_0^2}{8}; \quad D = \frac{A}{B}; \quad B = d_0 \sin\left(\frac{\omega}{2}\right)$$

ω	A	B	D	z
1				0.023
4				2.72
2.5				0.67
3.0				1.2
3.2				1.47
3.11	1.54	2	0.77	1.35

$$U = \frac{Q}{A}$$

Ex. 3.6 Trapezoidal; $b = 6\ m$; $s = 2\ m$; $Q = 14\ m^3/s$; $\alpha = 1$

Bisection

$$f(h) = 1 - Fr^2 = 1 - \frac{2c^2}{2^2} = 1 - \frac{\alpha Q^2}{gA^2D} = 1 - \frac{\alpha Q^2 B}{gA^3}; \quad B = b + 2sh$$

$$= 6 + 2 \times 2 \times h$$

$$= 6 + 4h$$

$$A = bh + sh^2$$

$$= 6h + 2h^2$$

h_l	h_u	h	$f(h)$
0	10	5	0.999
0	5	2.5	0 98
0	2.5	1.25	0 82
0	1.25	0.625	−0.825
0.625	1.25	0.938	0.516
0.625	0.938	0.782	0.12
0.625	0.782	0.704	−0.245
0.704	0.782	0.743	−0.042
0.743	0.782	0.763	0.043
0.743	0.763	0.753	0.003
0.743	0.753	0.748	−0.019
0.748	0.753	0.751	−0.008

$\therefore h_c = 0.751\ m$

Ex. 3.7 Trapezoidal channel

$b = 6m$; $s = 2$; $Q = 14\ m^3/s$; $\alpha = 1$

Newton Raphson

$$1 - Fr^2 = 0 \Rightarrow 1 - \frac{2c^2}{2^2} = 0 \Rightarrow 1 - \frac{\alpha Q^2 B}{g A^3} = 0$$

$$\Rightarrow A^3 - \frac{\alpha Q^2 B}{g} = 0 \longrightarrow f(h)$$

$$f'(h) = 3A^2 B - \frac{\alpha Q^2}{g}\frac{dB}{dh}$$

$A = bh + sh^2 = 6h + 2h^2$ $\qquad \frac{dB}{dh} = 4$

$B = b + 2sh = 6 + 4h$

$$\therefore f(h) = (6h+2h^2)^3 - \frac{14^2}{9.81}[6+4h] = (6h+2h^2)^3 - 19.98(6+4h)$$

$$f'(h) = 3[6h+2h^2]^2[6+4h] - 79.92$$

h	$f(h)$	$f'(h)$	$h' = h - \frac{f(h)}{f'(h)}$
1	312.19	1840	0.83
0.83	70.77	1050.27	0.76
0.76	6.06	805.91	0.75
0.75	-1.84	774.38	0.75

$\therefore h_c = 0.75\ m$

Ex. 3.7 (nep) Trapezoidal; $b = 6m$; $s = 2$; $Q = 14\ m^3/s$; $\alpha = 1$

Newton – Raphson

$$1 - Fr^2 = 0 \Rightarrow 1 - \frac{Zc^2}{Z^2} = 0 \Rightarrow 1 - \frac{\alpha Q^2 B}{g A^3} = 0$$

$$\Rightarrow A^3 - \frac{\alpha Q^2}{g} B = 0$$

$$f(h) = A^3 - \frac{\alpha Q^2 B}{g}$$

$$f'(h) = 3A^2 B - \frac{\alpha Q^2}{g} \frac{dB}{dh}$$

$$A = bh + sh^2 = 6h + 2h^2 \qquad \frac{dB}{dh} = 4$$

$$B = b + 2sh = 6 + 4h$$

$$\therefore f(h) = (6h + 2h^2)^3 - 19.98\,(6 + 4h)$$

$$f'(h) = 3\,[6h + 2h^2]^2\,[6 + 4h] - 79.92$$

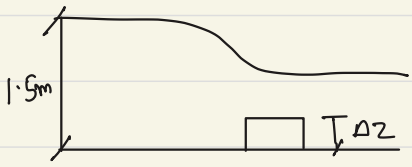
Trial:

h	$h' = h - \frac{f(h)}{f'(h)}$
2	0.83
0.83	0.76
0.76	0.75
0.75	0.75

Ex. 3.8 $V = 1\ m/s$; $d = 1.5\ m$; $B = 3\ m$; $\alpha = 1$

(a) $h_c = \sqrt[3]{\frac{\alpha Q^2}{gB^2}} = 0.61\ m$

$E_c = 1.5 h_c = 0.92\ m$



$E_1 = E_c + \Delta z$

$1.5 + \frac{1^2}{2 \times 9.81} = 0.92 + \Delta z$

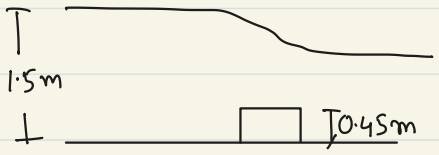
$\Rightarrow \Delta z = 0.63\ m$

$Q = AV = 3 \times 1.5 \times 1 = 4.5\ m^3/s$

$F_{r1} = \frac{V}{\sqrt{gD}} = 0.26$ [sub]

Drop $= 1.5 - 0.61 - 0.63 = 0.26\ m$

(b) (i) upward step $0.45\ m$



$E_1 = 1.5 + \frac{1^2}{2 \times 9.81} = 1.55\ m$

$1.55 = E_2 + (\Delta z = 0.45\ m)$

$\Rightarrow E_2 = 1.1\ m$ [$> E_c$]

$1.1 = h + \frac{Q^2}{2gA^2}$

$1.1 = h + \frac{4.5^2}{2 \times 9.81 \times (3h)^2}$

$\Rightarrow h = 0.41\ m,\ 0.98\ m$

Since u/s sub, $h = 0.98\ m$

Drop $= 1.5 - 0.45 - 0.98 = 0.07\ m$

(ii) Upward, 0.80m

$E_1 = 1.55\,m$

$E_2 = E_1 - \Delta z = 1.55 - 0.80 = 0.75 \quad (< E_c)$

$\therefore E_1' = E_c + \Delta z = 0.92 + 0.80 = 1.72\,m$

$h_2 = h_c = 0.61\,m$

$$1.72 = h_1 + \frac{v^2}{2g} = h_1 + \frac{\frac{Q^2}{A^2}}{2g}$$

$$\therefore 1.72 = h_1 + \frac{4.5^2/(3h_1)^2}{2\times 9.81}$$

$$\therefore h_1 = 0.28\,m,\ 1.68\,m$$

$$\therefore h_1 = 1.68\,m$$

$$\text{Drop} = 1.68 - 0.8 - 0.61 = 0.27\,m$$

(iii) Downward, 0.45 m

$E_1 = 1.55\,m$

$E_2 = E_1 + \Delta z$

$= 1.55 + 0.45 = 2.0\,m$



$$2 = h_2 + \frac{4.5^2}{2\times 9.81\times(3h_2)^2}$$

$$\therefore h_2 = 1.97\,m,\ 0.26\,m$$

$$\text{Rise} = 1.97 - 1.5 - 0.45 = 0.02\,m$$

Ex. 3.9 $v = 1\ m/s;\ h = 1.5\ m;\ b = 3\ m$

$Q = (1.5 \times 3) \times 1 = 4.5\ m^3/s$

(a) $E_1 = 1.5 + \frac{1^2}{2 \times 9.81} = 1.55\ m$

$$E_2 = E_c = 1.55 = \sqrt[3]{\frac{4.5^2}{9.81 \times b_c^2}} + \frac{4.5^2}{2 \times 9.81 \times \left[b_c \times \sqrt[3]{\frac{4.5^2}{9.81 \times b_c^2}}\right]^2}$$

$$\Rightarrow b_c = 1.37\ m$$

Alt: $E_c = 1.5 h_c \Rightarrow h_c = 1.03\ m$

$$1.55 = 1.03 + \frac{4.5^2}{2 \times 9.81 \times (b_c \times 1.03)^2} \Rightarrow b_c = 1.37\ m$$

(b) (i) $b_2 = 2\ m$

$$E_2 = E_1 = 1.55 = h_2 + \frac{4.5^2}{2 \times 9.81 \times (2 \times h_2)^2} \Rightarrow h_2 = 0.49\ m,\ 1.42\ m$$

$$h_c = \sqrt[3]{\frac{4.5^2}{9.8 \times 2^2}} = 0.80\ m$$

$\therefore h_2 = 1.42\ m$

Drop $= 1.5 - 1.42 = 0.08\ m$

(ii) $b_2 = 1\ m$

$$E_2 = E_1 = 1.55 = h_2 + \frac{4.5^2}{2\times 9.81\times(1\times h_2)^2}$$

$$\Rightarrow h_2 = -0.68\ m \ ; \quad E_1 \text{ to increase}$$

$$E_2 = E_c = 1.5\,h_c \longrightarrow 1.27\ m$$

$$= 1.5\left(\sqrt[3]{\frac{4.5^2}{9.81\times 2^2}}\right) = 1.9\ m$$

$$E_1 = E_2 = 1.9 = h_1' + \frac{4.5^2}{2\times 9.81\times(h_1'\times 3)^2}$$

$$\Rightarrow h_1' = 0.26\ m,\ 1.87\ m$$

$$h_2 = h_c = 1.27\ m$$

$$\therefore \text{Rise} = 1.87 - 1.27 = 0.6\ m$$

(iii) $b_2 = 4m$

$$E_2 = E_1 = 1.55 = h_2 + \frac{4.5^2}{2\times 9.8\times(h_2\times 4)^2}$$

$$\Rightarrow h_2 = 0.22\ m,\ 1.52\ m$$

$$\text{Drop} = 0.02\ m$$

Ex. 3.10 $b = 6m$; $h = 2m$; $U = 2\ m/s$; $\alpha = 1$

$Q = 24\ m^3/s$

$b_2 = 3m$;

$$E_1 = 2 + \frac{2^2}{2 \times 9.81} = 2.2\ m = E_2$$

$$h_c = \sqrt[3]{\frac{24^2}{9.81 \times 3^2}} = 1.87\ m$$

$$E_c = 1.5 \times 1.87 = 2.81\ m$$

$\because E_c > E_2$, channel bed need to be lowered

$$E_c = E_2 + \Delta z$$

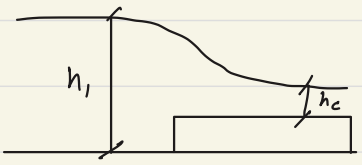
$$\Rightarrow \Delta z = 2.81 - 2.2 = 0.61\ m$$

Flow measurements:

Broad crested: $h_c = \sqrt[3]{\frac{Q^2}{gb^2}}$

$\Rightarrow h_c^3 \times g b^2 = Q^2$

$\Rightarrow Q = b\sqrt{g}\, h_c^{1.5}$



$Q = \left(\frac{2}{3}\right)^{1.5} b\sqrt{g}\, h_1^{1.5}$ $\quad [h_1 = 1.5 h_c]$

$V_1 \approx V_c$

$h_1 = h_c + \frac{V_c^2}{2g} = h_c + \frac{h_c}{2} = 1.5 h_c$

w/ approach vel, $Q = \left(\frac{2}{3}\right)^{1.5} b\sqrt{g}\left(h_1 + \frac{V_1^2}{2g}\right)^{1.5}$

$\frac{V_c^2}{2g} = \frac{Q^2}{2g b^2 h_c^2}$

$= \frac{h_c^3}{2h_c^2} = \frac{h_c}{2}$

Ex. 3.11 $b = 2\,m$; $D_2 = 1.2\,m$

$h_1 = 0.8\,m$

$\therefore Q = \left(\frac{2}{3}\right)^{1.5} b\sqrt{g}\,(h_1)^{1.5}$

$= \left(\frac{2}{3}\right)^{1.5} \times 2 \times \sqrt{9.81} \times (0.8)^{1.5}\ m^3/s$

$= 2.44\ m^3/s$

Chapter 4

Uniform flow: fluid chars remain same along its length

Steady flow: " " " " at a point throughout time.

Chars: (i) Chars remain same along len

(ii) $S_f = S_w = S_0$

Reqs: (i) Have friction

(ii) Downward slope

(iii) Net ext. force zero

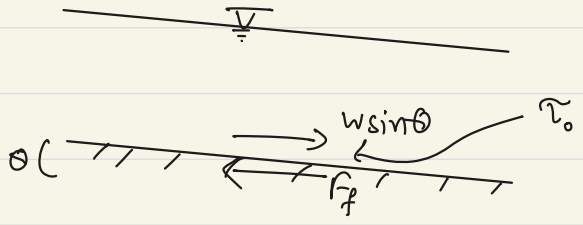
(iv) Not have adverse slope where dir of gravity and friction force are same and/or opposite of flow dir.

$$\rho Q(\beta V_2 - \beta V_1) = F_{P1} - F_{P2} + W\sin\theta - F_f$$

$$\Rightarrow W\sin\theta = F_f$$

Velocity distribution

Shear stress,



$$W\sin\theta = F_f$$

$$\gamma AL(\sin\theta \simeq \tan\theta = S_0) = \tau_0 PL$$

$$\Rightarrow \tau_0 = \frac{A}{P}\gamma S_0$$

$$= R\rho g S_0$$

$$\therefore \tau_0 = \rho g R S_0$$

Drag velocity, $u^* = \sqrt{\tau_0/\rho} = \sqrt{gRS_0}$

$\tau_0 = \rho u^{*2}$

Laminar sublayer:

Thickness, $\delta_v = \frac{11.6\,\nu}{u^*}$

Boundaries: $\frac{K_s u^*}{\nu} \rightarrow$

≤ 5	≥ 70	$(5, 70)$
Smooth	Rough	Transition

Velocity distribution along a vertical

$\frac{u}{u^*} = \frac{1}{K} \ln \frac{z}{z_0}$; $K = 0.4$

z_0:

$0.11 \frac{\nu}{u^*}$	$0.033 K_s$	$0.11 \frac{\nu}{u^*} + 0.033 K_s$
Smooth	Rough	Transition

Depth avg vel: $U = \frac{v_{0.2} + v_{0.8}}{2}$ or $v_{0.6}$

Cross-sectional mean vel

$\frac{U}{u^*} =$

$5.75 \log \frac{3.64\,u^* R}{\nu}$	$5.75 \log \frac{12.2R}{K_s}$	$5.75 \log \frac{12.2R}{K_s + 335 \nu/u^*}$
Smooth	Rough	Transition

Formulas:

$$\tau_0 = \rho g R S_0$$

$$u^* = \sqrt{gRS_0}$$

$$\delta_v = \frac{11.6\nu}{u^*}$$

Component	Smooth	Rough	Transition
Boundary	$\frac{u^* k_s}{\nu} \leq 5$	$\frac{u^* k_s}{\nu} \geq 70$	$\frac{u^* k_s}{\nu}$ btwn $(5, 70)$
velocity along vert.	$\frac{u_z}{u^*} = \frac{1}{K} \ln \frac{z}{z_0}$		
z_0	$0.11 \frac{\nu}{u^*}$	$0.033 k_s$	$0.11 \frac{\nu}{u^*} + 0.033 k_s$
Cross-sectional mean vel, $\frac{U}{u^*}$	$5.75 \log \frac{3.64R}{\nu/u^*}$	$5.75 \log \frac{12.2R}{k_s}$	$5.75 \log \frac{12.2R}{k_s + 3.35\frac{\nu}{u^*}}$

Ex. 4.1 Rect.; $b = 6m$; $S_0 = 0.0025$; $k_s = 2mm = 0.002m$

$h = 0.50m$

Boundary: $\frac{u^* k_s}{\nu} = \frac{0.103 \times 0.002}{10^{-6}}$

$= 206 \gg 70 \rightarrow$ Rough

$u^* = \sqrt{gRS_0}$

$= \sqrt{9.81 \times 0.429 \times 0.0025}$

$= 0.103$

Mean vel:

$\frac{U}{u^*} = 5.75 \log\left(\frac{12.2R}{k_s}\right)$

$\Rightarrow U = 5.75 \log\left(\frac{12.2 \times 0.429}{0.002}\right) \times 0.103$ m/s

$= 2.02$ m/s

$A = bh = 3 m^2$

$P = b + 2h = 7m$

$R = A/P = 0.429$

Uniform flow formulas

Chezy's formula:

Two assumptions: (i) $w \sin\theta = F_f$

(ii) $F_f = k P L U^2$ [Resistance stress $\propto U^2$]

$\therefore w \sin\theta = k P L U^2$

$\Rightarrow \gamma A L S_f = k P L U^2$

$\Rightarrow \frac{\gamma}{k} \frac{A}{P} S_f = U^2$

$\Rightarrow U = C R^{1/2} S_f^{1/2}$; $C = \sqrt{\frac{\gamma}{k}}$

$C \rightarrow 30 \sqrt{m}/s$ to $80 \sqrt{m}/s$

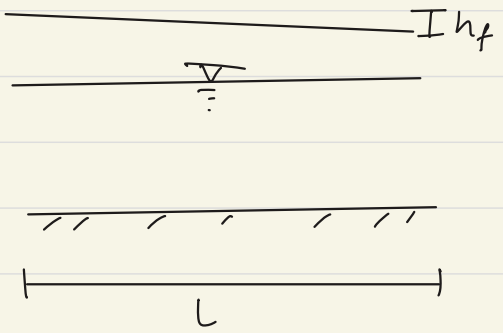
$\rightarrow 50 \sqrt{m}/s$

Darcy - weisbach formula:

$$h_f = f \frac{L}{d_0} \frac{v^2}{2g}$$

$$d_0 = 4R; \quad S_f = \frac{h_f}{L}$$

$$v = \sqrt{\frac{8g}{f}} R^{1/2} S_f^{1/2}$$



Manning's formula:

$$v = \frac{1}{n} R^{2/3} S_f^{1/2} \longrightarrow m/s$$

$$= \frac{1.486}{n} R^{2/3} S_f^{1/2} \longrightarrow ft/s$$

Relations among the factors (Q same)

$$\frac{C R^{1/2} S_f^{1/2}}{\sqrt{\frac{8g}{f}} R^{1/2} S_f^{1/2}} = 1 \Rightarrow \sqrt{\frac{8g}{f}} = C$$

$$\frac{C R^{1/2} S_f^{1/2}}{\frac{1}{n} R^{2/3} S_f^{1/2}} = 1 \Rightarrow \frac{1}{n} R^{1/6} = C$$

$$\frac{\sqrt{\frac{8g}{f}} R^{1/2} S_f^{1/2}}{\frac{1}{n} R^{2/3} S_f^{1/2}} = 1 \Rightarrow \frac{1}{n} R^{1/6} = \sqrt{\frac{8g}{f}}$$

$$\frac{v}{u^*} = \frac{C}{\sqrt{g}} = \sqrt{\frac{8}{f}}$$

$$\left[\begin{array}{l} u^* = \sqrt{gRS} \\ \frac{v}{u^*} = \frac{C}{\sqrt{g}} \end{array} \right]$$

Estimation of C: → $18 \log\left(\frac{11.4 Re}{C}\right)$

C:	$18 \log \frac{3.64 R}{\sqrt{/u^*}}$	$18 \log \frac{12.2 R}{k_s}$	$18 \log \frac{12.2 R}{k_s + 3.35 \sqrt{/u^*}}$
	Smooth	Rough	Transition

$$\left[\frac{C}{\sqrt{g}} = 5.75 \log[\cdots] \Rightarrow C = 18 \log[\cdots] \right]$$

Estimation of n:

$$n = \frac{d_{50}^{1/6}}{21.1}$$

Factors affecting n: n correlates to resistance to flow

n ↑ when there are more eff. resistance

(i) Bed roughness
(ii) vegetation
(iii) Channel alignment
(iv) Channel irregularities
(v) Silting (n ↓), scouring (↑)
(vi) Obstructions
(vii) Stage and discharge (n ↑)
(viii) Suspended mats & bed ld

Ex. 4.2 $d_{50} = 1.5\ mm = 0.0015\ m$

$S_0 = 0.1\% = 0.001$

Trapezoidal: $b = 6m;\ s = 2;\ h = 2m$

$$n = \frac{d_{50}^{1/6}}{21.1} = 0.016$$

$$Q = \frac{1}{n} A R^{2/3} S^{1/2};$$

$$= \frac{1}{0.016} \times 20 \times (1.34)^{2/3} \times (0.001)^{1/2}$$

$$\therefore 48.04\ m^3/s$$

$A = bh + sh^2 = 6 \times 2 + 2 \times 2^2 = 20\ m^2$

$P = b + 2h\sqrt{1+s^2} = 6 + 2 \times 2\sqrt{1+4}$

$= 14.94\ m$

$R = A/P = 1.34$

$$C: \frac{C R^{1/2} S^{1/2}}{\frac{1}{n} R^{2/3} S^{1/2}} = 1$$

$$\Rightarrow C = \frac{1}{n} R^{1/6} = 65.62$$

$$\frac{C}{\sqrt{g}} = \sqrt{\frac{8}{f}} \Rightarrow f = 0.018$$

Normal depth — depth of uniform flow, h_n

Section factor — $AR^{2/3}$, $AR^{1/2}$

Conveyance — measure of carrying capacity of a channel

$Q_n = K_n S_n^{1/2}$; $K_n = \frac{1}{n} A_n R_n^{2/3}$ [for manning]

$Q_n = K_n S_n^{1/2}$; $K_n = C A_n R_n^{1/2}$ [for chezy]

Hydraulic exponent for uniform flow

$$K^2 = C_2 h^N$$

$$\Rightarrow 2 \ln K = \ln C + N \ln h$$

$$\Rightarrow \frac{d}{dh} \ln K = \frac{1}{2} N \frac{1}{h} = \frac{N}{2h}$$

Again, $K = \frac{1}{n} A R^{2/3} = \frac{1}{n} A \left(\frac{A}{P}\right)^{2/3}$

$$= \frac{1}{n} A^{5/3} \frac{1}{P^{2/3}}$$

$$\Rightarrow \ln K = \frac{5}{3} \ln A - \frac{2}{3} \ln P - \ln(n)$$

$$\Rightarrow \frac{d}{dh} \ln K = \frac{5}{3} \frac{1}{A} \frac{dA}{dh} - \frac{2}{3} \frac{1}{P} \frac{dP}{dh}$$

$$\Rightarrow \frac{N}{2h} = \frac{1}{3A} \left[5B - 2R \frac{dP}{dh}\right]$$

$$\Rightarrow N = \frac{2}{3} \frac{h}{A} \left[5B - 2R \frac{dP}{dh}\right] \longrightarrow \text{in Manning's formula}$$

$$N = \frac{h}{A} \left[3B - R \frac{dP}{dh}\right] \longrightarrow \text{in Chezy's formula}$$

Ex. 4.3 Rect; $A = bh$; $B = b$; $P = b + 2h$; $R = A/P$; $\frac{dP}{dh} = 2$

$$N = \frac{2}{3}\frac{h}{A}\left[5B - 2R\frac{dP}{dh}\right]$$

$$= \frac{2}{3}\frac{1}{b}\left[5b - \frac{2bh}{b+2h} \times 2\right]$$

$$= \frac{2}{3}\left[5 - \frac{4h}{b+2h}\right] = \frac{2}{3}\left[5 - \frac{4(h/b)}{1+2(h/b)}\right]$$

(i) wide channel $\rightarrow$ $h/b \approx 0$

$$\therefore N = \frac{2}{3}[5] = 3.33$$

(ii) Narrow channel $\rightarrow$ $h/b \approx \infty$

$$\therefore N = \frac{2}{3}\left[5 - \frac{4 \times 2}{1/(h/b) + 2}\right]$$

$$= \frac{2}{3}\left[5 - \frac{4}{2}\right] = 2$$

Ex. 4.4 Trapezoidal; $b = 6.1\,m$; $S = 2$; $h = 2\,m$

$$N = \frac{2}{3}\frac{h}{A}\left[5B - 2R\frac{dP}{dh}\right]$$

$$= \frac{2}{3}\frac{2}{20.2}\left[5 \times 14.1 - 2 \times 1.34 \times 4.47\right]$$

$$= 3.86$$

$$A = bh + Sh^2$$

$$= 20.2\,m$$

$$B = b + 2Sh = 14.1\,m$$

$$P = b + 2h\sqrt{1+S^2} = 15.04\,m$$

$$R = A/P = 1.34$$

$$\frac{dP}{dh} = 2\sqrt{1+2^2} = 4.47$$

Computation of Normal depth

Analytical method :

Ex. 4.5 wide; $S_0 = 0.0025$; $q = 3\ m^2/s$;

(i) $n = 0.020$

$$Q = \frac{1}{n} A_n R_n^{2/3} \sqrt{S_0}$$

$$\Rightarrow q = \frac{1}{n} \frac{A_n}{b} (h_n)^{2/3} \sqrt{S_0}$$

$$\Rightarrow 3 = \frac{1}{0.020} \times h_n \times (h_n)^{2/3} \sqrt{0.0025}$$

$$\Rightarrow h_n = 1.12\ m$$

$$v_n = \frac{q}{h_n} = 2.68\ m/s$$

$$R = \frac{A}{B} = \frac{bh}{b+2h} \approx \frac{bh}{b} \approx h$$

(ii) $C = 45\sqrt{m}/s$

$$q = C \frac{A_n}{b} R^{1/2} S^{1/2}$$

$$\Rightarrow 3 = 45 \times h_n \times \sqrt{h_n \times 0.0025}$$

$$\Rightarrow h_n = 1.21\ m$$

$$v = \frac{q}{h_n} = 2.48\ m/s$$

Ex. 4.6 Triangular; $S = 2$; $S_0 = 0.0016$; $n = 0.015$; $Q = 10\ m^3/s$

$$Q = \frac{1}{n} A R^{2/3} \sqrt{S_0}$$

$$\Rightarrow 10 = \frac{1}{0.015} \times 2h_n^2 \times (0.447 h_n)^{2/3} \times \sqrt{0.0016}$$

$$\Rightarrow h_n = 1.55\ m$$

$$A_n = S h_n^2 = 2h_n^2$$

$$P_n = 2h_n \sqrt{1+S^2} = 2h_n \sqrt{1+2^2} = 4.47 h_n$$

$$R_n = A_n / P_n = \frac{2h_n^2}{4.47 h_n} = 0.447 h_n$$

Trial & error method

Ex. 4.7 Rect. $b = 6.0\ m$; $n = 0.025$; $S_0 = 0.0025$; $Q = 20\ m^3/s$

$$A_n R_n^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.025 \times 20}{\sqrt{0.0025}} = 10$$

h	$A = bh$	$P = b + 2h$	$A_n R_n^{2/3}$
1	6	8	4.95
2	12	10	13.55
1.8	10.8	9.6	11.68
1.7	10.2	9.4	10.77
1.6			9.88
1.62	9.72	9.24	10.05

$\therefore h_n = 1.62$

$$V_n = \frac{Q}{A_n} = 2.06$$

Ex. 4.8 Trapezoidal; $b = 6m$; $S = 2$; $n = 0.025$; $S_0 = 0.001$

$Q = 14\ m^3/s$

$$AR^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.025 \times 14}{\sqrt{0.001}} = 11.07$$

h	$A = bh + sh^2$	$P = b + 2h\sqrt{1+s^2}$	$AR^{2/3}$
1	8	10.47	6.69
2	20	14.94	24.29
1.3	11.18	11.81	10.77
1.35	11.75	12.04	11.55
1.32	11.40	11.9	11.08

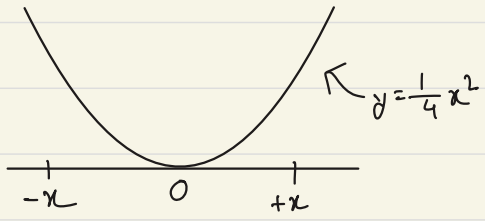
$h_n = 1.32\ m$

$V_n = Q/A_n = 14/11.4 = 1.23\ m^3/s$

Ex. 4.9 Parabolic; $Q = 20\ m^3/s$; $n = 0.025$; $S_0 = 0.0025$; $y^2 = 4z$

$\Rightarrow z = \frac{1}{4}y^2$

$$AR^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.025 \times 20}{\sqrt{0.0025}} = 10$$



$h = 1/4\, x^2 \Rightarrow x = \sqrt{4h}$

$B = 2x\,|\sqrt{4h}|$

$A = Bh - \int_{-\sqrt{4h}}^{+\sqrt{4h}} 0.25x^2\, dx$

$P = \int_{-\sqrt{4h}}^{+\sqrt{4h}} \sqrt{1 + \left(\frac{d}{dx} 0.25x^2\right)^2}\, dx$

$= \int_{-\sqrt{4h}}^{+\sqrt{4h}} \sqrt{1 + (0.5x)^2}\, dx$

h	A	R	$AR^{2/3}$
1	2.67	0581	1.86
3	13.86	1.45	17.74
2	7.54	1.05	7.79
2.3	9.3	1.17	10.36

Ex. 4.9 Re Parabolic; $Q = 20\ m^3/s$; $n = 0.025$; $S_0 = 0.0025$; $y^2 = 4z$

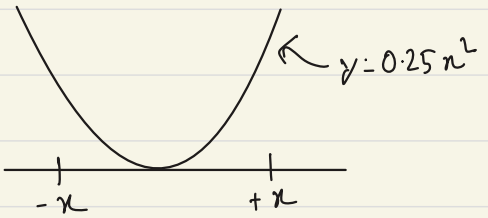
$$A_n R_n^{4/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.025 \times 20}{\sqrt{0.0025}} = 10$$

$\Rightarrow z = 0.25 y^2$

$\rightarrow y = 0.25 x^2$

$h = 0.25 x^2 \Rightarrow x = \sqrt{4h}$

$B = 2\sqrt{4h}$



$$A = Bh - \int_{-\sqrt{4h}}^{\sqrt{4h}} 0.25 x^2 \, dx$$

$$P = \int_{-\sqrt{4h}}^{+\sqrt{4h}} \sqrt{1 + \left\{\frac{d}{dx}(0.25x^2)\right\}^2} \, dx$$

$$= \int_{-\sqrt{4h}}^{+\sqrt{4h}} \sqrt{1 + (0.5x)^2} \, dx = \int_{-\sqrt{4h}}^{+\sqrt{4h}} \sqrt{1 + 0.25x^2} \, dx$$

$R = A/P$

h	A	$R = \frac{A}{P}$	$AR^{2/3}$
1.	2.67	0.581	1.86

- . -

4.10 Circular; $d_0 = 2m$; $S_0 = 0.001$; $Q = 4\ m^3/s$; $n = 0.013$

$$A_n R_n^{2/3} = \frac{nQ}{\sqrt{S_0}} = 1.64$$

w	A	P	R	$AR^{2/3}$
1	0.079	1	0.079	0.015
2				0.229
4	2.38	4	0.595	1.68

$$h_n = d_0 \left(1 - \cos\frac{w}{2}\right)/2 = \cdots$$

Ex. 4.11 Bisection method

Trapezoidal; $b = 6\,m$; $s = 2$; $n = 0.025$; $S_0 = 0.001$; $Q = 14\ m^3/s$

$$f(h) = AR^{2/3} - A_n R_n^{2/3} = AR^{2/3} - \frac{nQ}{\sqrt{S}} = 0$$

$$\Rightarrow f(h) = A\left(\frac{A}{P}\right)^{2/3} - \frac{0.025 \times 14}{\sqrt{0.001}}$$

$$\Rightarrow f(h) = \frac{A^{5/3}}{P^{2/3}} - 11.07 = 0$$

$$A = bh + sh^2 = 6h + 2h^2$$

$$P = b + 2h\sqrt{1+s^2} = 6 + 2h\sqrt{5}$$

h_l	h_h	h	$f(h)$
0	10	5	148.64
0	5	2.5	26.56
0	2.5	1.25	−1.04
1.25	2.5	1.875	10.38
1.25	1.875	1.563	4.10
1.25	1.563	1.407	1.396
1.25	1.407	1.329	0.146
1.25	1.329	1.290	−0.453
1.290	1.329	1.310	−0.148
1.310	1.329	1.32 ✓	0.006

Ex. 4.12 Trapezoidal; $b = 6m$; $S = 2$; $n = 0.025$; $S_0 = 0.001$; $Q = 14\ m^3/s$

$$f(h) = AR^{2/3} - \frac{nQ}{\sqrt{S_0}} = 0$$

$$\Rightarrow A^{5/3} - \frac{nQ}{\sqrt{S_0}} P^{2/3} = 0$$

$$f'(h) = \frac{5}{3} A^{2/3} \frac{dA}{dh} - \frac{nQ}{\sqrt{S_0}} \frac{2}{3} P^{-1/3} \frac{dP}{dh} = 0$$

$$= \frac{5}{3} A^{2/3} B - \frac{2nQ}{3\sqrt{S_0}} P^{-1/3} \frac{dP}{dh} = 0$$

Here, $\frac{nQ}{\sqrt{S_0}} = 11.068$; $\frac{2nQ}{3\sqrt{S_0}} = 7.379$

$$A = bh + sh^2 = 6h + 2h^2$$

$$P = b + 2h\sqrt{1+s^2} = 6 + 2h\sqrt{5}$$

$$\frac{dP}{dh} = 2\sqrt{5}$$

$$B = b + 2sh = 6 + 4h$$

Trial:

h	$f(h)$	$f'(h)$	$h' = h - \frac{f(h)}{f'(h)}$
1	−20.08	51.58	1.407
1.407	7.491	89.533	1.323
1.323	0.327	81.096	1.319
1.319	0.00354	80.703	1.319

Restrict depth of circular section to $0.80d$:

For sections having gradually closing top, the section factor at first increases then decreases w/ depth. As a result it is possible to have to alt normal depth for the same discharge and small disturbance may cause the water to seek alt depth.

Circular sec falls in this category, and this is noticeable at depth $> 0.82d_0$. Thus, restrict to $\leq 0.80d_0$

Normal slope: Longitudinal slope required to maintain uniform flow

Critical slope: Longitudinal slope required to maintain both uniform and critical flow

Ex. 4.13 Rect.; $b = 6m$; $\alpha = 1.12$; $n = 0.020$

(i) $h_n = 1m$, $Q = 11\ m^3/s$

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$\Rightarrow 11 = \frac{1}{0.02} \times 6 \times 0.75^{2/3} \sqrt{S}$$

$$\Rightarrow S = 0.002$$

$A = bh = 6 \times 1 = 6\ m^2$

$P = b + 2h = 8\ m$

$R = A/P = 6/8 = 0.75\ m$

(ii) $Q = 11\ m^3/s$

$$h_c = \sqrt[3]{\frac{\alpha Q^2}{g b^2}} = \sqrt[3]{\frac{1.12 \times 11^2}{9.81 \times 6^2}} = 0.73\ m$$

$$Q = \frac{1}{n} A R^{2/3} \sqrt{S}$$

$$\Rightarrow 11 = \frac{1}{0.02} \times 4.2 \times (0.587)^{2/3} \sqrt{S_c}$$

$$\Rightarrow S_c = 0.0056$$

$A = bh = 6 \times 0.73 = 4.38\ m^2$

$P = b + 2h = 7.46\ m$

$R = 0.587\ m$

(iii) $h_c = h_n = 1 = \sqrt[3]{\frac{\alpha Q^2}{g B^2}}$

$$\Rightarrow 1 = \sqrt[3]{\frac{1.12 \times Q^2}{9.81 \times 6^2}} = 17.76\ m^3/s$$

$$Q = \frac{1}{n} A_c R_c^{2/3} \sqrt{S_c}$$

$$\Rightarrow 17.76 = \frac{1}{0.02} \times 6 \times (0.75)^{2/3} \sqrt{S_c}$$

$$\Rightarrow S_c = 0.0051$$

$A_c = 6 \times 1 = 6\ m^2$

$P = b + 2h = 8\ m$

$R = 6/8 = 0.75\ m$

Channel section with composite roughness

$$n = \left(\frac{\sum P_i n_i^{3/2}}{P} \right)^{2/3}$$

$$= \left(\frac{P_1 n_1^{3/2} + P_2 n_2^{3/2}}{P} \right)^{2/3}$$

Ex. 4.14 Flume; side: $n = 0.010$, $P = 0.4\ m$

Bottom: $n = 0.015$, $P = b = 1\ m$

$b = 1\ m$

$S = 0.001$

$h_n = 0.4\ m$

$$n = \left(\frac{\sum P_i n_i^{3/2}}{P} \right)^{2/3}$$

$$= \left(\frac{2 \times 0.4 \times (0.010)^{3/2} + 1 \times (0.015)^{3/2}}{1 + 2 \times 0.4} \right)^{2/3}$$

$$= 0.0129$$

$$Q = \frac{1}{n} A R^{2/3} \sqrt{S}$$

$$= \frac{1}{0.0126} \times 0.4 \times (0.222)^{2/3} \sqrt{0.001}$$

$$= 0.368\ m^3/s$$

$A = bh = 1 \times 0.4 = 0.4\ m^2$

$P = b + 2h = 1.8\ m$

$R = A/P = \frac{0.4}{1.8} = 0.222\ m$

Compound cross-section

$$Q_i = \frac{1}{n_i} A_i R_i^{2/3} \sqrt{S_i}$$

$$Q = \Sigma Q_i$$

$$Q = \frac{1}{n} A R^{2/3} \sqrt{S_0} \Rightarrow n = \frac{A R^{2/3} \sqrt{S_0}}{Q}$$

$$\text{or, } n = \frac{A R^{2/3}}{\Sigma K_i} \; ; \quad K_i = \frac{1}{n_i} A_i R_i^{2/3}$$

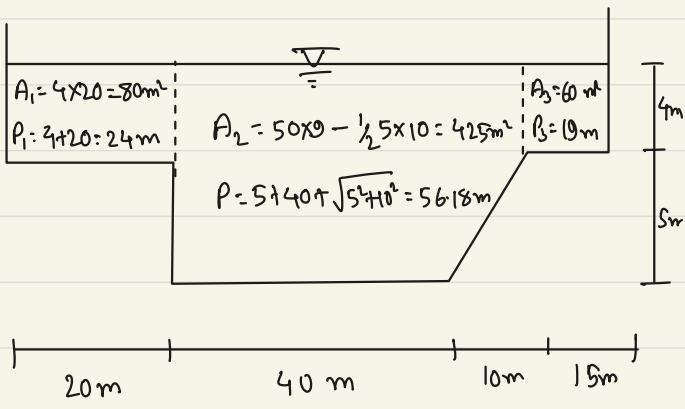
$$\alpha = \frac{\Sigma \alpha_i \, K_i^3 / A_i^2}{K^3 / A^2}$$

$$\beta = \frac{\Sigma \beta_i \, K_i^2 / A_i}{K^2 / A}$$

Ex. 4.5 $n = 0.025, 0.035$

$S_0 = 0.0002$; $\alpha = 1.12$

$\beta = 1.04$



$$Q_1 = \frac{1}{n_1} A_1 R_1^{2/3} \sqrt{S_0}$$

$$= \frac{1}{0.035} \times 80 \times \left(\frac{80}{24}\right)^{2/3} \sqrt{0.0001}$$

$$= 51 \; m^3/s$$

$Q_2 = 655.12 \; m^3/s$

$Q_3 = 36.90 \; m^3/s$

$\therefore Q = \Sigma Q_i = 743 \; m^3/s$

$\therefore U = \frac{Q}{\Sigma A_i} = \frac{743}{80 + 425 + 60} = 1.315 \; m/s$

$K_1 = \frac{1}{n_1} A_1 R_1^{2/3} = 5100$

$K_2 = 65512$

$K_3 = 3690$

$P = \Sigma P_i = 99.18$

$A = \Sigma A_i = 565$

$\therefore n = \frac{A R^{2/3}}{\Sigma K_i} = 74302$

$= 0.0243$

$$\alpha = \frac{\sum \alpha_i \, K_i^3/A_i^2}{K^3/A^2} = \frac{1.12\left[\frac{5100^3}{80^2} + \frac{65512^3}{425^2} + \frac{3690^3}{60^2}\right]}{\frac{74302^3}{565^2}}$$

$$= 1.39$$

$$\beta = \frac{\sum \beta_i \frac{K_i^2}{A_i}}{K^2/A} = 1.13$$

Alt: $n = 0.025,\ 0.035$

$S_0 = 0.0001$, $\alpha = 1.12$; $\beta = 1.04$

Section	A	P	$K = \frac{1}{n} A\left(\frac{A}{P}\right)^{2/3}$	$Q = K\sqrt{S_0}$
Left	80	24	5100	51.00
Middle	425	56.18	65512	655.12
Right	60	19	3690	36.90
	565	99.18	74302	743.02

— — —

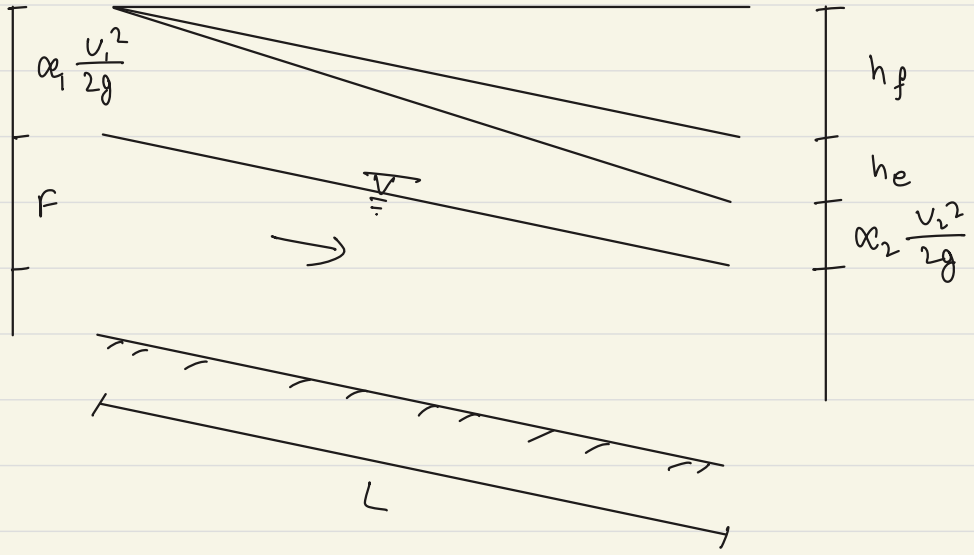
Flood discharge by slope-area method

Applicable when:

(i) Changes in stage & discharge are sufficiently gradual
(ii) Sufficient data not available to justify other methods

selection of suitable reach:

(i) Good-quality high water marks available
(ii) Straight & uniform reach. Prefer contracting over expanding reach. Change in conveyance $\leq 30\%$.
(iii) Length $\geq$ 75 time avg depth of flow



$$F + \alpha_1 \frac{v_1^2}{2g} = \alpha_2 \frac{v_2^2}{2g} + h_e + h_f \rightarrow h_f = \text{- - -}$$

$$h_e = k \left[\alpha_1 \frac{v_1^2}{2g} - \alpha_2 \frac{v_2^2}{2g} \right];$$

$k \rightarrow 0 - 0.1$ gradually contracting
$\rightarrow 0 - 0.2$ expanding
$\rightarrow 0.5$ abrupt

Ex. 4.16 $L = 850\,m$, $F = 0.76\,m$

$A_1 = 10350\ m^2$, $P_1 = 2035\,m$, $n_1 = 0.03$, $\alpha_1 = 1.15$

$A_2 = 9275\ m^2$, $P_2 = 1965\,m$, $n_2 = 003$, $\alpha_2 = 1.18$

$$K_1 = \frac{1}{n_1} A_1 R_1^{2/3} = \frac{1}{0.03} \times 10350 \times \left(\frac{10350}{2035}\right)^{2/3} = 1020320$$

$$K_2 = \frac{1}{n_2} A_2 R_2^{2/3} = \frac{1}{0.03} \times 9275 \times \left(\frac{9275}{1965}\right)^{2/3} = 869949$$

$$\therefore K = \sqrt{K_1 K_2} = 942139$$

$$F + \alpha_1 \frac{v_1^2}{2g} = \alpha_2 \frac{v_2^2}{2g} + \cancel{he}^{0} + h_f \Rightarrow h_f = F + \alpha_1 \frac{v_1^2}{2g} - \alpha_2 \frac{v_1^2}{2g}$$

$$= F + \alpha_1 \frac{Q^2}{2gA_1^2} - \alpha_2 \frac{Q^2}{2gA_2^2}$$

$$\text{Init } S_f = \frac{F}{L} = \frac{0.76}{850} = 0.000894$$

Trial #	Q	h_f	$S_f = \frac{h_f}{L}$	$Q' = K\sqrt{S_f}$
0	—	—	0.00089	28170
1	28170	0.639	0.00075	25840
2	25840	0.659	0.00077	26224
3	26224	0.656	0.00077	26163
4	26163	0.656	0.00077	26173
5	26173	0.656	0.00077	26171
6	26171	0.656	0.00077	26172
7	26172	0.656	0.00077	26171

$$\therefore Q = 26171\ m^3/s$$

Def $K = \sqrt{K_1 K_2}$ → formulate h_f → find init $h_f = \frac{F}{L}$

Trial $Q \rightarrow h_f \rightarrow S_f = h_f/L \rightarrow Q = K\sqrt{S_f}$