GEPM 375

Project Planning and Construction Management

Linear Programming

Linear Programming Class-1

What we are going to learn today?

- **What is optimization?**
- Definition of Linear Programming
- **Example of a linear programming problem**.
- Formulating a real life problem in LP problem.
- **Common Terminologies**
- Steps and Process of LP

Optimization, is the way of life:

- We all have **finite** **resources and time** and we **want to make** **the most out of them.**

- From **using your time**, **productively** to solving **supply chain problems** for your company – **everything uses optimization**.

- It **starts with** **simple problems**, but can get very complex.

For example:

- **Sharing chocolates** among siblings is a simple optimization problem. We don't think in mathematical term while solving it.

Question: What is optimization in project management? Describe with example.

- On the other hand **devising inventory and warehousing strategy** can be very complex.

- **Millions of SKUs** with **different popularity** in **different regions** to be **delivered in defined time and resources**.

(A **stock keeping unit- SKU**, is **a store or catalog's product** and **service identification code**; it is often portrayed as a machine-readable bar code)

Inventory refers to the goods, materials, and products a business **holds for sale, production, or use**





Linear Programming (LP)

- LP is **one of the simplest ways to perform optimization.**
- Helps **to solve some very complex optimization problems** by **making a few simplifying assumptions**.
- As an engineer you are **bound to come across different applications and problems** to be solved by Linear Programming.

Question: Define linear programming in project management perspective.

What is Linear Programming?

- A **simple technique** where we **depict** **complex relationships through linear functions** **and** then **find** the **optimum points/solutions**.

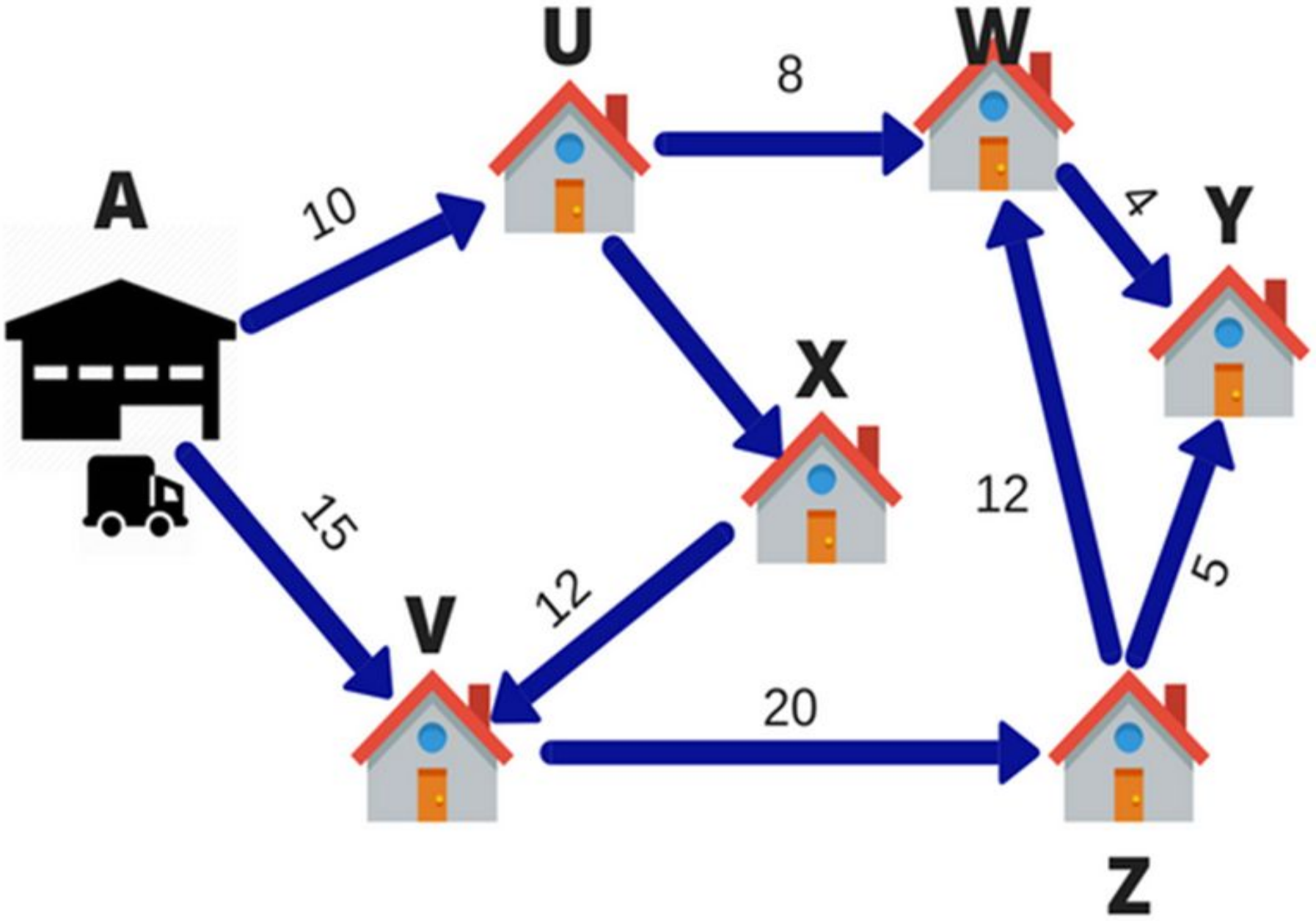
Important word is **depict**.

- **Real relationships might be much more complex** – but we **can simplify them to linear relationships.**

- A **linear function** has one independent variable and one dependent variable.)

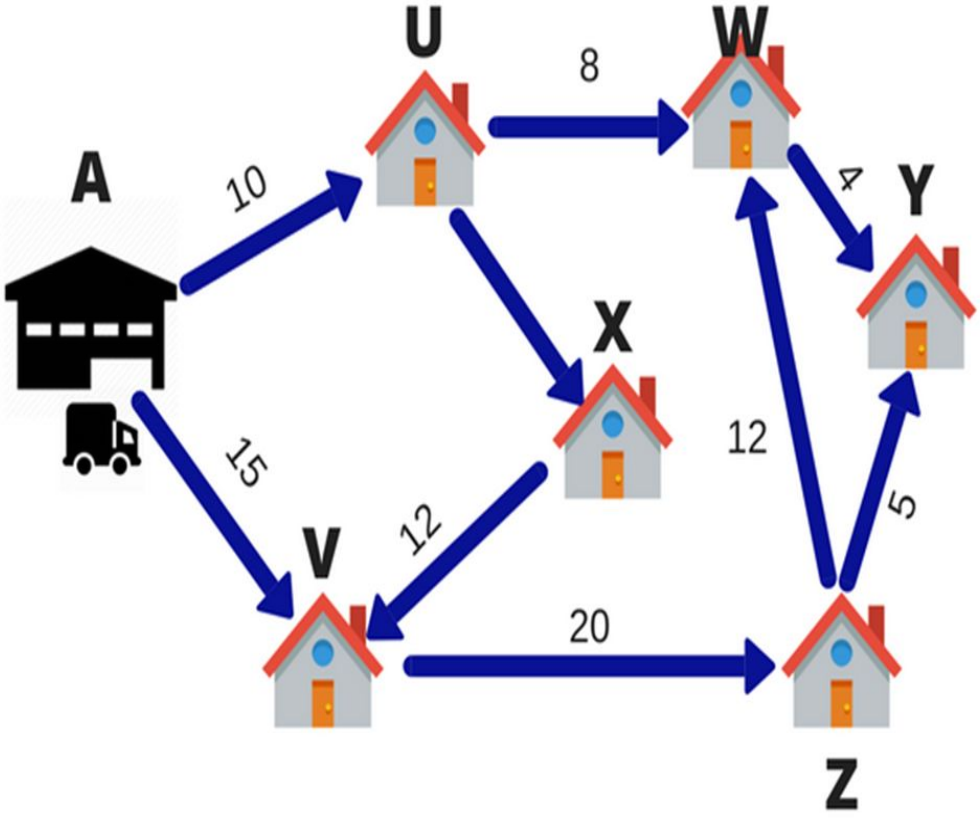
Applications of linear programming are everywhere around you.

- You use LP at **personal and professional fronts.**
- You are using LP when you are **driving from home to work and want to take the shortest route**.
- Or when you have a **product delivery,** you **make strategies to make your team work efficiently** for **on time** delivery.


A
U
W
Y
X
V
Z
10
8
4
15
12
12
5
20

Example of a linear programming problem

- Let's say a FedEx delivery man has **6 packages to deliver in a day.**
- Warehouse is located **at point A**.
- **6 delivery destinations** are given by U, V, W, X, Y and Z.
- **Numbers** on the lines **indicate the distance** between the destinations, cities.
- **Save on fuel and time** the delivery person wants to take the **shortest route**.



- Delivery person **will calculate different routes** for going to all the 6 destinations and then **come up with the shortest route**.

- **This technique of choosing the shortest route is called linear programming.**

Question: Explain linear programming problem with an example.

Here,
Objective of the delivery person is to **deliver the parcel on time** at all 6 destinations.

Process of choosing the best route is called **Operation Research**.

Operation research is **an approach to decision-making**, which involves a **set of methods to operate a system.**

In the above example, the **system is** as the **Delivery model**.

Concept of using LP Method

- **LP is used for obtaining the most optimal solution for a problem with given constraints**.

- In LP, **formulation of real life problem into a mathematical model**.

- And involves an **objective function**, **linear inequalities** with subject **to constraints**.

Question: In linear programming, formulation of real life problem in to a mathematical model. Describe with example.

Example of LP in real life problem:

- **Linear representation of the 6 points** is the representation of **real world**.

- It is **oversimplification** as the **real route would not be a straight line**.

- It would likely have **multiple turns**, **U turns**, **signals** and **traffic jams**.

- But **with a simple assumption**, we have **reduced** the complexity of the problem drastically and are **creating a solution** which should work in most scenarios.

Formulating a real problem into LP:

Let's manufacture some chocolates:

Example: Consider a chocolate manufacturing company which produces only **two types of chocolate** – A and B. Both the chocolates require **Milk and Choco** only. To manufacture each unit of A and B, following quantities are required:

- **Each unit of A** requires **1 unit of Milk and 3 units of Choco**
- **Each unit of B** requires 1 unit of Milk and **2** units of Choco

Question: How do you formulate a real world problem for linear programming? Explain with example.

Linear Programming

- Company **kitchen has** a total of **5 units of Milk and 12 units of Choco**.

- **On each sale**, the company makes a profit of

  Taka 6.00 per unit A sold.
  Taka 5.00 per unit B sold.

- Now, the company wishes to **maximize its profit**.

- **How many units of A and B should it produce respectively**?

Linear Programming

Solution:

- First thing, is to **represent the problem in a tabular form** for better understanding.

	Milk	Choco	Profit per unit
A	1	3	Taka 6.00
B	1	2	Taka 5.00
Total	**5**	**12**	

- **Let** the **total number of units** produced of **A be = X**
- Let the **total number of units** produced of **B be = Y**
- Now, **the total profit** is represented by **Z**

•Total profit the company makes is given by the total number of units of A and B produced multiplied by its per unit profit Tk 6 and Tk 5 respectively.

Total Profit: $Z = 6X+5Y$

which means we have to **maximize Z**.

•Company will try to **produce as many units of A and B to maximize the profit.**

•But the **resources Milk and Choco are available in limited amount.**

Milk:

•As per the above table, each unit of A and B requires **1 unit of Milk**. The **total amount of Milk available is 5 units**.

•To represent this **mathematically,**

$X+Y \leq 5$

Choco:

•Also, each unit of A and B requires 3 units & 2 units of Choco respectively. The **total amount of Choco available is 12 units**.

•To represent this **mathematically**,

$$3X+2Y \leq 12$$

•Also, the **values for units of A and B can only be integers**.

•So we have **two more constraints**, $X \geq 0$ & $Y \geq 0$

For the company to make **maximum profit**, the **above inequalities have to be satisfied**.

This is called formulating a real-world problem into a mathematical model.

Question: What is called the formulating a real time problem in to mathematical model? Explain with example.

Parameters used in Linear Programming

- **Decision Variables**
- **Objective functions**
- **Constraints**
- **Non-negative restrictions.**

Decision Variables:

- Decision variables are the variables **which will decide the output.**

- **Represent the ultimate solution**.

- To solve any problem, it is needed **first to identify the decision variables.**

- For the above example, the **total number of units for A and B denoted** by **X & Y respectively are the decision variables.**

Objective Function:

- Defined as the **objective of making decisions**.

- For the above example, the **company wishes to increase the total profit represented by Z.**

- So, **profit Z is the objective function.**

Constraints:

- Constraints are the **restrictions or limitations on the decision variables.**

- Usually **limit the value of the decision variables**.

- In the above example, the **limit on the availability of resources Milk and Choco are the constraints**.

- Milk for A is 1 and Choco for A is 3 is constraints.
- Milk for B is 1 and Choco for B is 2 is constraints.

Non-negativity restriction:

•For all linear programs, the **decision variables should always take non-negative values**.

•Which means the values for decision variables should be **greater than or equal to 0**.

•Previous example, $X \geq 0$ & $Y \geq 0$

Process/steps to define a Linear Programming problem

Let us look at the **steps of defining a Linear Programming** problem generically:

Step-1:
•Identify the decision variables
Step-2
•Write the objective function
Step-3
•Mention the constraints
Step-4
•Explicitly state the non-negativity restrictions.

For a problem to be a linear programming problem, the **decision variables, objective function and constraints all have to be linear functions.**

- If the all the above conditions are satisfied, it is called a **Linear Programming Problem**.

Question: What is linear programming problem? Describe the different steps of defining a linear programming problem generically.

Thank you