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Open Channel Flow Solutions: Problem Set from Chapter 1.6

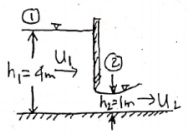
Course: Water Resource Engineering II (CE373) 4 documents
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Open Channel Flow

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① Solutions to Problems of Chap. 1

1.6 Rectangle $b = 6\,m$
$h_1 = 4m$, $h_2 = 1\,m$,
$U_1 = 2m$, $U_2 = 8\,m/s$



Upstream

$A_1 = bh_1 = 24\,m^2$, $P_1 = b + 2h = 14m$, $R_1 = A/P_1 = 1.714m$

$Re_1 = \frac{U_1 R_1}{\nu} = \frac{2 \times 1.714}{10^{-6}} = 3.43 \times 10^6 > 12,500$

$Fr_1 = \frac{U_1}{\sqrt{gD_1}} = \frac{U_1}{\sqrt{gh_1}} = \frac{2}{\sqrt{9.81 \times 4}} = 0.32 < 1$

Hence, the flow upstream of the sluice gate is subcritical turbulent.

Downstream (at the vena contracta)

$A_2 = bh_2 = 6m^2$, $P_2 = b + 2h_2 = 8m$, $R_2 = \frac{A_2}{P_2} = 0.75m$

$Re_2 = \frac{U_2 R_2}{\nu} = \frac{8 \times 0.75}{10^{-6}} = 6 \times 10^6 > 12,500$

$Fr_2 = \frac{U_2}{\sqrt{gD_2}} = \frac{U_2}{\sqrt{gh_2}} = \frac{8}{\sqrt{9.81 \times 1}} = 2.55 > 1$

Hence, the flow downstream of the sluice gate is supercritical turbulent.

(b) $A = 33,500\,m^2$, $Q = 56,200\,m^3$, $B = 3820\,m$
The river is wide.

$U = Q/A = 56200/33500 = 1.68\,m/s$

$h = R = D = A/B = 33500/3820 = 8.77\,m$

$Re = \frac{UR}{\nu} = \frac{Uh}{\nu} = \frac{1.68 \times 8.77}{10^{-6}} = 14.73 \times 10^6 > 12,500$

$Fr = \frac{U}{\sqrt{gD}} = \frac{U}{\sqrt{gh}} = \frac{1.68}{\sqrt{9.81 \times 8.77}} = 0.18 < 1$

Hence the [illegible]

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② 1.7 $h = 1\,m$ $U = 3\,m/s$

i) Wide

$A = h = 1\,m^2$ $R = D = h = 1m$

$q = Uh = 3 \times 1 = 3\,m^3/s/m$ or m^2/s

$Re = \frac{UR}{\nu} = \frac{Uh}{\nu} = \frac{3 \times 1}{10^{-6}} = 3 \times 10^6 > 12,500$

$Fr = \frac{U}{\sqrt{gD}} = \frac{U}{\sqrt{gh}} = \frac{3}{\sqrt{9.81 \times 1}} = 0.96 < 1$

Hence, the flow is subcritical turbulent.

$c = \sqrt{gD} = \sqrt{gh} = \sqrt{9.81 \times 1} = 3.13\,m/s$

∴ Speed of wave front upstream $= c - U$
$= 3.13 - 3 = 0.13\,m/s$

Speed of wave front downstream $= c + U$
$= 3.13 + 3 = 6.13\,m/s$

ii) Rectangle $b = 6m$

$A = bh = 6 \times 1 = 6\,m^2$ $P = b + 2h = 6 + 2 \times 1 = 8$

$R = A/P = 6/8 = 0.75\,m$ $D = A/B = h = 1\,m$

$Q = AU = 6 \times 3 = 18\ m^3/s$

$Re = \frac{UR}{\nu} = \frac{3 \times 0.75}{10^{-6}} = 2.25 \times 10^6 > 12{,}500$

$Fr = \frac{U}{\sqrt{gD}} = \frac{U}{\sqrt{gh}} = \frac{3}{\sqrt{9.81 \times 1}} = 0.96 < 1$

Hence, the flow is subcritical turbulent.

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$c = \sqrt{gD} = \sqrt{gh} = \sqrt{9.81 \times 1} = 3.13\ m/s$

∴ Speed of wave front upstream $= c - U$
$= 3.13 - 3 = 0.13\ m/s$

Speed of wave front downstream $= c + U$
$= 3.13 + 3 = 6.13\ m/s$

iii) Trapezoid $b = 6m$ $s = 2$

$A = (b + sh)h = (6 + 2 \times 1) \times 1 = 8\ m^2$

$P = b + 2h\sqrt{1 + s^2} = 6 + 2 \times 1 \times \sqrt{1 + 2^2} = 10.47\ m$

$B = b + 2sh = 6 + 2 \times 2 \times 1 = 10\ m$

$R = A/P = 8/10.47 = 0.76\ m$

$D = A/B = 8/10 = 0.8\ m$

$Q = AU = 8 \times 3 = 24\ m^3/s$

$Re = \frac{UR}{\nu} = \frac{3 \times 0.76}{10^{-6}} = 2.28 \times 10^6 > 12{,}500$

$Fr = \frac{U}{\sqrt{gD}} = \frac{3}{\sqrt{9.81 \times 0.8}} = 1.07 > 1$

Hence, the flow is supercritical turbulent.

$c = \sqrt{gD} = \sqrt{9.81 \times 0.8} = 2.80\ m/s < U$

Hence, both the wave fronts move downstream

Speed of one wave front $= U - c = 3 - 2.80$
$= 0.20\ m/s$

Speed of another wave front $= U + c$
$= 3 + 2.80 = 5.80\ m/s$

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iv) Triangle $s = 2$

$A = sh^2 = 2 \times 1^2 = 2\ m^2$ $P = 2h\sqrt{1 + s^2} = 2 \times 1 \times \sqrt{1 + 2^2}$
$= 4.47\ m$

$B = 2sh = 2 \times 2 \times 1 = 4m$

$R = A/P = 2/4.47 = 0.45m$ $D = A/B = 2/4 = 0.50$

$Q = AU = 2 \times 3 = 6\ m^3/s$

$Re = \frac{UR}{\nu} = \frac{3 \times 0.45}{10^{-6}} = 1.35 \times 10^6 > 12{,}500$

$Fr = \frac{U}{\sqrt{gD}} = \frac{3}{\sqrt{9.81 \times 0.50}} = 1.35 > 1$

Hence, the flow is supercritical turbulent

$c = \sqrt{gD} = \sqrt{9.81 \times 0.50} = 2.21\ m/s < U$

Hence, both the wave fronts move downstream

Speed of one wave front $= U - c = 3 - 2.21 = 0.7$

Speed of another wave front = $U+c = 3+2.21$
$= 5.21$ m/s

vi) Parabola $B = 4$ m, $h = 1$ m, $x = \frac{4h}{B} = \frac{4\times1}{4} = 1$

$A = \frac{2}{3}Bh = \frac{2}{3}\times4\times1 = 2.667$ m^2

$P = \frac{B}{2}\left[\sqrt{1+x^2} + \frac{1}{x}\ln\left(x+\sqrt{1+x^2}\right)\right]$

$= \frac{4}{2}\left[\sqrt{1+1^2} + 1\times\ln\left(1+\sqrt{1+1^2}\right)\right] = 4.591$ m

$R = A/P = 2.667/4.591 = 0.58$ m

$D = A/B = 2.667/4 = 0.667$ m

$Q = AU = 2.667\times3 = 8$ m^3/s

$Re = \frac{UR}{\nu} = \frac{3\times0.58}{10^{-6}} = 1.74\times10^6 > 12,500$

$Fr = \frac{U}{\sqrt{gD}} = \frac{3}{\sqrt{9.81\times0.667}} = 1.17 > 1$

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Hence, the flow is supercritical turbulent

$c = \sqrt{gD} = \sqrt{9.81\times0.667} = 2.56$ m/s $< U$

Hence, both the wave fronts move downstream

Speed of one wave front = $U-c = 3-2.56 = 0.44$

Speed of another wave front = $U+c = 3+2.56$
$= 5.56$ m/s

vii) Circle $d_0 = 2.5$ m

$\omega = 2\cos^{-1}\left(1-\frac{2h}{d_0}\right) = 2\cos^{-1}\left(1-\frac{2\times1}{2.5}\right) = 2.739$ rad

$A = \frac{1}{8}(\omega-\sin\omega)d_0^2 = \frac{1}{8}(2.739-\sin2.739)2.5^2 = 1.833$ m^2

$P = \frac{1}{2}\omega d_0 = \frac{1}{2}\times2.739\times2.5 = 3.424$ m

$B = \left(\sin\frac{\omega}{2}\right)d_0 = \left(\sin\frac{2.739}{2}\right)\times2.5 = 2.449$ m

$R = A/P = 1.833/3.424 = 0.535$ m

$D = A/B = 1.833/2.449 = 0.748$ m

$Q = AU = 1.833\times3 = 5.50$ m^3/s

$Re = \frac{UR}{\nu} = \frac{3\times0.535}{10^{-6}} = 1.61\times10^6 > 12,500$

$Fr = \frac{U}{\sqrt{gD}} = \frac{3}{\sqrt{9.81\times0.748}} = 1.11 > 1$

Hence, the flow is supercritical turbulent.

$c = \sqrt{gD} = \sqrt{9.81\times0.748} = 2.71$ m/s $< U$

Hence, both the wave fronts move downstream.

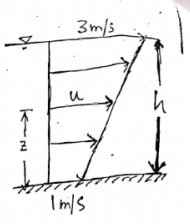
Speed of one wave front = $U-c = 3-2.71 = 0.29$ m/s

Speed of another wave front = $U+c = 3+2.71 = 5.71$ m/s

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1.8(a) $u = 1+2z/h$, $h = 5$m

The velocity varies linearly from 1m/s at the channel bottom ($z=0$) to 3m/s at the water surface ($z=h$).



$\therefore \bar{u} = \frac{1+3}{2} = 2$ m/s

(i) $q = \bar{u}h = 2\times5 = 10$ m^2/s

(ii) $Re = \frac{\bar{u}R}{\nu} = \frac{\bar{u}h}{\nu} = \frac{2\times5}{10^{-6}} = 10\times10^6 > 12,500$

$$Fr = \frac{\bar{u}}{\sqrt{gD}} = \frac{\bar{u}}{\sqrt{gh}} = \frac{2}{\sqrt{9.81\times 5}} = 0.29 < 1$$

Hence the flow is subcritical turbulent.

(iii) $$\alpha = \frac{1}{\bar{u}^3 h}\int_0^h u^3 dz = \frac{1}{\bar{u}^3 h}\int_0^h \left[1+2z/h\right]^3 dz$$

$$= \frac{1}{\bar{u}^3 h}\int_0^h \left[1 + 3.1.\frac{2z}{h} + 3.1.\frac{4z^2}{h^2} + \frac{8z^3}{h^3}\right] dz$$

$$= \frac{1}{\bar{u}^3 h}\int_0^h \left[1 + \frac{6z}{h} + \frac{12z^2}{h^2} + \frac{8z^3}{h^3}\right] dz$$

$$= \frac{1}{\bar{u}^3 h}\left[z + \frac{6z^2}{2h} + \frac{12z^3}{3h^2} + \frac{8z^3}{4h^3}\right]_0^h$$

$$= \frac{1}{\bar{u}^3 h}\left(h + 3h + 4h + 2h\right)$$

$$= \frac{10h}{\bar{u}^3 h} = \frac{10}{\bar{u}^3} = \frac{10}{2^3} = \frac{10}{8} = 1.250$$

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1.8(a) (iii) Contd.

$$\beta = \frac{1}{\bar{u}^2 h}\int_0^h u^2 dz = \frac{1}{\bar{u}^2 h}\int_0^h \left[1+2\frac{z}{h}\right]^2 dz$$

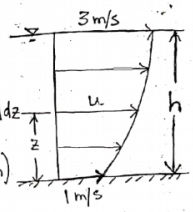
$$= \frac{1}{\bar{u}^2 h}\int_0^h \left[1 + 2.1.\frac{2z}{h} + \frac{4z^2}{h^2}\right] dz = \frac{1}{\bar{u}^2 h}\int_0^h \left[1 + \frac{4z}{h} + \frac{4z^2}{h^2}\right] dz$$

$$= \frac{1}{\bar{u}^2 h}\left(z + \frac{4z^2}{2h} + \frac{4z^3}{3h^2}\right)_0^h = \frac{1}{\bar{u}^2 h}\left(h + 2h + \frac{4}{3}h\right)$$

$$= \frac{1}{\bar{u}^2 h} \times \frac{13h}{3} = \frac{13}{3\bar{u}^2} = \frac{13}{3\times 2^2} = \frac{13}{12} = 1.083$$

$$\therefore \frac{\alpha - 1}{\beta - 1} = \frac{1.250 - 1}{1.083 - 1} = \frac{0.250}{0.083} = 3$$

1.8(b) $u = 1 + 2(z/h)^{1/2}$, $h = 5$m



$$\bar{u} = \frac{1}{h}\int_0^h u\, dz = \frac{1}{h}\int_0^h \left[1 + 2(z/h)^{1/2}\right] dz$$

$$= \frac{1}{h}\left[z + \frac{2z^{3/2}}{(3/2)h^{1/2}}\right]_0^h = \frac{1}{h}\left(h + \frac{4}{3}h\right)$$

$$= 1 + \frac{4}{3} = \frac{7}{3} = 2.33 \text{ m/s}$$

(i) $$q = \bar{u}h = 2.33 \times 5 = 11.67 \text{ m}^2/\text{s}$$

(ii) $$Re = \frac{\bar{u}R}{\nu} = \frac{\bar{u}h}{\nu} = \frac{2.33\times 5}{10^{-6}} = 11.67\times 10^6 > 12{,}500$$

$$Fr = \frac{\bar{u}}{\sqrt{gD}} = \frac{\bar{u}}{\sqrt{gh}} = \frac{2.33}{\sqrt{9.81\times 5}} = 0.33 < 1$$

Hence, the flow is subcritical turbulent.

(iii) $$\alpha = \frac{1}{\bar{u}^3 h}\int_0^h u^3 dz = \frac{1}{\bar{u}^3 h}\int_0^h \left[1 + 2(z/h)^{1/2}\right]^3 dz$$

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1.8(b)(iii) Contd.

$$\alpha = \frac{1}{\bar{u}^3 h}\int_0^h \left[1 + \frac{6z^{1/2}}{h^{1/2}} + \frac{12z}{h} + \frac{8z^{3/2}}{h^{3/2}}\right] dz$$

$$= \frac{1}{\bar{u}^3 h}\left[z + \frac{6z^{3/2}}{(3/2)h^{1/2}} + \frac{12z^2}{2h} + \frac{8z^{5/2}}{(5/2)h^{3/2}}\right]_0^h$$

$$= \frac{1}{\bar{u}^3 h}\left[h + 4h + 6h + \frac{16}{5}h\right] = \frac{1}{\bar{u}^3}\left[1 + 4 + 6 + \frac{16}{5}\right]$$

$= \frac{71}{5U^3} = \frac{71\times3\times3\times3}{5\times7\times7\times7} = 1.118$

$\beta = \frac{1}{U^2h}\int_0^h u^2\,dz = \frac{1}{U^2h}\int_0^h \left[1+2(z/h)^{1/2}\right]^2 dz$

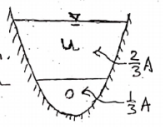
$= \frac{1}{U^2h}\int_0^h \left[1+\frac{4z^{1/2}}{h^{1/2}}+\frac{4z}{h}\right]dz$

$= \frac{1}{U^2h}\left[z+\frac{4z^{3/2}}{(3/2)h^{1/2}}+\frac{4z^2}{2h}\right]_0^h = \frac{1}{U^2h}\left[h+\frac{8}{3}h+2h\right]$

$= \frac{1}{U^2h}\times\frac{17h}{3} = \frac{17}{3U^2} = \frac{17\times3\times3}{3\times7\times7} = 2.89 = 1.041$

$\therefore \frac{\alpha-1}{\beta-1} = \frac{1.118-1}{1.041-1} = \frac{0.118}{0.041} = 2.89$

1.9 Let u be the uniform velocity over the two-thirds of the cross-section, and U be the mean velocity over the entire cross-section. Then,



$U = \frac{0\times\frac{1}{3}A + u\times\frac{2}{3}A}{A} = \frac{2}{3}u$

$\alpha = \frac{\Sigma u^3\,dA}{U^3A} = \frac{0^3\times\frac{1}{3}A + u^3\times\frac{2}{3}A}{\left(\frac{2}{3}u\right)^3\times A} = \frac{\frac{2}{3}\times u^3A}{\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times u^3A} = \frac{9}{4}$

$\beta = \frac{\Sigma u^2\,dA}{U^2A} = \frac{0^2\times\frac{1}{3}A + u^2\times\frac{2}{3}A}{\left(\frac{2}{3}u\right)^2A} = \frac{\frac{2}{3}\times u^2A}{\frac{2}{3}\times\frac{2}{3}\times u^2A} = \frac{3}{2} = 1.50$ $= 2.25$

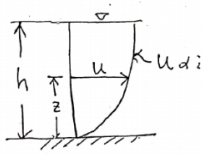
$\therefore \alpha = 2.25$ and $\beta = 1.50$ $\left(\frac{\alpha-1}{\beta-1} = \frac{2.25-1}{1.50-1} = \frac{1.25}{0.5} = 2.5\right)$

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1.10 $u = kz^n$, k = constant

$\bar{U} = \frac{1}{h}\int_0^h u\,dz = \frac{1}{h}\int_0^h kz^n\,dz$

$= \frac{k}{h}\,\frac{z^{n+1}}{n+1}\Big]_0^h = \frac{kh^n}{n+1}$



$\alpha = \frac{1}{U^3h}\int_0^h u^3\,dz = \frac{1}{U^3h}\int_0^h k^3z^{3n}\,dz$

$= \frac{k^3}{U^3h}\,\frac{z^{3n+1}}{3n+1}\Big]_0^h = \frac{k^3}{U^3h}\,\frac{h^{3n+1}}{3n+1}$

$= \frac{(n+1)^3k^3}{k^3h^{3n}h}\cdot\frac{h^{3n+1}}{3n+1} = \frac{(n+1)^3}{3n+1}$

$\beta = \frac{1}{U^2h}\int_0^h u^2\,dz = \frac{1}{U^2h}\int_0^h k^2z^{2n}\,dz$

$= \frac{k^2}{U^2h}\,\frac{z^{2n+1}}{2n+1}\Big]_0^h = \frac{k^2}{U^2h}\,\frac{h^{2n+1}}{2n+1}$

$= \frac{(n+1)^2k^2}{k^2h^{2n}h}\cdot\frac{h^{2n+1}}{2n+1} = \frac{(n+1)^2}{2n+1}$

$\therefore \frac{\alpha-1}{\beta-1} = \frac{\frac{(n+1)^3}{3n+1}-1}{\frac{(n+1)^2}{2n+1}-1} = \frac{\frac{(n+1)^3-(3n+1)}{3n+1}}{\frac{(n+1)^2-(2n+1)}{2n+1}}$

$= \frac{(n^3+3n^2+3n+1-3n-1)(2n+1)}{(n^2+2n+1-2n-1)(3n+1)} = \frac{(2n+1)(n+}{3n+1}$

When $n = \frac{1}{7}$, $\alpha = 1.045$, $\beta = 1.016$ and $\frac{\alpha-1}{\beta-1} = 2.83$

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1.11 Example 1.4 by Simpson's rule

$$A = \frac{d}{3}\left[(O_0+O_n)+2(O_2+O_4+\cdots)+4(O_1+O_3+\cdots)\right]$$

To apply the Simpson's rule, no. of ordinates must be odd and no. of strips must be even.

In this problem, no. of strips, $n = 6$ and no. of ordinates, $n+1 = 7$. Hence, the Simpson's rule can be used directly.

$$q = \frac{1}{3}\left[(0+4.51)+2(3.31+3.95)+4(2.95+3.62+4.12)\right]$$

$$= \frac{1}{3}\times 61.79 = 20.597 \text{ m}^2/\text{s}$$

$$\bar{U} = q/h = 20.597/6 = 3.433 \text{ m/s}$$

$$\alpha = \frac{1}{3.433^3\times 6}\times\frac{1}{3}\left[(0^3+4.51^3)+2(3.31^3+3.95^3)+4(2.95^3+3.62^3+4.12^3)\right]$$

$$= \frac{859.702}{3.433^3\times 6\times 3} = 1.181$$

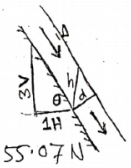
$$\beta = \frac{1}{3.433^2\times 6}\times\frac{1}{3}\left[(0^2+4.51^2)+2(3.31^2+3.95^2)+4(2.95^2+3.62^2+4.12^2)\right]$$

$$= \frac{228.5825}{3.433^2\times 6\times 3} = 1.078$$

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1.12(a) Slope = 1H : 3V, $h = 1$ m

$\tan\theta = 3$, $\theta = 71.56^\circ$, $\cos\theta = 0.3162$



$\therefore p_{bed} = 1000\times 9.81\times 1\times 0.3162 = 980.83 \text{ N/m}^2$

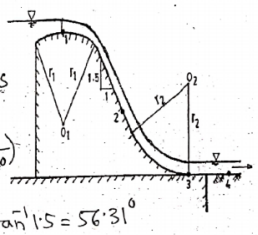
$F = \frac{1}{2}\times 1000\times 9.81\times 1^2\times 0.3162 = 155.07$ N

$O.M. = \frac{1}{6}\times 1000\times 9.81\times 1^3\times 0.3162 = 16.35$ m-N

(b) $r_1 = r_2 = 20$ m, $q = 6.5$ m^2/s

$h_1 = h_2 = h_3 = h_4 = 1$ m

$U = \frac{q}{h} = \frac{6.5}{1} = 6.5$ m/s



(i) $p_1 = \gamma h\left(1-\frac{U^2}{gr}\right) = 1000\times 9.81\times 1\times\left(1-\frac{6.5^2}{9.81\times 20}\right)$

$= 7,697.50 \text{ N/m}^2$

(ii) $\tan\theta = \frac{1.5}{1} = 1.5$ $\therefore \theta = \tan^{-1}1.5 = 56.31^\circ$

$p_2 = \gamma h\cos^2\theta = 9810\times 1\times\cos^2 56.31^\circ = 3,018.46 \text{ N/m}^2$

(iii) $p_3 = \gamma h\left(1+\frac{U^2}{gr}\right) = 9810\times 1\times\left(1+\frac{6.5^2}{9.81\times 20}\right)$

$= 11,922.50 \text{ N/m}^2$

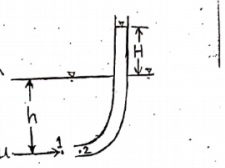
(iv) $p_4 = \gamma h = 9810\times 1 = 9810 \text{ N/m}^2$

(Note that at point 1 the centrifugal force acts against the hydrostatic force. Point 2 lies on the slope of the spillway. At point 3 the centrifugal force adds to the hydrostatic force. Point 4 lies on the horizontal.)

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Chapter 2

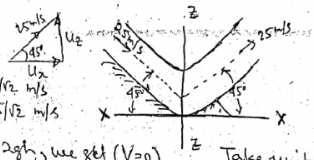
2.4 Applying the energy equation between points 1 and 2 and neglecting the energy loss, we get



$$0 + h + \frac{u^2}{2g} = 0 + (h+H) + 0$$

or, $\frac{u^2}{2g} = H \quad \therefore u = \sqrt{2gH}$ (Proved)

2.5



$u_z = 25 \sin 45° = 25/\sqrt{2}$ m/s

$u_x = 25 \cos 45° = 25/\sqrt{2}$ m/s

From $V^2 = u^2 - 2gh$, we get ($V=0$) Take unit width

$$h = \frac{u_z^2}{2g} = \frac{25^2}{2\times2\times9.81} = 15.93 \text{ m}$$

Force in the z-z direction (i.e. vertical force)

= mass × change in velocity

= ρQ (change in velocity)

= $1000 \times 0.5 \times 1 \times 2.5 \times [i5/\sqrt{2} - (-i5/\sqrt{2})]$

= 44194 N (acting downward) per unit width

[$\rho Q.2u_z = \frac{kgm}{m^3} \times \frac{m^3}{s} \times \frac{m}{s} = kgm\text{-}m/s^2 =$ force]

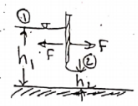
Force in the X-X direction (i.e. horizontal force)

= mass × change in velocity

= $\rho Q(u_x - u_x) = 0$

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2.6 (a) Consider unit width of the channel. The force exerted on the fluid by the gate is F which is equal and opposite to that exerted by the fluid on the gate. Assume that (i) section 1 is sufficiently u/s from the gate, and (ii) section 2 is at the vena contracta, (iii) $\alpha_1 = \alpha_2 = 1$, (iv) $\beta_1 = \beta_2 = 1$, (v) energy losses between the sections is negligible, and (vi) the bottom friction force F_f between the sections is negligible. Then writing the momentum equation between sections 1 and 2



$$\rho q(u_2 - u_1) = F_{p1} - F_{p2} - F$$

or, $\rho q\left(\frac{q}{h_2} - \frac{q}{h_1}\right) = \frac{1}{2}\gamma h_1^2 - \frac{1}{2}\gamma h_2^2 - F$

or, $F = \frac{1}{2}\gamma h_1^2 - \frac{1}{2}\gamma h_2^2 - \rho q^2\left(\frac{1}{h_2} - \frac{1}{h_1}\right)$

$= \frac{1}{2}\gamma h_1^2 - \frac{1}{2}\gamma h_2^2 - \rho q^2\left(\frac{h_1 - h_2}{h_1 h_2}\right)$

$= \frac{1}{2}\gamma\left[(h_1^2 - h_2^2) - \frac{2q^2}{g}\left(\frac{h_1 - h_2}{h_1 h_2}\right)\right]$

$= \frac{1}{2}\gamma\left(\frac{h_1 - h_2}{h_1 h_2}\right)\left[h_1 h_2 (h_1 + h_2) - \frac{2q^2}{g}\right]$ (1)

The energy equation between sections 1 & 2 gives

$h_1 + \frac{u_1^2}{2g} = h_2 + \frac{u_2^2}{2g}$ or $h_1 + \frac{q^2}{2gh_1^2} = h_2 + \frac{q^2}{2gh_2^2}$

or, $\frac{q^2}{2g}\left[\frac{1}{h_2^2} - \frac{1}{h_1^2}\right] = h_1 - h_2$

or, $\frac{q^2}{2g}\left[\frac{h_1 + h_2}{h_1 h_2}\right] = 1$ or $\frac{q^2}{g} = \frac{2h_1 h_2}{h_1 + h_2}$ (2)

Combining Eqs (1) and (2)

$$F = \frac{1}{2}\gamma\left(\frac{h_1-h_2}{h_1h_2}\right)\left[h_1h_2(h_1+h_2) - \frac{4h_1h_2}{h_1+h_2}\right]$$

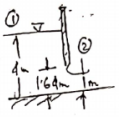
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or, $F = \frac{1}{2}\gamma\left(\frac{h_1-h_2}{h_1h_2}\right)\left[\frac{h_1h_2(h_1+h_2)^2 - 4h_1h_2}{h_1+h_2}\right]$

$= \frac{1}{2}\gamma\frac{(h_1-h_2)^3}{h_1+h_2}$ (3)

The force on the gate is equal and opposite to F.

2.6 (b)(i) $h_1 = 4m$, $h_2 = 1m$, $b = 6m$



Applying the continuity equation between sections 1 and 2

$A_1U_1 = A_2U_2$ or, $bh_1U_1 = bh_2U_2$ or $4U_1 = U_2$ ①

Applying the energy equation between sections ① & ②

$h_1 + \frac{U_1^2}{2g} = h_2 + \frac{U_2^2}{2g}$ or, $4 + \frac{U_1^2}{2g} = 1 + \frac{U_2^2}{2g}$

or, $4 + \frac{U_1^2}{2g} = 1 + \frac{16U_1^2}{2g}$ (using ①)

or, $15\frac{U_1^2}{2g} = 4-1 = 3$ $\therefore U_1 = \sqrt{\frac{2\times9.81\times3}{15}} = 1.98 m/s$

$\therefore Q = A_1U_1 = bh_1U_1 = 6\times4\times1.98 = 47.52 m^3/s$

ii) $Q = 47.52 m^3/s$ $U_1 = 1.98 m/s$ $U_2 = 7.92 m/s$

$\rho Q(U_2-U_1) = \frac{1}{2}\gamma bh_1^2 - \frac{1}{2}\gamma bh_2^2 - F$

or, $F = \frac{1}{2}\gamma bh_1^2 - \frac{1}{2}\gamma bh_2^2 - \rho Q(U_2-U_1)$

$= \frac{1}{2}\times1000\times9.81\times6(4^2-1^2) - 1000\times47.52(7.92-1.98)$

$= 441,450 - 282,383.6 = 159,062.4 N = 159.06 kN$

When the formula is used

$F = \frac{1}{2}\gamma b\frac{(h_1+h_2)^3}{h_1+h_2} = \frac{1}{2}\times1000\times9.81\times6\frac{(4-1)^3}{4+1} = 158922 N$

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2.6 (contd.) Assuming that the pressure distribution is hydrostatic

$h_g = \frac{h_2}{c_e} = \frac{1}{0.61} = 1.64 m$

$F = \frac{1}{2}\gamma b(h_1-h_g)^2$

$= \frac{1}{2}\times1000\times9.81\times6(4-1.64)^2$

$= 163913 N = 163.91 kN$

Also intensity of pressure on the C.G. of the gate $= \gamma\bar{h}$

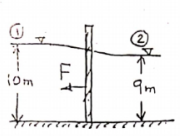
$\therefore F = \gamma\bar{h}A = \rho g\bar{h}A = 1000\times9.81\times(4-1.64)/2 \times(4-1.64)\times6$

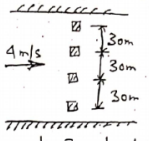
$= 11595.8\times14.16 = 163913.3 N = 163.91 kN$

The force on the gate is equal and opposite to F.

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2.7





$b = 30m, h_1 = 10m, h_2 = 9m, U_1 = 4 m/s$

$q = h_1 U_1 = 10 \times 4 = 40 \ m^2/s$

$U_2 = q/h_2 = 40/9 = 4.44 \ m/s$

Since the piers are placed @ 30m c/c, we consider a width of 30m and apply the steady 1-D momentum eqn. assuming that $\beta_1 = \beta_2 = 1$.

$\rho Q(U_2 - U_1) = F_{p1} - F_{p2} + W\sin\theta^{0} - F_f^{0} - F$

(F is the force on a pier)

$\therefore F = F_{p1} - F_{p2} - \rho Q(U_2 - U_1)$

$= \frac{1}{2}\gamma b(h_1^2 - h_2^2) - \rho b q(U_2 - U_1)$

$= \frac{1}{2} \times 1000 \times 9.81 \times 30(10^2 - 9^2) - 1000 \times 40 \times 30 \times (4.44 - 4)$

$= 1000 \times 30\left(\frac{1}{2} \times 9.81 \times 19 - 40 \times 0.44\right)$

$= 1000 \times 30 \times 75.595 = 2268 \times 10^3 \ N$

$= 2268 \ kN$

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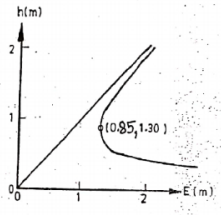
Chapter 3

3.7(a) Rectangular channel, $b = 6m$, $Q = 15 m^3/s$

$E = h + \frac{U^2}{2g} = h + \frac{Q^2}{2gA^2} = h + \frac{Q^2}{2gb^2h^2}$

$= h + \frac{15^2}{2 \times 9.81 \times 6^2 \times h^2} = h + \frac{0.3186}{h^2}$

h	E	h	E
0	∞	1.0	1.32
0.2	8.16	1.2	1.42
0.4	2.39	1.5	1.64
0.6	1.48	2.0	2.08
0.7	1.35	5.0	5.02
0.8	1.30	10.0	10.00
0.9	1.29		



The specific energy curve is plotted as shown.

From the plot, we see that $h_c = 0.85$m and $E_c = 1.30$m

(b) Rectangular channel, $b = 6$m, $E = 3$m

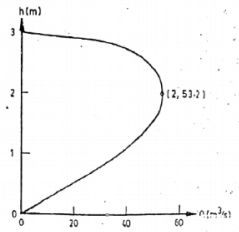
$Q^2 = 2gA^2(E-h)$

$= 2gb^2h^2(E-h)$

$= 2\times 9.81\times 6^2\times h^2(3-h)$

$= 706.32\,h^2(E-h)$

h	Q	h	Q
0	0	1.6	50.31
0.2	8.89	1.8	52.40
0.4	17.19	2.0	53.15
0.6	24.70	2.2	52.29
0.8	31.53	2.4	49.40
			43.70



The discharge-depth curve is plotted as shown. From the

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3.8 Trapezoidal channel, $b = 6$m, $s = 2$, $Q = 20$ m³/s, $h = 1$ m $h/b = 1/6 = 0.167$

$$M = \frac{3[1+2s(h/b)]^2 - 2s(h/b)[1+s(h/b)]}{[1+2s(h/b)][1+s(h/b)]}$$

$$= \frac{3(1+2\times2\times0.167)^2 - 2\times2\times0.167\times(1+2\times0.167)}{(1+2\times2\times0.167)(1+2\times0.167)}$$

$$= \frac{8.333 - 0.891}{2.222} = 3.35$$

Alternative solution

$A = (b+sh)h = (6+2\times1)\times1 = 8$ m²

$B = b+2sh = 6+2\times2\times1 = 10$ m

$D = A/B = 8/10 = 0.8$ m

$dB/dh = 2s = 2\times2 = 4$

$$M = \frac{h}{A}\left(3B - D\frac{dB}{dh}\right) = \frac{1}{8}(3\times10 - 0.8\times4)$$

$= 26.8/8 = 3.35$

3.9 (i) $\alpha\frac{U_c^2}{2g} = \frac{D_c}{2}$ or, $\frac{\alpha Q^2}{2gA_c^2} = \frac{h_c}{2}$ (for a rectangular channel, $D_c = h_c$)

or, $\frac{\alpha Q^2}{2gb^2h_c^2} = \frac{h_c}{2}$ $\therefore h_c = \sqrt[3]{\frac{\alpha Q^2}{gb^2}} = \sqrt[3]{\frac{\alpha q^2}{g}}$

(ii) $\alpha\frac{U_c^2}{2g} = \frac{D_c}{2} = \frac{h_c}{2}$ or, $U_c^2 = \frac{gh_c}{\alpha}$

$\therefore U_c = \sqrt{\frac{gh_c}{\alpha}}$

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3.9 (iii) $E_c = h_c + \frac{U_c^2}{2g} = h_c + \frac{D_c}{2} = h_c + \frac{h_c}{2} = 1.5h_c$

(iv) $Q = A_cU_c = bh_c\sqrt{gh_c} = \sqrt{g}\,bh_c^{1.5}$

3.10 (i) Wide channel, $q = 4$ m²/s, $\alpha = 1.12$

$$h_c = \sqrt[3]{\frac{\alpha q^2}{g}} = \sqrt[3]{\frac{1.12\times4^2}{9.81}} = 1.222\text{ m}$$

$$U_c = \sqrt{\frac{gh_c}{\alpha}} = \sqrt{\frac{9.81\times1.222}{1.12}} = 3.372\text{ m/s}$$

(ii) Rectangular channel, $b=6m, Q=35 m^3/s, \alpha=1.12$

$$h_c=\sqrt[3]{\frac{\alpha Q^2}{gb^2}}=\sqrt[3]{\frac{1.12\times35^2}{9.81\times6^2}}=1.083m$$

$$U_c=\sqrt{\frac{gh_c}{\alpha}}=\sqrt{\frac{9.81\times1.083}{1.12}}=3.079\ m/s$$

(iii) Triangular channel, $s=1, Q=5 m^3/s, \alpha=1.12$

$$h_c=\sqrt[5]{\frac{2\alpha Q^2}{gs^2}}=\sqrt[5]{\frac{2\times1.12\times5^2}{9.81\times1^2}}=1.417\ m$$

$$U_c=\frac{Q}{A_c}=\frac{Q}{sh^2}=\frac{5}{1\times1.417^2}=\frac{5}{2.008}=2.490\ m/s$$

(iv) Parabolic channel, $y^2=5z \rightarrow z=\frac{1}{5}y^2 \rightarrow c=\frac{1}{5}=0.$

$Q=25 m^3/s, \alpha=1.12$

$$h_c=\sqrt[4]{\frac{27\alpha cQ^2}{32g}}=\sqrt[4]{\frac{27\times1.12\times0.20\times25^2}{32\times9.81}}=1.863\ m$$

$$A_c=\frac{4h_c^{1.5}}{3\sqrt{c}}=\frac{4\times1.863^{1.5}}{3\sqrt{0.2}}=7.580\ m^2$$

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3.11 (a) Trapezoidal channel, $b=6m, s=2, Q=30 m^3/s$, $\alpha=1.12$

(i) Trial-and-error method

$$Z_c=\frac{Q}{\sqrt{g}\alpha}=\frac{30}{\sqrt{9.81}/1.12}=10.137$$

$A=(6+2h)h$, $B=6+4h$, $\therefore D=A/B$, $Z=A\sqrt{D}$

h	A	B	D	Z
1.00	8.00	10.00	0.800	7.155
2.00	20.00	14.00	1.428	23.905
1.22	10.297	10.88	0.946	10.017
1.23	10.406	10.92	0.953	10.158 ← Nearest

$\therefore h_c=1.23m$ $\quad U_c=\frac{Q}{A_c}=\frac{30}{10.406}=2.883\ m/s$

(ii) Bisection method $\quad A=(6+2h)h \quad B=6+4h$

$$f(h)=1-\frac{\alpha Q^2 B}{gA^3}=1-\frac{1.12\times30^2\times B}{9.81\times A^3}=1-\frac{102.75B}{A^3}$$

hmin	hmax	hmean	A	B	f(h)	Root lies between
0	10	5	80.00	260	0.995	0 and 5
0	5	2.5	27.50	16.0	0.921	0 and 2.5
0	2.5	1.25	10.625	11.0	0.058	0 and 1.25
0	1.25	0.625	4.531	8.5	−8.311	0.625 & 1.25
0.625	1.25	0.9375	7.3828	9.75	−1.490	0.9375 & 1.25
0.9375	1.25	1.09375	8.955	10.375	−0.486	1.09375 & 1.25
1.0938	1.25	1.1719	9.778	10.688	−0.175	1.17 & 1.25
1.17	1.25	1.21	10.19	10.84	−0.057	1.21 & 1.25
1.21	1.25	1.23	10.41	10.92	0.004	1.21 & 1.23

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3.11(a)(ii) Bisection method (contd.)

hmin	hmax	hmean	A	B	f(h)	Root lies between
1.21	1.23	1.22	10.297	10.88	−0.023	1.22 and 1.2
1.22	1.23	1.225	10.351	10.90	−0.009	1.225 and 1.2

∴ The critical depth, $h_c = 1.23\,m$

$A_c = (6 + h)h_c = (6 + \quad \times 1.23) \times 1.23 = 10.4058\,m^2$

$\therefore V_c = \frac{Q}{A_c} = \frac{30}{10.4058} = 2.883\ m/s$

(ii) Newton-Raphson method

$A = (6+2h)h \quad B = 6+4h \quad dB/dh = 2s = 2\times 2 = 4$

$f(h) = A^3 - \frac{\alpha Q^2 B}{g} = [(6+2h)h]^3 - \frac{1.12\times 30^2 \times (6+4h)}{9.81}$

$= [(6+2h)h]^3 - 102.75(6+4h)$

$f'(h) = 3A^2B - \frac{\alpha Q^2}{g}\frac{dB}{dh} = 3[(6+2h)h]^2(6+4h) - \frac{1.12\times 30^2 \times 4}{9.81}$

$= 24(3+2h)[(3+2h)h]^2 - 411.9$

h	f(h)	f'(h)	$\Delta h = -\frac{f(h)}{f'(h)}$	$h = h + \Delta h$
1.00	−515.52	1508.99	0.34	1.34
1.34	406.26	4199.50	−0.10	1.24
1.24	36.49	3224.51	−0.01	1.23
1.23	4.69	3136.26	−0.001	1.23

∴ The critical depth, $h_c = 1.23\,m$

$A_c = (6+4h_c)h_c = (6+4\times 1.23)\times 1.23 = 10.40$

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3.11 (b) (i) Circular channel, $d_o = 3\,m$, $Q = 5\,m^3/s$, $\alpha = 1$

$Z_c = \frac{Q}{\sqrt{g/\alpha}} = \frac{5}{\sqrt{9.81/1}} = 1.596$ $\quad A = \frac{1}{8}(\omega - \sin\omega)d_o^2$ $\quad B = d_o \sin\frac{\omega}{2}$

ω (rad)	A (m²)	B (m)	D (=A/B)	Z (=A√D)
1	0.178	1.438	0.014	0.124
2	1.227	2.524	0.486	0.855
3	3.216	2.992	1.075	3.339
2.40	1.940	2.796	0.694	1.616
2.39	1.921	2.791	0.688	1.593 ⇐

$\therefore \omega_c = 2.39$ rad $\quad h_c = \frac{d_o}{2}\left(1 - \cos\frac{\omega_c}{2}\right) = \frac{3}{2}\left(1 - \cos\frac{2.39}{2}\right)$

$= 0.950\,m$

$A_c = 1.921\,m^2 \quad V_c = \frac{Q}{A_c} = \frac{5}{1.921} = 2.600\ m/s$

(ii) Circular section, $d_o = 3\,m$, $Q = 5\,m^3/s$, $\alpha = 1.12$

$Z_c = \frac{Q}{\sqrt{g/\alpha}} = \frac{5}{\sqrt{9.81/1.12}} = 1.689$

ω (rad)	A (m²)	B (m)	D (=A/B)	Z (=A√D)
1	0.178	1.438	0.014	0.124
2	1.227	2.524	0.486	0.855
3	3.216	2.992	1.075	3.339
2.43	1.999	2.812	0.711	1.686 ⇐
2.44	2.019	2.817	0.717	1.709

$\therefore \omega_c = 2.43$ rad $\quad h_c = \frac{d_o}{2}\left(1 - \cos\frac{\omega_c}{2}\right) = \frac{3}{2}\left(1 - \cos\frac{2.43}{2}\right)$

$= 0.977\,m$

$A_c = 1.999\,m^2 \quad V_c = Q/A_c = 5/1.999 = 2.501\ m/s$

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3.12 The alternate depths ... the depths ...

The alternate depths are the two depths for the same value of the specific energy. So, we can write

$$E = h_1 + \frac{U_1^2}{2g} = h_2 + \frac{U_2^2}{2g}$$

or, $h_1 + \frac{Q^2}{2gA_1^2} = h_2 + \frac{Q^2}{2gA_2^2}$ $(\because Q = A_1U_1 = A_2U_2)$

or, $h_1 + \frac{Q^2}{2gb^2h_1^2} = h_2 + \frac{Q^2}{2gb^2h_2^2}$ $(\because$ For a rectangular channel, $A = bh$ and $A_2 = b_2h_2)$

or, $h_1 - h_2 = \frac{Q^2}{2gb^2h_2^2} - \frac{Q^2}{2gb^2h_1^2}$

or, $h_1 - h_2 = \frac{Q^2}{2gb^2}\left(\frac{1}{h_2^2} - \frac{1}{h_1^2}\right)$

or, $h_1 - h_2 = \frac{Q^2}{2gb^2}\left(\frac{h_1^2 - h_2^2}{h_1^2h_2^2}\right) = \frac{Q^2}{2gb^2} \cdot \frac{(h_1+h_2)(h_1-h_2)}{h_1^2h_2^2}$

or, $1 = \frac{Q^2}{2gb^2}\left(\frac{h_1+h_2}{h_1^2h_2^2}\right)$ (i)

Now, the critical depth for a rectangular channel is given by

$$h_c^3 = \frac{Q^2}{gb^2} \quad \text{or,} \quad \frac{Q^2}{2gb^2} = \frac{h_c^3}{2} \quad \text{(ii)}$$

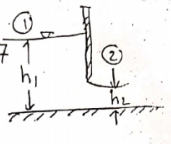
Combining (i) and (ii), we have

$$1 = \frac{h_c^3}{2}\left(\frac{h_1+h_2}{h_1^2h_2^2}\right)$$

or, $\frac{2h_1^2h_2^2}{h_1+h_2} = h_c^3$ (Proved)

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3.13 Rectangle, $b = 6m$, $h_1 = 2m$, $Q = 30.67\ m^3/s$



$$U_1 = \frac{Q}{A_1} = \frac{Q}{bh_1} = \frac{30.67}{6\times2} = 2.556\ m/s$$

Applying the energy equation between sections 1 and 2, neglecting the frictional head loss and taking the channel bed as datum, we get

$$0 + h_1 + \frac{U_1^2}{2g} = 0 + h_2 + \frac{U_2^2}{2g}$$

or, $2 + \frac{2.556^2}{2\times9.81} = h_2 + \frac{Q^2}{2gb^2h_2^2}$

or, $2.333 = h_2 + \frac{30.67^2}{2\times9.81\times6^2\times h_2^2} = h_2 + \frac{1.332}{h_2^2}$

By trial, we get $h_2 = 1m$ and $h_2 = 2m$,

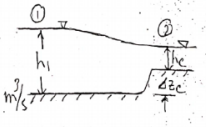
When $h_2 = 1m$, $U_2 = 5.112\ m/s$, $Fr_2 = \frac{5.112}{\sqrt{9.81\times1}} = 1.63 > 1$

When $h_2 = 2m$, $U_2 = 2.556\ m/s$, $Fr_2 = \frac{2.556}{\sqrt{9.81\times2}} = 0.562 < 1$

Since the sluice gate acts as a control, the downstream flow must be supercritical.

$\therefore h_2 = 1m$ (Ans)

3.14 $U_1 = 2m/s$, $h_1 = 2.5m$, $b = 6m$



$Q = A_1U_1 = bh_1U_1 = 6\times2.5\times2 = 30\ m^3/s$

$$h_c = \sqrt[3]{\frac{\alpha Q^2}{gb^2}} = \sqrt[3]{\frac{1\times30^2}{9.81\times6^2}} = 1.366m$$

$$U_c = \frac{Q}{A_c} = \frac{Q}{bh_c} = \frac{30}{6\times1.366} = 3.660\ m/s$$

Applying the energy equation between sections 1 and 2, we get

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$$h_1 + \frac{U_1^2}{2g} = \Delta z_c + h_c + \frac{U_c^2}{2g}$$

$$\therefore \Delta z_c = h_1 + \frac{U_1^2}{2g} - h_c - \frac{U_c^2}{2g} = 2.5 + \frac{2^2}{2\times 9.81} - 1.366 - \frac{3.660^2}{2\times 9.81}$$

$$= 0.655 \text{ m}$$

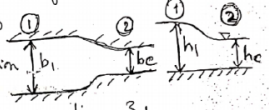
Drop in water level $= h_1 - h_c - \Delta z_c = 2.5 - 1.366 - 0.655$

$$= 0.479 \text{ m}$$

3.15 $U_1 = 2 \text{ m/s}$, $h_1 = 2.5 \text{ m}$, Rectangular, $b = 6 \text{ m}$

$Q = A_1 U_1 = b h_1 U_1 = 6 \times 2.5 \times 2 = 30 \text{ m}^3/\text{s}$

Applying the energy equation between sections 1 and 2



$$h_1 + \frac{U_1^2}{2g} = h_c + \frac{U_c^2}{2g} = h_c + \frac{D_c}{2} = h_c + \frac{h_c}{2} = \frac{3}{2} h_c$$

$$\therefore h_c = \frac{2}{3}\left(h_1 + \frac{U_1^2}{2g}\right) = \frac{2}{3}\left(2.5 + \frac{2^2}{2\times 9.81}\right) = \frac{2}{3} \times 2.70 = 1.80 \text{ m}$$

$$\therefore U_c = \sqrt{g D_c/\alpha} = \sqrt{g h_c/\alpha} = \sqrt{9.81 \times 1.80/1} = 4.20 \text{ m/s}$$

Since $Q = A_c U_c = b_c h_c U_c$, we have

$$b_c = \frac{Q}{h_c U_c} = \frac{30}{1.80 \times 4.20} = 3.96 \text{ m}$$

Drop in water level $= h_1 - h_c = 2.50 - 1.80 = 0.70 \text{ m}$

3.16 $b = 10 \text{ km} = 10{,}000 \text{ m}$, $Q = 1{,}00{,}000 \text{ m}^3/\text{s}$, $h = 10 \text{ m}$

$$U = Q/A = Q/bh = 1{,}00{,}000/(10{,}000 \times 10) = 1 \text{ m/s}$$

The length of the bridge will be minimum when the width of the river is minimum. The width of the river is minimum when the flow at the bridge section is critical (without affecting the upstream flow). Hence, applying the energy equation between an upstream section and the bridge section, we get

$$h + \frac{U^2}{2g} = h_c + \frac{U_c^2}{2g} = h_c + \frac{D_c}{2} = h_c + \frac{h_c}{2} = \frac{3}{2} h_c$$

$$\therefore h_c = \frac{2}{3}\left(h + \frac{U^2}{2g}\right) = \frac{2}{3}\left(10 + \frac{1^2}{2\times 9.81}\right) = \frac{2}{3} \times 10.051 = 6.701 \text{ m}$$

$$U_c = \sqrt{g h_c} = \sqrt{9.81 \times 6.701} = 8.108 \text{ m/s}$$

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Chapter 4 Uniform Flow

Solutions to the problems

4.10 $A = 33{,}500 \text{ m}^2$, $Q = 56{,}200 \text{ m}^3/\text{s}$, $B = 3820 \text{ m}$

$$S_o = 4 \text{ cm/km} = 4 \times 10^{-2}/10^3 = 4 \times 10^{-5} = 0.00004$$

Since the river is wide, we have

$$R = D = h = \frac{A}{B} = \frac{33{,}500}{3820} = 8.77 \text{ m}$$

$$U = Q/A = 56{,}200/33{,}500 = 1.68 \text{ m/s}$$

Since $Q = \frac{1}{n} A R^{2/3} S_o^{1/2}$, we obtain

$$n = \frac{A R^{2/3} S_o^{1/2}}{Q} = \frac{33500 \times 8.77^{2/3} \times 0.0004^{1/2}}{56{,}200} = 0.016$$

$$C = \frac{1}{n} R^{1/6} = \frac{1}{0.016} \times 8.77^{1/6} = 89.57 \text{ m}^{1/2}/\text{s}$$

$$f = \frac{8g}{C^2} = \frac{8 \times 9.81}{89.57^2} = 0.010$$

4.11 (a) $h = 1 \text{ m}$, Use Manning formula

i) Rectangle, $b = 6 \text{ m}$, $h = 1 \text{ m}$, $h/b = 1/6 = 0.167$

$$\therefore N = \frac{2}{3}\left(5 - \frac{4 \times 0.167}{1 + 2 \times 0.167}\right) = 3$$

ii) Trapezoid, $b = 6 \text{ m}$, $s = 2$, $h = 1 \text{ m}$, $h/b = 0.167$

$$\therefore N = \frac{10}{3}\frac{1 + 2 \times 2 \times 0.167}{1 + 2 \times 0.167} - \frac{8}{3}\frac{\sqrt{5} \times 0.167}{1 + 2\sqrt{5} \times 0.167} =$$

$$= 4.167 - 0.569 = 3.598$$

Alternatively $A = 8 \text{ m}^2$, $P = 10.47 \text{ m}$, $R = 0.76 \text{ m}$

$B = 10m$, $Ar/an = 2\times1+3^2 = 2\sqrt{5}$

$\therefore N = \frac{2+1}{3\times8}(5\times10 - 2\times0.76\times2\sqrt{5}) = 3.600$

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4.11(b) Use Chezy formula

i) Rectangle

$N = 3 - \frac{2\times0.167}{1+2\times0.167} = 2.75$

ii) Trapezoid

$N = 3\,\frac{1+2\times2\times0.167}{1+2\times0.167} - \frac{2\sqrt{5}\times0.167}{1+2\sqrt{5}\times0.167} = 3.323$

Alternatively

$N = \frac{1}{8}(3\times10 - 0.76\times2\sqrt{5}) = 3.32$

4.12(a) $n=0.025$, $S_0=0.0025$, $q = 3\,m^2/s$

$h_n = \left(\frac{nq}{\sqrt{S_0}}\right)^{3/5} = \left(\frac{0.025\times3}{\sqrt{0.0025}}\right)^{3/5} = 1.275m$

$U_n = \frac{q}{h_n} = \frac{3}{1.275} = 2.35\ m/s$

(b) $S_0 = 0.006$, $C = 50\ m^{1/2}/s$, $\therefore = 4\ m^2/s$

$h_n = \left(\frac{q}{C^2S_0}\right)^{1/3} = \left(\frac{4}{50^2\times0.006}\right)^{1/3} = 1.022\ m$

$U_n = \frac{q}{h_n} = \frac{4}{1.022} = 3.915\ m/s$



4.13 Triangular, $s=1$, $S_0=0.001$, $n=0.015$, $h_n=1m$

$A_n = sh_n^2 = 1\times1^2 = 1\,m^2$

$P_n = 2h_n\sqrt{1+s^2} = 2\times1\times\sqrt{1+1^2} = 2\sqrt{2} = 2.828\ m$

$R_n = A/P = 1/2.828 = 0.35\ m$

$\therefore Q = \frac{1}{n}A_nR_n^{2/3}S_0^{1/2} = \frac{1}{0.015}\times1\times0.35^{2/3}\times0.001^{1/2}$

$= 1.05\ m^3/s$

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4.13 Triangular, $s=1$, $n=0.025$, $S_0=0.0025$, $Q=5m^3/s$

$h_n = \frac{2^{1/4}(1+s^2)^{1/8}}{s^{5/8}}\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/8}$

$= \frac{2^{1/4}(1+1^2)^{1/8}}{1^{5/8}}\left(\frac{0.025\times5}{\sqrt{0.0025}}\right)^{3/8} = 1.83\ m$

$A_n = sh_n^2 = 1\times1.83^2 = 3.34\ m^2$

$\therefore U_n = \frac{Q}{A_n} = \frac{5}{3.34} = 1.50\ m/s$

4.14 $h_n = 1m$, $S_0=0.0001$, $n=0.015$, $Q_n = ?$

i) $Q_n = \frac{1}{0.015}\times(6\times1)\times\left(\frac{6\times1}{6+2\times1}\right)^{2/3}\times0.0001^{1/2} = 3.30\ m^3/s$

ii) $A = (6+2\times1)\times1 = 8m^2$, $P = 6+2\times1\times\sqrt{1+1^2} = 8.824\ m$

$R = A/P = 8/8.824 = 0.906m$

$Q_n = \frac{1}{015}\times8\times0.906^{2/3}\times0.0001^{1/2} = 4.99\ m^3/s$

iii) $s=1.5$, $A = sh^2 = 1.5\times1^2 = 1.5\,m^2$, $P = 2\times1\times\sqrt{1+1^2} = 3.606m$

$R = A/P = 1.5/3.606 = 0.416\ m$

$Q_n = \frac{1}{0.015}\times1.5\times0.416^{2/3}\times0.0001^{1/2} = 0.56\ m^3/s$

iv) $y^2 = 4z$ $\therefore z = (1/4)y^2$ $\therefore c = 1/4$

$A = \frac{4}{3\sqrt{c}}h^{3/2} = \frac{8}{3}h^{3/2} = 2.667\,m^2$, $B = \frac{2}{\sqrt{c}}h^{1/2} = 4\ m$, $x = \frac{4\times1}{4} = 1$

$P = \frac{4}{1}\left[\sqrt{1+1^2} + \frac{1}{1}\times\ln\left(1+\sqrt{1+1^2}\right)\right] = 4.591\ m$

$Q_n = \frac{1}{0.015} \times 2.667 \times \left(\frac{2.667}{9.591}\right)^{2/3} \times 0.0001^{1/2} = 1.24\ m^3/s$

v) $d_o = 1.5m \quad \omega = 2\cos^{-1}\left(1 - \frac{2.41}{1.5}\right) = 3.821\ rad$

$A = (\omega - \sin\omega) d_o^2/8 = (3.821 - \sin 3.821) \times 1.5^2/8 = 1.2515 m^2$

$P = \frac{1}{2}\omega d_o = \frac{1}{2} \times 3.821 \times 1.5 = 2.866 m, \quad R = 1.2515/2.866 = 0.437$

$Q_n = \frac{1}{0.015} \times 1.2515 \times 0.437^{2/3} \times 0.0001^{1/2} = 0.48\ m^3/s$

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4.15 $n = 0.025 \quad S_o = 0.0025$

i) Rectangular channel, $b = 8m, Q = 22 m^3/s$

$$AR^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.025 \times 22}{\sqrt{0.0025}} = 11.00$$

h	A (=bh)	P (=b+2h)	R (=A/P)	$AR^{2/3}$
1	8.00	10.00	0.800	6.894
2	16.00	12.00	1.333	19.383
1.36	10.88	10.72	1.015	10.928
1.37	10.96	10.74	1.020	11.109

$\therefore h_n = 1.36m, \quad U_n = \frac{Q}{A_n} = \frac{22}{10.88} = 2.022\ m/s$

ii) Trapezoidal channel, $b = 6m, s = 2, Q = 30 m^3/s$

$$AR^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.025 \times 30}{\sqrt{0.0025}} = 15.00$$

$A = (b+sh)h = (6+2h)h$

$P = b + 2h\sqrt{1+s^2} = 6 + 2\sqrt{5}h$

h	A	P	R	$AR^{2/3}$
1	8.00	10.472	0.764	6.685
2	20.00	14.944	1.338	24.288
1.55	14.105	12.932	1.091	14.946
1.56	14.227	12.966	1.096	15.127

$\therefore h_n = 1.55m$

$U_n = \frac{Q}{A_n} = \frac{30}{14.105} = 2.127\ m/s$

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4.15 iii) Parabolic channel, $y^2 = 5z, Q = 15 m^3/s$

$$AR^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.025 \times 15}{\sqrt{0.0025}} = 7.50$$

$z = h, B = 2y$

h	y	B	A	P	R	$AR^{2/3}$
1	2.236	4.447	2.981	4.984	0.598	2.166
2	3.162	6.325	8.433	7.742	1.089	8.927
1.83	3.025	6.050	7.381	7.304	1.011	7.433
1.84	3.033	6.066	7.441	7.330	1.015	7.516

$\therefore h_n = 1.84m \quad U_n = \frac{Q}{A_n} = \frac{15}{7.441} = 2.016\ m/s$

iv) Circular channel, $d_o = 2m, Q = 3 m^3/s$

$$AR^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.025 \times 3}{\sqrt{0.0025}} = 1.5$$

$A = \frac{1}{8}(\omega - \sin\omega) d_o^2$

$P = \frac{1}{2}\omega d_o$

ω (rad)	A	P	R	$AR^{2/3}$
1	0.079	1.00	0.079	0.015
2	0.595	2.00	0.273	0.229
3	1.429	3.00	0.476	0.871
4	2.338	4.00	0.595	1.681
3.75	2.162	3.75	0.577	1.500

$\therefore h_n = \frac{d_0}{2}\left(1-\cos\frac{\omega}{2}\right) = \frac{2}{2}\left(1-\cos\frac{3.75}{2}\right) = 1.30\ m$

$U_n = \frac{Q}{A_n} = \frac{3}{2.162} = 1.387\ m/s$

(30)

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(6)

4.16(a) Rectangular, $n=0.025$, $S_0=0.0001$, $Q=6 m^3/s$, $h_n=1.5m$

$A_n R_n^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.025\times 6}{\sqrt{0.0001}} = 15$

$A=bh$, $P=b+2h$

b	A	P	R	$AR^{2/3}$
5	7.5	8	0.9375	7.18
10	15	13	1.1538	16.50
9	13.5	12	1.125	14.60
9.20	13.80	12.2	1.131	14.98
9.21	13.815	12.21	1.131	15.00

$\therefore b = 9.21\ m$

(b) Trapezoidal, $s=1.5$, $n=0.020$, $S_0=0.0002$, $Q=25 m^3/s$, $h_n=2m$

$A=(b+sh)h$, $P=b+2h\sqrt{1+s^2}$

$A_n R_n^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.020\times 25}{\sqrt{0.0002}} = 35.36$

b	A	P	R	$AR^{2/3}$
5	16	12.211	1.3103	19.16
10	26	17.211	1.5107	34.23
11	28	18.211	1.5375	37.30
10.37	26.74	17.581	1.5210	35.37
10.36	26.72	17.571	1.5265	35.33

$\therefore b = 10.37\ m$

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(7)

4.17 Trapezoidal channel, $b=6m$, $s=1.5$, $n=0.025$, $\alpha=1.12$

i) $h_n = 1m$, $Q = 20 m^3/s$

$A = (6+1.5\times1)\times1 = 7.5 m^2$

$P = 6+2\times1\times\sqrt{1+1.5^2} = 9.6055\ m$

$R = A/P = 7.5/9.6055 = 0.7808\ m$

$\therefore S_n = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.025\times 20}{7.5\times 0.7808^{2/3}}\right)^2 = 0.0062$

ii) $Q = 20 m^3/s$ $\quad Z_c = \frac{Q}{\sqrt{gA}} = \frac{20}{\sqrt{9.81/1.12}} = 6.758$

n	n	$\sqrt{S}$	D	$Z = A\sqrt{D}$
1.00	7.50	9.00	0.833	6.847
0.98	7.32	8.94	0.819	6.624
0.99	7.41	8.97	0.826	6.735

$\therefore h_c = 0.99m = h_n$, $A = 7.41\ m^2$

$P = 6 + 2\times0.99\times\sqrt{1+1.5^2} = 10.923m$, $R = A/P = 0.711\ m$

$\therefore S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.025\times20}{7.41\times0.711^{2/3}}\right)^2 = 0.0072$

iii) $h_n = 1m = h_c$, $A = 7.5m^2$, $B = 6+2\times1.5\times1 = 9m$

$D = A/B = 0.833m$, $R = 0.7808m$

$U = \sqrt{9D/\alpha} = \sqrt{9.81\times0.833/1.12} = 2.701\ m/s$

~~$Q = AU = 7.5\times2.701 = 20.26\ m^3/s$~~

$\therefore S_c = \left(\frac{nU}{n^{2/3}}\right)^2 = \left(\frac{0.025\times2.701}{0.7808^{2/3}}\right)^2 = 0.0063$

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4.18 Trapezoidal, $b = 6m$, $n = 0.025$, $s = 1$, $h_n = 2m$, $S_o = 0.0005$

$A = (b+sh)h = (6+1\times2)2 = 16\ m^2$

$P = b + 2\sqrt{1+s^2}\,h = 6 + 2\sqrt{1+1^2}\,2 = 11.65685\ m$

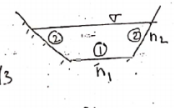
$R = A/P = 1.373\ m$

$Q = \frac{1}{n}AR^{2/3}S^{1/2} = \frac{1}{.025}\times16\times1.373^{2/3}\times0.0005^{1/2}$

$= 17.68\ m^3/s$

i) $P_1 = 6m$ $P_2 = 5.65685m$

$n_1 = 0.025$ $n_2 = 0.013$



$n = \left[\frac{P_1 n_1^{1.5} + P_2 n_2^{1.5}}{P}\right]^{2/3}$

$= \left(\frac{6\times0.025^{1.5} + 5.65685\times0.013^{1.5}}{11.65685}\right)^{2/3} = 0.01965$

$\therefore Q = \frac{1}{0.01965}\times16\times1.373^{2/3}\times0.0005^{1/2} = 22.49\ m^3/s$

ii) $P_1 = 6m$ $P_2 = 5.65685m$ $n_1 = 0.013$ $n_2 = 0.025$

$n = \left(\frac{6\times0.013^{1.5} + 5.65685\times0.025^{1.5}}{11.65685}\right)^{2/3} = 0.01930$

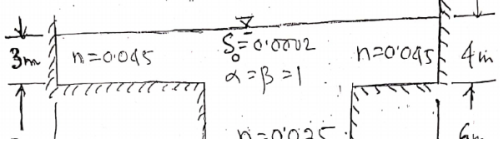
$\therefore Q = \frac{1}{0.01930}\times16\times1.373^{2/3}\times0.0005^{1/2} = 22.90\ m^3/s$

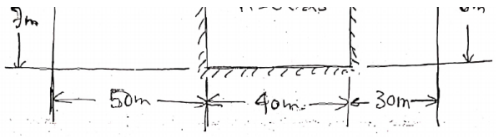
iii) $n = 0.013$

$\therefore Q = \frac{1}{0.013}\times16\times1.373^{2/3}\times0.0005^{1/2} = 33.99\ m^3/s$

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4.19





Section	A	P	R	Q	K
Main	400	53	7.547	870.61	61562
Left	150	53	2.830	94.32	6669
Right	120	34	3.529	87.42	6182
Σ	670	140		1052.35	74413

$$U = \frac{Q}{A} = \frac{1052.35}{670} = 1.571 \text{ m/s}$$

$$n = AR^{2/3} S_o^{1/2}/Q = 670 \times (670/140)^{2/3} \times .0002^{1/2}/1052.35$$
$$= 0.0256 \simeq 0.026$$

$$\alpha = \frac{1 \times 61562^3/400^2 + 1 \times 6669^3/150^2 + 1 \times 6182^3/120^2}{74913^3/670^2}$$
$$= 1.621$$

$$\beta = \frac{1 \times 61562^2/400 + 1 \times 6669^2/150 + 1 \times 6182^2/120}{74913^2/670}$$

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Chapter 5 Design of channels

5.3(i) Circular section

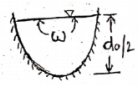
$$A = \frac{1}{8}(\omega - \sin\omega) d_o^2 \quad \therefore d_o^2 = 8A/(\omega - \sin\omega)$$

$$P = \frac{1}{2}\omega d_o \quad \therefore P^2 = \frac{1}{4}\omega^2 d_o^2$$

$$\text{or, } P^2 = \frac{1}{4}\omega^2 \, 8A/(\omega - \sin\omega) = \frac{2A\omega^2}{\omega - \sin\omega}$$

$$\therefore 2P\frac{dP}{d\omega} = 2A\,\frac{(\omega - \sin\omega)2\omega - \omega^2(1-\cos\omega)}{(\omega - \sin\omega)^2} = 0$$

$$\therefore (\omega - \sin\omega)2\omega - \omega^2(1-\cos\omega) = 0$$



$$\text{or, } 2\omega^2 - 2\omega\sin\omega - \omega^2 + \omega^2\cos\omega = 0$$

$$\text{or, } \omega^2 - 2\omega\sin\omega + \omega^2\cos\omega = 0$$

LHS: 0, 0, $8A^2$, $\frac{8A^2}{2(\pi-2)}$

By trial, we get $\omega = \pi$, i.e. the best hydraulic circular section is a semi-circle.

5.3(iii) Triangular section

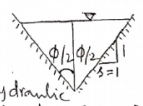
$$A = sh^2 \quad \therefore h^2 = A/s$$

$$P = 2\sqrt{1+s^2}\,h \quad \therefore P^2 = 4(1+s^2)h^2 = 4(1+s^2)\frac{A}{s}$$

$$\text{or, } P^2 = 4\left(\frac{1}{s} + s\right)A \quad \therefore 2P\frac{dP}{ds} = 4\left(-\frac{1}{s^2} + 1\right)A = 0$$

$$\therefore -\frac{1}{s^2} + 1 = 0 \quad \text{or, } s^2 = 1 \quad \therefore s = 1$$

Hence, the side slope is 1:1,



i.e. $\phi/2 = \tan^{-1}(1) = 45°$ $\therefore \phi = 90°$

This means that the best hydraulic triangular section is one-half of a square.

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5.3(v) Parabolic section

$$A = \frac{2}{3}Bh \quad \therefore B = \frac{3A}{2h}$$

$$P = B + \frac{8}{2B}h^2 = \frac{3A}{2h} + \frac{16}{9}\frac{h^3}{A}$$

$$\therefore \frac{dP}{dh} = -\frac{3A}{2h^2} + \frac{16}{9A}\cdot 3h^2 = -\frac{3A}{2h^2} + \frac{16}{3A}h^2 = 0$$

$$\therefore h^4 = \frac{3A}{2}\cdot\frac{3A}{16} = \frac{9A^2}{32} = \frac{9}{32}\cdot\frac{4}{9}B^2h^2$$

$$\therefore h^2 = \frac{1}{8}B^2 \quad \therefore B^2 = 8h^2 \Rightarrow B = 2\sqrt{2}h \text{ (Proved)}$$

5.7 $A = 40\ m^2$

i) Rectangular section

$A = 2h^2 = 40 \quad \therefore h^2 = 20 \quad \therefore h = 4.472\ m$

$\therefore P = 4h = 4 \times 4.472 = \underline{17.889\ m}$

$R = A/P = 40/17.889 = 2.236\ m$

ii) Triangular section

$A = h^2 = 40 \quad \therefore h = 6.325\ m$

$\therefore P = 2\sqrt{2}h = 2\sqrt{2} \times 6.325 = \underline{17.889\ m}$

$R = A/P = 40/17.889 = 2.236\ m$

iii) Trapezoidal section

$A = \sqrt{3}h^2 = 40 \quad \therefore h = 4.806\ m$

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5.7 iii) (Contd.)

$P = 2\sqrt{3}h = 2\sqrt{3} \times 4.806 = \underline{16.647\ m}$

$R = A/P = 40/16.647 = 2.403\ m$

iv) Circular section

$A = \frac{\pi}{2}h^2 = 40 \quad \therefore h = 5.046\ m$

$\therefore P = \pi h = \pi \times 5.046 = \underline{15.853\ m}$ ←

$R = A/P = 40/15.853 = 2.523\ m$

v) Parabolic section

$A = \frac{4}{3}\sqrt{2}h^2 = 40 \quad \therefore h = 4.606\ m$

$\therefore P = \frac{8}{3}\sqrt{2}h = \frac{8}{3}\sqrt{2} \times 4.606 = \underline{17.369\ m}$

$R = A/P = 40/17.369 = 2.303\ m$

Hence, for a given area, the circular section has the minimum wetted perimeter.

5.2(b) For the best hydraulic rectangular section,

$A = 2h^2 \quad \therefore h = \sqrt{A/2}$

$\therefore P = 4h = 4\sqrt{A/2} = 2\sqrt{2}\sqrt{A} \quad (i)$

For the best hydraulic triangular section,

$A = h^2 \quad \therefore h = \sqrt{A}$

$\therefore P = 2\sqrt{2}h = 2\sqrt{2}\sqrt{A} \quad (ii)$

Hence, for a given area A, the best hydraulic rectangular and triangular sections have the same wetted perimeter.

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5.8 $Q = 20\ m^3/s$, $S_0 = 1$ in 2500, $s = 1$, $n = 0.012$

For the best hydraulic trapezoidal section,

$R = h/2$

$b = 2(\sqrt{1+s^2} - s)h = 2(\sqrt{1+1^2} - 1)h = 0.828h$

$A = (b + sh)h = (0.828h + h)h = 1.828h^2$

$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$

$\therefore 20 = \frac{1}{0.012} \times 1.828h^2 \times (h/2)^{2/3} \times \left(\frac{1}{2500}\right)^{1/2}$

$\therefore h^{8/3} = 10.92 \quad \therefore h = 2.408\ m$

$\therefore b = 0.828h = 0.828 \times 2.408 = 1.994\ m$

(b) $Q = 10\ m^3/s$, $s = 2$, $n = 0.015$, $h = 1\ m$

For the best hydraulic trapezoidal section

$R = h/2 = 1/2 = 0.5\ m$

$\therefore b = 2(\sqrt{1+s^2} - s)h = 2(\sqrt{1+2^2} - 2) \times 1 = 0.472\ m$

$A = (b + sh)h = (0.472 + 2 \times 1) \times 1 = 2.472\ m^2$

$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$

$\therefore 10 = \frac{1}{0.015} \times 2.472 \times 0.5^{2/3} \times S_0^{1/2}$

$\therefore S_0 = \left(\frac{0.015 \times 10}{2.472 \times 0.5^{2/3}}\right)^2 = 9.238 \times 10^{-3}$

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5.9 $n = 0.015$; $S_0 = 1$ in 2000, $s = 1.5$

(i) Triangular section, $Q = 40\ m^3/s$, rounded corner

$A = h^2(\phi + \cot\phi)$, $P = 2h(\phi + \operatorname{cosec}\phi)$, $R = A/P = h/2$

$\cot\phi = s = 1.5 \quad \therefore \phi = 0.588$ rad

$A = h^2(0.588 + 1.5) = 2.088h^2$

$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$

$\therefore 40 = \frac{1}{0.015} \times 2.088h^2 \times (h/2)^{2/3} \times (1/2000)^{1/2}$

$\therefore h^{8/3} = 20.3996 \quad \therefore h = 3.098$ m

(ii) Trapezoidal section, $Q = 80\ m^3/s$, rounded corner, $U_{max} = 2$ m/s

$A = bh + h^2(\phi + \cot\phi) = Q/U_{max}$

$\therefore bh + 2.088h^2 = 80/2 = 40 \quad$ (i)

From the Manning formula $U = \frac{1}{n} R^{2/3} S_0^{1/2}$, we get

$2 = \frac{1}{0.015} \times R^{2/3} \times (1/2000)^{1/2} \Rightarrow R = 1.559$ m.

$\therefore P = A/R = 40/1.559 = 25.74$ m

or, $b + 2h(\phi + \cot\phi) = 25.74$

or, $b + 2h(0.588 + 1.5) = 25.74 \Rightarrow b + 4.176h = 25.74$ (ii)

From (ii), using $b = 25.74 - 4.176h$ in (i), we get

$h(25.74 - 4.176) + 2.088h^2 = 0$

or, $2.088h^2 - 25.74h + 40 = 0$

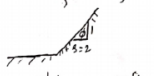
$\therefore h = \frac{25.74 \pm \sqrt{25.74^2 - 4 \times 2.088 \times 40}}{2 \times 2.088} = 10.50, 1.824$

When $h = 10.50$m, $b = -18.108$m | We except $h = 1.824$m, $b = 18.128$m | Ans

(12a)

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5.10 $Q = 30\,m^3/s$, $d_{50} = 2.5$ cm, $d_{75} = 3$ cm, $n = 0.025$, $\theta = 32°$,
Trapezoidal section, $s = 2$, $S_o = 0.001$



i) Using the method of Lane

$d_{75} = 3/2.54 = 1.18$ inch, $s = 2$ $\therefore \phi = \tan^{-1}\frac{1}{2} = 26.56°$.

Assume $b/h_n = 5$.

The maximum unit tractive force on sides (Fig. 5.4(a)) $= 0.76\gamma h_n S_o = 0.76 \times 9810 \times h_n \times 0.001 = 7.455 h_n$

$K = \sqrt{1 - \sin^2\phi/\sin^2\theta} = \sqrt{1 - \sin^2 26.56°/\sin^2 32°} = 0.537$

$\tau_b = 0.40 d_{75} = 0.40 \times 1.18 = 0.472\,lb/ft^2 = 22.62\ N/m^2$

$\tau_s = K\tau_b = 0.537 \times 22.62 = 12.15\ N/m^2$ $(1\,lb/ft^2 = 47.86\ N/m^2)$

$\therefore 7.455 h_n = 12.15$ $\therefore h_n = 1.61$ m

$\therefore b = 5h_n = 5 \times 1.61 = 8.06$ m

$\therefore A = (b + sh_n)h_n = (8.06 + 2 \times 1.61) \times 1.61 = 18.19\ m^2$

$P = b + 2\sqrt{1+s^2}\,h = 8.06 + 2\sqrt{1+2^2} \times 1.61 = 15.27$ m

$R = A/P = 18.19/15.27 = 1.19$ m

$\therefore Q = (1/0.025) \times 18.19 \times 1.19^{2/3} \times 0.001^{1/2} = 25.85\ m^3/s$

which is less than the design discharge of $30\,m^3/s$.

Computation for other values of b/h_n is as follows.

b/h_n	h_n	b	A	P	R	Q
5.9	1.61	9.50	20.53	16.72	1.228	29.77
6.0	1.61	9.67	20.79	16.88	1.231	30.21

Hence, we accept $b/h_n = 6.0$, $h_n = 1.61$ m and $b = 9.67$ m.

With $s = 2$ and $b/h_n = 6.0$, the maximum unit tractive force on bottom (Fig. 5.4(b)) is $0.99\gamma h_n S_o = 0.99 \times 9810 \times 1.61 \times 0.001 = 15.64\ N/m^2$ which is less than $22.62\ N/m^2$, the permissible tractive force on bottom. Hence, the design is acceptable.

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(7)

5.10(ii) Using modified Yalin-Karahan curve

$$R_*^* = \left(\frac{s_s - 1}{s}\right)^{1/3} \times \frac{g^{1/3} d_{50}}{\nu^{2/3}} = 1.65^{1/3} \times \frac{9.81^{1/3} \times 0.025}{(10^{-6})^{2/3}} = 632.40$$

The corresponding value of τ_c^* from Fig. 5.9 is 0.045.

$\therefore \tau_b = \tau_c = \tau_c^* \gamma (s_s - 1) d_{50} = 0.045 \times 9810 \times 1.65 \times 0.025 = 18.21\ N/m^2$

$\therefore \tau_s = K\tau_b = 0.537 \times 18.21 = 9.78\ N/m^2$

$\therefore 7.455 h_n = 9.78$ $\therefore h_n = 1.30$ m

$\therefore b = 5h_n = 6.47$ m

$\therefore A = (6.47 + 2 \times 1.30) \times 1.30 = 11.74\ m^2$ | $R = A/P = 0.956$ m

$P = 6.47 + 2\sqrt{1+2^2} \times 1.30 = 12.26$ m

$\therefore Q = (1/0.025) \times 11.74 \times 0.956^{2/3} \times 0.001^{1/2} = 14.11\ m^3/s$

which is far less than the design discharge of $30\,m^3/s$.

Computation for other values of b/h_n is as follows.

b/h_n	h_n	b	A	P	R	Q
10.00	1.30	12.95	20.12	18.39	1.094	26.69
12.00	1.30	15.59	23.98	21.33	1.101	31.66
11.40	1.30	14.76	22.93	20.55	1.093	30.17

Hence, we accept $b/h_n = 11.40$, $h_n = 1.30$ m and $b = 14.76$ m.

With $s = 2$ and $b/h_n = 11.40$, the maximum unit

tractive force on level bottom (Fig. 5.4(b)) is $\gamma h S_0$
$= 9810 \times 1.30 \times 0.001 = 12.75\ N/m^2$ which is less than
$18.21\ N/m^2$, the permissible tractive force on bottom.
Hence, the design is acceptable.

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5.11 Lacey method, $d = 1.5$ mm, $Q = 25\ m^3/s$, $S = 0.5$

$f_s = 1.76\sqrt{d} = 1.76\sqrt{1.5} = 2.156$

$S_e = \dfrac{f_s^{5/3}}{3340 Q^{1/6}} = \dfrac{2.156^{5/3}}{3340 \times 25^{1/6}} = 6.30 \times 10^{-4}$

$R = 0.47 (Q/f_s)^{1/3} = 0.47 (25/2.156)^{1/3} = 1.064\ m$

$P = 4.75\sqrt{Q} = 4.75 \times \sqrt{25} = 23.75\ m$

$A = PR = 23.75 \times 1.064 = 25.27\ m^2$

$\therefore P = 23.75 = b + 2\sqrt{1+0.5^2}\, h = b + 2.236h$ (i)

$A = 25.27 = (b + 0.5h)h$ (ii)

From (i) we have, $b = 23.75 - 2.236h$

$\therefore$ From (ii), $25.27 = (23.75 - 2.236h + 0.5h)h$

$= 23.75h - 1.736h^2$

$\therefore 1.736h^2 - 23.75h + 25.27 = 0$

$\therefore h = \dfrac{23.75 \pm \sqrt{23.75^2 - 4 \times 1.736 \times 25.27}}{2 \times 1.736}$

$= 12.518,\ 1.163$

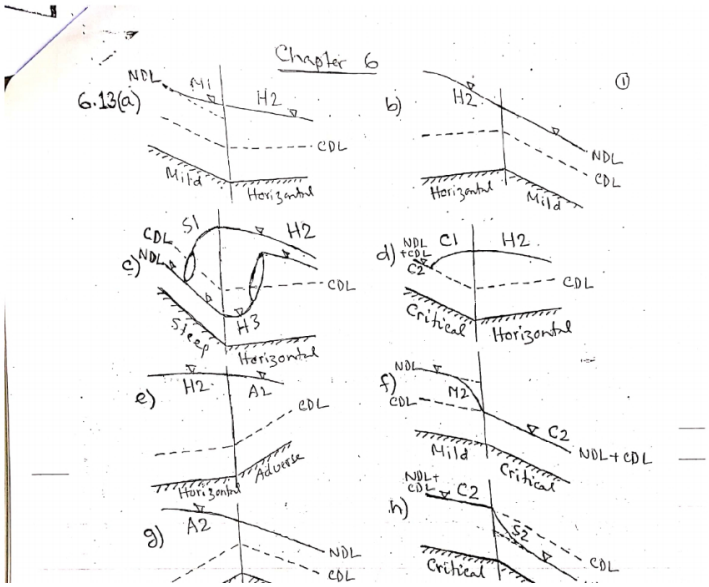
When $h = 12.518$m, $b = -4.240$ m

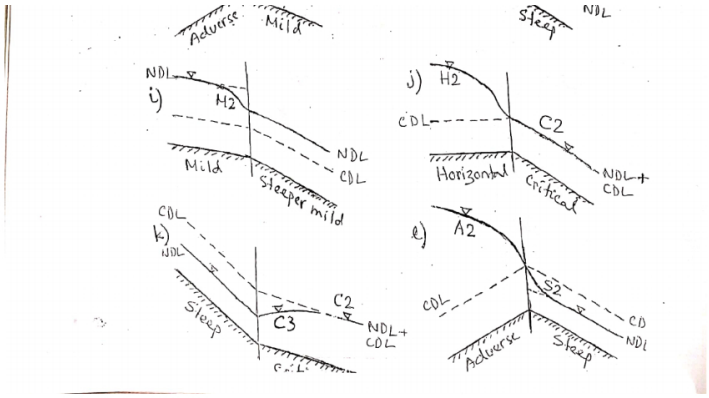
When $h = 1.163$ m, $b = 21.150$ m

Hence we accept $h = 1.163$ m and $b = 21.150$ m.

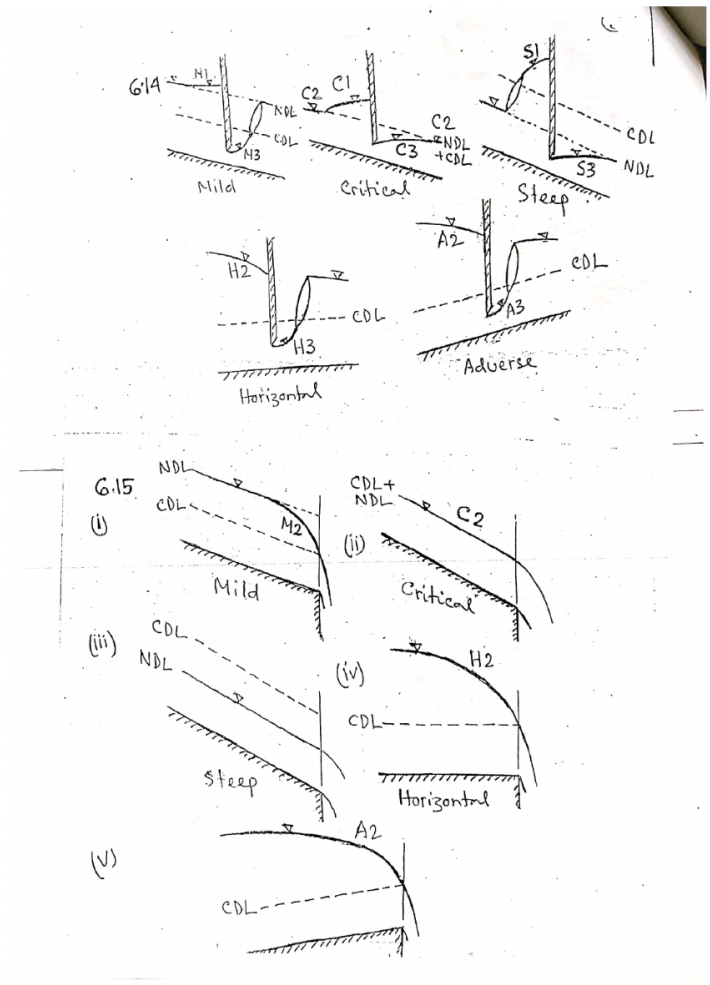
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Chapter 6

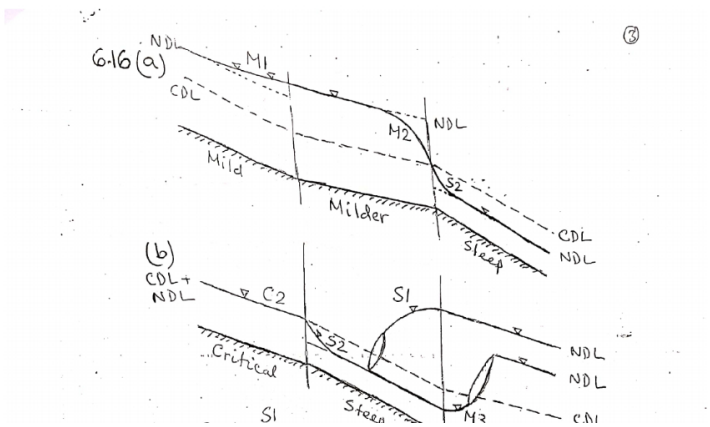


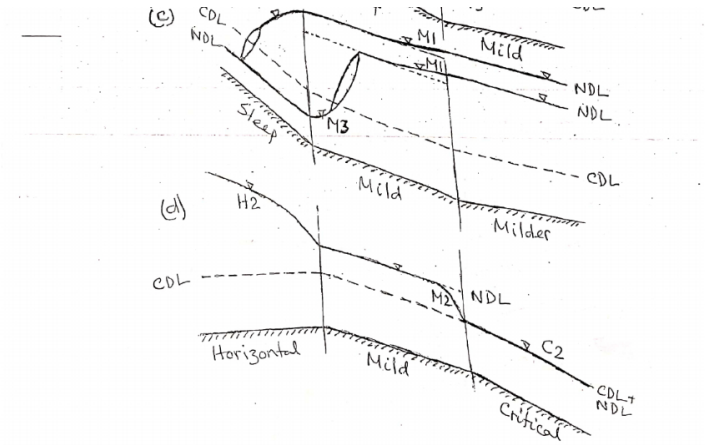


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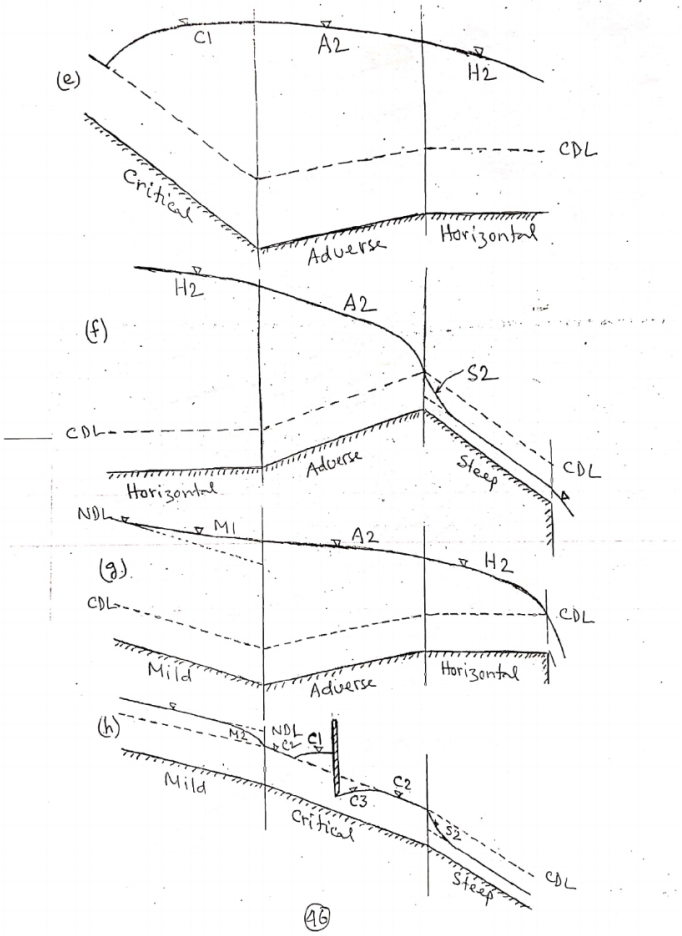


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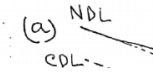


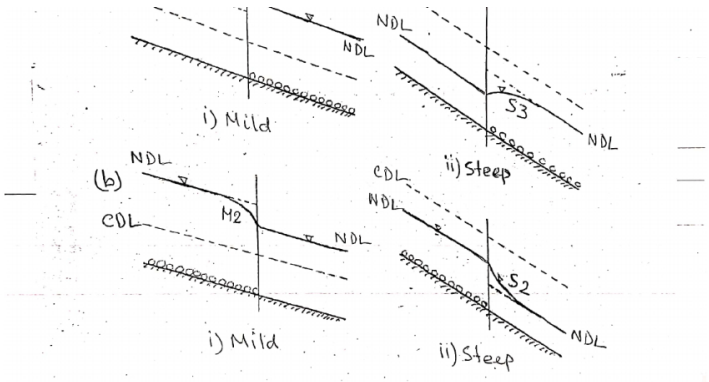
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6.17 For given channel section, discharge and bottom slope, the normal depth h_n increases as a result of increase in surface surface and vice versa. With increase in surface roughness, the numerical value of n increases.





6.9 Rectangle, $b = 6m$, $n = 0.020$, $Q = 24 m^3/s$

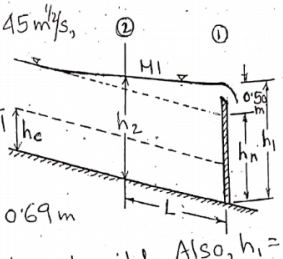
$$h_n = h_c = \sqrt[3]{\frac{Q^2}{gb^2}} = \sqrt[3]{\frac{24^2}{9.81 \times 6^2}} = 1.177 m$$

$A = bh = 6 \times 1.177 = 9.063 m^2$, $P = b + 2h = 8.359 m$

$R = A/P = 0.895 m$

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6.18 Wide channel, $C = 45 m^{1/2}/s$, $S_0 = 0.0001$, $q = 1.8 m^2/s$



$$h_n = \sqrt[3]{\frac{q^2}{C^2 S_0}} = \sqrt[3]{\frac{1.8^2}{45^2 \times 0.0001}} = 2.52 m$$

$$h_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{1.8^2}{9.81}} = 0.69 m$$

Since $h_n > h_c$, the slope is mild. Also, $h_1 = 2.52 + 0.50 = 3.02 m$ and $h_2 = 2.80 m$. Since h_1 or $h_2 > h_n > h_c$, the profile is M1.

$u_1 = h_1/h_n = 3.02/2.52 = 1.20$, $u_2 = h_2/h_n = 2.80/2.52 = 1.11$

$$\phi = \frac{1}{6} \ln \frac{u_1^2 + u_1 + 1}{(u_1 - 1)^2} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}}{2u + 1}$$

$$\therefore \phi_1 = \frac{1}{6} \ln \frac{1.20^2 + 1.20 + 1}{(1.20 - 1)^2} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}}{2 \times 1.20 + 1} = 0.47$$

$$\phi_2 = \frac{1}{6} \ln \frac{1.11^2 + 1.11 + 1}{(1.11 - 1)^2} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}}{2 \times 1.11 - 1} = 0.652$$

$$\therefore L = \frac{h_n}{S_0} \left[(u_2 - u_1) - \left(1 - \frac{h_c^3}{h_n^3}\right)(\phi_2 - \phi_1) \right]$$

$$= \frac{2.52}{0.0001} \left[(1.11 - 1.20) - \left(1 - \frac{0.69^3}{2.52^3}\right)(0.652 - 0.479) \right]$$

$$= 2.52 \times 10^4 \left[-0.09 - 0.979 \times 0.173 \right]$$

$$= -6484.22 m$$

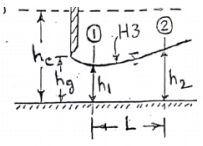
(18)

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6.19 Rectangular channel, $C_c = 0.61$, $h_g = 0.60m$, $Q = 27 m^3/s$, $b = 6m$.

CDL

$n = 0.013$



Depth at the vena contracta, h_1

$= C_c \times h_g = 0.61 \times 0.60 = 0.366\,m$

$h_c = \sqrt[3]{\frac{Q^2}{gb^2}} = \sqrt[3]{\frac{27^2}{9.81 \times 6^2}} = 1.273\,m$

$h_2 = 0.50\,m$

Since h_1 or $h_2 < h_c$, the profile is H3.

The critical slope is obtained using the Manning formula with $h_n = h_c = 1.273\,m$. Then,

$A = 6 \times 1.273 = 7.638\,m^2$; $P = 6 + 2 \times 1.273 = 8.546\,m$

and $R = A/P = 7.638/8.546 = 0.894\,m$ so that

$$S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.013 \times 27}{7.638 \times 0.894^{2/3}}\right)^2 = 2.453 \times 10^{-3}$$

For a rectangular channel, $M = 3$. The value of N is obtained for the average depth, $\bar{h} = \frac{h_1 + h_2}{2}$

$= \frac{0.366 + 0.5}{2} = 0.433\,m$ so that $\bar{h}/b = 0.072$.

$$\therefore N = \frac{2}{3}\left[5 - \frac{4(h/b)}{1 + 2(h/b)}\right] = \frac{2}{3}\left[5 - \frac{4 \times 0.072}{1 + 2 \times 0.072}\right] = 3.165$$

Therefore, $N - M + 1 = 1.165$, $N + 1 = 4.165$, $p_1 = h_1/h_c = 0.366/1.273 = 0.2875$ and $p_2 = h_2/h_c = 0.50/1.273 = 0.3928$

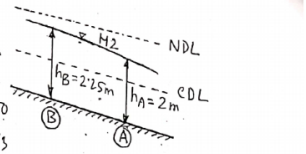
$$\therefore L = \frac{h_c}{S_c}\left[\frac{p_2^{N-M+1} - p_1^{N-M+1}}{N-M+1} - \frac{p_2^{N+1} - p_1^{N+1}}{N+1}\right]$$

$$= \frac{1.273}{2.453 \times 10^{-3}}\left[\frac{0.3928^{1.165} - 0.2875^{1.165}}{1.165} - \frac{0.3928^{4.165} - 0.2875^{4.165}}{4.165}\right]$$

$= 518.96 \times (0.08806 - 0.00356) = 43.85\,m$

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6.20 Rectangular channel, $b = 6\,m$, $n = 0.025$, $S_o = 0.0025$, $Q = 40\,m^3/s$, $h_A = 2\,m$



$h_c = \sqrt[3]{\frac{Q^2}{gb^2}} = \sqrt[3]{\frac{40^2}{9.81 \times 6^2}} = 1.65\,m$

$AR^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.025 \times 40}{\sqrt{0.0025}} = 20.00$

h	A	P	R	$AR^{2/3}$
2.65	15.90	11.30	1.407	19.96
2.66	15.96	11.32	1.410	20.07

$\therefore h_n = 2.65\,m$

Since $h_n > h_c$, the slope is mild. Since $h_n > 2h_A > h_c$, the profile is M2.

a) Suppose the depth at section B, $h_B = 2.25\,m$. Since M2 is a drawdown profile and $h_B > h_A$, section B is located upstream of section A. Since the M2 profile represents subcritical flow, we have to start computation at section A and proceed towards B. Since the distance AB is required, using the direct step method, the computation is shown below.

h	A	P	R	U	$\alpha U^2/2g$
2.00	12.00	10.00	1.200	3.333	0.5663
2.25	13.50	10.50	1.286	2.963	0.4475

$\Rightarrow$

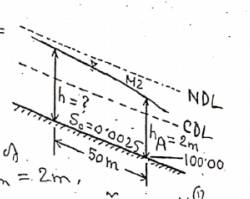
E	ΔE	S_f	$\bar{S}_f$	$S_o - \bar{S}_f$	Δx
2.5863	—	0.005946	—	—	—
2.6975	0.1313	0.003924	0.004685	−0.002185	−60.09

$\Rightarrow$

$\therefore$ The section B where the depth is 2.25 m is located 60.09 m upstream from section A.

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6.20(b) Rectangular channel, b = 6m, n = 0.025, S_0 = 0.0025, Q = 40 m^3/s, h_n = 2.65m, h_c = 1.65m, Mild slope, Profile M2, h_A = 2m. Trial value of depth at the upstream section = 2m,



Station	h	zw	A	U	$U^2/2g$	H_1 ①
0	2.00	102.000	12.00	3.333	0.5663	102.5663
~~50~~	~~2.00~~	~~102.125~~	~~12.00~~	~~3.333~~	~~0.5663~~	~~102.6913~~ ⇒
~~50~~	~~2.24~~	~~102.365~~	~~13.44~~	~~2.976~~	~~0.4515~~	~~102.8165~~
50	2.22	102.345	13.22	3.003	0.4596	102.8046

P	R	S_f	$\bar{S}_f$	Δx	h_f	h_e	H_1 ②
10.00	1.200	0.005446	—	-50	—	0	102.5663
~~10.00~~	~~1.200~~	~~0.005446~~	~~0.005446~~	~~-50~~	~~-0.2723~~	~~0~~	~~102.8386~~
~~10.48~~	~~1.282~~	~~0.003973~~	~~0.004710~~	~~-50~~	~~-0.2355~~	~~0~~	~~102.8018~~
10.44	1.266	0.004144	0.004795	-50	-0.2398	0	102.8061

Trial improvement

1. $F(h_2) = 102.6913 - 102.8386 = -0.1473$ m, B_2 = 6m, D_2 = 12/6 = 2m, $Fr_2 = U_2/\sqrt{gD_2} = \frac{3.333}{\sqrt{9.81\times 2}} = 0.5663$, $N_2 = \frac{2}{3}\left[5 - \frac{4(h_2/b)}{1+2(h_2/b)}\right]$
$= \frac{2}{3}\left[5 - \frac{4\times 0.333}{1+2\times 0.333}\right] = 2.8$ $(\because h_2/B = 2/6 = 0.333)$

$$\therefore \Delta h \text{ (or } \Delta z_w) = -\frac{F(h_2)}{1 - Fr_2^2 - N_2 S_{f2}\Delta x/2h_2}$$

$$= -\frac{-0.1473}{1-0.5663-2.80\times 0.005446\times(-50)/(2\times 2)} = +0.2359\text{m}$$

$\therefore h_2^* = h_2 + \Delta h = 2.00 + 0.2359 = \underline{2.24\text{m}}$

2. F = 102.8165 - 102.8018 = 0.0147
B_2 = 6m, D_2 = 2.24m, Fr_2 = 0.4030
N_2 = 2.7633
Δh = -0.0204
h = 2.24 - 0.02 = 2.22 m

3. F = 102.8046 - 102.8061 = -0.0015
B_2 = 6m, D_2 = 2.20 m
Fr_2 = 0.4172, N_2 = 2.77
Δh = +0.0022 m

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6.20(c) Rectangular, b = 6m, n = 0.025, S_0 = 0.0025, Q = 40 m^3/s

(i) Euler method

h_1 = 2m, A_1 = 12.00 m^2, P_1 = 10.00m, R_1 = 1.20m, S_{f1} = 0.005446, B_1 = 6m, Δx = -50m

$$\left(\frac{dh}{dx}\right)_1 = \frac{S_0 - S_f}{1 - \frac{\alpha Q^2 B}{gA^3}} = \frac{0.0025 - 0.005446}{1 - \frac{1\times 40^2\times 6}{9.81\times 12^3}} = -6.793\times 10^{-3}$$

$$\therefore h_2 = h_1 + \Delta x\left(\frac{dh}{dx}\right)_1 = 2.00 + (-50)\times(-6.793\times 10^{-3})$$

$= 2.3396$ m

(ii) 4th-order Runge-Kutta method

From the Euler method, $(dh/dx)_1 = f(x_1, h_1) = -6.793\times 10^{-3}$

$$\therefore K_1 = \Delta x\left(\frac{dh}{dx}\right)_1 = \Delta x f(x_1, h_1) = -50\times(-6.793\times 10^{-3})$$

$= 0.33965$

$\therefore h_1 + \frac{1}{2}K_1 = 2.00 + \frac{1}{2}\times 0.33965 = 2.169825$

Then, A = 13.01895 m^2, P = 10.33965m, R = 1.25913m, $R^{2/3}$ = 1.16604, B = 6m, $S_f = 4.339316\times 10^{-3}$

$$\therefore f(x_1 + 0.5\Delta x, h_1 + \frac{K_1}{2}) = \frac{0.0025 - 4.339316\times 10^{-3}}{1 - \frac{1\times 40^2\times 6}{9.81\times 13.01895^3}} = -3.30503\times 10^{-3}$$

$$\therefore K_2 = \Delta x f(x_1 + 0.5\Delta x, h_1 + \frac{K_1}{2}) = -50\times(-3.30503\times 10^{-3})$$

$= 0.165252$

$\therefore h_1 + \frac{1}{2}K_2 = 2.00 + \frac{1}{2}\times 0.165252 = 2.08263$

Then, A = 12.49575 m^2, P = 10.16525m, R = 1.22926m, $R^{2/3}$ = 1.147527, B = 6m, $S_f = 4.86350\times 10^{-3}$

$$\therefore f(x + 0.5\Delta x, h_1 + \frac{1}{2}K_2) = \frac{0.0025 - 4.86350\times 10^{-3}}{1\times 40^2\times 6} = -4.79121$$

$1 - \frac{\quad}{9.81 \times 12.495375^3}$ $\times 10^{\quad}$

(5a)

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6.20(c)(ii) Contd. (11)

$\therefore K_3 = \Delta x\, f\left(x+0.5\Delta x, h_1+\frac{K_2}{2}\right) = -50\times(-4.74171\times10^{-3})$

$= 0.237086$

$\therefore h_1 + K_3 = 2.00 + 0.237086 = 2.237086$

Then, $A = 13.42251\ m^2$, $P = 10.474171\ m$, $R = 1.28149\ m$

$R^{2/3} = 1.179803$, $B = 6m$, $S_f = 3.98762\times10^{-3}$

$\therefore f(x_1+0.5\Delta x, h_1+K_3) = \frac{0.0025 - 3.98762\times10^{-3}}{1 - \frac{1\times40^2\times6}{9.81\times13.42251^3}} = -2.4988 \times 10^{-3}$

$\therefore K_4 = \Delta x\, f(x+0.5\Delta x, h_1+K_3) = -50\times(-2.49881\times10^{-3})$

$= 0.12494$

$\therefore h_2 = h_1 + \frac{1}{6}(K_1+2K_2+2K_3+K_4)$

$= 2.00 + \frac{1}{6}(0.33965 + 2\times0.165252 + 2\times0.237086 + 0.12494)$

$= 2.00 + 0.211544 = 2.211544\ m$

$= 2.21\ m$

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Chapter 7

7.8 (i) $F_1 = F_2$ or, $\frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2$

or, $\frac{Q^2}{g}\left(\frac{1}{A_1} - \frac{1}{A_2}\right) = \bar{z}_2 A_2 - \bar{z}_1 A_1 \Rightarrow \frac{Q^2}{g}\left(\frac{1}{bh_1} - \frac{1}{bh_2}\right) = \frac{bh_2^2}{2} - \frac{bh_1^2}{2}$

or, $\frac{Q^2}{g}\left(\frac{1}{bh_1} - \frac{1}{bh_2}\right) = \frac{b}{2}(h_2^2 - h_1^2) \Rightarrow \frac{Q^2}{gb^2}\left(\frac{h_2 - h_1}{h_1 h_2}\right) = \frac{(h_2-h_1)(h_2+h_1)}{2}$

or, $\frac{Q^2}{gb^2} = \frac{h_1 h_2}{2}(h_2+h_1)$ (A)

or, $\frac{b^2 h_1^2 U_1^2}{gb^2} = \frac{h_1 h_2}{2}(h_2+h_1)$ ($\because Q = A_1U_1 = bh_1U_1$)

or, $\frac{U_1^2}{gh_1} = \frac{h_2}{2h_1^2}(h_2+h_1)$ (Dividing by h_1^3)

or, $Fr_1^2 = \frac{1}{2}\frac{h_2}{h_1}\left(\frac{h_2}{h_1}+1\right)$ ($\because Fr_1^2 = \frac{U_1^2}{gD_1} = \frac{U_1^2}{gh_1}$)

or, $2Fr_1^2 = \left(\frac{h_2}{h_1}\right)^2 + \frac{h_2}{h_1} \Rightarrow \left(\frac{h_2}{h_1}\right)^2 + \frac{h_2}{h_1} - 2Fr_1^2 = 0$

$\therefore \frac{h_2}{h_1} = \frac{-1 \pm \sqrt{1^2 - 4.1(-2Fr_1^2)}}{2.1} = \frac{-1+\sqrt{1+8Fr_1^2}}{2}$

$= \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right)$ (Ignoring − sign) (7.10)

$\Rightarrow$ Proved

(ii) From (A), using $Q = A_2U_2 = bh_2U_2$, we get

$\frac{b^2 h_2^2 U_2^2}{gb^2} = \frac{h_1 h_2}{2}(h_2+h_1) \Rightarrow \frac{U_2^2}{gh_2} = \frac{h_1}{2h_2^2}(h_1+h_2)$ (Dividing by h_2^3)

or, $Fr_2^2 = \frac{1}{2}\frac{h_1}{h_2}\left(\frac{h_1}{h_2}+1\right) \Rightarrow 2Fr_2^2 = \left(\frac{h_1}{h_2}\right)^2 + \frac{h_1}{h_2}$

or, $\left(\frac{h_1}{h_2}\right)^2 + \frac{h_1}{h_2} - 2Fr_2^2 = 0$

$\therefore \frac{h_1}{h_2} = \frac{1}{2}\left(\sqrt{1+8Fr_2^2}-1\right)$ (As before ignoring − sign)

$\Rightarrow$ Proved (7.11)

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7.8 (ii) (Contd.) $h_L = H_1 - H_2 = E_1 - E_2 = \left(h_1 + \frac{U_1^2}{2g}\right) - \left(h_2 + \frac{U_2^2}{2g}\right)$

$= (h_1 - h_2) + \frac{Q^2}{2gb^2}\left(\frac{1}{h_1^2} - \frac{1}{h_2^2}\right)$

$= (h_1 - h_2) + \frac{Q^2}{2gb^2}\,\frac{(h_1+h_2)(h_2-h_1)}{h_1^2 h_2^2}$ (i)

Again from $F_1 = F_2$ we have

$\frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2$

Since $\bar{z}_1 = h_1/2$ and $\bar{z}_2 = h_2/2$

$\frac{Q^2}{gbh_1} + \frac{h_1}{2} bh_1 = \frac{Q^2}{gbh_2} + \frac{h_2}{2} bh_2$

$\frac{Q^2}{gb}\left(\frac{1}{h_1} - \frac{1}{h_2}\right) = \frac{b}{2}\left(h_2^2 - h_1^2\right)$

$\frac{2Q^2}{gb^2} = h_1 h_2 (h_1 + h_2)$ (ii)

From (i) and (ii) we get

$h_L = (h_1 - h_2) + \frac{h_1 h_2}{4}(h_1+h_2)\cdot\frac{(h_1+h_2)(h_2-h_1)}{h_1^2 h_2^2}$

$= \frac{(h_2-h_1)}{4h_1h_2}\left[(h_1+h_2)^2 - 4h_1h_2\right]$

$= \frac{(h_2-h_1)}{4h_1h_2}(h_2-h_1)^2 = \frac{(h_2-h_1)^3}{4h_1h_2}$ (7.15)

$\rightarrow$ Proved

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7.8 (iii) (Contd.) $E_1 = h_1 + \frac{U_1^2}{2g} \Rightarrow \frac{E_1}{h_1} = 1 + \frac{U_1^2}{2gh_1} = 1 + \frac{1}{2}Fr_1^2$

$\therefore \frac{E_1}{h_1} = \frac{2+Fr_1^2}{2}$ (i)

$A_1U_1 = A_2U_2$
$h_1U_1 = h_2U_2$
$U_2 = U_1\frac{h_1}{h_2}$

$E_2 = h_2 + \frac{U_2^2}{2g} \Rightarrow \frac{E_2}{h_1} = \frac{h_2}{h_1} + \frac{U_2^2}{2gh_1}$

$\frac{E_2}{h_1} = \frac{h_2}{h_1} + \frac{U_1^2}{2gh_1}\cdot\frac{h_1^2}{h_2^2} = \frac{h_2}{h_1} + \frac{Fr_1^2}{2}\cdot\left(\frac{h_1}{h_2}\right)^2$

$= \frac{1}{2}\left(\sqrt{1+8Fr_1^2}-1\right) + \frac{Fr_1^2}{2}\left[\frac{1}{\frac{1}{2}\left(\sqrt{1+8Fr_1^2}-1\right)}\right]^2$

$= \frac{1}{2}\left(\sqrt{1+8Fr_1^2}-1\right) + 2Fr_1^2\,\frac{\left(\sqrt{1+8Fr_1^2}+1\right)^2}{(8Fr_1^2)^2}$

$= \frac{\frac{1}{2}\left[16Fr_1^2\sqrt{1+8Fr_1^2} - 16Fr_1^2 + 1 + 8Fr_1^2 + 2\sqrt{1+8Fr_1^2} + 1\right]}{16Fr_1^2}$

$= \frac{1}{2}\left[\frac{\left(\sqrt{1+8Fr_1^2}\right)^{3/2} - 4Fr_1^2 + 1}{}\right]$ (ii)

$-2L \qquad 8Fr_1^2$

From (i) & (ii)

$$\frac{E_2}{E_1} = \frac{(8Fr_1^2+1)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2(2+Fr_1^2)} \Rightarrow \text{Proved} \quad (7.18)$$

(v) $h_j = h_2 - h_1 \qquad \frac{E_1}{h_1} = \frac{2+Fr_1^2}{2}$ (i) (as before)

$$\therefore \frac{h_j}{h_1} = \frac{h_2}{h_1} - 1 = \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) - 1$$

$$= \frac{\sqrt{1+8Fr_1^2} - 3}{2} \quad \text{(ii)}$$

$\therefore$ From (i) and (ii), $\frac{h_j}{E_1} = \frac{\sqrt{1+8Fr_1^2} - 3}{2+Fr_1^2} \Rightarrow$ Proved (7.20)

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7.8 (i) (Central) $\frac{\gamma q}{g}(\beta_2 u_2 - \beta_1 u_1) = F_{p1} - F_{p2} + W\sin\theta - F_f$ (4)

$q = u_1 d_1 \quad u_2 = \frac{u_1 d_1}{d_2} \quad F_{p1} = \frac{1}{2}\gamma d_1^2 \cos\theta$

$F_{p2} = \frac{1}{2}\gamma d_2^2 \cos\theta \quad F_f = 0 \quad \beta_1 = \beta_2 = 1$

$W = \frac{1}{2}\Gamma\gamma L_j (d_1+d_2) \quad Fr_1 = \frac{u_1}{\sqrt{g d_1}}$

$$\therefore \frac{\gamma q}{g}(u_2 - u_1) = \frac{1}{2}\gamma\cos\theta(d_1^2 - d_2^2) + \frac{1}{2}\Gamma\gamma L_j (d_1+d_2)\sin\theta$$

$$\frac{q}{g}\left(\frac{u_1 d_1}{d_2} - u_1\right) = \frac{1}{2}\cos\theta(d_1^2 - d_2^2) + \frac{1}{2}\Gamma L_j (d_1+d_2)\sin\theta$$

$$\frac{u_1 d_1}{g}\left(\frac{u_1 d_1}{d_2} - u_1\right) = \cdots$$

$$\frac{u_1^2 d_1^2}{g d_2} - \frac{u_1^2 d_1}{g} = \cdots$$

$$\frac{Fr_1^2 \cdot d_1^3}{d_2} - Fr_1^2 d_1^2 = \frac{1}{2}(d_1+d_2)\{\cos\theta(d_1-d_2) + \Gamma L_j \sin\theta\}$$

$$Fr_1^2\left(\frac{d_1^3}{d_2} - d_1^2\right) =$$

$$\therefore d_1^2\left(\frac{d_1}{d_2} - 1\right) = \frac{d_1+d_2}{2}\left\{\frac{\cos\theta(d_1-d_2) + \Gamma L_j \sin\theta}{Fr_1^2}\right\}$$

$$d_1^2\left(\frac{d_1-d_2}{d_2}\right) =$$

$$\frac{d_1^2}{d_2} = \frac{d_1+d_2}{2}\left\{\frac{\cos\theta + \Gamma L_j \sin\theta/(d_1-d_2)}{Fr_1^2}\right\}$$

$$= \frac{d_1+d_2}{2}\left[\frac{\cos\theta - \frac{\Gamma L_j \sin\theta}{d_2-d_1}}{Fr_1^2}\right] = \frac{d_1+d_2}{2G^2}$$

where $G^2 = \frac{Fr_1^2}{\cos\theta - \frac{\Gamma L_j \sin\theta}{d_2-d_1}}$

$$\therefore -\frac{d_1^2}{d_2} + \frac{d_1+d_2}{2G^2} = 0 \Rightarrow -d_1^2(2G^2) + d_2(d_1+d_2) = 0$$

$$-2d_1^2 G^2 + d_1 d_2 + d_2^2 = 0 \Rightarrow d_2^2 + d_1 d_2 - 2d_1^2 G^2 = 0$$

$$\therefore d_2 = \frac{-d_1 \pm \sqrt{d_1^2 + 8d_1^2 G^2}}{2} \Rightarrow \frac{d_2}{d_1} = \frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1+8G^2} - 1\right) \Rightarrow \text{Proved} \quad (7.23)$$

(+ sign only)

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or, $8+8Fr_1^2 - (1+8Fr_1^2) + 3\sqrt{1+8Fr_1^2} = 0$

or, $7 - 4Fr_1^2 = -3\sqrt{1+8Fr_1^2}$

By trial, $Fr_1 = 2.37$

Using $Fr_1 = 2.37$, we get

$h_j/E_1 = \frac{\sqrt{1+8\times2.37^2} - 3}{2+2.37^2}$

$= 0.51$

Fr_1	R.H.S.	L.H.S.
2	-9	-13.23
3	-29	-25.63
2.7	-22.16	-23.11
2.37	-23.69	-23.69

7.9 (a) $U_1 = 6.1$ m/s $\quad h_1 = 1$m $\quad b = 6.1$m, rectangle

$\therefore Fr_1 = \frac{u_1}{\sqrt{g h_1}} = \frac{6.1}{\sqrt{9.81\times1}} = 1.95 \qquad \boxed{Fr_1^2 = 3.793}$

i) $\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) = \frac{1}{2}\left(\sqrt{1+8\times1.95^2} - 1\right) = 2.30$

$\therefore h_2 = 2.30\times1 = 2.30$m

ii) Since $Fr_1 = 1.95$, the jump is a weak jump.

iii) $h_j = h_2 - h_1 = 2.30 - 1.00 = 1.30\,m$

iv) $L_j/h_2 = 4.2 \quad \therefore L_j = 4.2 \times 2.30 = 9.66\,m$

Also, $L_j = 9.75 h_1 (Fr_1 - 1)^{1.01} = 9.75 \times 1 \times 0.95^{1.01} = 9.28\,m$

v) $h_L = \frac{(h_2 - h_1)^3}{4 h_1 h_2} = \frac{(2.30-1)^3}{4 \times 1 \times 2.30} = 0.238 \ \frac{m\text{-}kg}{kg}$

$\therefore$ hp dissipated $= \frac{7.22 Q h_L}{550} = \frac{7.22 \times 1000 \times 61 \times 1 \times 61 \times 0.238}{550}$

$= 116.2$

vi) $E_1 = h_1 + \frac{U_1^2}{2g} = 1 + \frac{6.1^2}{2 \times 9.81} = 2.8965$

$E_2 = h_1 + \frac{U_1^2}{2g} = 2.30 + \frac{2.652}{2 \times 9.81} = 2.6585$

$\therefore$ Efficiency $= E_2/E_1 = 2.6585/2.8965 = 0.9178 = 91.78\%$

or $\frac{E_2}{E_1} = \frac{(1+8Fr_1^2)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2(2+Fr_1^2)} = \frac{(1+8\times3.793)^{3/2} - 4\times3.793 + 1}{8\times3.793(2+3.793)} = 0.9178 = 91.78\%$

$U_2 = \frac{Q}{A_2} = \frac{bh_1U_1}{bh_2} = \frac{h_1U_1}{h_2} = \frac{1 \times 6.1}{2.30} = 2.652\ m/s$

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10) $h_1 = 1\,m$ trapezoidal $b = 5\,m$ $s = 1$ $Q = 30\,m^3/s$

$A_1 = (b + s h_1) h_1 = 6\,m^2$

$B_1 = b + 2s h_1 = 7\,m$

$D_1 = A_1/B_1 = 0.857\,m$

$U_1 = Q/A_1 = 30/6 = 5\,m/s$

$Fr_1 = \frac{U_1}{\sqrt{gD_1}} = \frac{5}{\sqrt{9.81 \times 0.857}} = 1.724$

$\bar{z}_1 = \frac{h_1}{6}\left(\frac{3b + 2s h_1}{b + s h_1}\right) = \frac{1}{6}\left(\frac{3\times5 + 2\cdot1\cdot1}{5+1\cdot1}\right) = \frac{17}{36} = 0.472\,m$

$\therefore F_1 = \frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{30^2}{9.81 \times 6} + 0.472 \times 6 = 18.12385$

h_2	A_2	$\bar{z}_2$	F_2	Remarks
3.00	24	1.3125	35.32	>
2.00	14	0.905	19.22	>
1.90	13.11	0.863	18.31	≃
1.89	13.0221	0.859	18.22	
1.88	12.9344	0.859	18.19	Closest

$\therefore h_2 = 1.88\,m$

$E_1 = h_1 + \frac{U_1^2}{2g} = 1 + \frac{5^2}{2 \times 9.81} = 2.27421\,m$

$U_2 = \frac{30}{12.9344} = 2.3193\,m/s$

$E_2 = h_L + \frac{U_2^2}{2g} = 1.88 + \frac{2.3193^2}{2 \times 9.81} = 2.15412\,m$

$\therefore h_L = E_1 - E_2 = 0.12\,m$

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11) Horizontal triangular, $s = 2$, $Q = 20\,m^3/s$, $h_1 = 1\,m$

$A_1 = s h_1^2 = 2\,m^2$ $B_1 = 2s h_1 = 4\,m$ $D_1 = 0.5\,m$

$U_1 = 10\,m/s$ $Fr_1 = \frac{U_1}{\sqrt{gD_1}} = \frac{10}{\sqrt{9.81 \times 0.5}} = 9.515$

$\bar{z}_1 = h_1/3 = 0.333$

$F_1 = \frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{20^2}{9.81 \times 2^2} + 0.333 \times 2 = 21.05$

h_2	A_2	$\bar{z}_2$	F_2
2.00	8.00	0.667	10.433
3.00	18.00	1.000	20.265
3.10	19.22	1.033	21.982
3.05	18.605	1.017	21.107

$\therefore h_2 = 3.05$ m

7.12 Horizontal parabolic $y^2 = 4z$ $Q = 10\ m^3/s$ $h_1 = 0.50$ m

$h_1 = 0.50$ m $y_1 = (4 \times 0.50)^{1/2} = 1.4142$ m $B_1 = 2y_1 = 2.828$ m

$A_1 = \frac{2}{3} B_1 h_1 = 0.942\ m^2$ $D_1 = 0.333$ m $U_1 = 10.616$ m/s

$Fr_1 = 5.873$ $\bar{z}_1 = 2h_1/5 = 0.20$ m

$F_1 = \frac{Q^2}{g A_1} + \bar{z}_1 A_1 = \frac{10^2}{9.81 \times 0.942} + 0.20 \times 0.942 = 11.010$

h_2	y_2	B_2	A_2	$\bar{z}_2$	F_2
2.00	2.828	5.657	7.542	0.800	9.385
3.00	3.464	6.928	13.856	1.200	17.363
2.50	3.162	6.325	10.541	1.000	11.508
2.95	3.130	6.261	10.226	0.980	11.019

$\therefore h_2 = 2.95$ m

(60)

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(61)

7.13(a) Rectangle, $b = 1.2$ m, $\theta = 3°$, $Q = 0.14\ m^3/s$, $h_1 = 0.02$ m

$A_1 = bh_1 = 0.024\ m^2$ $U_1 = 5.833$ m/s $Fr_1 = 13.16$

$k_1 = 10^{0.0270} = 1.205$ $G = k_1 Fr_1^2 = 251.5$

$h_2^*/h_1 = d_2/d_1 = \frac{1}{2}(\sqrt{1+8G} - 1) = 21.93$

$\therefore h_2^* = 21.93 \times 0.02 = 0.438$ m

(b) Rectangle, $b = 6$ m, $\theta = 3.5°$, $Q = 0.75\ m^3/s$, $h_1 = 0.02$ m

$h_t = 0.95$ m

$A_1 = 0.12\ m^2$ $U_1 = 6.25$ m/s $Fr_1 = 14.11$

$\frac{h_2}{h_1} = \frac{1}{2}(\sqrt{1+8Fr_1^2} - 1) = 19.96$ $\therefore h_2 = 0.3992$ m

Since $h_t < h_2$, the depth h_2^* must be calculated.

$k_1 = 10^{0.0270} = 1.243$ $G = k_1 Fr_1^2 = 309.65$

$\therefore \frac{d_2}{d_1} = \frac{h_2^*}{h_1} = \frac{1}{2}(\sqrt{1+8G} - 1) = 24.3$

$\therefore h_2^* = 0.486$ m

Since $h_2^* > h_t$, i.e. $0.486 > 0.95$ (h_t is 1m), hence

~~This jump is A B type~~.

7.13 Rectangle, $b = 6$m, $\theta = 5°$, $Q = 20\ m^3/s$, $h_t = 3.20$ m

i) $d_1 = 0.20$ m

$A_1 = bd_1 = 1.2\ m^2$ $U_1 = Q/A_1 = 16.67$ m/s

$Fr_1 = U_1/\sqrt{gD_1} = U_1/\sqrt{gd_1} = 11.90$

The sequent depth for a jump corresponding to d_1 in a horizontal channel is given by

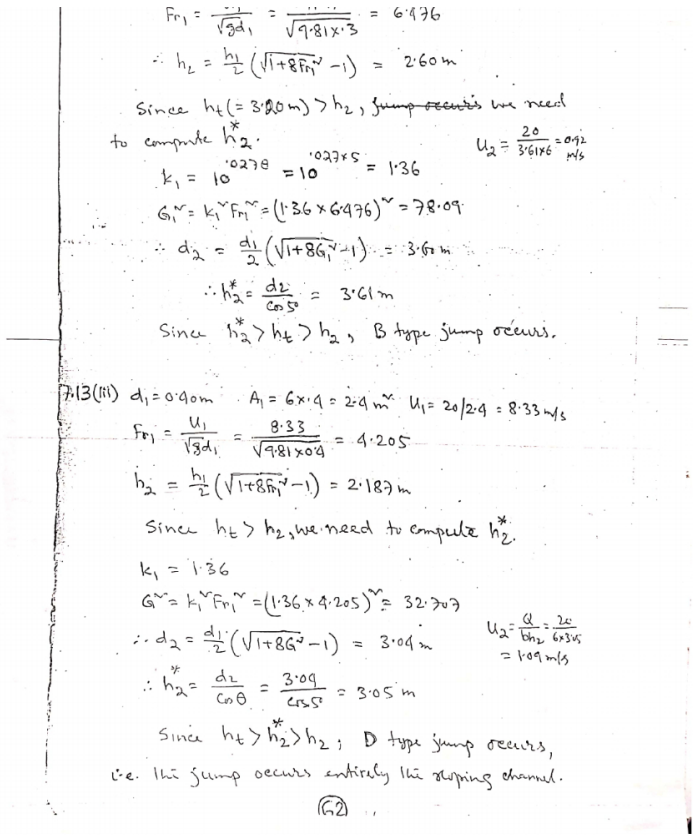
$h_2 = \frac{h_1}{2}(\sqrt{1+8Fr_1^2} - 1) = \frac{0.2}{2}(\sqrt{1+8\times 11.9^2} - 1) = 3.27$ m

Since $h_t < h_2$, jump type A occurs, i.e. the jump is on the horizontal floor

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7.13(ii) $d_1 = 0.30$ m

$A_1 = 6 \times 3 = 1.8\ m^2$ $U_1 = \frac{20}{1.8} = 11.11$ m/s



$Fr_1 = \frac{\cdot}{\sqrt{g d_1}} = \frac{\cdot}{\sqrt{9.81 \times 3}} = 6.976$

$\therefore h_2 = \frac{h_1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) = 2.60\,m$

Since $h_t (= 3.20\,m) > h_2$, ~~jump occurs~~ we need to compute h_2^*.

$k_1 = 10^{0.027\theta} = 10^{0.027 \times 5} = 1.36$

$U_2 = \frac{20}{3.61 \times 6} = 0.92\ m/s$

$G_1 = k_1 Fr_1 = (1.36 \times 6.976)^2 = 78.09$

$\therefore d_2 = \frac{d_1}{2}\left(\sqrt{1+8G_1^2} - 1\right) = 3.62\,m$

$\therefore h_2^* = \frac{d_2}{\cos 5^\circ} = 3.61\,m$

Since $h_2^* > h_t > h_2$, B type jump occurs.

7.13(iii) $d_1 = 0.9\,m$ $A_1 = 6 \times 4 = 24\,m^2$ $U_1 = 20/2.4 = 8.33\ m/s$

$Fr_1 = \frac{U_1}{\sqrt{g d_1}} = \frac{8.33}{\sqrt{9.81 \times 0.9}} = 4.205$

$h_2 = \frac{h_1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) = 2.187\,m$

Since $h_t > h_2$, we need to compute h_2^*.

$k_1 = 1.36$

$G_2 = k_1 Fr_1 = (1.36 \times 4.205)^2 = 32.709$

$\therefore d_2 = \frac{d_1}{2}\left(\sqrt{1+8G_2^2} - 1\right) = 3.09\,m$

$U_2 = \frac{Q}{Th_2} = \frac{20}{6 \times 3.5} = 1.09\ m/s$

$\therefore h_2^* = \frac{d_2}{\cos\theta} = \frac{3.09}{\cos 5^\circ} = 3.05\,m$

Since $h_t > h_2^* > h_2$; D type jump occurs,
i.e. the jump occurs entirely the sloping channel.

(G2)

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